Discrete Mathematics for the Fostering of Mathematically Talented Children

Nadine Benstein

Bergische Universität Wuppertal

November 14, 2015

Aim and Motivation

MIKADU

Mathematisch interessierte Kinder
an der Bergischen Universität Wuppertal

- problem tasks taken from discrete mathematics play an essential role in our enrichtment project for mathematically gifted children
- the children that participate in our project are between 11 and 13 years old; their prior knowledge has to be considered
- problems from number theory (divisibility) and combinatorics
- NEW: graph theory, discrete optimization

Aim and Motivation

MIKADU

Mathematisch interessierte Kinder
an der Bergischen Universität Wuppertal

- problem tasks taken from discrete mathematics play an essential role in our enrichtment project for mathematically gifted children
- the children that participate in our project are between 11 and 13 years old; their prior knowledge has to be considered
- problems from number theory (divisibility) and combinatorics
- NEW: graph theory, discrete optimization

Aim and Motivation

MIKADU

Mathematisch interessierte Kinder
an der Bergischen Universität Wuppertal

- problem tasks taken from discrete mathematics play an essential role in our enrichtment project for mathematically gifted children
- the children that participate in our project are between 11 and 13 years old; their prior knowledge has to be considered
- problems from number theory (divisibility) and combinatorics
- NEW: graph theory, discrete optimization

Aim and Motivation

MIKADU

Mathematisch interessierte Kinder
an der Bergischen Universität Wuppertal

- problem tasks taken from discrete mathematics play an essential role in our enrichtment project for mathematically gifted children
- the children that participate in our project are between 11 and 13 years old; their prior knowledge has to be considered
- problems from number theory (divisibility) and combinatorics
- NEW: graph theory, discrete optimization

Aim and Motivation

MIKADU

Mathematisch interessierte Kinder
an der Bergischen Universität Wuppertal

- problem tasks taken from discrete mathematics play an essential role in our enrichtment project for mathematically gifted children
- the children that participate in our project are between 11 and 13 years old; their prior knowledge has to be considered
- problems from number theory (divisibility) and combinatorics
- NEW: graph theory, discrete optimization

Aim and Motivation

Can tasks taken from graph theory and discrete optimization foster the mathematically talented children in our project?

Aim and Motivation

Can tasks taken from graph theory and discrete optimization foster the mathematically talented children in our project?

Discrete mathematics

Aim and Motivation

Can tasks taken from graph theory and discrete optimization foster the mathematically talented children in our project?

Mathematical giftedness	Discrete mathematics

Aim and Motivation

Can tasks taken from graph theory and discrete optimization foster the mathematically talented children in our project?

The project MIKADU	Mathematical giftedness	Discrete mathematics

The Project MIKADU

Mathematical Giftedness

Discrete Mathematics and MIKADU
- Shortest Paths: Vacation
- Parking Lots

The Project


MIKADU
Mathematisch interessierte Kinder
an der Bergischen Universität Wuppertal

Basic Principles

- meetings every other week, 90 minutes:
  - introduction (5-10 minutes)
  - work phase (55-60 minutes)
  - presentation and discussion (20-30 minutes)
- free choice of working methods
- project seminar for prospective teacher students of mathematics
- problem solving tasks
- *enrichment*

The Project MIKADU

Mathematical Giftedness

Discrete Mathematics and MIKADU
- Shortest Paths: Vacation
- Parking Lots

Features of Mathematical Giftedness according to Käpnick (1998)

Mathematically specific features

- mathematical sensitivity
- originality and fantasy
- capacity of memory
- ability to structure
- ability to change the form of represenation
- ability of transfer

Features of Mathematical Giftedness according to Käpnick (1998)

Mathematically specific features

- mathematical sensitivity
- originality and fantasy
- capacity of memory
- ability to structure
- ability to change the form of represenation
- ability of transfer

Features of Mathematical Giftedness according to Käpnick (1998)

General personality traits that support mathematical giftedness

- mental vigour
- intellectual curiosity
- readiness for efforts, achievement motivation
- pleasure in problem solving
- ability to concentrate
- persistence
- self-reliance
- ability to cooperate

Features of Mathematical Giftedness according to Käpnick (1998)

General personality traits that support mathematical giftedness

- mental vigour
- intellectual curiosity
- readiness for efforts, achievement motivation
- pleasure in problem solving
- ability to concentrate
- persistence
- self-reliance
- ability to cooperate

Problem Solving Strategies according to Fuchs (2013)

Differentiation of different procedure syles :

- alternate trying and reflecting
- persistent trying
- intuitive approach
- systematic approach

Aims of MIKADU

- children should experience the diversity of mathematical action
- appreciation of different approaches and talents

Aims of MIKADU

- children should experience the diversity of mathematical action
- appreciation of different approaches and talents

So, the tasks/problems...

- should have the potential to foster as many features of mathematical giftedness as possible.

Aims of MIKADU

- children should experience the diversity of mathematical action
- appreciation of different approaches and talents

So, the tasks/problems...

- should have the potential to foster as many features of mathematical giftedness as possible.
- should be solvable in varied ways and with the help of different problem solving strategies.

Aims of MIKADU

- children should experience the diversity of mathematical action
- appreciation of different approaches and talents

So, the tasks/problems...

