the teaching and learning of modelling motion

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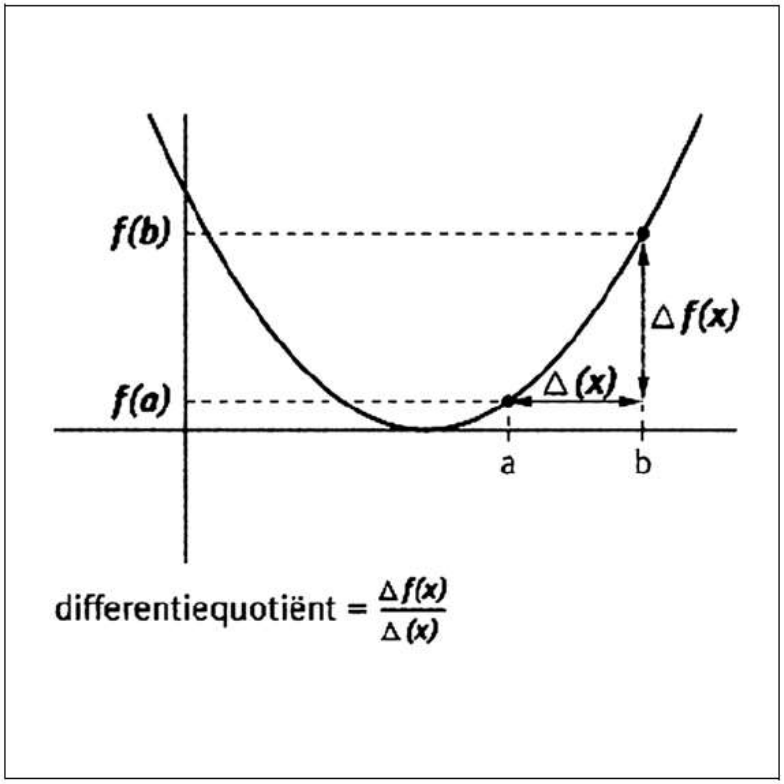
Structure

- Basic principles of calculus in secondary education (15’)
- Graphing motion in primary education (15’)
- Experiencing embodiment and graphing (15’)
- Discussion (15’)
  - What are the basic concepts?
  - To what extent is a ‘discrete’ preparation needed?
  - What can embodied experiences add to learning?

Basic principles of calculus in secondary education



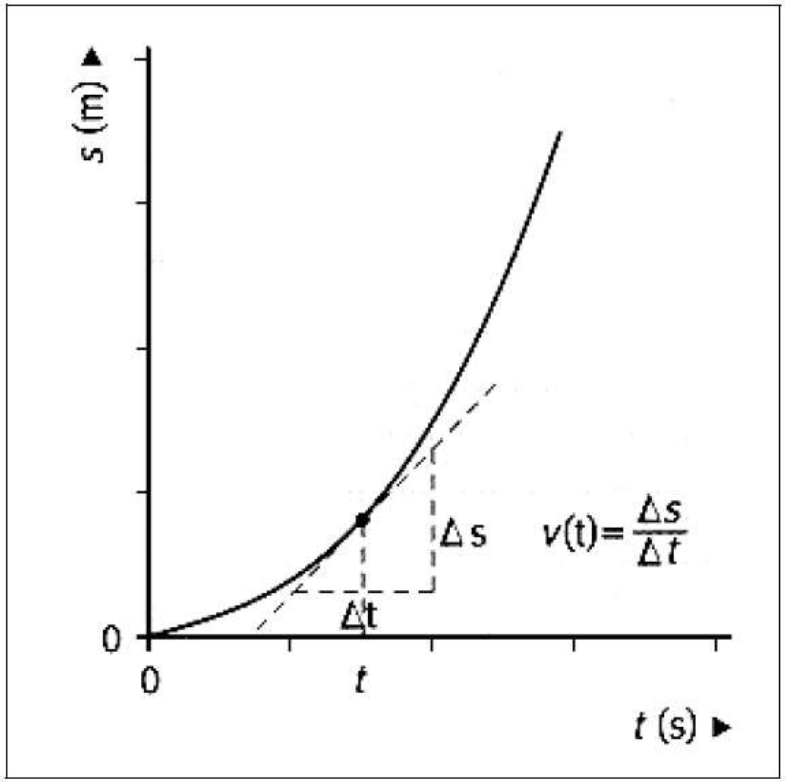
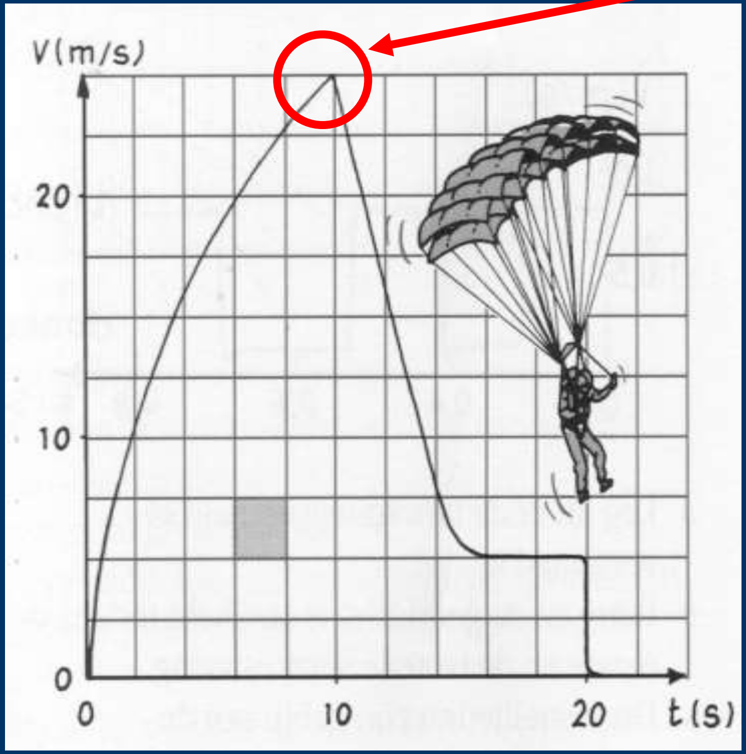
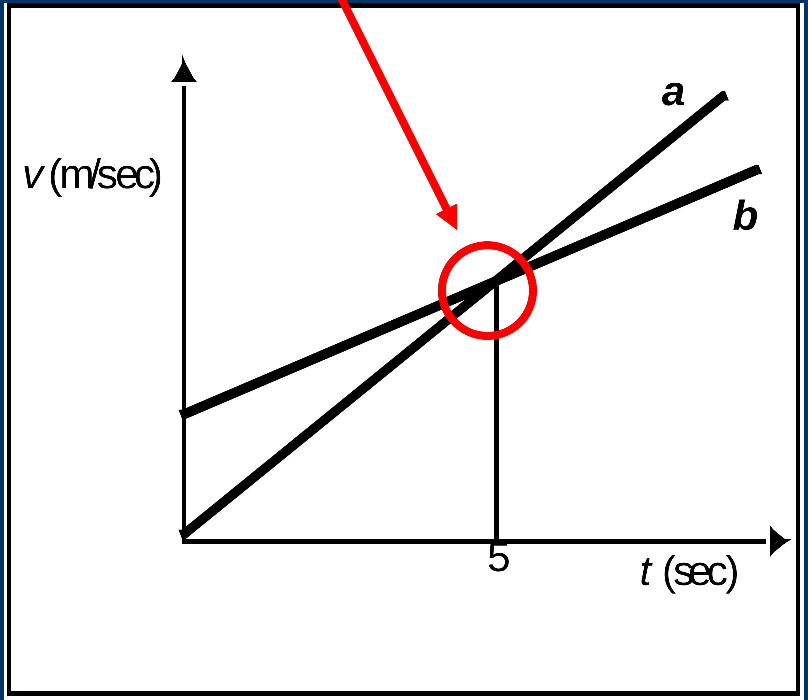


figure 2.7 On the left: mathematics textbook (Staal et al., 1998, p. 189)
On the right: physics textbook (Kortland et al., 1998, p. 197)

Students' difficulties

What is happening at these points?





Continuous graphs are not as 'transparent' as we think

Example: Calculus

Research orientations

- What are the basic concept(s)
- How were these concepts developed?
- What are intuitive ways of reasoning? (in students' reality)
- How can 'realistic' contexts offer access?

