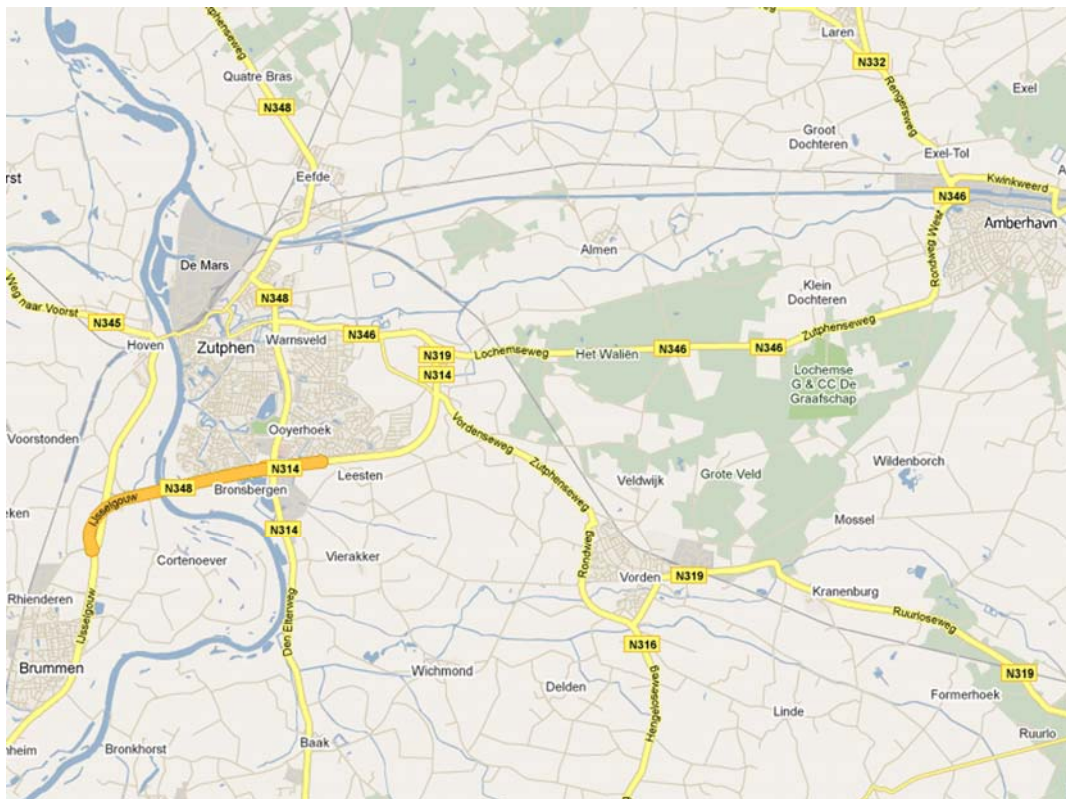# FAIR COMPETITION?

Preliminary round assignment 21st Mathematics A-lympiad

November 20, 2009

**Instructions for the preliminary round assignment for the
Mathematics A-lympiad 2009/2010**

This Mathematics A-lympiad assignment consists of three introductory assignments, and two final assignments.

General advice for working on this assignment

- First read the entire text of the assignment so that you'll know everything you have to do.
- **Keep an eye on the time you spend on the introductory and follow-up assignments, make sure you have enough time left for the final assignment.** Divide tasks where possible and confer with each other when needed.
- If you have divided your tasks, discuss the results of the introductory assignments with each other before you start on the final assignment.
- For the final assignment it is important that you formulate a real advice with a substantiated plan of evacuation, if necessary complemented by an alternative plan.
- The answers to the introductory and follow-up assignments should not be included in the elaboration of the final assignment. Add the results of the introductory and follow-up assignments as an appendix.

Hand in:

- A clear and detailed procedure (assignment 5) and the answer to assignment 4, supported by calculations and arguments.
- As appendices: the results of the introductory assignments 1, 2 and 3.