- should have the potential to foster as many features of mathematical giftedness as possible.
- should be solvable in varied ways and with the help of different problem solving strategies.
- should also serve as a diagnostic tool.

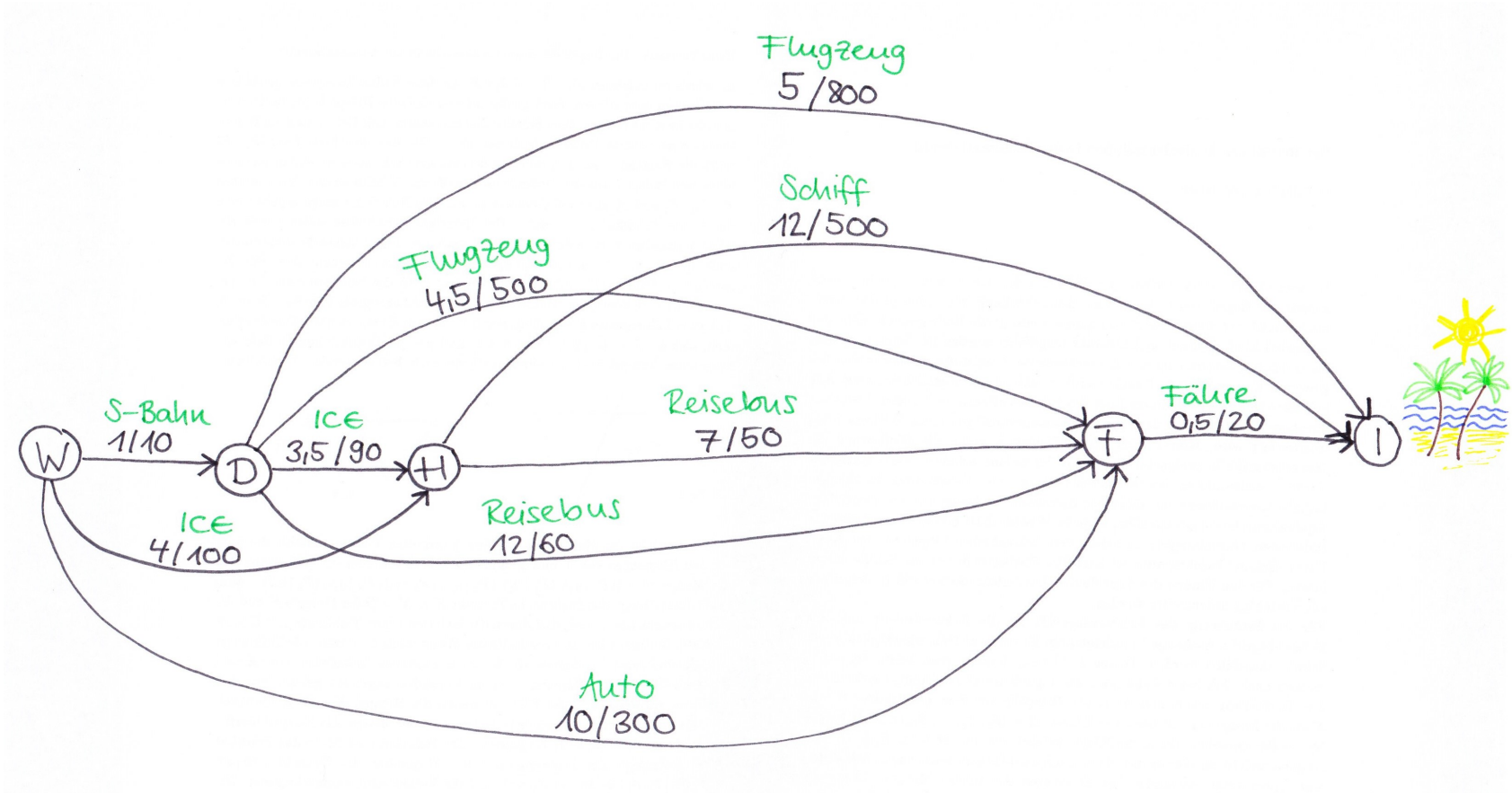
The Project MIKADU

Mathematical Giftedness

Discrete Mathematics and MIKADU
- Shortest Paths: Vacation
- Parking Lots

Task

What is the fastest/cheapest/best way?