History: grasping speed

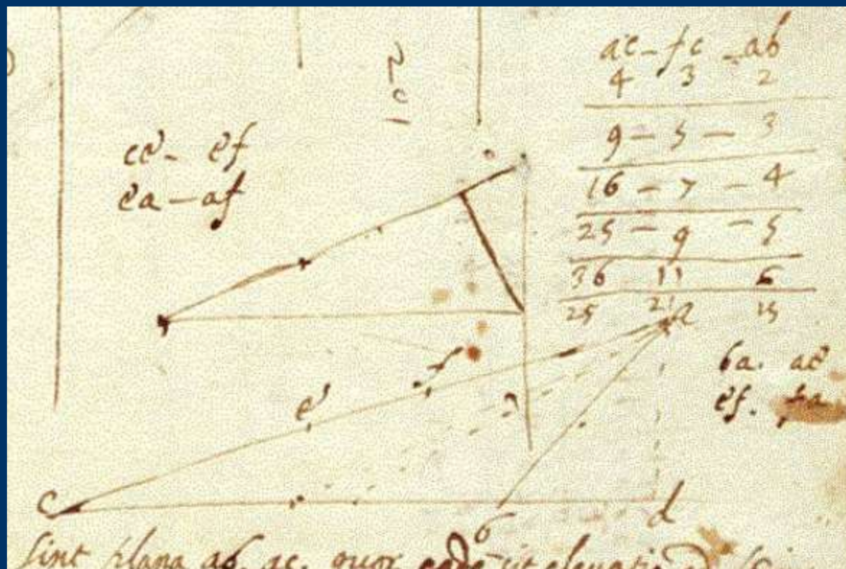
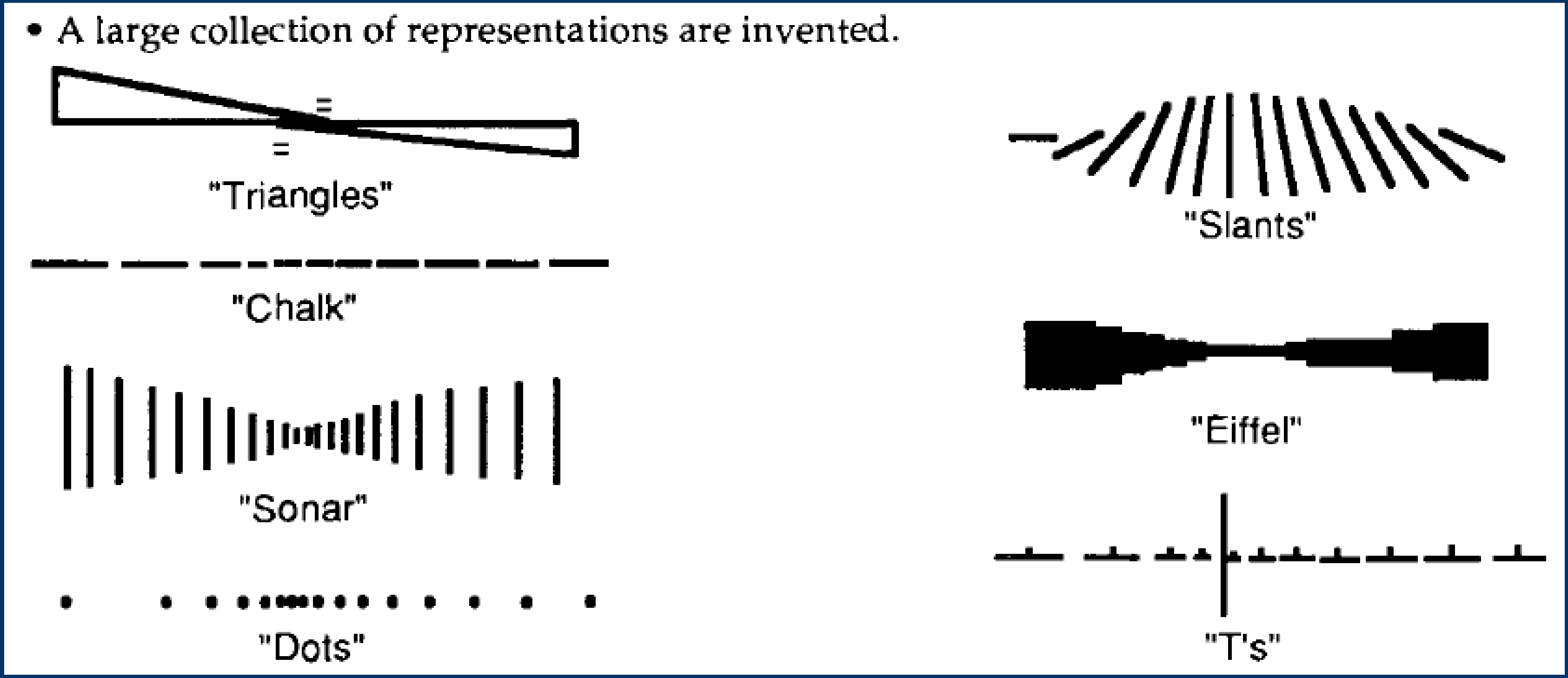


Figure 1. Drawings from Oresme's "De configurationibus qualitatum."

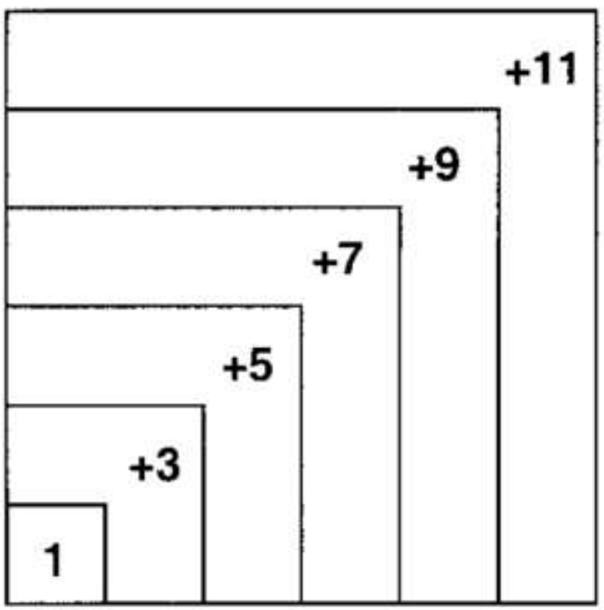
- C. 350 BC Aristotle: falling speed ~ heaviness
- 13th century Albert of Sachsen: falling speed ~ falling distance
- 14th century Oresme: time graph of changing velocity
- 16th century Stevin: experiment with leaden balls
- 17th century Galileo: falling speed ~ falling time
- Newton & Leibniz

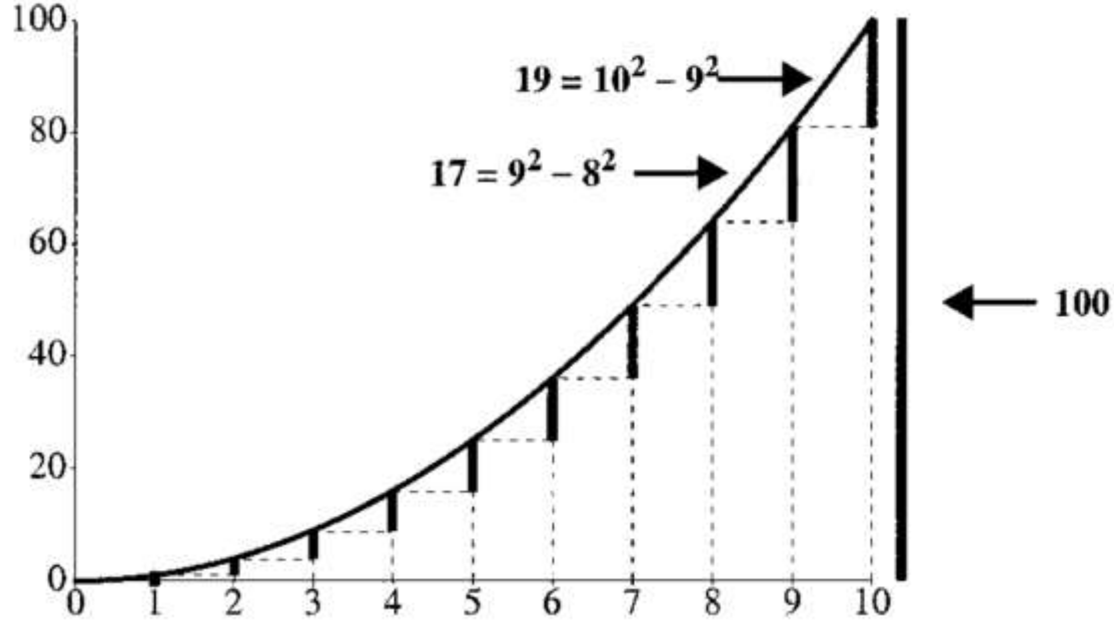
Intuitive ways of reasoning

- diSessa, A., Hammer, D., Sherin, B., & Kolpakowski, T. (1991). Inventing graphing: meta-representational expertise in children. Journal of Mathematical Behavior, 10, 117-160.