The jury will be given copies of your work. Of course these copies must be legible. Therefore write and draw with a black pen, only print on A4 paper, and do not use pencil for any of your drawings. If in doubt, make a test copy yourself!

Assessment

Among other things, the jury will pay attention to:

- legibility and clarity of the final assignment,
- whether the work is complete,
- the use of mathematics,
- the arguments used and justification of choices (realism may play a part here),
- the level to which the assignment has been done,
- presentation: form, legibility, structure, use and function of appendices, etc.

Have fun and good luck!

Fair competition?

Usually, in competitions between sports clubs from different towns the clubs play against each other once. That means that, over the course of a season, each club will play several away matches, as well as home matches.

In this assignment, we will take a closer look at a number of different competition setups.

Introductory assignments - Part 1

We will take a look at the UEFA Cup for the season 2004 - 2005. This is an international football competition, organised by UEFA (Union of European Football Associations), where the participating teams are divided into groups of five to play a mutual competition. In this competition the teams all play each other once. In addition all teams play two home matches and two away matches.

We will look at the 2004-2005 competition between the following teams:

> Parma FC from Parma, Italy
> Besiktas from Istanbul, Turkey
> Atletico Bilbao from Bilbao, Spain
> Steaua Bucharest from Bucharest, Romania
> Standard Liège from Liège, Belgium

The schedule for this group looked as follows:

1st round (October 21, 2004)
Steaua Bucharest - Standard Liège
Atletico Bilbao - Parma

2nd round (November 4, 2004)
Parma - Steaua Bucharest
Besiktas - Atletico Bilbao

3rd round (November 25, 2004)
Steaua Bucharest - Besiktas
Standard Liège - Parma

4th round (December 1, 2004)
Besiktas - Standard Liège
Atletico Bilbao - Steaua Bucharest

5th round (December 16, 2004)
Parma - Besiktas
Standard Liège - Atletico Bilbao

The match was played at the first of each pair of teams mentioned. So, in the fourth round Besiktas played at home against Standard Liège.

The distances (in kilometres) between the various towns are listed in the table below:

	Liège	Bucharest	Bilbao	Parma	Istanbul
Liège		1700	1100	800	2100
Bucharest	1700		2300	1300	500
Bilbao	1100	2300		1100	2600
Parma	800	1300	1100		1600
Istanbul	2100	500	2600	1600	

Assignment 1

Check for each team how much they had to travel overall for the given schedule.

In the schedule provided not every team travels the same distance; it would have been a coincidence if they had! But you could make the competition 'more fair' by letting all teams travel more or less the same distance.

Assignment 2

Make a new schedule using <u>travel times</u> to make the competition 'more fair'. The only requirement in the new schedule is that the teams still play each other once. Clearly indicate your choices and why you made them!

Introductory assignments - Part 2

The KNVB (Koninklijke Nederlandse Voetbalbond) organises football competitions for junior teams everywhere in the Netherlands. In each of these competitions a number of teams will play every other team once. In this part of the assignment we will look at one of the competitions, which consists of twelve F teams from the north of Gelderland:

Name of the team	City of the team	Starting time
AZC F2	Zutphen	8:45
Be Quick Z. F2	Zutphen	10:30
Be Quick Z. F3	Zutphen	10:00
Brummen F5	Brummen	9:00
Eefde SP F1	Eefde	9:00
Eerbeekse Boys F5	Eerbeek	9:00
Epse F1	Epse	9:30
Harfsen F1	Harfsen	10:00
SBC '05 F1	Wichmond	9:30
Steenderen F2	Steenderen	11:00
Vorden F2	Vorden	9:00
Warnsveldse Boys F3	Warnsveld	8:30

An F team consists of boys and girls of 7 and 8 years old. It is therefore logical that the parents accompany the players to the matches. Transport is (usually) by car, but for (very) short distances it might be possible to cycle instead.