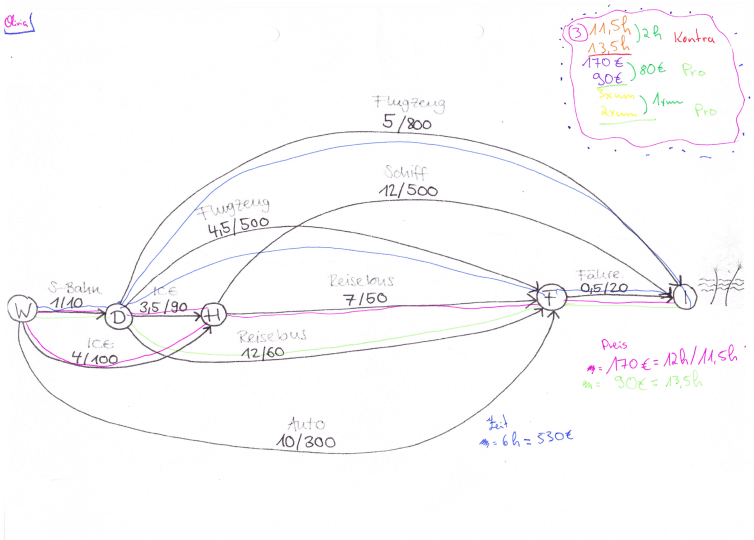
Students' Solutions

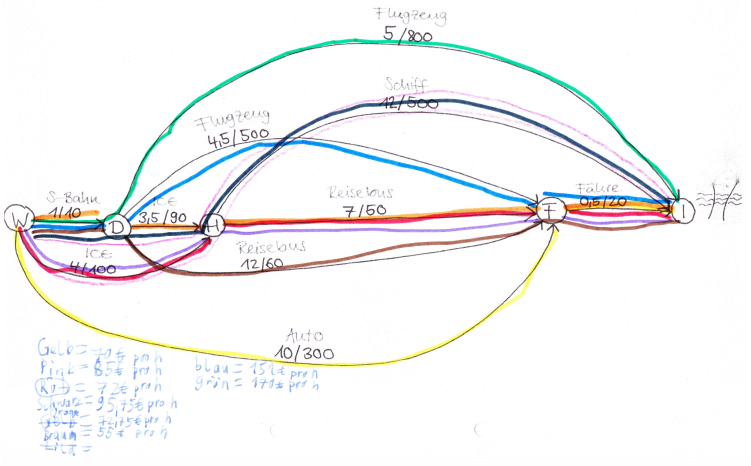
The cheapest and the fastest way:

- access to the form of representation
- successive comparison of single paths and/or edges
- immediate exclusion of certain edges

Students' Solutions

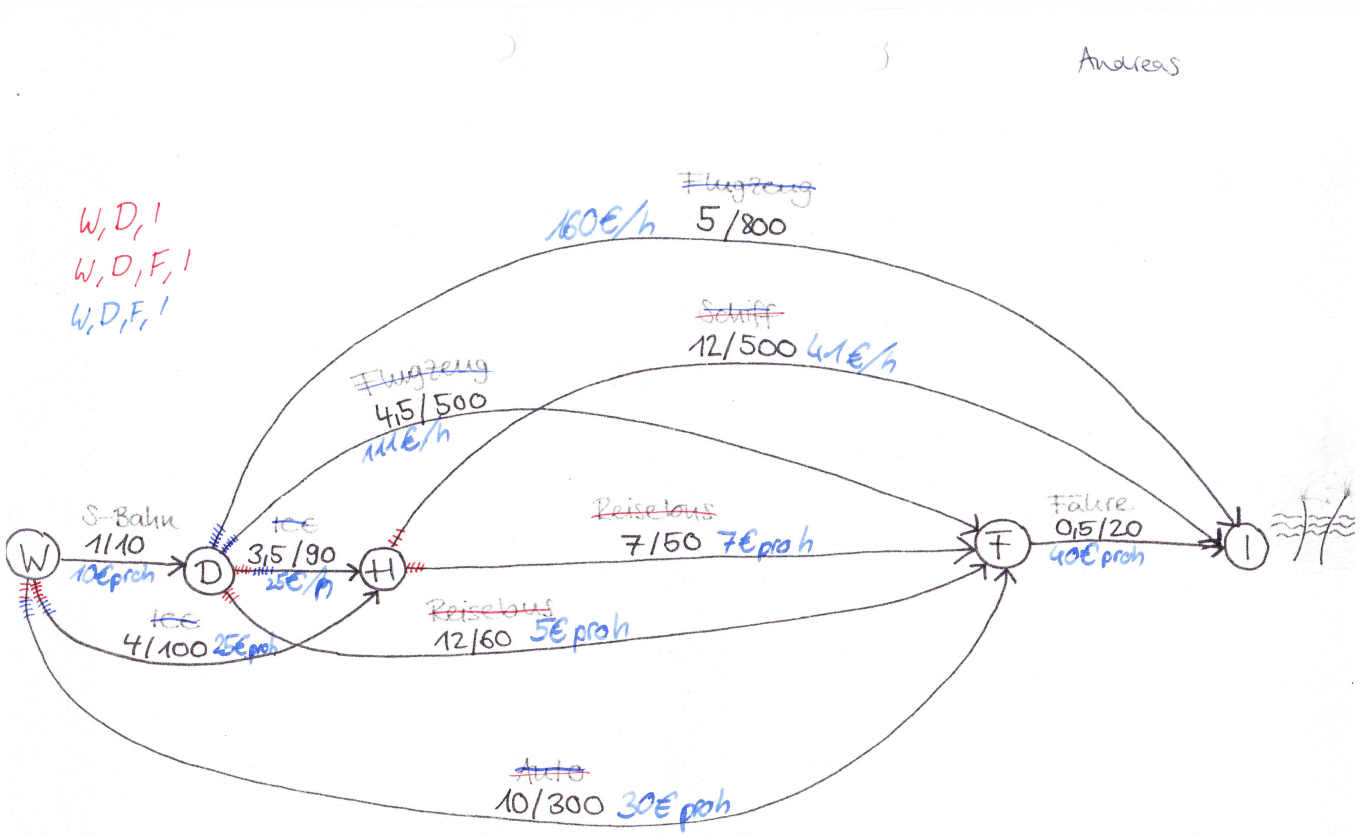
Coping with the new form of representation: tracing paths



