

COUNT DOWN 2012



Preliminary round assignment 23rd Mathematics A-lympiad

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Colophon

The Mathematics A-lympiad is an initiative of the Freudenthal Instituut, Universiteit Utrecht

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Instructions for the preliminary round assignment for the Mathematics A-lympiad 2011/2012

This Mathematics A-lympiad assignment consists of three introductory assignments, three follow-up assignments and a final assignment.

General advice for working on this assignment

- First read the entire text of the assignment so that you'll know everything you have to do.
- Keep an eye on the time you use for the introductory and follow-up assignments. Make sure you have enough time left for the final assignment. Divide tasks where possible and confer with each other when needed.
- If you have divided your tasks, discuss the results of the introductory assignments with each other before you start on the final assignment.
- **Research questions**
In several questions you can read *Investigate whether ...*. Always mention in an accurate way *what* you investigated, if necessary research smaller problems, consider alternatives, go beyond only answering the question. The results of these questions will be assessed with these criteria.

Handing in:

- The fully detailed final assignment
- As appendices: the results of the introductory and follow-up assignments

The jury will be given copies of your work. Of course these copies must be legible. Therefore write and draw with a black pen, only print on A4 paper, and do not use pencils for your drawings. If in doubt, make a test copy!

Assessment:

Among the criteria that the jury will take into account are:

- legibility and clarity of the final assignment,
- whether the work is complete,
- the use of mathematics,
- the arguments used and justification of choices (realism may play a part here),
- the level to which the assignment has been done,
- presentation: form, legibility, structure, use and function of appendices, etc.
- (mathematical) creativity in completing the assignments

Have fun and good luck!

Count Down 2012

Introduction

The imaginary town Amberhavn wants to change its television tower into a 350 m high Christmas tree. In order to finance this for the next years, Amberhavn has issued a scratch card lottery called Amberhavn Count Down 2012.

On the right you see a picture of the Count Down 2012 card. It is a lottery scratch card with 31 lights that can be scratched off.

The Count Down card costs € 5,00.

Two million copies are produced, it is assumed that all inhabitants of Amberhavn will like to participate.



The appendix contains an image of the Count Down card at almost true size. The intention is to scratch off a light for that particular day each day in December. So a month of excitement! If you scratch off a light a symbol will appear. There are nine different symbols:



SANTA



PRESENT



SNOWMAN



SLED



CANDLE



SKATE



MITTENS



BELL



STAR

By the end of December you will have scratched off 31 lights, and maybe won a prize. Or more than one!

If, for example, you end up finding three Santas, you will win € 3,00. The further distribution of prizes can be found on the back of the Count Down 2012 card:

AMBERHAVN COUNT DOWN 2012 FIRST PRIZES: € 100.000! 1 IN 2 CHANCE TO WIN

Number of symbols	Prize
3 x	€ 3
4 x	€ 5
5 x	€ 10
6 x	€ 20
7 x	€ 50
8 x	€ 100
9 x	€ 200
10 x	€ 5.000
11 x	€ 100.000

There are never more symbols on a card than you need to win a prize.
 For instance, there are no Count Down 2012 cards with four (or more) Santas!

Introductory assignments

Assignment 1

Give an example of a Count Down 2012 card with not a single prize win on it, but which does contain all nine different symbols.

Assignment 2

Assume that the data given above are the only rules for the Count Down 2012 lottery. Investigate the Count Down 2012 lottery for the following aspects, explaining every time how you did it:

- the maximum number of prizes that a Count Down 2012 card can win
- the maximum amount of money that a Count Down 2012 card can win
- the possible relation between the number of symbols needed to win a prize and the amount of that prize.










Assignment 3

Investigate whether it is possible that all two million copies of the Count Down 2012 card are different.

Follow-up assignments

Assignment 4

After the explorations in the introductory assignments you will now look at a Count Down 2012 card that is already partly scratched off. Below the state of affairs for one particular card on December 28th:

Number of symbols	Mark the symbols you found here	Prize
3 x 	X ██████████	€ 3
4 x 	X X X ██████████	€ 5
5 x 	X X X X X ██████████	€ 10
6 x 	X ██████████	€ 20
7 x 	██████████	€ 50
8 x 	██████████	€ 100
9 x 	X X X X X X X ██████████	€ 200
10 x 	X X X ██████████	€ 5.000
11 x 	X X X X X X X X X X X ██████████	€ 100.000

Almost there for the hundred thousand euros... and only three days to go!

The owner of the Count Down 2012 card above says: *“Because there are nine different symbols, it means that every day there is a chance of one in nine to scratch a star. So with only three days to go, there is a fairly big chance that I will win the 100 000 euros!”*

Give your reaction to this statement (with clear arguments and calculations).

Assignment 5





































Of course it's fun if it remains exciting for as long as possible, and you can keep hoping for a (big) prize until the end. This aspect will have to be taken into account in producing the Count Down 2012 cards.

Describe how you might do this, and provide a few other criteria that an appealing Count Down 2012 card will have to meet.

