

Final assignment for the

# A-lympiad 2010

Garderen,
March 12 and 13 2010









#### **GUIDE FINAL MATHEMATICS A-LYMPIAD 2010**

#### IN ADVANCE:

- First read the full text of the assignment so you will know what you have to do this weekend.
- Read the rules together and check whether everybody in your team interprets them the same way
- Divide tasks where possible and consult when needed.

#### TIME MANAGEMENT:

- Keep an eye on the time when you work on the different parts.
- Divide the different roles in the final tournament on Friday afternoon.
- Start working on the strategy card in time; you need to hand it in before the rest of the project.
- You have to hand in the report on Saturday afternoon before 13:00 lunchtime.

#### HANDING IN:

Saturday afternoon 12.00 o'clock: the strategy card, the specifications are in the assignment.

Saturday afternoon 13.00 o'clock: the report; again, check the assignment for the specifications.

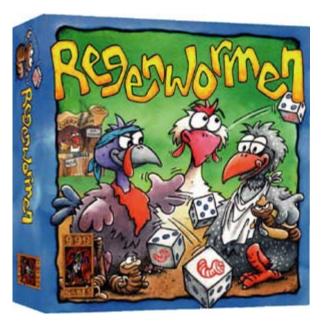
The jury will receive copies of your work. Of course the copies must be legible. So use a black pen for writing and only print on A4 size paper. Any drawings that you hand in, must copy well, so it is better not to use a pencil to make them.

#### JUDGING:

Among other things, the following points are important for the jury:

- How complete the answers for the various parts are;
- the analysis of the game (Assignment A);
- the strategies that have been designed, summarised in the complete game strategy;
- the use of math;
- the argumentation used and how choices that have been made are justified;
- the depth to which the various assignments have been answered;
- the style of presentation: form, legibility, (copyable) illustrations etc;
- originality and creativity.

#### Introduction



In this assignment you will investigate how the game *Earthworms* (Regenwormen) works. You will look at opportunities and chances within the game, and you will see the choices you may run into while playing and what the consequences are. You will think about game strategies in the different phases of the game and in the end you will formulate an optimal winning strategy.

## Preparation

Friday morning we will play the game *Earthworms* for an hour with all teams in the central area. You will play the game *Earthworms* with four people, in pairs from two different team who speak the same language. Before you start, carefully read the rules of the game. The purpose is to get a grip on the rules and on possible game scenarios. This is important for being able to complete the assignments successfully. While you're playing, you can of course ask questions from the members of the committee who are present.

## Part A: Why is the game the way it is?

## Sub-assignment A



Research why the game and the rules have been designed the way they are and present your findings in a coherent report. Try to give substantiated answers to questions such as:

- Why are there eight dice, why not more or less?
- Why are the numbers 21 to 36 on the tiles (and not smaller or larger ones)?
- Why is each turn no longer than six throws?
- Why have the worms been divided over the tiles in the way they have been in the game?
- Why are you allowed to close tiles after a certain point in the game?
- And all other questions that are raised by the game while you're playing.

## Part B: Final- and stop strategy

In this part you will look into the positions towards the end of a turn, and especially whether it's worth stopping your turn. You will calculate some examples and try to draw general conclusions from the results.

Generally speaking, you have several throws in one turn, until you can take a tile, or until you're out. Carefully read the rules before you start.

#### Assignment 1

You probably realised while you were playing that the higher tiles are difficult to take. Find out how many ways there are to take tile 36 during a turn. Present these ways clearly.

#### **Assignment 2**

You have taken out the following dice: 1 - 2 - 2 - 3 - 4 - 4.

So you have two dice left. Work out how large the chance is that you can take a tile in this turn (based on the assumption that all tiles are still open).

#### **Assignment 3**

In his turn a player has had five throws. He has taken out dice with an *earthworm* and with 1, 2, 3 and 5 dots.

Give all possible results (tiles *and* scores) if you know that he has exactly 19 points.

Then calculate for each of these results the chance that the player survives the next throw. Also indicate each time what his minimum and maximum final scores are.

#### **Assignment 4**

A player has had four throws in his turn. He has taken out dice with an *earthworm* and with 1, 3 and 5 dots. You also know that this player has scored a *maximum* of 20 points in the first four throws.

Give all possible results (tiles *and* scores) for the player in the first four throws and then give the chance for each possible result that the player survives the next throw.

#### **Assignment 5**

A player has had four throws in his turn and has taken out a total of six dice. Calculate the chance that he'll be out in the next throw because he throws one of the results he has already cast with the remaining dice.

## Sub-assignment B

Formulate a number of guidelines for a good (winning) **endgame** strategy. At the least include something about a **stopping strategy**. That is a strategy where you determine after each throw whether to stop or play on.

Indicate whether, how and why you use the results from assignments 1 to 5 for this. You will elaborate further in the final assignment.

## Part C: Start strategy - choices and effects

You make choices at all kinds of points in the game. You will now look into the possible effects of these choices at the start of your turn.

<u>Example</u>: If you throw **5 - R - 4 - 4 - 3 - 3 - 2** in your first throw, you can take out two 4s, or the *earthworm* (R for 'Regenworm'), or the 5, or the 2 or the three 3s. Your choice to do any of these already determines to some extent how many points you can score and what your chances are.

#### **Assignment 6**

Your first throw is 1 - 3 - 2 - 1 - 4 - 3 - R - 4

Your goal is to gain as high a score as possible in your turn. You have to make your choices with this goal in mind. Describe which dice you would take out and give your considerations.