Opportunities created by a discrete preparation





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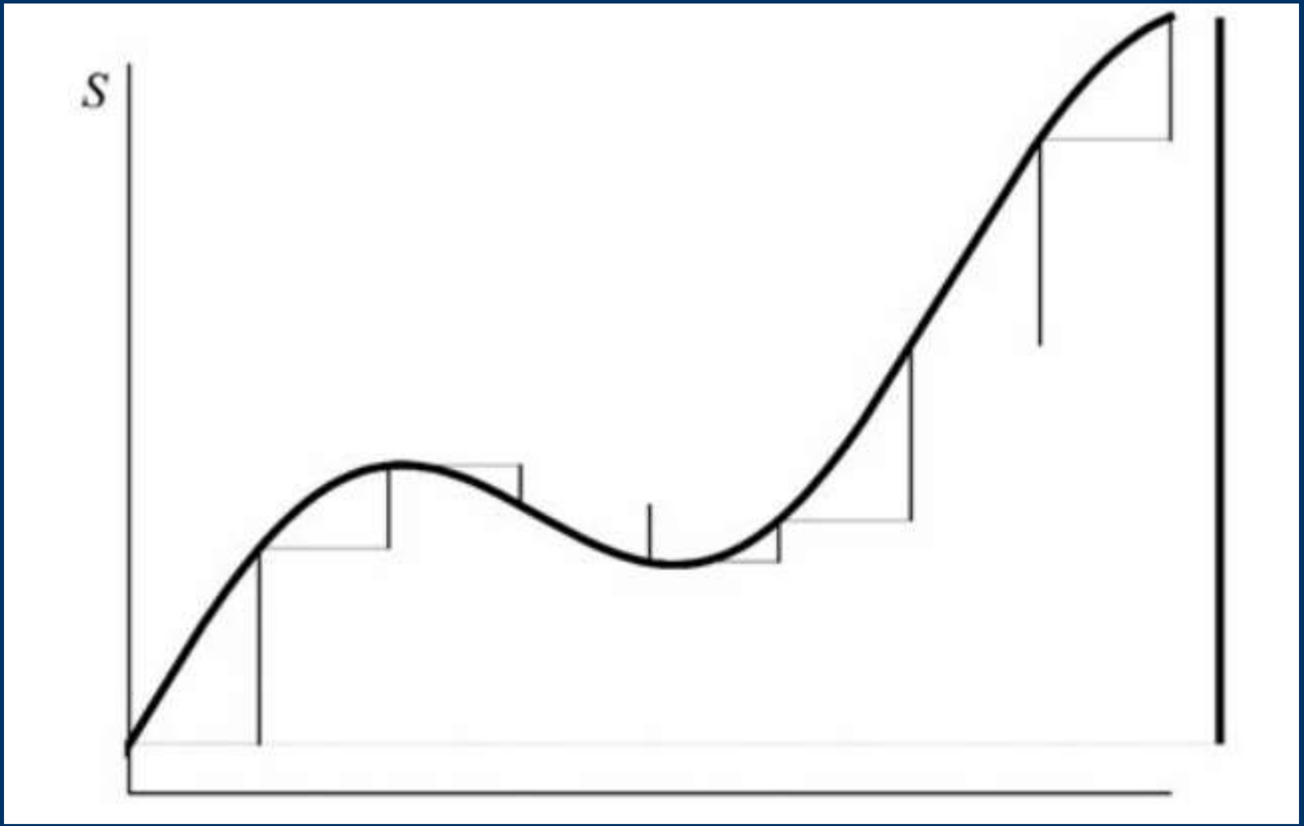


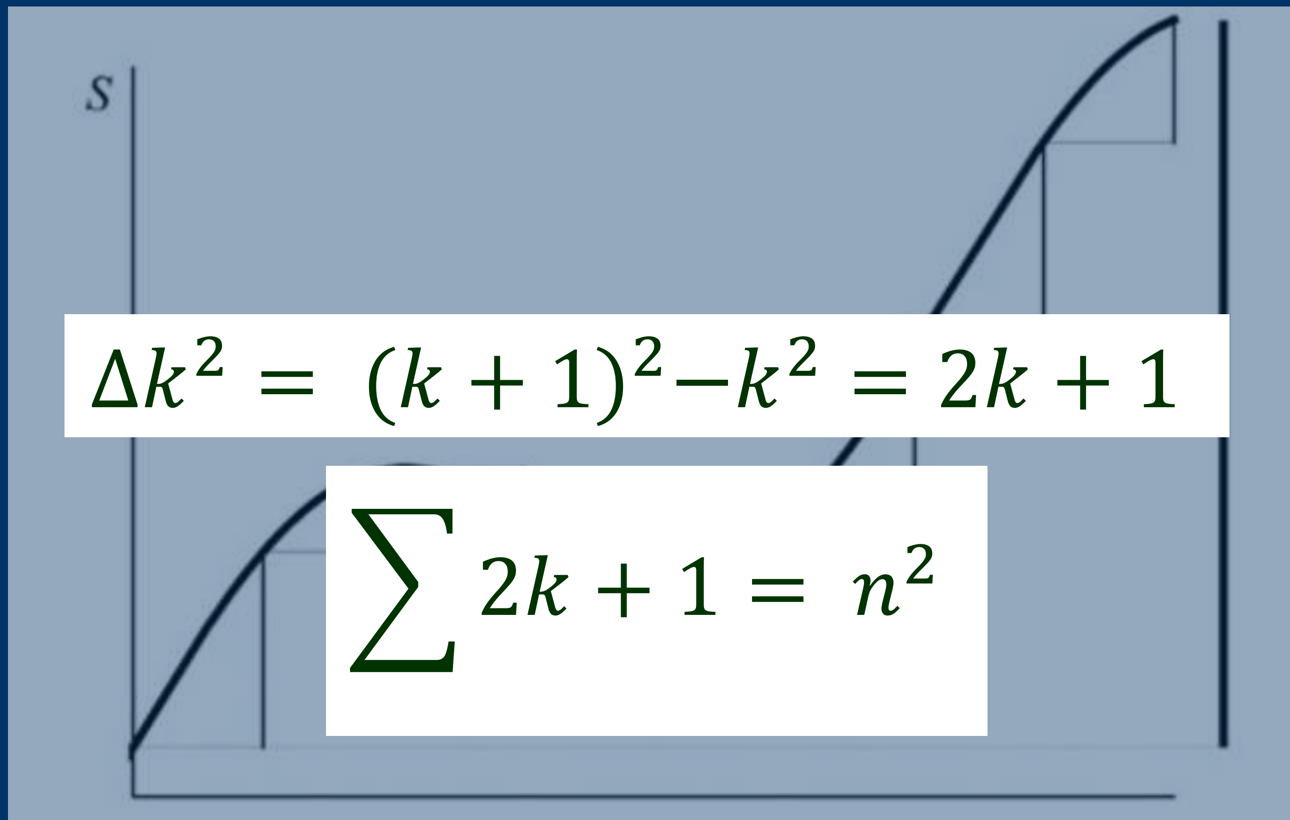
From the graph it can be seen that the sum of all displacements

$$D(0) + D(1) + \ldots + D(n-1) = S(n) - S(0).$$

In general the discrete case of the main theorem of calculus is:

