

# FOUR IN A ROW



Preliminary round assignment 24<sup>th</sup> Mathematics A-lympiad

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## Colophon

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## **Instructions for the preliminary round assignment for the Mathematics A-lympiad 2012/2013**

This Mathematics A-lympiad assignment consists of an introductory assignment, two follow-up assignments and a final assignment.

### ***General advice for working on this assignment***

- First read the entire text of the assignment so that you'll know everything you have to do.
- Keep an eye on the time you use for the introductory and follow-up assignments. Make sure you have enough time left for the final assignment. Divide tasks where possible and confer with each other when needed.
- If you have divided your tasks, discuss the results of the introductory assignments with each other before you start on the final assignment.
- **Research questions**  
In several questions you can read *Investigate whether ...*  
Always mention in an accurate way *what* you investigated, if necessary research smaller problems, consider alternatives, go beyond only answering the question. The results of these questions will be assessed with these criteria.

### ***Handing in:***

- The fully detailed final assignment
- As appendices: the results of the introductory and follow-up assignments

**The jury will be given copies of your work. Of course these copies must be legible. Therefore write and draw with a black pen, only print on A4 paper, and do not use pencils for your drawings. If in doubt, make a test copy!**

### ***Assessment:***

Among the criteria that the jury will take into account are:

- legibility and clarity of the final assignment,
- whether the work is complete,
- your use of mathematics,
- the arguments used and justification of choices,
- the level to which the assignment has been done,
- presentation: form, legibility, structure, use and function of appendices, etc.
- (mathematical) creativity in completing the assignments

**Have fun and good luck!**














## Four in a row

### Introduction

A national supermarket chain has a special arrangement: if you are the fourth person queuing for the checkout you will get your purchases for free!

This arrangement is a good reason to look at the number of customers in a supermarket and the number of checkouts that is needed to let them pay quickly. And of course without having to hand out free groceries too often.

To start with, we set out some conditions as mentioned on the supermarket chain's website:

	Checkout 1	Checkout 2	Checkout 3
Customer at checkout			
Customer in line 1			
Customer in line 2			
Customer in line 3			
Customer in line 4		Free! 	

1. If you are the fourth customer in line, you will get all your shopping for free.
2. The customer at the checkout doesn't count for being in line, so the fourth customer in line is the fifth customer at the checkout.
3. The arrangement is only valid if you cannot choose another checkout with a shorter line.
4. As soon as you are the fourth customer in line, you have to notify the checkout worker. Do not wait until it is your turn at the checkout, because it may not be possible to establish afterwards whether your claim is valid.

These rules are the starting point for all assignments in this A-lympiad.

## Introductory assignment

### A small supermarket

A small local supermarket has two checkouts. The owner is considering introducing the same system as the national supermarket chain. However, having to give away trolley loads of shopping is expensive. Therefore, the owner wants to look at the situation first, and only during this trial will he give the fourth person in line their shopping for free.

On the day of the trial there is one checkout open at 08:00 hr. The owner writes down the times that customers get into the checkout line. You will find these times in the chart on this page.

On average, it takes three minutes to pay for each customer.

### Assignment 1

Look at the development of the checkout line between 08:00 and 09:00 hrs. Clearly describe how you went about your investigation. Calculate the average waiting time per customer. Are there any customers who get their shopping for free during this hour?

Time of arrival (for the queue) for the checkout	Number of customers getting in line
8.01	1
8.02	2
8.07	1
8.14	2
8.15	1
8.16	1
8.22	1
8.24	1
8.25	1
8.29	1
8.31	1
8.33	1
8.34	1
8.39	1
8.45	1
8.50	1
8.56	1

## Follow-up assignments

### The second checkout

From 09:00 hr. on the day of the trial two checkouts are open.

Again, the owner has written down the times. You can find the details in the chart on this page. We've assumed that customers will join the shortest queue.

### Assignment 2

Look at how the line develops at both checkouts between 09:00 and 10:00 hrs. Visualise this clearly. Also, clearly describe how you went about your investigation.

Calculate the average waiting time per customer. Are there any customers who get their shopping for free during this hour?