#### **Assignment 7**

We assume a turn with the following series of throws:

Die <sup>2</sup>	1 Die2	Die3	Die4	Die5	Die6	Die7	Die8
5	2	2	4	1	5	R	R
4	3	5	R	R	2	4	2
R	4	4	5	R	1	4	3
4	2	R	R	2	3	2	3
1	1	R	3	3	1	3	2
5	R	1	2	4	4	5	5

Find out what the minimum and maximum scores in this turn are. Give the run of the game that leads to this result each time. Use the explanation below.

Explanation about using the throws: Because you take out dice every time, you will only use a part of the throw after the second one: namely only the dice that you have left. If for example you take out the two 2s in the first throw, you will no longer use the results for die 2 in the next throw (Die2) and die 3 (Die3). You will find several copies of these throws in appendix 1.

## Sub-assignment C

What exactly do you do at the start of your turn? How do you determine which dice you take out? What is the goal of your turn? Formulate a number of underpinned guidelines for this: the **starting strategy**.

Indicate whether, how and why you use the results from assignment 6 and 7 for this. You may, if you want, use appendix 2 which has some simulations of turns, to further examine your strategy.

You will elaborate this sub-assignment further in the final assignment.

## FINAL ASSIGNMENTS

#### **FINAL ASSIGNMENT 1**

In sub-assignment A you wrote a text about the game *Earthworms*. Next you have designed partial strategies for the game in sub-assignments B and C, namely an endgame and stop strategy (B) and a start strategy (C).

Combine your findings from sub-assignments B and C to formulate usable rules for a complete game strategy. Give a short (mathematical) underpinning (you may also do this by referring to previous answers).

#### Report

You will hand in a report that can be read on its own, consisting of:

- The elaboration of sub-assignment A
- The underpinned complete game strategy, based on sub-assignments B and C.

- The answers to assignments 1 to 7 (as an appendix)
- The original texts for sub-assignments B and C can be included as appendixes, but it isn't necessary.

This report you will hand in to the organisation Saturday 13:00 o'clock at the latest.

#### **FINAL ASSIGNMENT 2**

We will end the finals weekend the way we started it: by playing *Earthworms*. Every participant will play using a prescribed strategy: one of the strategies thought up by the participating teams.

Make a clearly legible instruction card for your own strategy, which has to meet the following conditions:

- the card has to be in English.
- the maximum size is one A4 sheet of paper, font Arial, size: 16pt.
- the strategy must be understandable for players who did *not* take part in the Alympiad.
- you can choose yourselves how to present your strategy.
- the name of your school must be included.

You must hand in this piece of paper to the organisation on Saturday at 12:00 o'clock at the latest (so we can make copies for the final *Earthworms* tournament).

#### CONCLUSION

The Alympiad committee organises the final tournament. The players will be yourselves, and additionally outside guests and members of the committee. Every game table needs a games master and an observer. Two players from each team will play with their own strategy.

The games master doesn't play, but explains the game to 'guest players', writes down the scores on the score form and decides in doubtful situations.

The observer doesn't play either, but keeps an eye on the time and will answer questions about the rules by the guest players. Teams with three members don't have to provide an observer.

So each member of the team is assigned a role:

- games master
- (observer)
- player

On Friday members of the committee will drop by to ask your team how you have divided these roles.

The team with the best scoring strategy will receive the public award during the prize giving ceremony on April 23 in Utrecht.



## Colophon

#### Alympiad committee:

Aldine Aaten, Lonneke Boels, Marcel Daems, Tom Goris, Dédé de Haan, Willem Hoekstra, Matthias Lippert, Johan van de Leur, Ruud Stolwijk and Monica Wijers.

#### Thanks to:

- 999Games for providing the games for the final weekend.Wietske de Blauw (999Games).

## Appendix 1: the throws in assignment 7

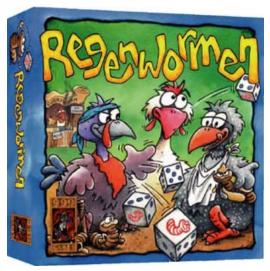
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## Appendix 2: 24 different turns

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Final assignment for the

# A-lympiad 2010

Garderen,
March 12 and 13 2010









#### **GUIDE FINAL MATHEMATICS A-LYMPIAD 2010**

#### IN ADVANCE:

- First read the full text of the assignment so you will know what you have to do this weekend.
- Read the rules together and check whether everybody in your team interprets them the same way
- Divide tasks where possible and consult when needed.

#### TIME MANAGEMENT:

- Keep an eye on the time when you work on the different parts.
- Divide the different roles in the final tournament on Friday afternoon.
- Start working on the strategy card in time; you need to hand it in before the rest of the project.
- You have to hand in the report on Saturday afternoon before 13:00 lunchtime.

#### HANDING IN:

Saturday afternoon 12.00 o'clock: the strategy card, the specifications are in the assignment.

Saturday afternoon 13.00 o'clock: the report; again, check the assignment for the specifications.

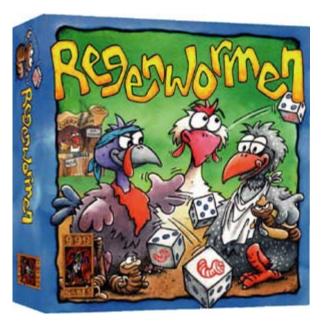
The jury will receive copies of your work. Of course the copies must be legible. So use a black pen for writing and only print on A4 size paper. Any drawings that you hand in, must copy well, so it is better not to use a pencil to make them.

#### JUDGING:

Among other things, the following points are important for the jury:

- How complete the answers for the various parts are;
- the analysis of the game (Assignment A);
- the strategies that have been designed, summarised in the complete game strategy;
- the use of math;
- the argumentation used and how choices that have been made are justified;
- the depth to which the various assignments have been answered;
- the style of presentation: form, legibility, (copyable) illustrations etc;
- originality and creativity.