Assignment 6

Assume that all 2 million Count Down 2012 cards were sold, at €5,00. Half of them won a prize. We are working from the following assumptions:

- about 60% of the lottery's income is paid out in prize money.
- The following combinations of prizes occur:

Combination of prizes on a card	
3x	
4x	
3x	 4x 
5x	
4x	 5x 
3x	 4x  5x 
6x	
5x	 6x 
4x	 5x  6x 
3x	 4x  5x  6x 
7x	
6x	 7x 
5x	 6x  7x 
4x	 5x  6x  7x 
8x	
7x	 8x 
9x	
10x	
11x	

Indicate in a clear way how many copies there are of each "winning" Count Down 2012 card so that the two assumptions mentioned above are met. Do not just present your final distribution, but also clearly explain how you found that distribution and which criteria played a part.

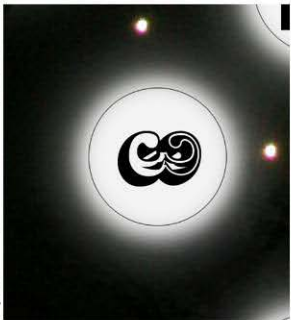
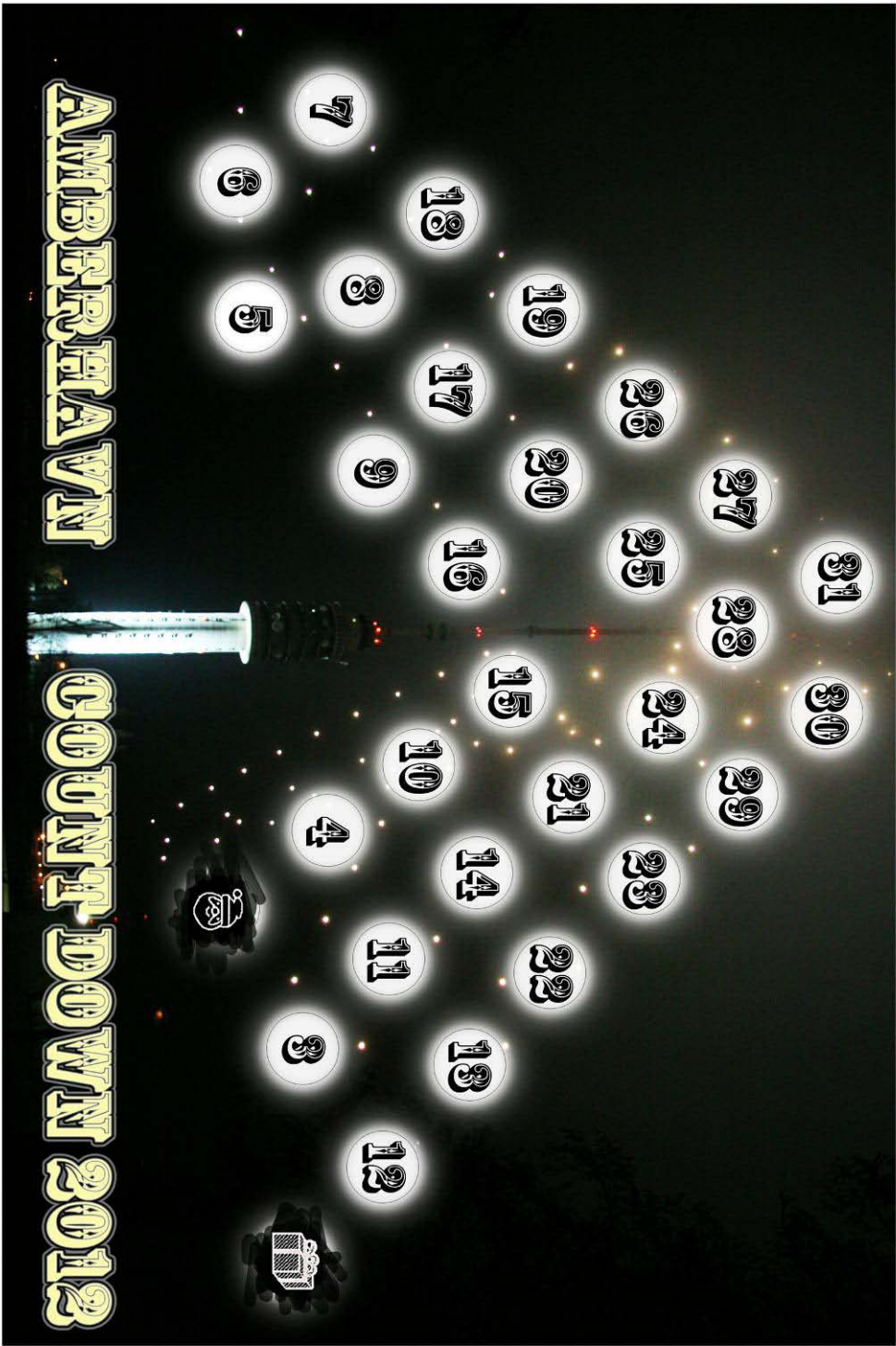
Final assignment

Inspired by the Amherhavn Count Down 2012 lottery the A-lympiad Committee felt it would be a nice idea to make an **A-lympiad weekly lottery card**. We could sell this scratch card in the week of preparations before the A-lympiad finals weekend. Assume a total of 100 lottery cards.

Your task is to design an attractive A-lympiad weekly lottery card (feel free to change the name...) To do this you will use your experiences from the assignments you have already done about the Count Down 2012 card. You must include criteria for making attractive lottery cards, for generating lottery cars, for how many different lottery cards there are, and a prize list,...

Of course you will provide arguments for all you decisions and calculations for all numerical aspects of the lottery cards!

Appendix



the light from december 3rd



scratching off the light



the symbol appears