### Assignment 3

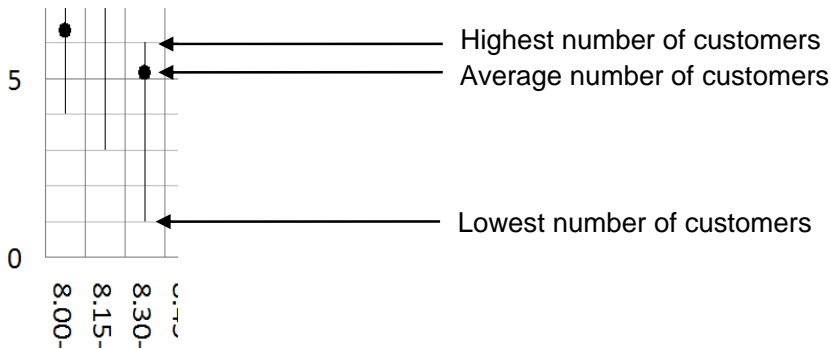
The supermarket owner wants to have only one checkout open when it's quiet, since that gives the other worker time to stock the shelves or other things that need doing. However, the owner also wants to avoid handing out free groceries. Therefore he decides to only open the second checkout from now on when there are, except for the customer currently paying, three more people in line. The next customer can start to pay immediately. The second checkout will close again as soon as possible.

Look at how the line would have developed at both checkouts between 09:00 and 10:00 hrs if the owner had used this system. Indicate at which times there would have been one checkout open and when there would have been two. Again, calculate the average waiting time per customer, and indicate whether there are any customers who get their shopping for free.

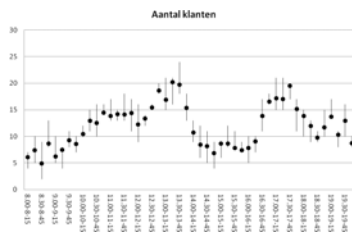
Time of arrival	Number of customers getting in line
9.00	1
9.01	1
9.02	1
9.04	1
9.08	1
9.09	1
9.10	2
9.11	1
9.16	1
9.19	1
9.21	2
9.25	1
9.27	1
9.32	2
9.33	3
9.34	1
9.35	3
9.36	1
9.37	1
9.39	1
9.43	1
9.48	1
9.50	1
9.53	1
9.57	1

## Final assignment

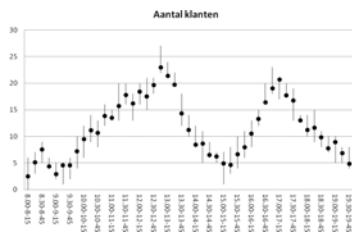
As a result of frequent counting, the development of the *checkout flow* (the number of customers arriving at the checkout) through the day is more or less a known quantity in larger supermarkets. By keeping track on a large number of 'normal' days (so no days around the festive season or around holidays) it's possible to calculate the average checkout flow per quarter of an hour. The greatest and smallest amount of customers is also known. These data are represented in a chart as follows (see the figure below):



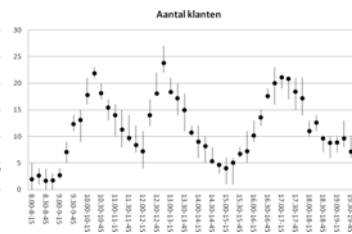
In appendices 1 to 3 you will find the following graphs at their actual size:



Graph 1



Graph 2



Graph 3

Based on this type of graph, a supermarket manager has to schedule how many checkouts should be manned throughout the day. Since giving away groceries is expensive, the manager wants a schedule that doesn't produce four people in line. On the other hand, there should be as few checkouts open as possible, for considerations of cost. Here too, it takes an average of three minutes to pay. The supermarket has eight checkouts.

By now there is so much data that the manager could have a different checkout flow graph for every single day.

It takes too much time to do an in-depth analysis of the graph for each day. The manager needs a method that allows him to draw up a schedule quickly.