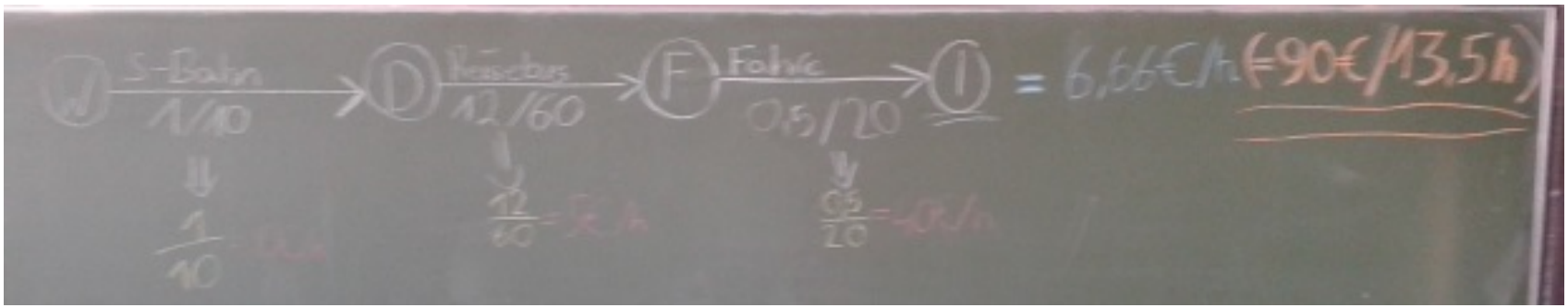


Students' Solutions

Coping with the new form of representation: tracing paths

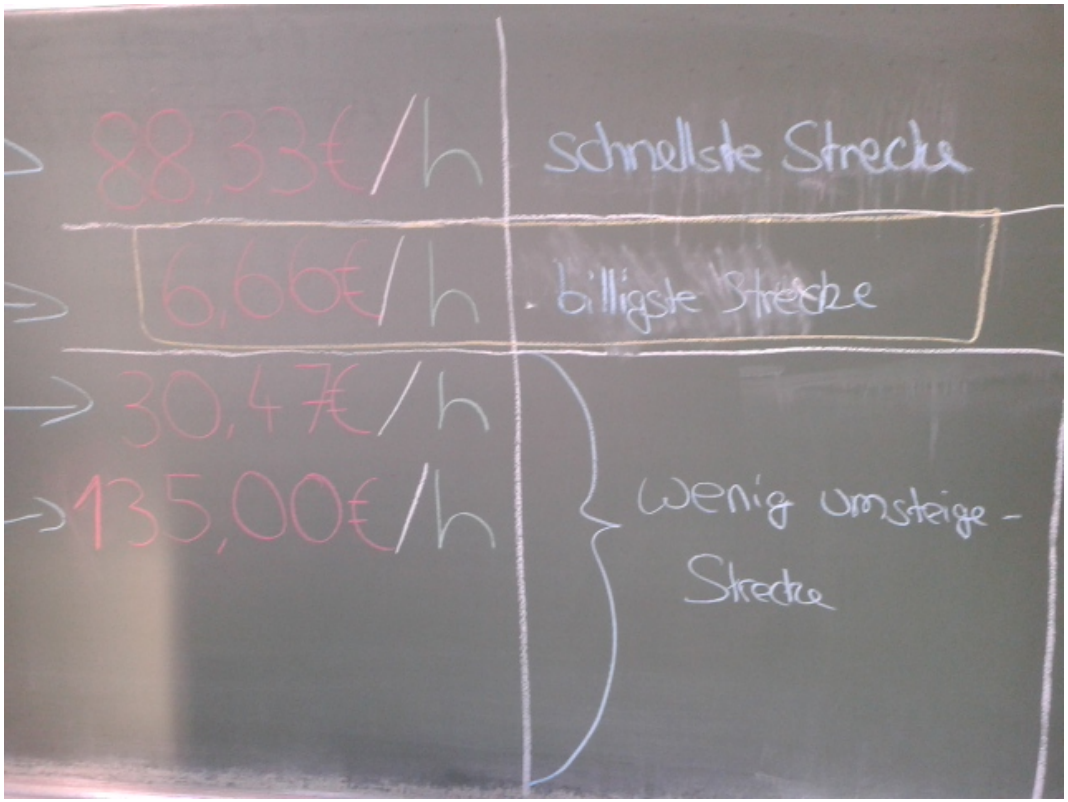


Students' Solutions



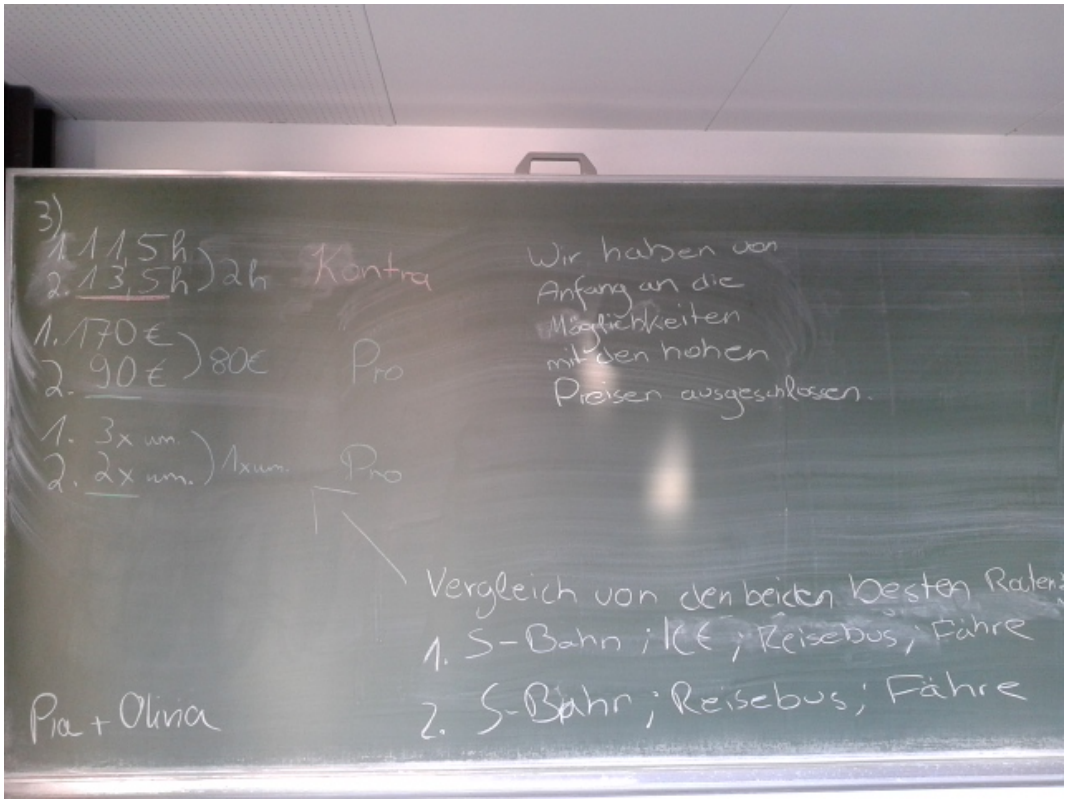
calculation of ratios

Students' Solutions



calculation of ratios

Students' Solutions



contentwise argumentation

Contemplation

- lively discussions about the meaning of the *best* way: comfort? number of transfers? reliability of the different means of transport?

Contemplation

- lively discussions about the meaning of the *best* way: comfort? number of transfers? reliability of the different means of transport?
- mathematically specific features observed/fostered:
  - change of form of representation
  - abillity to transfer
  - originality and fantasy
  - (capacity of memory)

Contemplation

- lively discussions about the meaning of the *best* way: comfort? number of transfers? reliability of the different means of transport?
- mathematically specific features observed/fostered: ✓
  - change of form of representation
  - abillity to transfer
  - originality and fantasy
  - (capacity of memory)

Contemplation

- lively discussions about the meaning of the *best* way: comfort? number of transfers? reliability of the different means of transport?
- mathematically specific features observed/fostered: ✓
  - change of form of representation
  - abillity to transfer
  - originality and fantasy
  - (capacity of memory)
- varied problem solving strategies possible

Contemplation

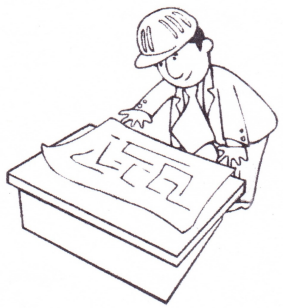
- lively discussions about the meaning of the *best* way: comfort? number of transfers? reliability of the different means of transport?
- mathematically specific features observed/fostered: ✓
  - change of form of representation
  - abillity to transfer
  - originality and fantasy
  - (capacity of memory)
- varied problem solving strategies possible ✓

Task

Parkplätze

Eine Baufirma soll für ein Unternehmen auf einem quadratischen Gelände einen Parkplatz bauen. Dabei müssen folgende Regeln beachtet werden:

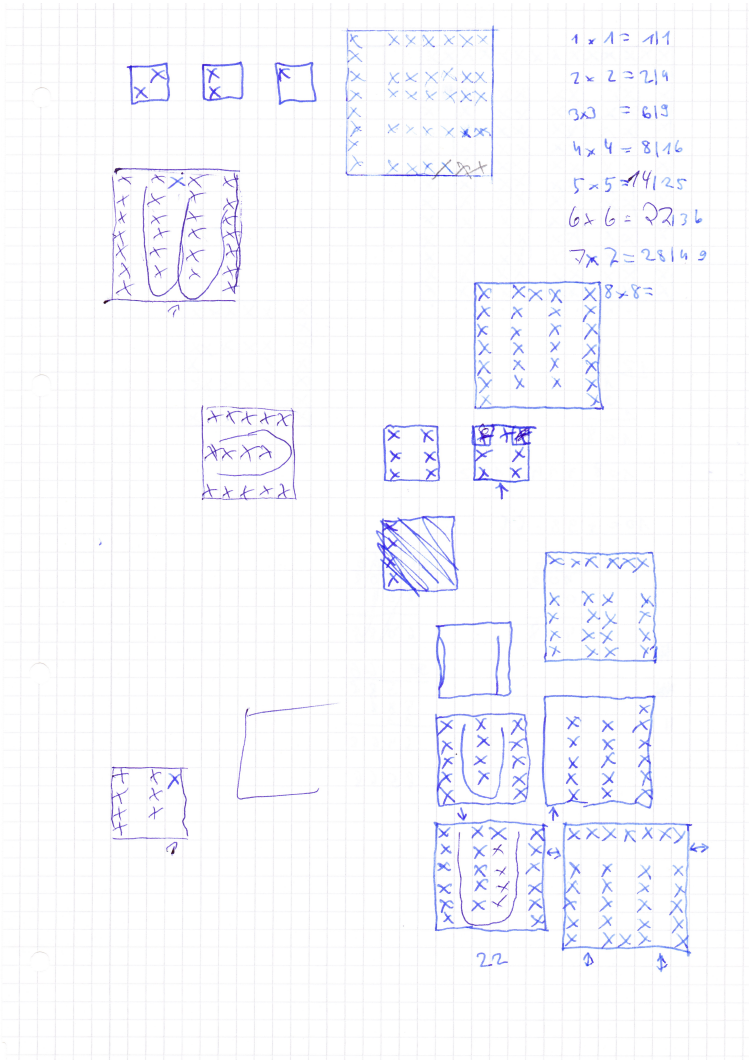
- Jedes Auto muss zu jedem Zeitpunkt auf den Parkplatz bzw. vom Parkplatz runter fahren können (es darf nicht durch ein anderes Auto blockiert, also zugeparkt werden).
- Es gibt nur eine Ein- bzw. Ausfahrt (an derselben Stelle können die Autos auf den Parkplatz fahren oder diesen verlassen).



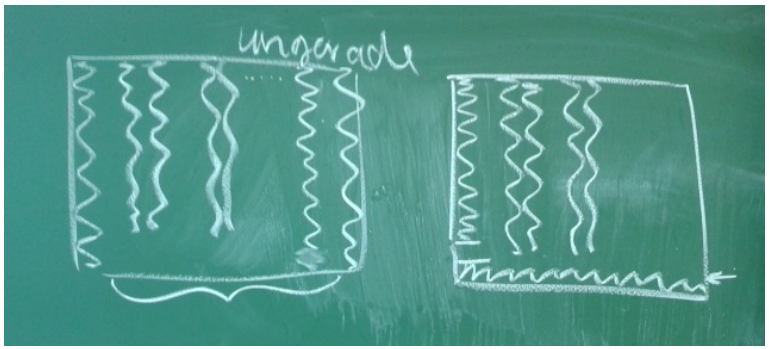
- didactical reduction: the shape of the parking lot is a square
- the rules for the parking lot construction are clarified with the help of a minimal example (2x2 grid on a transparency foil)

Students' Solutions

The first intuition of most of the students was to find as many *long lines* as possible.

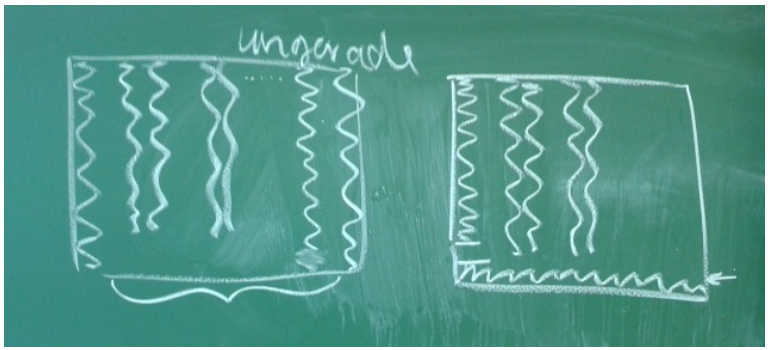


Students' Solutions



In the case of 100x100, 34 rows stay free and therefore there are 66 left. Then, 98×66=6468, because I have 98 vertical lines. Now, I take the last line and add it to the rest 6468+100=6568.