#### Introduction



In this assignment you will investigate how the game *Earthworms* (Regenwormen) works. You will look at opportunities and chances within the game, and you will see the choices you may run into while playing and what the consequences are. You will think about game strategies in the different phases of the game and in the end you will formulate an optimal winning strategy.

## Preparation

Friday morning we will play the game *Earthworms* for an hour with all teams in the central area. You will play the game *Earthworms* with four people, in pairs from two different team who speak the same language. Before you start, carefully read the rules of the game. The purpose is to get a grip on the rules and on possible game scenarios. This is important for being able to complete the assignments successfully. While you're playing, you can of course ask questions from the members of the committee who are present.

## Part A: Why is the game the way it is?

## Sub-assignment A



Research why the game and the rules have been designed the way they are and present your findings in a coherent report. Try to give substantiated answers to questions such as:

- Why are there eight dice, why not more or less?
- Why are the numbers 21 to 36 on the tiles (and not smaller or larger ones)?
- Why is each turn no longer than six throws?
- Why have the worms been divided over the tiles in the way they have been in the game?
- Why are you allowed to close tiles after a certain point in the game?
- And all other questions that are raised by the game while you're playing.

## Part B: Final- and stop strategy

In this part you will look into the positions towards the end of a turn, and especially whether it's worth stopping your turn. You will calculate some examples and try to draw general conclusions from the results.

Generally speaking, you have several throws in one turn, until you can take a tile, or until you're out. Carefully read the rules before you start.

#### Assignment 1

You probably realised while you were playing that the higher tiles are difficult to take. Find out how many ways there are to take tile 36 during a turn. Present these ways clearly.

#### **Assignment 2**

You have taken out the following dice: 1 - 2 - 2 - 3 - 4 - 4.

So you have two dice left. Work out how large the chance is that you can take a tile in this turn (based on the assumption that all tiles are still open).

#### **Assignment 3**

In his turn a player has had five throws. He has taken out dice with an *earthworm* and with 1, 2, 3 and 5 dots.

Give all possible results (tiles *and* scores) if you know that he has exactly 19 points.

Then calculate for each of these results the chance that the player survives the next throw. Also indicate each time what his minimum and maximum final scores are.

#### **Assignment 4**

A player has had four throws in his turn. He has taken out dice with an *earthworm* and with 1, 3 and 5 dots. You also know that this player has scored a *maximum* of 20 points in the first four throws.

Give all possible results (tiles *and* scores) for the player in the first four throws and then give the chance for each possible result that the player survives the next throw.

#### **Assignment 5**

A player has had four throws in his turn and has taken out a total of six dice. Calculate the chance that he'll be out in the next throw because he throws one of the results he has already cast with the remaining dice.

## Sub-assignment B

Formulate a number of guidelines for a good (winning) **endgame** strategy. At the least include something about a **stopping strategy**. That is a strategy where you determine after each throw whether to stop or play on.

Indicate whether, how and why you use the results from assignments 1 to 5 for this. You will elaborate further in the final assignment.

## Part C: Start strategy - choices and effects

You make choices at all kinds of points in the game. You will now look into the possible effects of these choices at the start of your turn.

<u>Example</u>: If you throw **5 - R - 4 - 4 - 3 - 3 - 2** in your first throw, you can take out two 4s, or the *earthworm* (R for 'Regenworm'), or the 5, or the 2 or the three 3s. Your choice to do any of these already determines to some extent how many points you can score and what your chances are.

#### **Assignment 6**

Your first throw is 1 - 3 - 2 - 1 - 4 - 3 - R - 4

Your goal is to gain as high a score as possible in your turn. You have to make your choices with this goal in mind. Describe which dice you would take out and give your considerations.

#### **Assignment 7**

We assume a turn with the following series of throws:

Die1	Die2	Die3	Die4	Die5	Die6	Die7	Die8
5	2	2	4	1	5	R	R
4	3	5	R	R	2	4	2
R	4	4	5	R	1	4	3
4	2	R	R	2	3	2	3
1	1	R	3	3	1	3	2
5	R	1	2	4	4	5	5

Find out what the minimum and maximum scores in this turn are. Give the run of the game that leads to this result each time. Use the explanation below.

Explanation about using the throws: Because you take out dice every time, you will only use a part of the throw after the second one: namely only the dice that you have left. If for example you take out the two 2s in the first throw, you will no longer use the results for die 2 in the next throw (Die2) and die 3 (Die3). You will find several copies of these throws in appendix 1.

## Sub-assignment C

What exactly do you do at the start of your turn? How do you determine which dice you take out? What is the goal of your turn? Formulate a number of underpinned guidelines for this: the **starting strategy**.

Indicate whether, how and why you use the results from assignment 6 and 7 for this. You may, if you want, use appendix 2 which has some simulations of turns, to further examine your strategy.

You will elaborate this sub-assignment further in the final assignment.

## FINAL ASSIGNMENTS

#### **FINAL ASSIGNMENT 1**

In sub-assignment A you wrote a text about the game *Earthworms*. Next you have designed partial strategies for the game in sub-assignments B and C, namely an endgame and stop strategy (B) and a start strategy (C).

Combine your findings from sub-assignments B and C to formulate usable rules for a complete game strategy. Give a short (mathematical) underpinning (you may also do this by referring to previous answers).

#### Report

You will hand in a report that can be read on its own, consisting of:

- The elaboration of sub-assignment A
- The underpinned complete game strategy, based on sub-assignments B and C.

- The answers to assignments 1 to 7 (as an appendix)
- The original texts for sub-assignments B and C can be included as appendixes, but it isn't necessary.

This report you will hand in to the organisation Saturday 13:00 o'clock at the latest.

#### **FINAL ASSIGNMENT 2**

We will end the finals weekend the way we started it: by playing *Earthworms*. Every participant will play using a prescribed strategy: one of the strategies thought up by the participating teams.