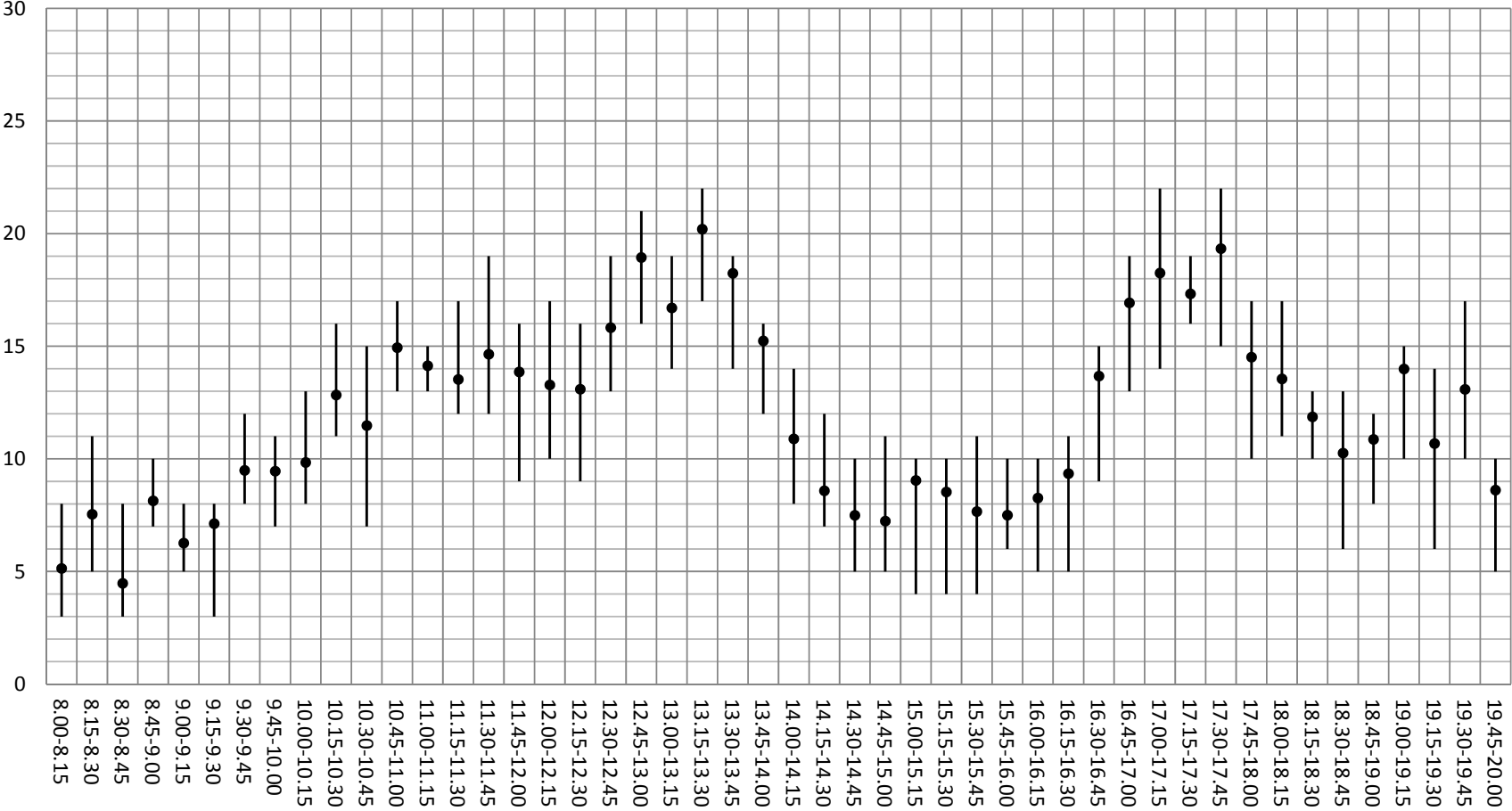
## Assignment 4

Make clear instructions that allow the manager to quickly draw up a schedule based on a random checkout flow graph, in which he or she can see immediately which checkouts should open or close at which times.

Show in an appendix, based on the three graphs provided, that the instructions work, and show how you verified this.

