Explo*RATIO* to learn fractions

Groupe d'enseignement mathématique

Workshop presented by
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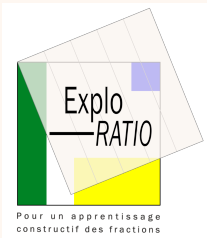
Haute École Galilée, Bruxelles

12 octobre 2019



Discovery of Exploratio

For all activities, the unit is the square shown on the first page and which appears on the vignettes.

Activity 1. Fiche 1. 1a and 1b.
a) Try to find "at a glance" [déterminez à l'oeil] the fractions which are represented.
Don't mesure, don't use the material now.

Discovery of Explo*RATIO*

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Activity 1. Fiche 1. 1a and 1b.
a) Try to find "at a glance" [déterminez à l'oeil] the fractions which are represented.
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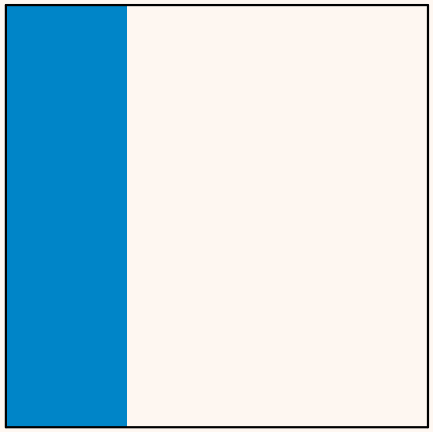
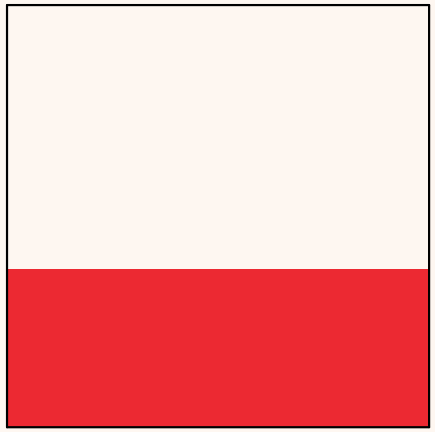
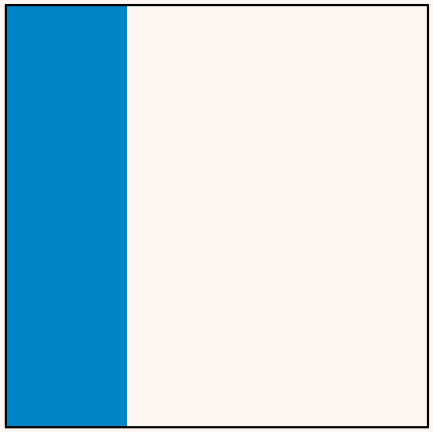
b) Check your answer using the material.