$$\Delta S(k) = D(k) \text{ and } \sum D(k) = S(n) - \mathrm{S}(0).$$

Opportunities created by a discrete preparation



$$\Delta k^2 = (k+1)^2 - k^2 = 2k+1$$

$$\sum 2k+1 = n^2$$

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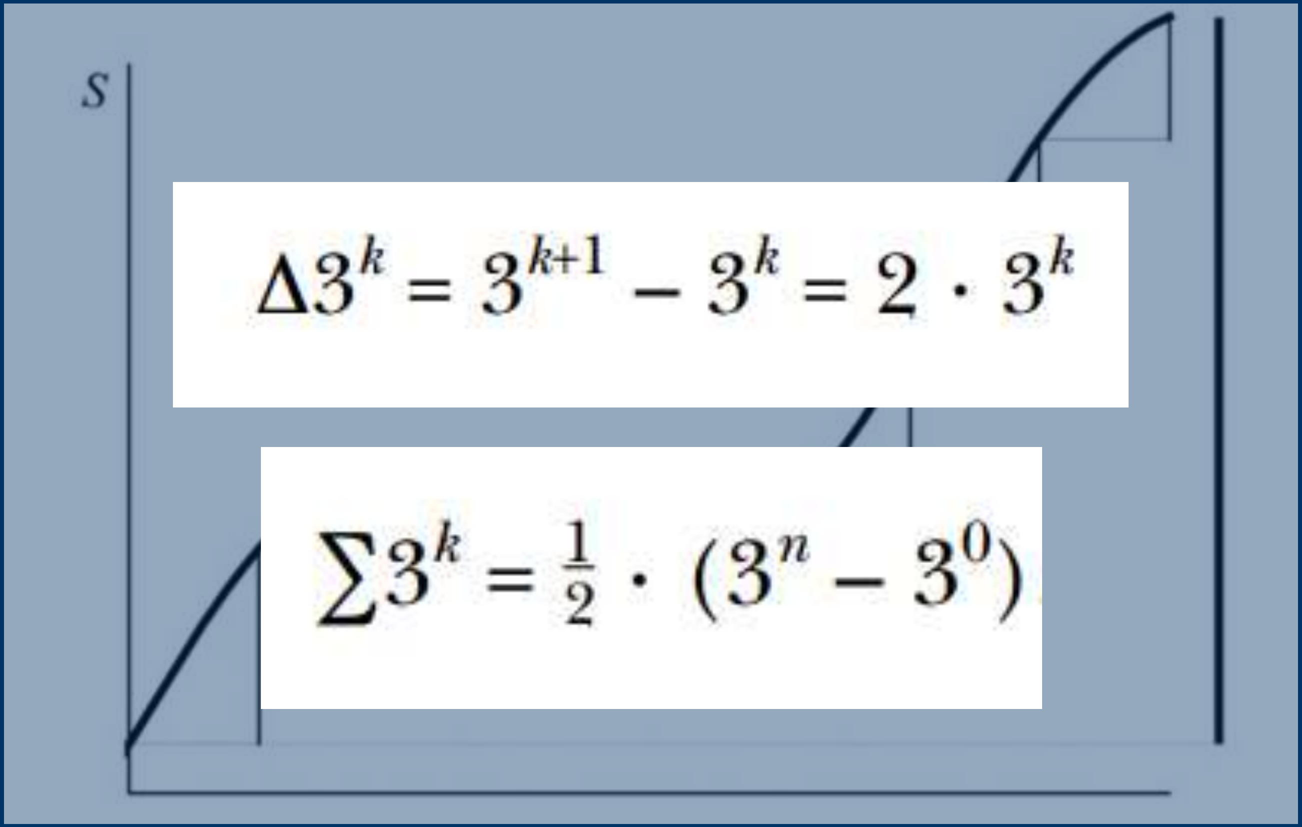

$$\Delta 3^k = 3^{k+1} - 3^k = 2 \cdot 3^k$$

$$\sum 3^k = \frac{1}{2} \cdot (3^n - 3^0)$$

From the graph it can be seen that the sum of all displacements

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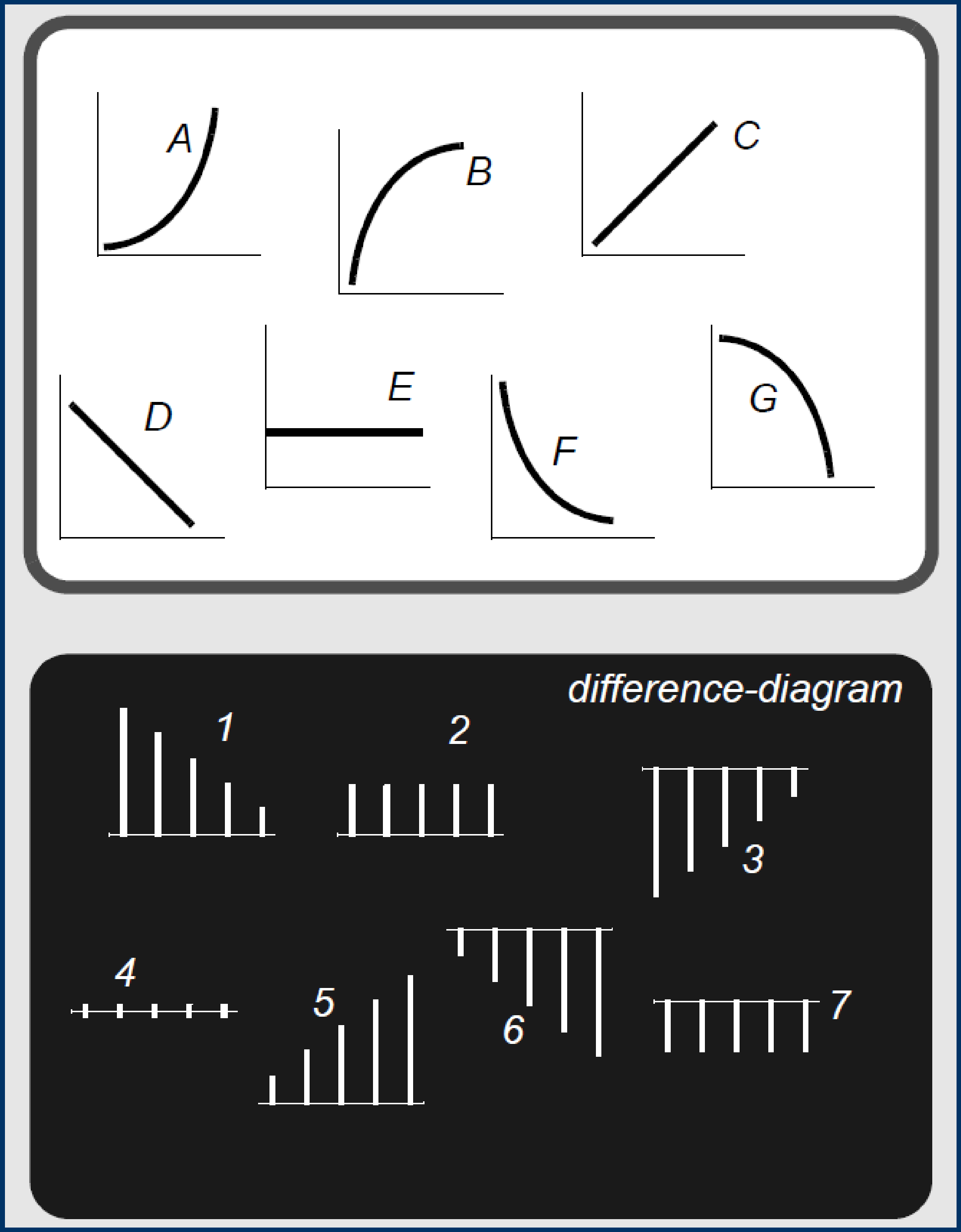
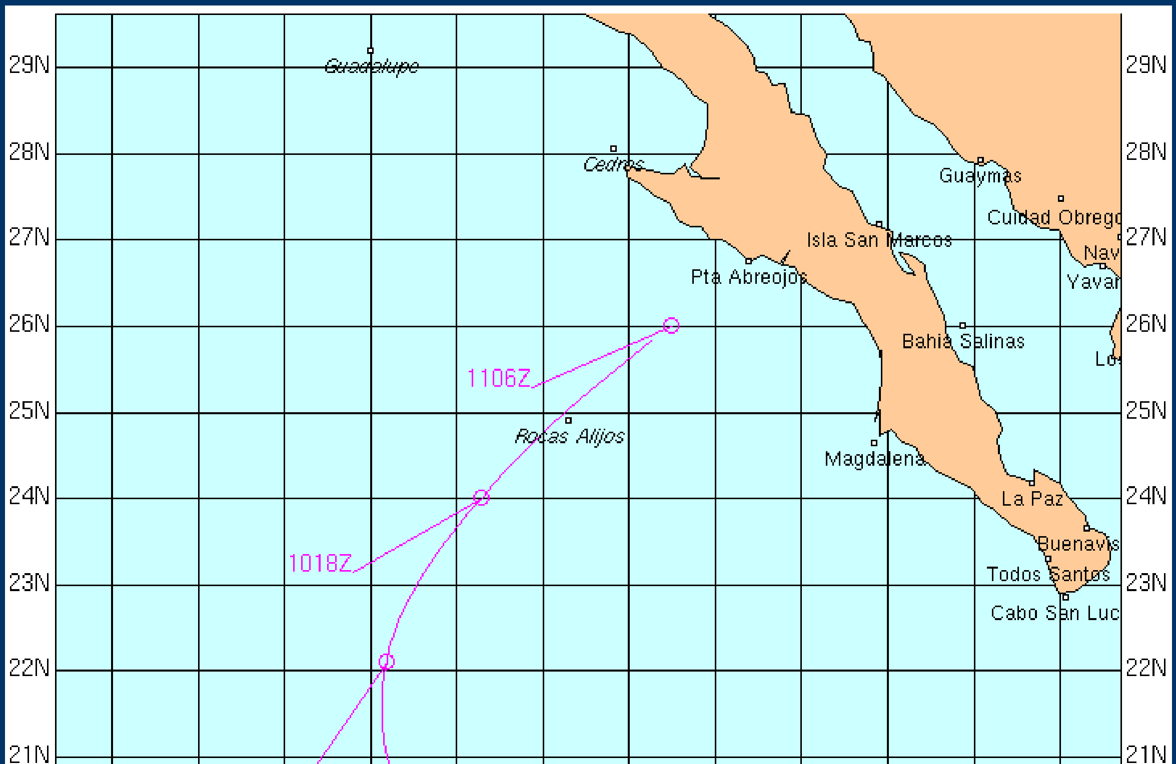