Make a clearly legible instruction card for your own strategy, which has to meet the following conditions:

- the card has to be in English.
- the maximum size is one A4 sheet of paper, font Arial, size: 16pt.
- the strategy must be understandable for players who did *not* take part in the Alympiad.
- you can choose yourselves how to present your strategy.
- the name of your school must be included.

You must hand in this piece of paper to the organisation on Saturday at 12:00 o'clock at the latest (so we can make copies for the final *Earthworms* tournament).

#### CONCLUSION

The Alympiad committee organises the final tournament. The players will be yourselves, and additionally outside guests and members of the committee. Every game table needs a games master and an observer. Two players from each team will play with their own strategy.

The games master doesn't play, but explains the game to 'guest players', writes down the scores on the score form and decides in doubtful situations.

The observer doesn't play either, but keeps an eye on the time and will answer questions about the rules by the guest players. Teams with three members don't have to provide an observer.

So each member of the team is assigned a role:

- games master
- (observer)
- player

On Friday members of the committee will drop by to ask your team how you have divided these roles.

The team with the best scoring strategy will receive the public award during the prize giving ceremony on April 23 in Utrecht.



## Colophon

#### Alympiad committee:

Aldine Aaten, Lonneke Boels, Marcel Daems, Tom Goris, Dédé de Haan, Willem Hoekstra, Matthias Lippert, Johan van de Leur, Ruud Stolwijk and Monica Wijers.

#### Thanks to:

- 999Games for providing the games for the final weekend.Wietske de Blauw (999Games).

## Appendix 1: the throws in assignment 7

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5	2	2	4	1	5	R	R
4	3	5	R	R	2	4	2
R	4	4	5	R	1	4	3
4	2	R	R	2	3	2	3
1	1	R	3	3	1	3	2
5 4 R 4 1 5	2 3 4 2 1 R	2 5 4 R R	R 5 R 3	R R 2 3	5 2 1 3 1 4	R 4 4 2 3 5	R 2 3 3 2 5
5 4 R 4 1 5	2 3 4 2 1 R	2 5 4 R R	4 R 5 R 3	1 R R 2 3 4	5 2 1	R 4 2 3 5	R 2 3 3 2 5
4	3	5	R	R	2	4	2
R	4	4	5	R	1	4	3
4	2	R	R	2	3 1 4	2	3
1	1	R	3	3	1	3	2
5	R	1	2	4	4	5	5
5	2	2	1	1	5	R	R
5 4 R 4 1 5	2 3 4 2 1 R	2 5 4 R R	4 R 5 R 3	P	2	1	R 2 3 3 2 5
	1	1	1\	R R 2 3 4	2	4 4 2 3 5	2
1	2	4 D	D	2	2	2	2
4	4	R	2	2	3	2	ა ე
<u> </u>		K	3	3	3 1 4	3	
5	K	1	2	4	4	5	5
5	2	2	4	1	5	R	R
4	3	5	R	R	2	4	2
R	4	4	5	R	2	4	3
4	2	R	R	2		2	3
1	1	R	3	3	3 1	3	2
5 4 R 4 1 5	2 3 4 2 1 R	2 5 4 R R	4 R 5 R 3	R R 2 3 4	4	R 4 2 3 5	R 2 3 3 2 5
5	2	2	4	1	5 2 1	R	R
4	3	5	R	R	2	4	2
R	4	4	5	R	1	4	3
4	2	R	R	2	3	2	3
1	1	R	3	3	3	3	2
5 4 R 4 1	2 3 4 2 1 R	2 5 4 R R	4 R 5 R 3	1 R R 2 3 4	4	R 4 2 3 5	R 2 3 2 5
5	2	2	4	1	5	R	R
		_	-	-	_		
4	3	5	R	R	2	4	2
R 4	4 2 1	4 R	5 R 3 2	R 2 3	1 3 1	4 2 3	3 2
4	2	K	K	2	3	2	3
1		R	3	3		3	2
5	R	1	2	4	4	5	5
5	2	2	4	1	5	R	R
4	2 3 4 2 1 R	2 5	R	R	2	4	R 2 3 3 2 5
R	4	4	R 5	R	2	4 4 2 3 5	3
4	2	R	R	2		2	3
R 4 1	1	R	R 3 2	3 4	3 1	2	2
5	 _		2	1	4	5	
၁	K	1		4	4	၁	၁

