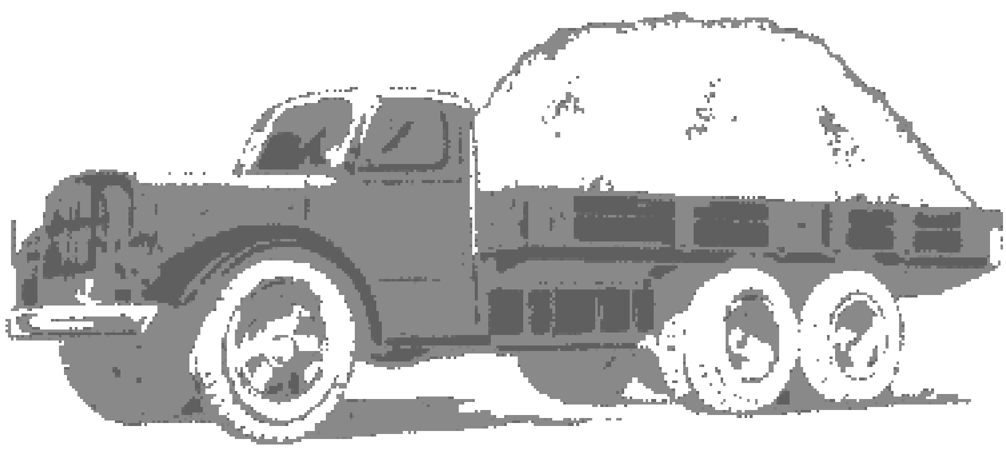
Final round Mathematics A-lympiad

2000-2001

Traffic jam caused by trucks



Final Mathematics A-lympiad 2000-2001

- Read through the complete problem.
- Use a time-schedule: the preinvestigation-part should be finished by Fridaynight
- As soon as you finish a part: print it!
- Answers may be given in several ways: using pictures and graphs, in words, etcetera.
- The most important part of problemsolving is the method you choose, the reasoning, the way of working, etcetera.
- Use a **black** pen, or a computer.
- Don’t forget to enumerate your pages.
- Write the names of your teammembers and your school on the cover.
- **The result of the main investigation is a report with appendices. One must be able to read this report as a coherent piece of work.**

Lots of fun & good luck!

Introduction

A contractor wants to transport 1000 m^3 of sand along a certain three-kilometer haul. Because of the slow pace of the loaded truck, this transport will cause traffic jam, for which the city council will charge costs. That's why the contractor calls in a consultancy, to estimate these costs.
Your team is this consultancy!

The consultancy is specialized in traffic-issues. To describe the traffic jam caused by a slowly driving vehicle, they use a certain model. In this model, data, provided by the contractor are incorporated in various graphs. Then, the traffic jam can be derived from the graphs.

Practice situations

Situation 1

The consultancy starts with a simplified version of the posed problem. They made the following assumptions:

- a 500-meter haul, on one lane, will be examined (after this haul, the truck leaves the road)
- there is one fully loaded truck driving with a pace of 30 km/h
- every 10 seconds, a car enters the haul with a pace of 60 km/h and stays behind the truck
- all changes in speed are sudden: one moment, a car drives 60 km/h; the next moment the car drives 30 km/h. Thus, no time is needed to decelerate or to accelerate.
- there is a passing restriction
- all drivers behave the same: they drive at the same pace, they keep the same distance to the car in front of them
- the distances between the cars depend on their speed: at a lower pace, the distance between cars is smaller of course than at a higher pace.

In *figure I* you can see the rides of the truck and of the first two cars that enter the haul 10 seconds after each other.
Study *figure I*. From this graph you can read how much the two cars are delayed.

By drawing the rides of all following cars that are still bothered by the truck in *figure I*, one can determine the *total amount of 'minutes of delay'*. This is the total amount of *extra* traveling time of all cars together, as a result from the truck. The city council uses this amount of minutes of delay to calculate the costs the contractor has to pay.

a. Calculate the total amount of minutes of delay in this situation.

The total amount of minutes of delay that will be caused by a slow transport depend on different factors. One of these factors is the *traffic-intensity*. This is the number of vehicles per minute that drives on the haul. Another factor is for example the length of the haul. In *figure I*, the traffic-intensity is 6 cars per minute and it's a 500-meter haul.