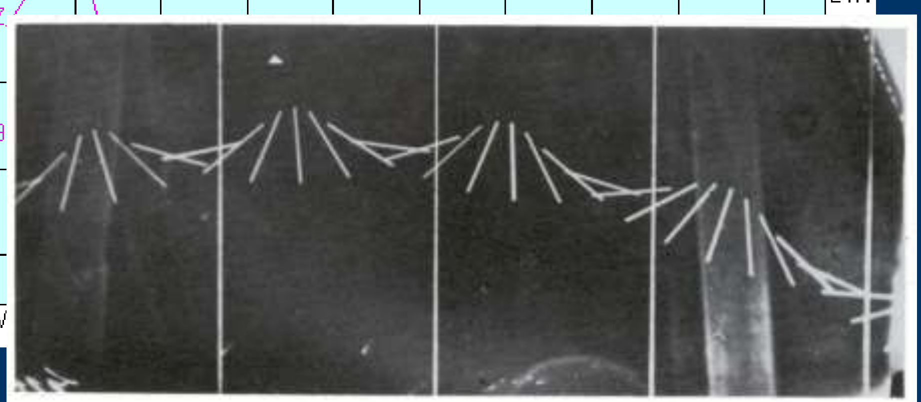
Example: Calculus

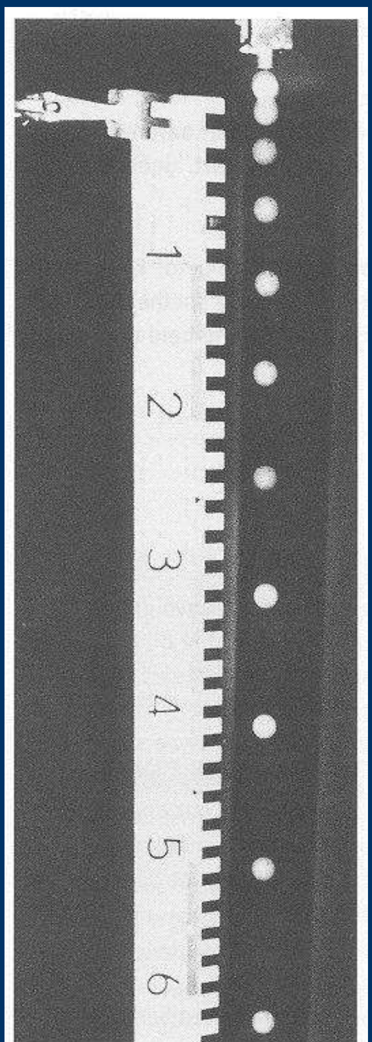
Research orientations

- What are the basic concept(s)
- How were these concepts developed?
- What are intuitive ways of reasoning? (in students' reality)
- How can 'realistic' contexts offer access?
  - Make students realize this is important
  - Help them to structure or model informally, intuitively, …
  - Can be connected to models or representations that support level raising

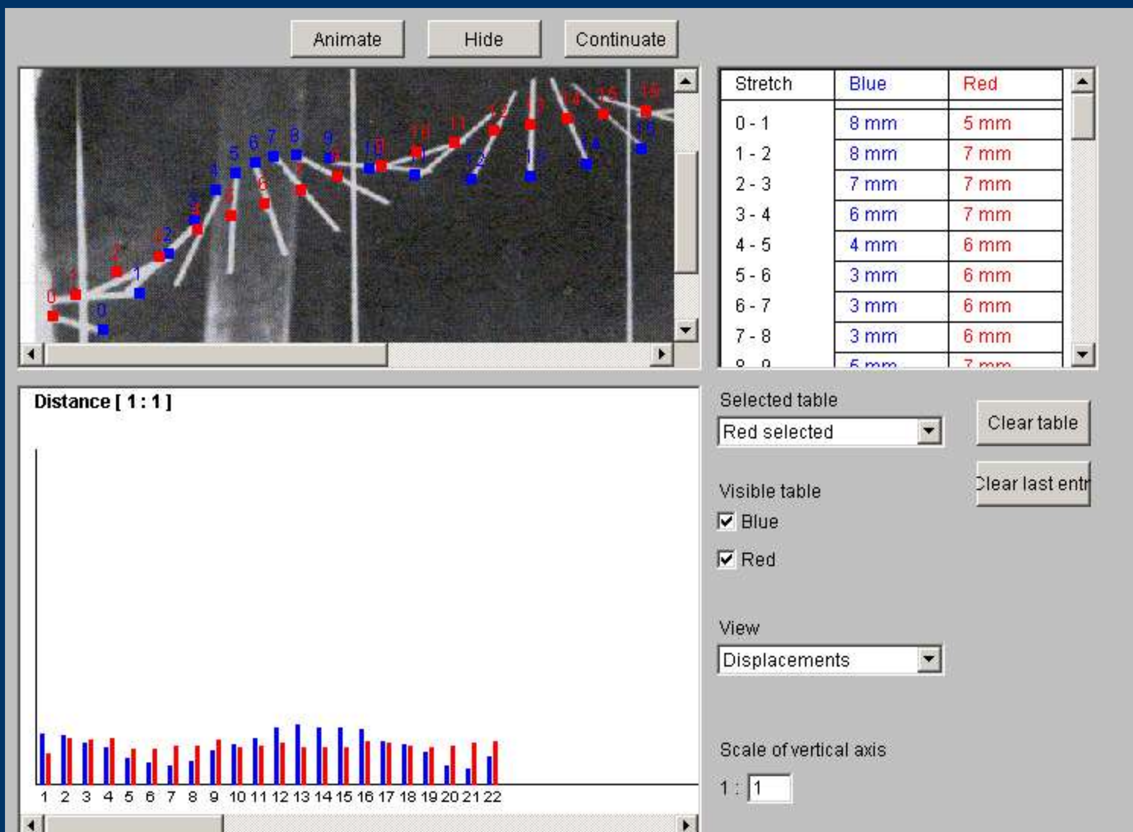
A Context that evokes the need to grasp change and that supports reasoning with intervals