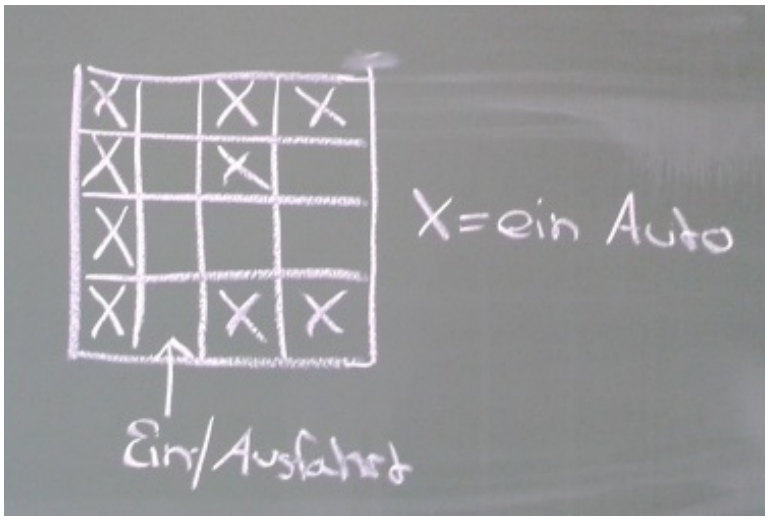
Students' Solutions



left: $64 \times 98 + 2 \times 100$

right: $66 \times 98 + 100$

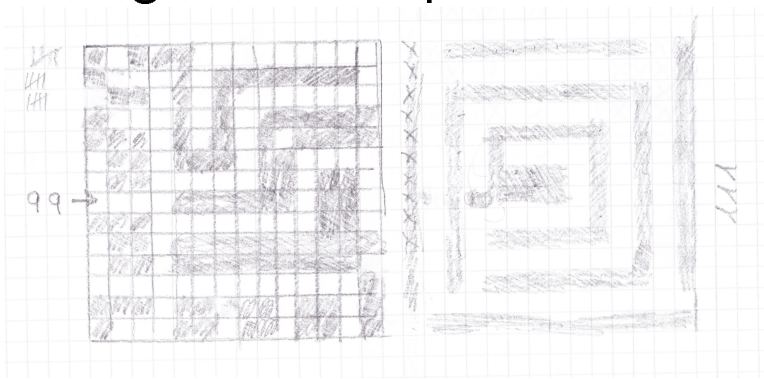
Students' Solutions

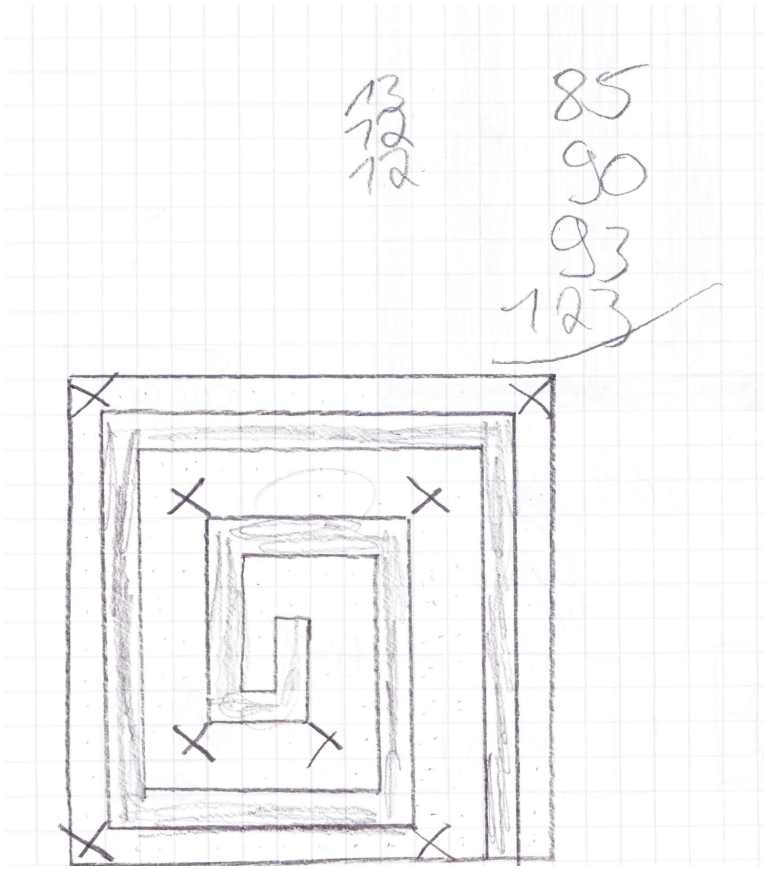


Then, one child proves with a counterexample that this strategy is not the *best*.

Students' Solutions

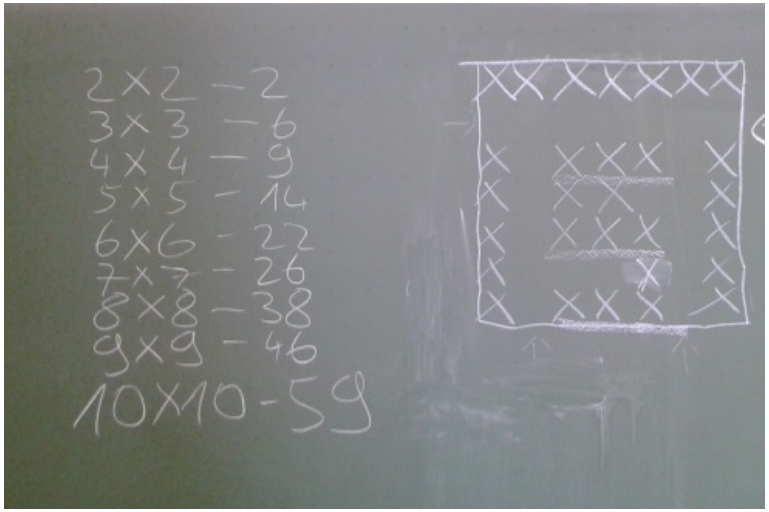
arrangement in spirals





Students' Solutions

The strategy was to find a *good* solution and then try to improve it, namely trying to add exactly one more parking lot.