5	2	2	4	1	5	R	R
4	3	5	R		2	4	2
R	4	4	5	R	1	4	3
4	2	R	R	2	٦	2	3
1	1	R	3	3	1	2	2
5 4 R 4 1	2 3 4 2 1 R	5 4 R R	4 R 5 R 3	R R 2 3	5 2 1 3 1 4	4 4 2 3 5	R 2 3 2 5
5 4 R 4 1 5	2 3 4 2 1 R	2 5 4 R R	4	1	5	R 4 4 2 3 5	R 2 3 3 2 5
4	3	5	R	R	2	4	2
R	4	4	5	R	1	4	3
4	2	R	R	2	3	2	3
1	1	R	R 5 R 3	R R 2 3 4	2 1 3 1 4	3	2
5	R	1	2	4	4	5	5
5	2	2	4	1	5	R	R
5 4 R 4 1 5	2 3 4 2 1 R	2 5 4 R R	4 R 5 R 3	R	5 2 1 3 1 4	4	R 2 3 2 5
R	4	4	5	R R 2 3	1	4 4 2 3 5	3
4	2	R	R	2	3	2	3
1	1	R	3	3	1	3	2
5	R	1	2	4	4	5	5
5	2	2	4	1	5	R	R
4	2	5	R	R	2	1	2
R	1	1	5	R	5 2 1 3	1	3
1	2	P	P	2	٦ .	2	3
1	1	P	3	3	1	<u>۲</u>	2
5 4 R 4 1 5	2 3 4 2 1 R	2 5 4 R R	4 R 5 R 3	1 R R 2 3	4	R 4 2 3 5	R 2 3 3 2 5
				4	4		
5 4 R 4 1 5	2 3 4 2 1 R	2 5 4 R R	4 R 5 R 3	1	5 2 1	R 4 2 3 5	R 2 3 3 2 5
4	3	5	R	R	2	4	2
R	4	4	5	R	1	4	3
4	2	R	R	2	3	2	3
1	1	R	3	1 R R 2 3 4	1	3	2
5	R	1	2	4	4	5	5
5	2	2	4	1	5	R	R
4	3	5	R	R	2	4	2
R	4	4	5	R	1	4	3
4	2	R	R	2	3	2	3
1	1	R	3	3	1	3	2
5 4 R 4 1 5	3 4 2 1 R	4 R R 1	5 R 3	R 2 3 4	2 1 3 1 4	4 4 2 3 5	2 3 3 2 5
5 4 R 4 1 5	2 3 4 2 1 R	2 5 4 R R	4	1	5 2 1 3 1 4	R 4 2 3 5	R 2 3 2 5
4	3	5	R	R	2	4	2
R	4	4	5	R	1	4	3
4	2	R	R	2	3	2	3
1	1	R	R 5 R 3	R R 2 3 4	1	3	2
5	R	1	2	4	4	5	5