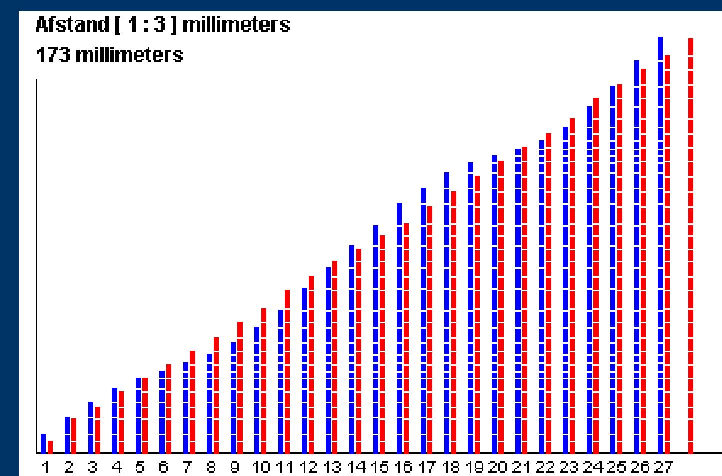






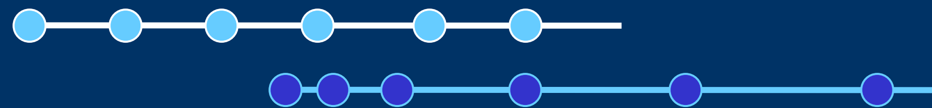
Modelling motion





Carefully introduced models

trace graphs



graphs of intervals

graphs of total distances traveled

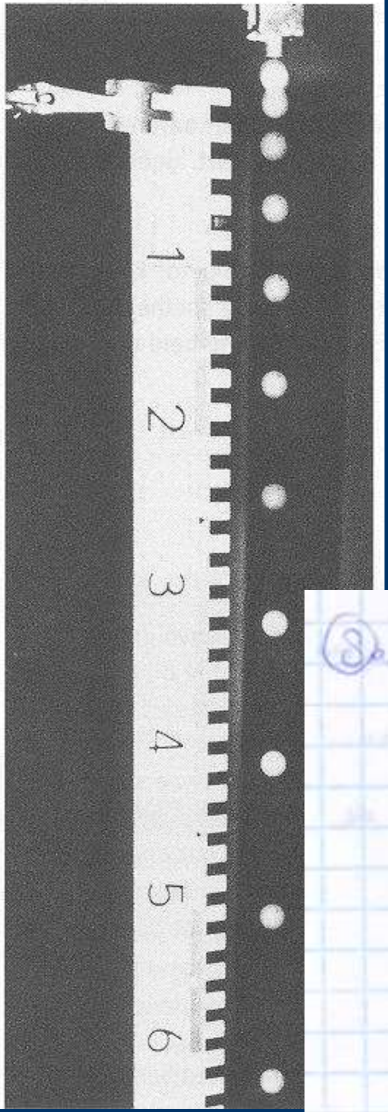
A sequence of models

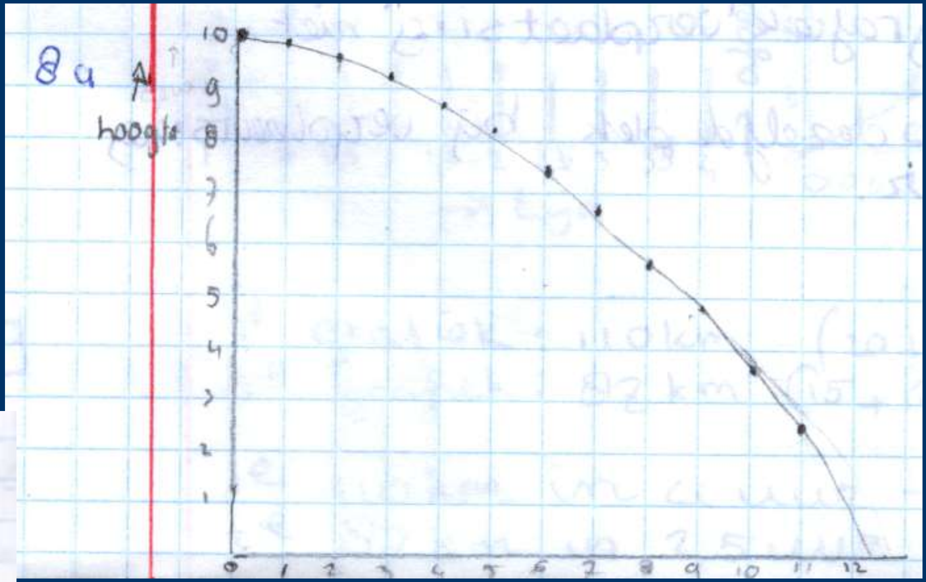
Change focus of graphing activity:

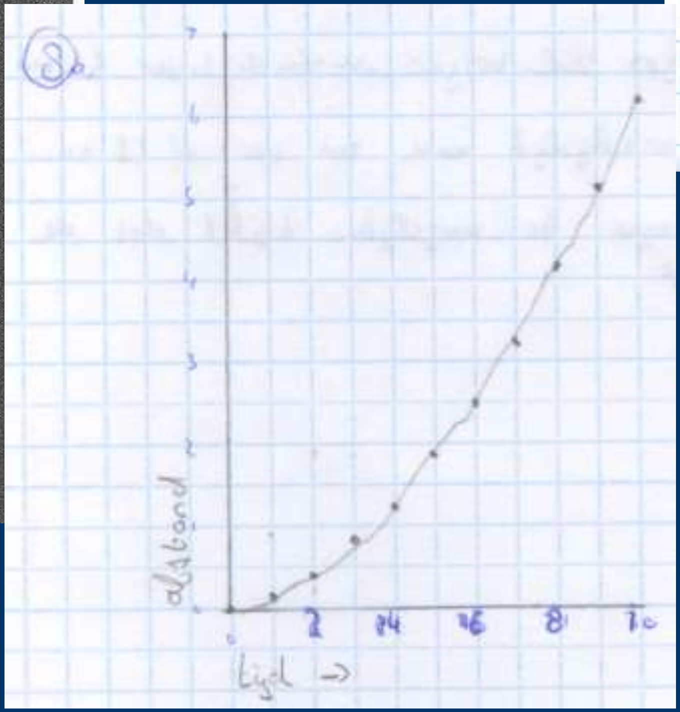
from: mathematizing subject matter
graphs describing and structuring situations
(graphs come to the for as a *model of* motion)

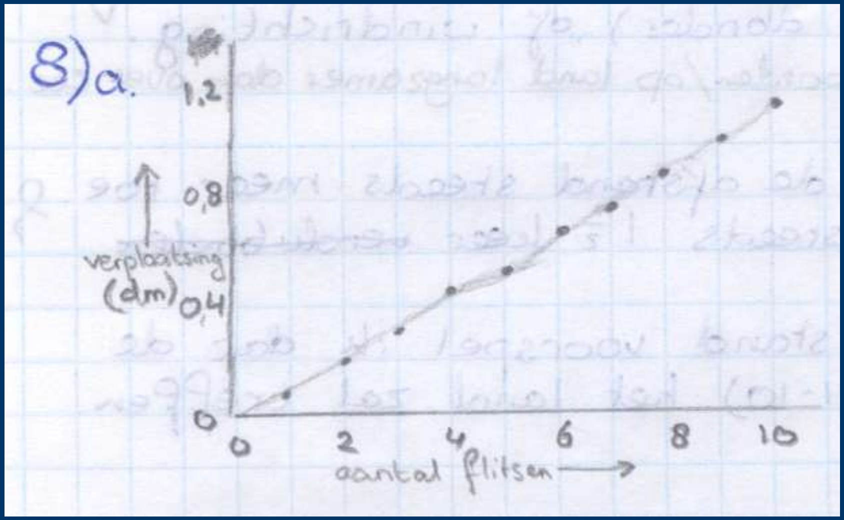
to: mathematizing the students' mathematical activity
reasoning about characteristics of graphs in relation with motion
(graphs serve as a *model for* math. & phys. reasoning about slope, area, …)

Experiences 1

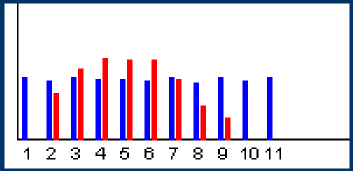


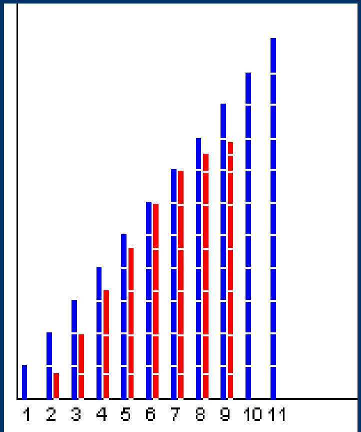






Experiences 2





O: So why did you choose the one for the total distance?