Contemplation

Contemplation

- the children distinguished between *even* and *uneven* squares → ask for *nxn* parking areas or 100x100 and 101x101

Contemplation

- the children distinguished between *even* and *uneven* squares → ask for *nxn* parking areas or 100x100 and 101x101
- individual strategies of optimization

Contemplation

- the children distinguished between *even* and *uneven* squares → ask for *nxn* parking areas or 100x100 and 101x101
- individual strategies of optimization
- mathematically specific features observed/fostered:
  - mathematical sensitivity
  - originality and fantasy
  - ability to change the form of representation

Contemplation

- the children distinguished between *even* and *uneven* squares → ask for *nxn* parking areas or 100x100 and 101x101
- individual strategies of optimization
- mathematically specific features observed/fostered: ✓
  - mathematical sensitivity
  - originality and fantasy
  - ability to change the form of representation

Contemplation

- the children distinguished between *even* and *uneven* squares → ask for *nxn* parking areas or 100x100 and 101x101
- individual strategies of optimization
- mathematically specific features observed/fostered: ✓
  - mathematical sensitivity
  - originality and fantasy
  - ability to change the form of representation
- varied problem solving strategies possible

Contemplation

- the children distinguished between *even* and *uneven* squares → ask for *nxn* parking areas or 100x100 and 101x101
- individual strategies of optimization
- mathematically specific features observed/fostered: ✓
  - mathematical sensitivity
  - originality and fantasy
  - ability to change the form of representation
- varied problem solving strategies possible ✓

Conclusion

- discrete optimization might serve as a strong tool to foster especially the ability to change the form of representation
- side effects: discussions about the *best* solution (mathematically internal and concerning the context); cooperations
- variation!
- optimization = realistic application of mathematics and contexts
- new plan/aim: encourage students to utilize graphs as a form of visualization
- students' perception of the notion of *optimal* $\rightarrow$ superlatives
- **suggestions?**

Conclusion

- discrete optimization might serve as a strong tool to foster especially the ability to change the form of representation
- side effects: discussions about the *best* solution (mathematically internal and concerning the context); cooperations
- variation!
- optimization = realistic application of mathematics and contexts
- new plan/aim: encourage students to utilize graphs as a form of visualization
- students' perception of the notion of *optimal* $\rightarrow$ superlatives
- **suggestions?**

Conclusion

- discrete optimization might serve as a strong tool to foster especially the ability to change the form of representation
- side effects: discussions about the *best* solution (mathematically internal and concerning the context); cooperations
- variation!
- optimization = realistic application of mathematics and contexts
- new plan/aim: encourage students to utilize graphs as a form of visualization
- students' perception of the notion of *optimal* $\rightarrow$ superlatives
- suggestions?

Conclusion

- discrete optimization might serve as a strong tool to foster especially the ability to change the form of representation
- side effects: discussions about the *best* solution (mathematically internal and concerning the context); cooperations
- variation!
- optimization = realistic application of mathematics and contexts
- new plan/aim: encourage students to utilize graphs as a form of visualization
- students' perception of the notion of *optimal* $\rightarrow$ superlatives
- suggestions?

Conclusion

- discrete optimization might serve as a strong tool to foster especially the ability to change the form of representation
- side effects: discussions about the *best* solution (mathematically internal and concerning the context); cooperations
- variation!
- optimization = realistic application of mathematics and contexts
- new plan/aim: encourage students to utilize graphs as a form of visualization
- students' perception of the notion of *optimal* $\rightarrow$ superlatives
- suggestions?

Conclusion

- discrete optimization might serve as a strong tool to foster especially the ability to change the form of representation
- side effects: discussions about the *best* solution (mathematically internal and concerning the context); cooperations
- variation!
- optimization = realistic application of mathematics and contexts
- new plan/aim: encourage students to utilize graphs as a form of visualization
- students' perception of the notion of *optimal* $\rightarrow$ superlatives
- suggestions?

Conclusion

- discrete optimization might serve as a strong tool to foster especially the ability to change the form of representation
- side effects: discussions about the *best* solution (mathematically internal and concerning the context); cooperations
- variation!
- optimization = realistic application of mathematics and contexts
- new plan/aim: encourage students to utilize graphs as a form of visualization
- students' perception of the notion of *optimal* $\rightarrow$ superlatives
- **suggestions?**

Fuchs, Mandy. 2013. Vorgehensweisen mathematisch potenziell begabter Dritt- und Viertklässler beim Probelmlösen: Empirische Untersuchungen zur Typisierung spezifischer Problemlösestile. *mathematica didactica 36*, 97–125.

Käpnick, Friedhelm. 1998. *Mathematisch begabte Kinder: Modelle, empirische Studien und Förderungsprojekte für das Grundschulalter*. Greifswaöder Studien zur Erziehungswissenschaft, vol. 5. Frankfurt am Main: Lang.

Verhulst, Ferdinand, & Walcher, Sebastian. 2010. *Das Zebra-Buch zur Geometrie*. Heidelberg: Springer.

Wittmann, G. Müller & E. Ch. 1984. *Der Mathematikunterricht in der Primarstufe*. Braunschweig/Wiesbaden: Vieweg.