## Appendix 2: 24 different turns

3         3         5         1         1         4         R         1         2         1         R         1         4         4         4         2         2         2         4         4         5         5         5         3         5         2         2         2         4         4         5         5         5         3         5         2         2         2         4         4         8         R         R         5         2         2         2         4         R         R         R         5         2         2         2         5         3         4         8         R         8         5         2         2         1         3         1         8         5         2         2         1         8         3         1         8         5         2         2         1         8         3         1         8         5         2         1         8         3         1         8         4         4         8         1         3         1         8         4         4         8         1         1         8         5         5         5	4	4	5	1	4	2	1	2		1	5	1	5	4	4	4	2
2       4       4       5       5       5       3       5         2       4       3       4       4       3       1       3         4       3       1       1       4       2       2       5         1       3       5       5       2       1       3       1       8       8       5         2       2       3       4       R       R       6       5       2       R       8       1       R       2       1       1       R       1       2       R       2       1       1       R       2       1       1       R       1       2       R       8       2       1       1       R       1       2       R       8       2       1       1       R       1       2       R       8       2       1       1       1       1       2       R       1       1       2       R       1       1       3       1       2       8       1       1       1       1       1       1       1       1       1       1       1       1       1       1       1	3	3	5	1	1	4	R	1		5	1	5	5	R	1	3	1
2       4       3       4       4       3       1       3         4       3       1       1       4       2       2       5         1       3       5       5       2       1       3       1       R       5         4       2       3       4       R       R       R       5         2       2       5       3       R       4       4       R         1       5       3       3       3       1       2       R       2       1       1       1       2       R       2       1       1       1       2       R       2       1       1       1       2       R       2       1       1       1       2       R       2       1       1       1       2       R       2       1       1       1       1       2       R       1       2       1       1       1       1       2       1       1       1       1       1       1       1       1       1       1       1       1       1       1       1       1       1       1       1 <td< td=""><td>2</td><td>1</td><td>R</td><td>1</td><td>4</td><td>4</td><td>4</td><td>2</td><td></td><td>R</td><td>5</td><td>3</td><td>R</td><td>R</td><td>5</td><td>2</td><td>2</td></td<>	2	1	R	1	4	4	4	2		R	5	3	R	R	5	2	2
4       3       1       1       4       2       2       5         1       3       5       5       2       1       3       1       R       5         2       2       5       3       R       4       4       R       2       1       1       1       2       1       1       1       2       1       1       1       1       2       1       1       1       1       2       1       1       1       1       2       1       1       1       1       1       2       1       1       1       1       2       1       1       1       1       2       1       1       1       1       2       1       1       1       1       2       1       1       1       1       2       1       1       1       2       1       1       1       2       1       1       1       1       2       1       1       4       1       1       2       1       1       4       4       1       1       2       1       1       4       4       1       1       2       1       1       1 <td>2</td> <td>4</td> <td>4</td> <td>5</td> <td>5</td> <td>5</td> <td>3</td> <td>5</td> <td></td> <td>4</td> <td>R</td> <td>R</td> <td>R</td> <td>5</td> <td>3</td> <td>4</td> <td>3</td>	2	4	4	5	5	5	3	5		4	R	R	R	5	3	4	3
1	2	4	3	4	4	3	1	3		4	5	4	3	2	1	R	3
4       2       3       4       R       R       R       5         2       2       5       3       R       4       4       R         1       5       3       3       3       1       2       5       1       4       2       2       3       5       5       5       5       5       5       5       5       5       5       5       5       5       7       8       2       2       3       5       3       4       4       3       6       5       3       3       8       5       5       3       3       8       5       5       5       1       1       3       4       8       8       8       8       8       8       8       8       8       8       8       8       8       8       8       8       8       9       8       8       9       8       9       8       9       8 <td>4</td> <td>3</td> <td>1</td> <td>1</td> <td>4</td> <td>2</td> <td>2</td> <td>5</td> <td><u></u></td> <td>4</td> <td>R</td> <td>4</td> <td>R</td> <td>3</td> <td>1</td> <td>R</td> <td>5</td>	4	3	1	1	4	2	2	5	<u></u>	4	R	4	R	3	1	R	5
4       2       3       4       R       R       R       5         2       2       5       3       R       4       4       R         1       5       3       3       3       1       2       5       1       4       2       2       3       5       5       5       5       5       5       5       5       5       5       5       5       5       7       8       2       2       3       5       3       4       4       3       6       5       3       3       8       5       5       3       3       8       5       5       5       1       1       3       4       8       8       8       8       8       8       8       8       8       8       8       8       8       8       8       8       8       9       8       8       9       8       9       8       9       8 <td></td> <td></td> <td></td> <td></td> <td></td> <td></td> <td></td> <td></td> <td>-</td> <td></td> <td></td> <td></td> <td></td> <td></td> <td>1</td> <td></td> <td></td>									-						1		
2       2       5       3       R       4       4       R         1       5       3       3       3       1       2       5       1       4       2       2       3       5        5       5       5       5       5       3       3       8       5       3       3       8       5       3       3       8       5       3       3       4       4       4       4       4       4       4       4       4       4       4       4       8       7       1	1	3	5	5	2	1	3	1	_								-
1       5       3       3       3       1       2         5       4       5       2       R       4       4       3         5       4       4       4       3       1       3       4       R       1       3       4       R       1       3       4       R       1       3       4       R       1       3       4       R       1       3       4       R       1       3       4       R       1       3       4       R       1       3       4       R       1       3       4       R       1       3       4       R       1       3       4       R       1 <td></td> <td></td> <td>3</td> <td></td> <td>R</td> <td>R</td> <td>R</td> <td></td> <td>_</td> <td></td> <td>2</td> <td></td> <td></td> <td></td> <td></td> <td></td> <td>-</td>			3		R	R	R		_		2						-
5       4       5       2       R       4       4       3       3       1       3       4       R       1       3       4       R       1       3       4       R       1       3       4       R       1       3       4       R       1       3       4       R       1       3       4       R       1       3       4       R       1       3       R       5       3       3       R       5       5       1       1       R       8       5       3       3       8       5       1       1       R       8       8       5       3       3       8       5       1       1       R       8       1       3       1       R       8       1       3       1	2	2				4	4		_			2					-
5       4       4       3       1       3       5       R         5       5       5       5       1       1       3       1       3         4       5       R       5       R       3       5       3         2       2       3       5       R       3       5       3         2       2       3       5       R       4       4       4       4       2       5       1         3       1       3       1       R       5       3       4       4       3       5       R       4       1       1       2       4       1       R       1 <td< td=""><td>1</td><td>5</td><td>3</td><td>3</td><td>3</td><td>3</td><td>1</td><td></td><td>_</td><td>5</td><td>1</td><td>4</td><td></td><td>_</td><td></td><td></td><td>-</td></td<>	1	5	3	3	3	3	1		_	5	1	4		_			-
5       5       5       1       1       3       1       3         4       5       R       5       R       3       5       3         2       2       3       5       R       4       4       1       1       2       4       1       R       1       1       R       1       1       R       1       1       1       1       1       1       R       1 <td>5</td> <td>4</td> <td>5</td> <td>2</td> <td>R</td> <td>4</td> <td>4</td> <td>3</td> <td>_</td> <td></td> <td></td> <td>_</td> <td></td> <td></td> <td></td> <td></td> <td>-</td>	5	4	5	2	R	4	4	3	_			_					-
4       5       R       5       R       3       5       3         2       2       3       5       3       4       4       3       5       R       4       R       1       3       5       R       4       R       1       3       1       R       1       3       1       R       5       3       4       4       3       5       R       4       R       1       3       2       1       3       8       5       1       1       3       4       8       1       3       1       R       1       3       4       4       5       1       1       3       2       1       1       3       8       8       1       1       3       2       4       4       4       5       3       2       R       4       4       4       2       8       4       4       5       3       1       1       1       1       1       1       1       1       1       1       1       2       8       4       4       1       2       8       4       4       2       3       1       1       2	5	4	4	3	1	3	5	R		2	3	5	R	3	R	5	3
4       5       R       5       R       3       5       3         2       2       3       5       3       4       4       3       5       R       4       R       1       3       5       R       4       R       1       3       1       R       1       3       1       R       5       3       4       4       3       5       R       4       R       1       3       2       1       3       8       5       1       1       3       4       8       1       3       1       R       1       3       4       4       5       1       1       3       2       1       1       3       8       8       1       1       3       2       4       4       4       5       3       2       R       4       4       4       2       8       4       4       5       3       1       1       1       1       1       1       1       1       1       1       1       2       8       4       4       1       2       8       4       4       2       3       1       1       2		_	_						Г	_	4	4	_	_	_	_	