J: Because it’s the total distance that they cover and then you can …

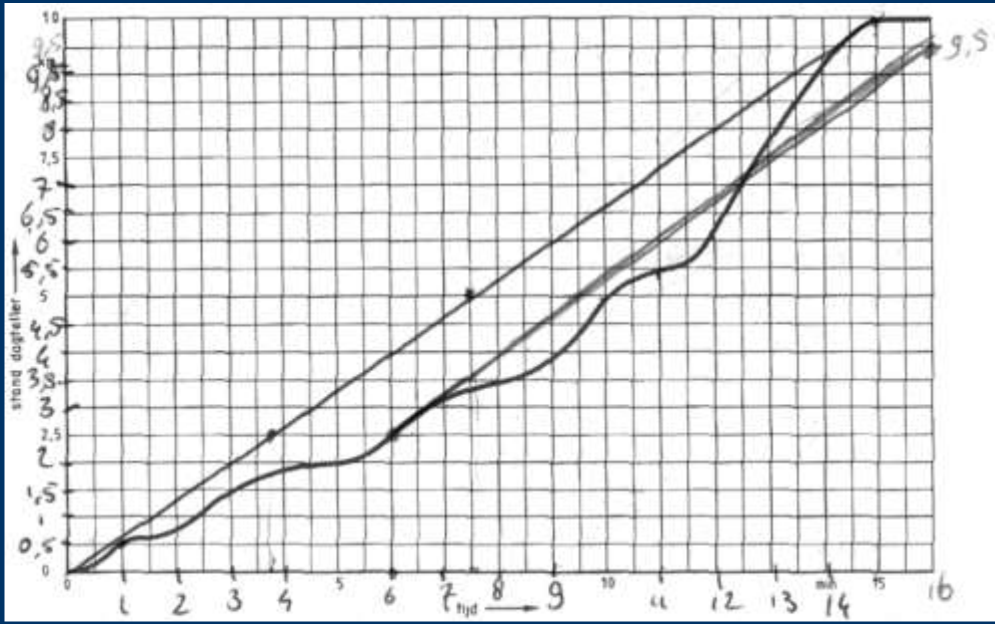
M: Then you can see if they catch up with each other.

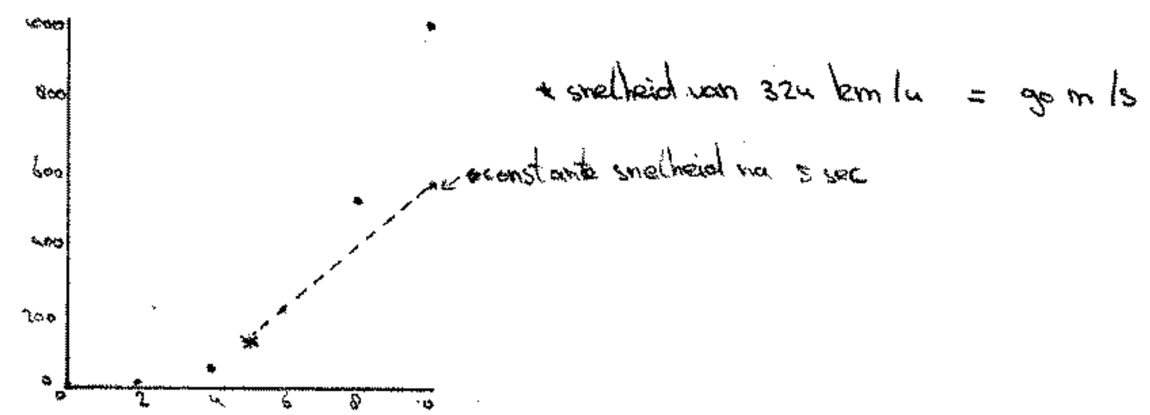
O: And can’t you see that in the other? There you can also see that the red catches up with blue?

J: Yes, but …

M: Yes, but that’s at one moment. That only means that it’s going faster at that moment but not that it’ll catch up with the zebra.

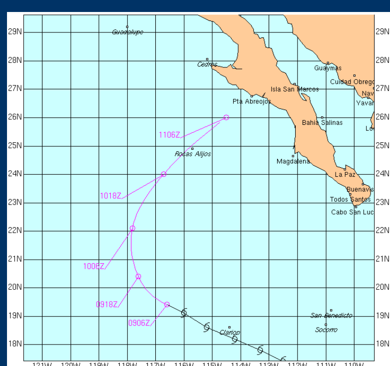
Experiences 3

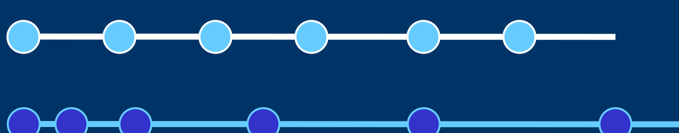


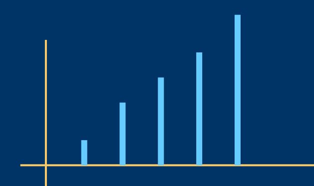


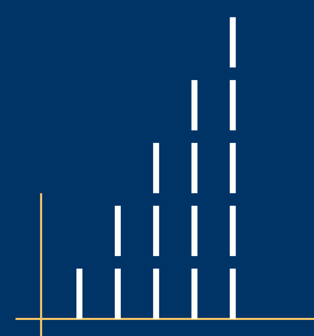
Conclusions

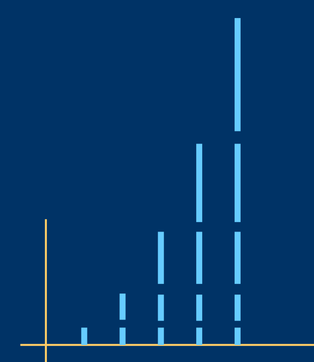
- The weather-prediction context
  - Motivated grasping change
  - Evoked reasoning with successive intervals
- The students were involved in the process of formalization from discrete to continuous











Discussion

- What can sensor-supported and embodied experiences add?
- At what age-group can you start?
- To what extent is a ‘discrete’ preparation needed?

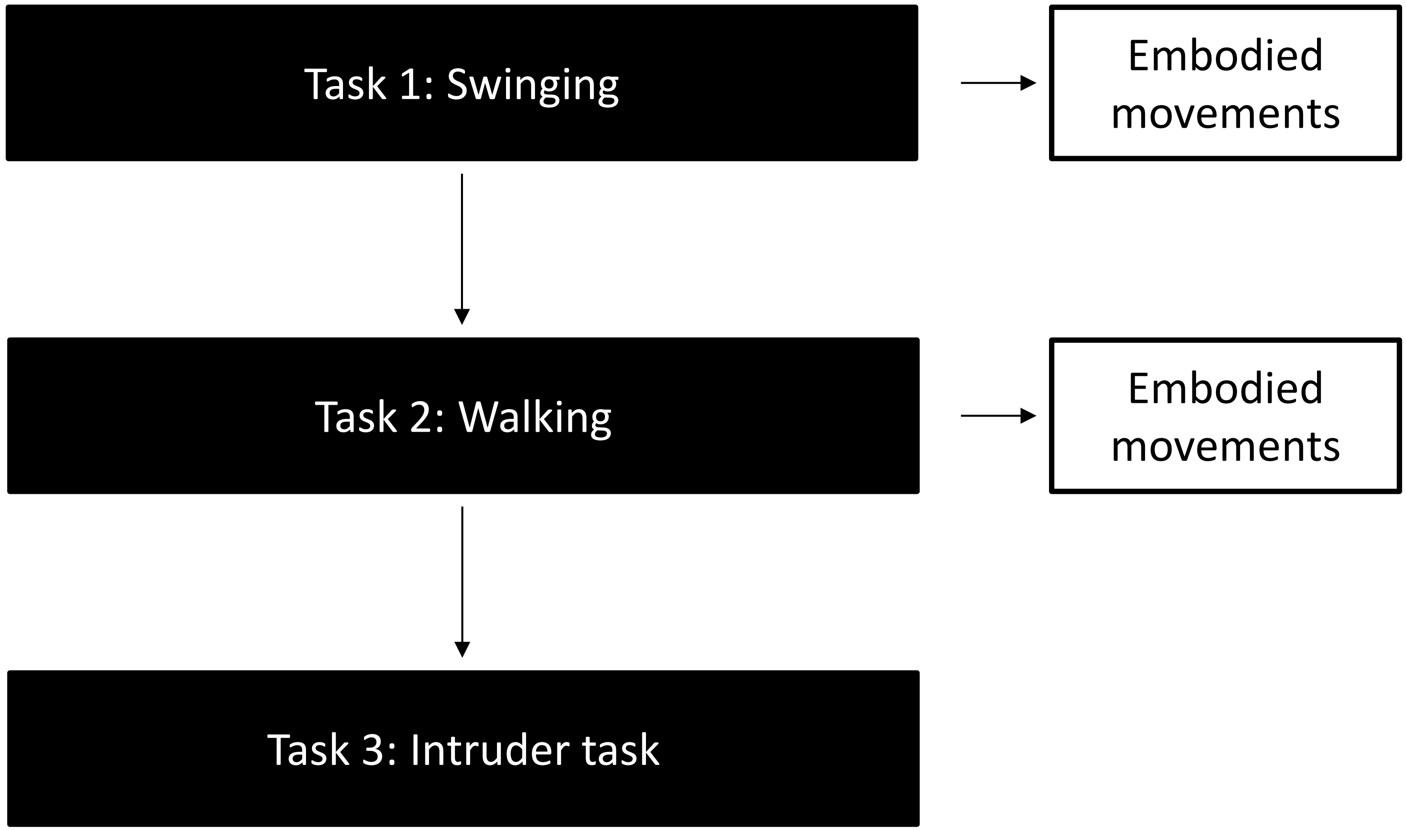
Beyond Flatland

Aim Flatland project:
Investigating whether and in what ways the domains of ***dynamic data modelling****, early algebra and probability do have potential to foster Higher-Order Thinking skills in primary school students.*