The following schedule is proposed:

Table 1: Schedule

		Away											
		AZC F2	Be Quick Z. F2	Be Quick Z. F3	Brummen F5	Eefde SP F1	Eerbeekse Boys F5	Epse F1	Harfsen F1	SBC '05 F1	Steenderen F2	Vorden F2	Warnsveldse Boys F3
Home	AZC F2	*	1	2	3	4	5	-	-	-	-	-	-
	Be Quick Z. F2	-	*	3	4	5	6	7	-	-	-	-	-
	Be Quick Z. F3	-	-	*	5	6	7	8	9	-	-	-	-
	Brummen F5	-	-	-	*	7	8	9	10	11	-	-	-
	Eefde SP F1	-	-	-	-	*	9	10	11	1	2	-	-
	Eerbeekse Boys F5	-	-	-	-	-	*	11	1	2	3	4	-
	Epse F1	6	-	-	-	-	-	*	2	3	4	5	1
	Harfsen F1	7	8	-	-	-	-	-	*	4	5	6	3
	SBC '05 F1	8	9	10	-	-	-	-	-	*	6	7	5
	Steenderen F2	9	10	11	1	-	-	-	-	-	*	8	7
	Vorden F2	10	11	1	2	3	-	-	-	-	-	*	9
	Warnsveldse Boys F3	11	2	4	6	8	10	-	-	-	-	-	*

<u>Explanation</u>: in the schedule you can see that there is a total of twelve teams, who all play each other once. Every team therefore plays eleven matches. The numbers indicate the rounds of play: as you can see, in the first round AZC F2 plays Be Quick Z F2 at home, and in the seventh round Eefde SP F1 plays away against Brummen F5.

Because each team plays eleven matches, it is not possible to have each team play home and away equally. You can try to make the competition as fair as possible by for instance having all teams play at home five times and away six times, or six times at home and five away.

We know from **Part 1** that travel time and distance can also play a role in making the schedule fairer. The journey time (in minutes by car) between the various sports centres where the teams play their home matches can be found in the following table:

Table 2: travel distances in minutes by car

	AZC F2	Be Quick Z. F2	Be Quick Z. F3	Brummen F5	Eefde SP F1	Eerbeekse Boys F5	Epse F1	Harfsen F1	SBC '05 F1	Steenderen F2	Vorden F2	Warnsveldse Boys F3
AZC F2	*	10	10	17	7	32	17	24	19	22	21	12
Be Quick Z. F2	10	*	0	15	9	32	20	27	10	14	17	11
Be Quick Z. F3	10	0	*	15	9	32	20	27	10	14	17	11
Brummen F5	17	15	15	*	21	20	31	38	27	16	27	23
Eefde SP F1	7	9	9	21	*	35	12	19	18	21	20	11
Eerbeekse Boys F5	32	32	32	20	35	*	26	38	38	32	43	40
Epse F1	17	20	20	31	12	26	*	14	28	31	30	21
Harfsen F1	24	27	27	38	19	38	14	*	33	38	33	26
SBC '05 F1	19	10	10	27	18	38	28	33	*	13	13	18
Steenderen F2	22	14	14	16	21	32	31	38	13	*	23	22
Vorden F2	21	17	17	27	20	43	30	33	13	23	*	18
Warnsveldse Boys F3	12	11	11	23	11	40	21	26	18	22	18	*

The schedule in table 1 only took the number of home and away matches into account. This will of course result in all kinds of unwanted situations: teams that have to play a lot of home matches in succession, teams that have to travel a lot, etcetera.

Assignment 3

Improve the schedule from table 1 to give as fair a schedule for this competition as possible. Describe all the criteria you take into account and show how you have used them.

For organisational reason, the KNVB decides to split the F competitions in the north of Gelderland into two partial competitions with six teams each.
Again, in these two partial competitions all teams still only play each other once.

Final assignments - Part 3

Assignment 4

Research which two groups you can best split the teams into to end up with the two most fair competitions. Explain your reasoning and the criteria you used. Finally, provide the most fair schedule for each of the two partial competitions.

Assignment 5

Describe a clear procedure for the KNVB (or any other sports association that organises competitions where the teams play each other once) to follow to put together the most fair schedule. Keep in mind that the number of teams may vary. Take care that your description is not longer than one sheet of A4 paper.