Situation 2

The consultancy investigates a second, simplified situation: another haul (1000 m) and another traffic-intensity. The graph of this situation is presented in *figure II.*

b. Describe extensively all you can read from figure II . Mention aspects like intensity, different paces, distances between cars, etc. Also calculates the total amount of minutes of delay.

Preinvestigation traffic-intensity

After the two practice situations, the consultancy decides to study a number of different traffic-situations on the original three-kilometer haul. Of course there are more factors, next to the traffic-intensity, that can influence the total amount of minutes of delay; in calculating the amount of minutes of delay, the consultancy only takes into account the variation in traffic-intensity, in this pre-investigation.

Drawing the graphs like in *figure II* is very labor-intensive. For the global calculation of the total amount of minutes of delay, not all details of the graph of *figure II* are necessary.

In *figures III up to VI* four different traffic-situations are presented on the 3-km haul. The traffic-intensity is different in each situation.
- line AB represents: the ride of the truck that drives along the 3-km haul with a pace of 30 km/h
- line AC represents: the ride of the last car that can drive along the haul with a pace of 60 km/h before delays are being caused by the truck
- line DE represents: the ride of the first car that drives along the haul without delay, after the truck has left the haul.

Comparable lines can be found in *figure II*.

a. Determine the total amount of minutes of delay in *figures III up to VI*. Describe clearly how you did that.
b. Describe how the total amount of minutes of delay depends upon the traffic-intensity and illustrate this using a graph.

The main investigation

After having done the pre-investigation, the consultancy (so this is you!) starts the real work.

First, they investigate the traffic-intensity on the 3-km haul in question. This intensity varies a lot during a working day. By means of traffic counting, this varying traffic-intensity was measured between 8 am and 4 pm. The results of an average working day can be found in *figure VII.*

The consultancy also used their model to draw a graph that represents the correlation between the traffic-intensity on the haul and the total amount of minutes of delay per ride of the truck. This is shown in *figure VIII*.

With the help of *figures VII and VIII*, the data on the traffic-situation on the haul and the data from the contractor, the consultancy is able to design several transport strategies and to calculate the costs.

The consultancy has to take the following costs into consideration:
- the contractor hires 1 truck;
- the costs of the rent of a truck (driver included) are fl. 500,= per shift or fl. 850,= per day;
- the truck can only be rented for a whole day, or for a shift (a shift is from 8 am to 12, or from 12 to 4pm);
- each minute of delay will cost the contractor fl 1,=.

The following facts of the contractor are also known:
- per truck, 12 m^3 of sand is transported;

- in total, 1000 m^3 of sand has to be transported;
- loading and unloading a truck will take 15 minutes per truck altogether;
- a full truck drives at a pace of 30 km/h; an empty one 60 km/h.

On the traffic situation on the haul, the following is known:
- it is a 3-km haul;
- cars drive at a pace of 60 km/h on this haul;
- there is a passing restriction on this haul.

The consultancy realizes this: if they choose to let the truck drive as many times as possible on one day, the traffic jam will be enormous because of the strongly varying traffic-intensities (see *figure VII*). Consequences are that the costs due to the minutes of delay will increase. On the other hand: spreading the transport on times that the traffic-intensity is low will be expensive too, because the truck has to be rented for a lot of shifts then.

Based on the data and graphs provided, the cheapest transport-strategy for the contractor can be determined.

a. Determine this cheapest strategy.

The cheapest strategy is strongly bounded by limiting conditions, like: there is just one truck; this truck is either full or empty; the speed is fixed, etc.

b. Investigate whether there is a cheaper strategy possible if you let go of the limiting conditions. Make realistic assumptions about facts and data (pace, cost, intensity) if you don't have the data; if necessary, make new graphs, but make your assumptions based on data you dó have!

c. Incorporate the results in a recommendation-report to the contractor. Incorporate at least the cheapest strategy within the limiting conditions. Next to that, you can say something about letting go of the limiting conditions and the consequences for the strategy. All should be provided with a solid foundation.
   Put calculations, graphs, etc. in appendices.