Dynamic Data Modelling

Exploring graphical representations of changing quantities over time

The HOT skills aimed at in this project include representing dynamic data related to motion, reflecting on these representations, refining them and using them for reasoning, hypothesizing and testing predictions


Task 1: Swinging
Embodied movements
Task 2: Walking
Embodied movements
Task 3: Intruder task

Experiencing embodiment and graphing

- Explore what happens
- Walk the graphs

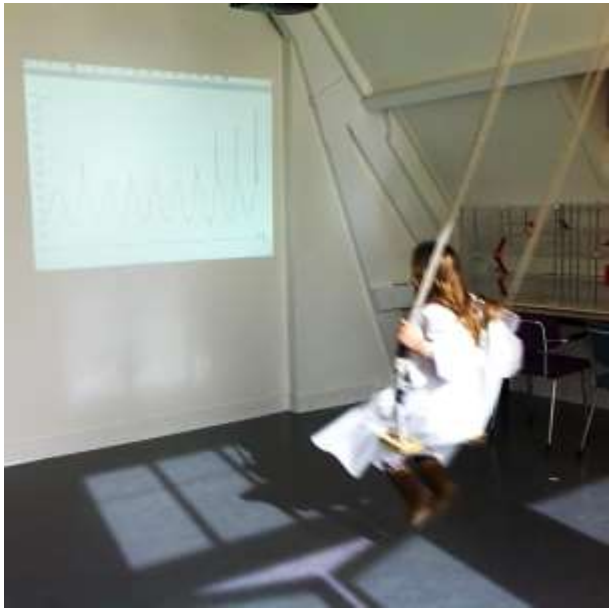
- What can sensor-supported and embodied experiences add to learning about graphs?
- To what extent is a ‘discrete’ preparation needed?

Task 1: Swinging

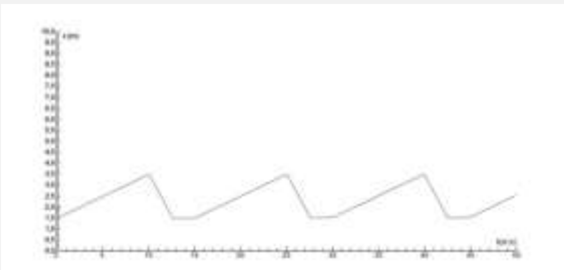
Question(s) for the children:

'Can you start swinging and just tell us what happens in the graph.'

'Can you change the graph?'



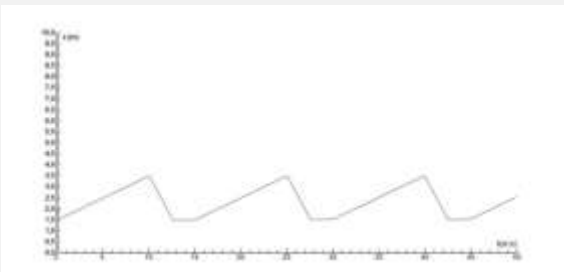
Task 2: Walking



‘This is an example of a graph, can you try to walk this graph?’

Bas walks the graph

Task 2: Walking



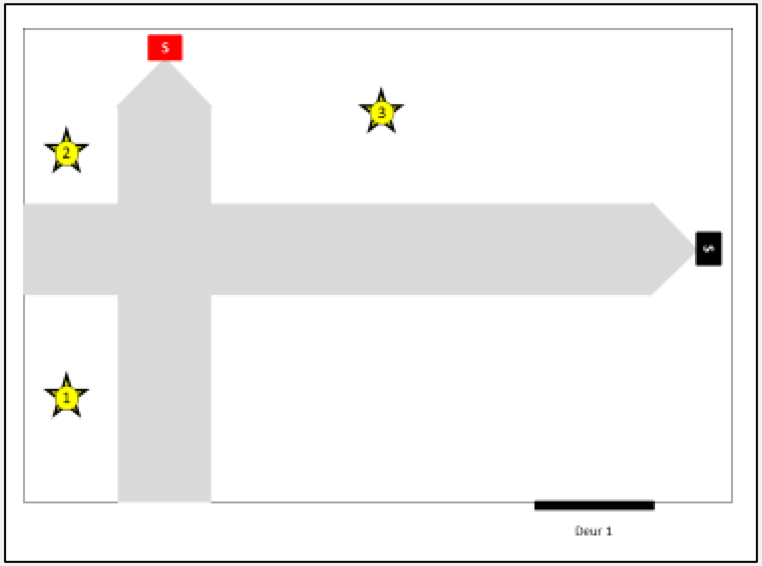
'This is an example of a graph, can you try to walk this graph?'

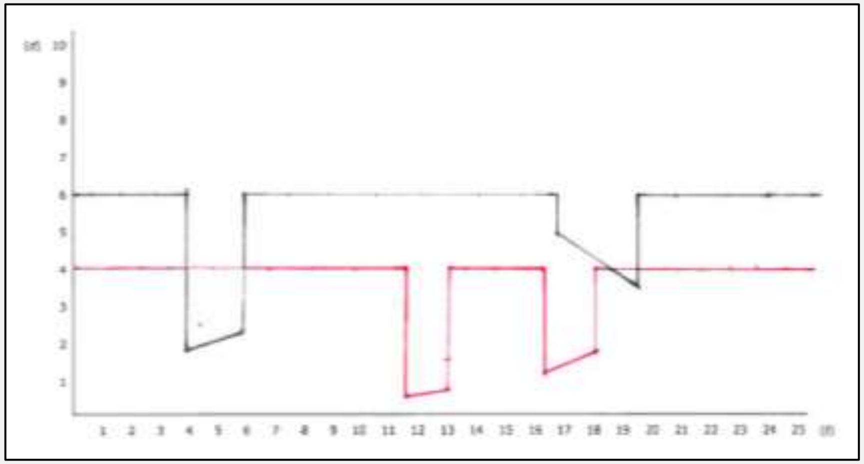
Timon walks the graph

Task 2: Walking

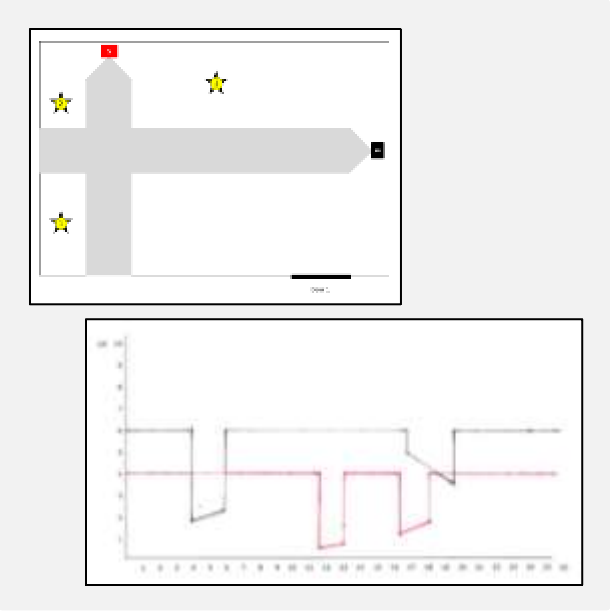
- In order to walk the graph, children have to vary in the speed and direction of their movements.
- Both children connect their movements to the graphical representation.
- Timon: Verbalising the aspect of the graphical representation he might change. Timon shows us that he knows which aspects of his movements he should alter – he should walk even slower.
    - Indication of higher order thinking.

Task 3: Intruder task





Task 3: Intruder task



Fragment: Tamar and Sophie are solving the intruder task

Task 3: Intruder task

- Sophie and Tamar solve the intruder task with trial ar
- Both sensors interact with each other in a complex m
- Tamar shows she understands what she sees very cle
  information and focusing on what is of use for solving
  route Sophie just walked.
    - Indication of higher order thinking.

Thank you

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Discussion

- What can sensor-supported and embodied experiences add to learning about graphs?
- To what extent is a ‘discrete’ preparation needed?