Appendix 1

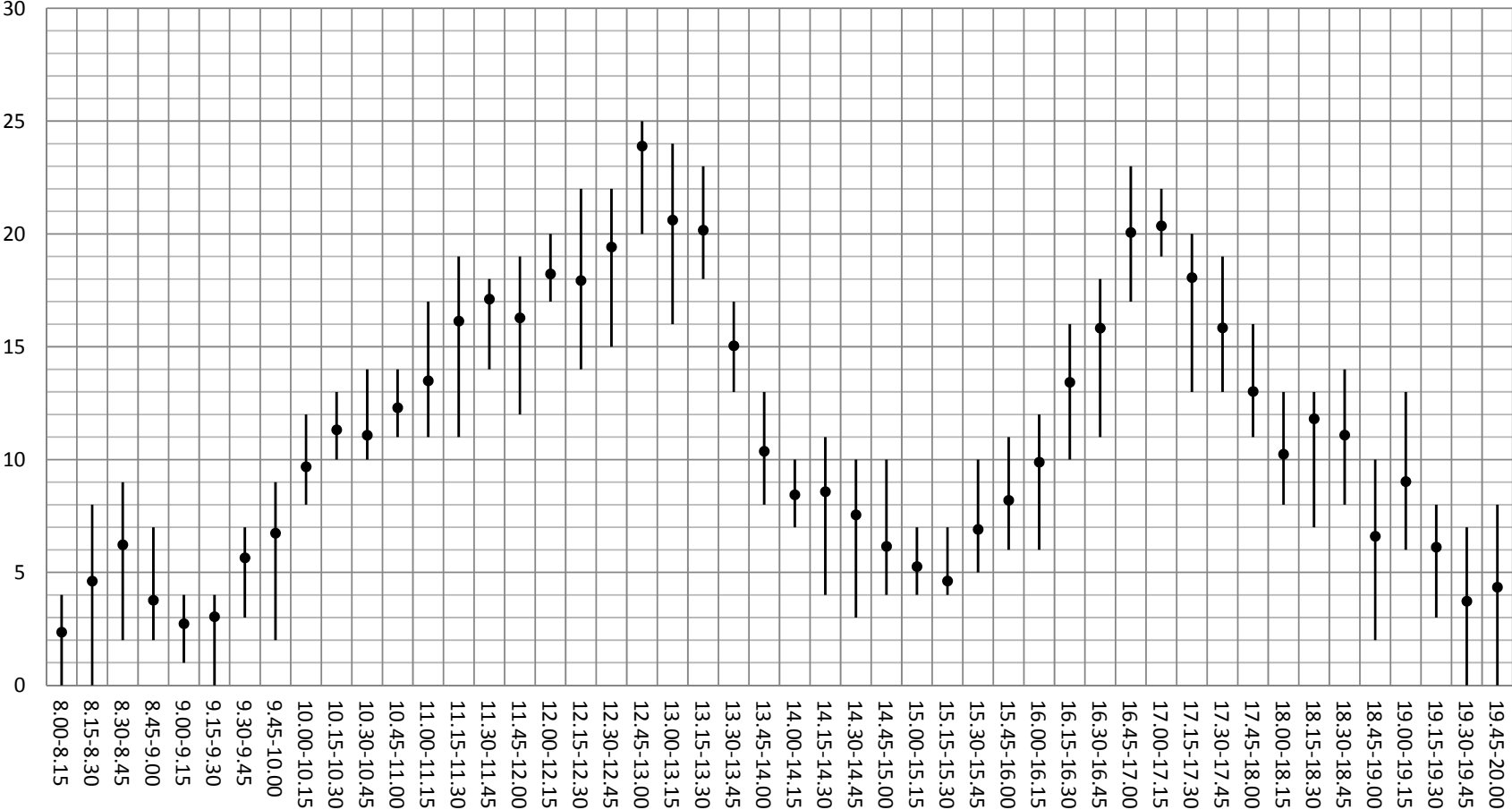
Checkout flow on weekdays





Appendix 2

Checkout flow on Fridays



Appendix 3

Checkout flow on Saturdays