2       2       3       5       3       4       4       3         R       1       3       1       R       5       3       4       R       1       3         2       4       3       5       1       3       4       R       1       3         3       5       3       4       3       2       4       1       3       3       R       4       5       1       1       3       1       1       3       1       1       3       1       1       3       1       1       3       1       1       3       1       1       3       1       1       3       1       1       1       3       1       1       1       2       1       3       1       1       1       2       1       3       1       1       1       2       5       3       1 </td <td></td> <td></td> <td>_</td> <td>_</td> <td></td> <td></td> <td></td> <td></td> <td>-</td> <td></td> <td></td> <td></td> <td></td> <td></td> <td></td> <td></td> <td>-</td>			_	_					-								-
R       1       3       1       R       5       3       4         2       4       3       5       1       3       4       R       4       5       1       3       R       R       4       5       1       3       R       R       4       5       1       3       R       R       4       8       4       8       4       2       2       R       4       7       8       8       4       8       4       8       4       2       8       8       4       4       8       4       8       4       2       8       8       4       4       8       4       8       4       2       8       8       4       8       5       7       5       5       3       1       1       1       2       5       5       3       1       1       1       2       5       3       1       1       1       2       1								_	-				_				
2       4       3       5       1       3       4       R       4       5       1       3       R       R       4       5       1       3       R       R       4       5       3       2       R       R       4       5       3       2       R       R       4       8       4       2         1       R       R       R       2       R       R       3       3       2       R       4       8       4       2       2       4       3       R       4       8       4       2       2       4       3       R       4       8       4       2       2       4       3       R       4       8       5       3       2       R       4       5       5       5       3       1	-								-				_				
3       5       3       4       3       2       4       1         1       R       R       4       2       R       R       3         3       1       R       3       R       5       R       5         5       3       1       4       2       1       5       5         3       3       2       2       3       4       4       5         5       3       2       R       2       5       2       1         3       3       3       4       4       5       5       3       1       1       1       1       1       1       1       2       3       1       <								_									_
1       R       R       4       2       R       R       3       R       5       R       4       1       2       R       4       3       R       4       1       2       R       4       3       R       4       1       2       R       4       1       2       5       3       1				5			4		_								_
3       1       R       3       R       5       R       5         5       3       1       4       2       1       5       5       3       1       1       2       5       3       1	3	5	3	4	3	2	4	1	L	3	2	R	R	4	R	4	2
3       1       R       3       R       5       R       5         5       3       1       4       2       1       5       5       3       1       1       2       5       3       1	4	Ъ	Ъ	4	2	Ь	ь	2	Г	2	1	2	2	1	2	Ь	1
5       3       1       4       2       1       5       5       3       1			_						-								
3       3       2       2       3       4       4       5         5       3       2       R       2       5       2       1         3       R       3       3       4       1       5       4         3       3       3       4       1       5       4         3       3       3       1       3       7       8       5         3       3       3       1       3       8       7       8       5         3       5       5       4       4       1       2       4       3       3       1       2       5       8       5       8       3       4       4       1       2       8       5       8       3       4       4       1       2       8       5       8       3       4       4       4       3       3       1       1       1       2       8       5       8       3       4       4       4       3       8       3       1       4       4       4       3       8       1       1       3       3       1       1       <		-															-
5     3     2     R     2     5     2     1       3     R     3     3     4     1     5     4     2     3     1     2     3     1     5       4     2     3     1     2     1     2     4     2     3     1     2     8     3     5       3     3     3     1     3     8     8     5     8     5     8     3     4     1     2     5     8     3     1     2     5     8     3     4     1     2     5     8     3     4     1     2     5     8     3     4     4     4     3     3     4 <td></td> <td></td> <td></td> <td></td> <td></td> <td></td> <td></td> <td></td> <td>-</td> <td></td> <td></td> <td></td> <td></td> <td></td> <td></td> <td></td> <td>-</td>									-								-
3     R     3     3     4     1     5     4       5     R     2     1     2     1     5     4       3     3     3     1     3     R     R     5       3     5     5     4     4     1     R     1       1     2     5     4     1     1     2     1     2     1     2     1     3     3     1     2     5     1     3     3     4     4     4     2     5     3     4 <td></td> <td></td> <td>_</td> <td></td> <td></td> <td></td> <td></td> <td>_</td> <td>-</td> <td></td> <td></td> <td>_</td> <td></td> <td></td> <td></td> <td></td> <td>-</td>			_					_	-			_					-
5       R       2       1       2       1       5       4         3       3       3       1       3       R       R       5         3       5       5       4       4       1       R       1         1       2       5       4       1       1       2       1       2       1       2       1       3       1       2       5       1       3       4       4       4       1       2       1       2       1       2       1       2       1       3       4 </td <td>_</td> <td></td> <td>_</td> <td></td> <td></td> <td></td> <td></td> <td>_</td> <td></td> <td></td> <td></td> <td></td> <td></td> <td></td> <td></td> <td></td> <td>_</td>	_		_					_									_
3       3       3       1       3       R       R       5         3       5       5       4       4       1       R       1         1       2       5       4       1       1       2       1       2       1       2       1       1       2       3       1       2       5       1       1       3       4       4       3       3       4       8 <td>S</td> <td>Κ</td> <td>S</td> <td>J</td> <td>4</td> <td>ı</td> <td>Э</td> <td>4</td> <td>L</td> <td>7</td> <td></td> <td>J</td> <td>ı</td> <td>_</td> <td>J</td> <td>۷</td> <td><u> </u></td>	S	Κ	S	J	4	ı	Э	4	L	7		J	ı	_	J	۷	<u> </u>
3       3       3       1       3       R       R       5         3       5       5       4       4       1       R       1         1       2       5       4       1       1       2       1       2       1       2       1       1       2       3       1       2       5       1       1       3       4       4       3       3       4       8 <td>5</td> <td>R</td> <td>2</td> <td>1</td> <td>2</td> <td>1</td> <td>5</td> <td>4</td> <td></td> <td>2</td> <td>4</td> <td>3</td> <td>5</td> <td>2</td> <td>R</td> <td>3</td> <td>5</td>	5	R	2	1	2	1	5	4		2	4	3	5	2	R	3	5
3       5       5       4       4       1       R       1         1       2       5       4       1       4       1       2         3       3       R       2       R       2       2       1       1       2       5       1       1       3       4       8       9       8       9       9       9 <td></td> <td></td> <td>_</td> <td></td> <td></td> <td></td> <td></td> <td>_</td> <td></td> <td></td> <td>1</td> <td></td> <td></td> <td></td> <td></td> <td></td> <td>-</td>			_					_			1						-
1       2       5       4       1       4       1       2         3       3       R       2       R       2       2       1       1       2       5       1       1       3       3       1       4       8       8       7       8       9       8       9       8       9				4							R	5	R		R		
3       3       R       2       R       2       2       1         5       4       3       4       2       3       R       3         R       1       2       4       4       2       5       3         3       R       3       1       2       1       3       3         3       R       3       1       2       1       3       3         8       1       5       2       1       8       1 <td< td=""><td>_</td><td></td><td>_</td><td>_</td><td></td><td></td><td></td><td></td><td></td><td>1</td><td>2</td><td>5</td><td>2</td><td></td><td>1</td><td></td><td>3</td></td<>	_		_	_						1	2	5	2		1		3
Triangle 1     Triangle 2     Triangle 3     Triangle 4     Triangle 4     Triangle 5     Triangle 4     Triangle 5     Triangle 5     Triangle 6     Triangle 6 <td></td> <td></td> <td></td> <td>-</td> <td></td> <td></td> <td></td> <td></td> <td></td> <td>1</td> <td>4</td> <td>4</td> <td>3</td> <td></td> <td>1</td> <td>4</td> <td>-</td>				-						1	4	4	3		1	4	-
R     1     2     4     4     2     5     3       3     R     3     1     2     1     3     3       R     1     5     2     1     R     3     3       1     5     2     3     3     2     5     5       1     5     2     3     3     2     5     5       2     7     8     5     5     5     5       2     8     5     2     5     5     5       2     8     5     2     5     5     5											3	3			R	2	R
3     R     3     1     2     1     3     3       R     1     5     2     1     R     3     3       1     5     2     3     3     2     5     5       1     5     2     4     5     5     2       2     R     5     2     5     5     2					<u> </u>		<u> </u>		<u>-</u>								
R     1     5     2     1     R     3     3       1     5     2     3     3     2     5     5       R     1     5     2     4     5     5     2       R     1     5     2     4     5     5     2	R	1	2	4	4	2	5	3		5	1	5	4	R	4	3	R
1     5     2     3     3     2     5     5       R     1     5     2     4     5     5     2       E     2     8     5     2     5     5     5     2	3	R	3	1	2	1	3	3		3	4	2	2	4	R	1	1
R 1 5 2 4 5 5 2 2 8 5 5 5 2	R	1	5	2	1	R	3	3		3	3	R	5	1	1	1	R
	1	5	2	3	3	2	5	5		5	2	5	3	4	4	5	3
1 R 2 R R 1 1 R 5 4 4 5 2 2 1 R	R	1	5	2	4	5	5	2		2	R	5	2	5	5	5	2
	1	R	2	R	R	1	1	R		5	4	4	5	2	2	1	R

1	D		2	1	2		1	2	2	1	1	1	1	1	2
4	R	5	2	1	3	5	1	3	3	4	4	4	1	4	2
1	4	5	1	2	5	R	1	R	2	5	1	5	5	5	2
2	5	5	3	5	5	1	3	4	5	5	3	2	R	5	2
1	4	3	R	R	4	4	1	2	1	3	1	3	3	4	5
4	4	R	1	4	3	5	3	3	R	3	4	3	3	3	3
4	3	5	4	R	3	2	3	2	2	1	3	2	2	1	2
2	1	1	1	4	2	2	4	1	2	2	R	3	1	2	R
4	2	R	5	4	R	4	3	3	4	5	2	3	R	R	5
5	4	1	R	3	4	5	R	4	1	4	R	2	5	R	3
4	R	4	R	5	3	1	5	5	3	4	2	4	4	2	2
1	5	2	4	5	R	3	1	R	R	3	R	1	4	1	R
4	R	1	1	3	2	2	5	1	R	2	2	2	4	5	2
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1	R	1	1	3	5	4	5	5	4	1	1	5	1	2	4
1	3	R	5	3	5	2	4	3	3	1	2	2	1	R	5
3	5	5	4	R	R	3	5	1	2	1	4	R	3	2	2
5	4	1	R	1	5	3	5	R	1	2	3	4	5	5	3
1	2	5	2	2	5	1	2	4	1	5	3	4	5	3	5
3	5	3	R	4	1	3	1	1	1	5	5	3	4	4	3
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2	4	2	5	2	5	5	2	4	4	4	5	3	3	2	4
3	4	R	3	2	5	R	R	1	2	R	R	1	5	2	4
5	1	2	3	R	5	2	3	5	4	1	2	5	5	2	R
R	R	4	4	R	R	2	2	4	R	R	R	R	5	R	4
4	1	1	2	3	4	3	3	R	5	1	R	3	1	5	2
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3	R	3	3	5	5	5	5	2	4	4	3	4	2	R	4
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5	R	R	R	5	R	2	3		2	5	2	5			
5 3	R R	R	R	2	2	2	R	2	4	4	1	5	1	R	1
5 3 4	R R 2	R 4	R 2	2	2	2	R 1	2	4	4	1 5	5 1	1 4	R 1	1 5
5 3	R R	R	R	2	2	2	R	2	4	4	1	5	1	R	1
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5 3 4 2	R R 2 1	R 4 5	R 2 5	2 3 2	2 2 5	2 3 2	R 1 5	2 2 R	4 1 1	4 4 2 3	1 5 R	5 1 1	1 4 3	R 1 2	1 5 R
5 3 4 2 R	R R 2 1	R 4 5 3 1	R 2 5 1 4	2 3 2 1 4	2 2 5 4 3	2 2 4	R 1 5 4 2	2 2 R	4 1 1 R R	4 4 2 3 2	1 5 R R	5 1 1 R 1	1 4 3 4 R	R 1 2 1 5	1 5 R 5 R
5 3 4 2 R R 4	R R 2 1 R R	R 4 5 3 1	R 2 5 1 4 1	2 3 2 1 4 2	2 2 5 4 3 5	2 3 2 2 4 4	R 1 5 4 2 4	2 R 2 4 4	4 1 1 1 R R R 3	4 4 2 3 2 R	1 5 R R 1 2	5 1 1 R 1 4	1 4 3 4 R 2	1 2 1 5 4	1 5 R 5 R
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