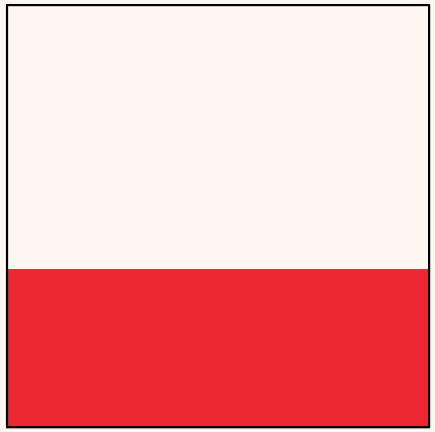
1a



1b

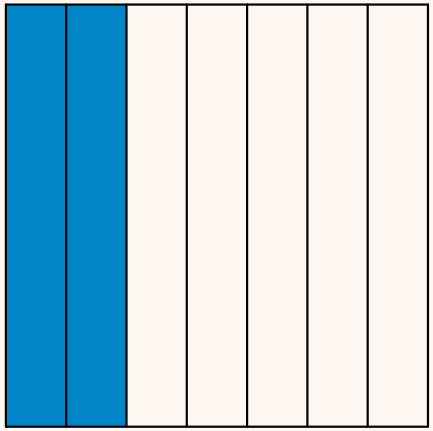


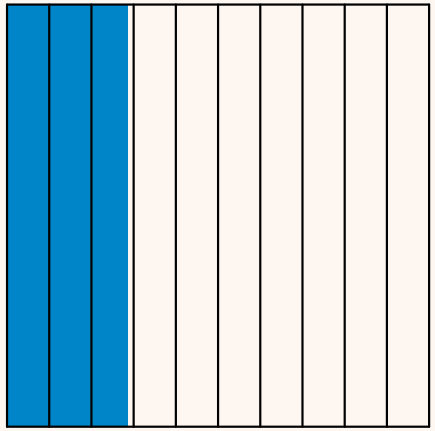
$\frac{2}{7}$? $\frac{3}{10}$?



$\frac{3}{8}$

We see that $\frac{3}{10} > \frac{2}{7}$.





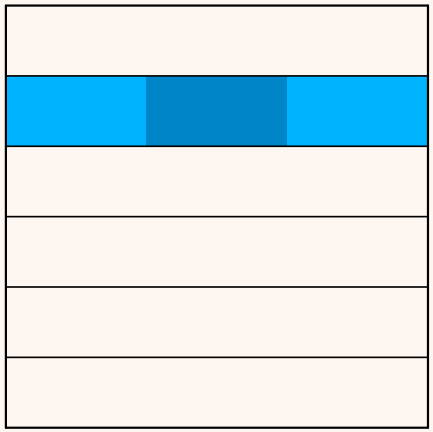
First discoveries I

Activity 2. Browse through some vignettes :
Fiche 1 : page 3,
Fiche 2 : 2c, 2d, 2e, 2f, 2p, 2s
Fiche 3 : 3a, 3b, 3g.
Find the fractions represented by each of the coloured parts.

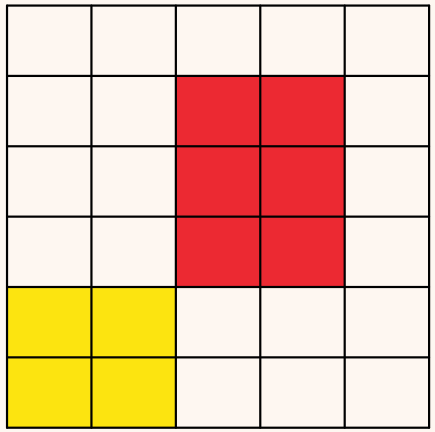
Some discoveries

Some discoveries

1. "Fraction of fraction", for example $\frac{1}{3}$ of $\frac{1}{6}$.

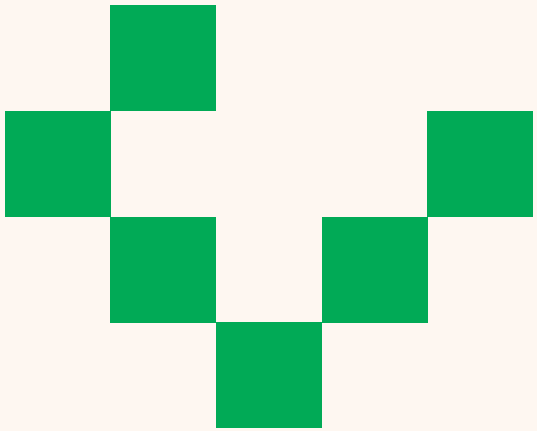


2d

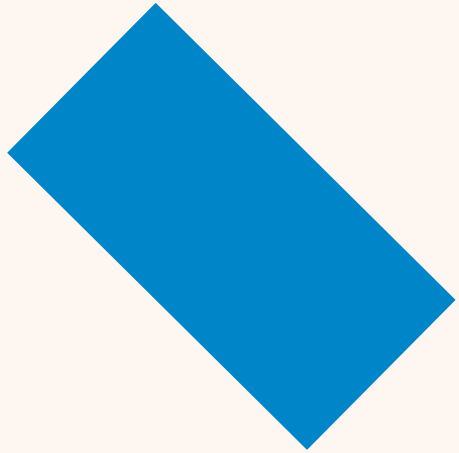


2h

2. Representations of fractions without the unit.

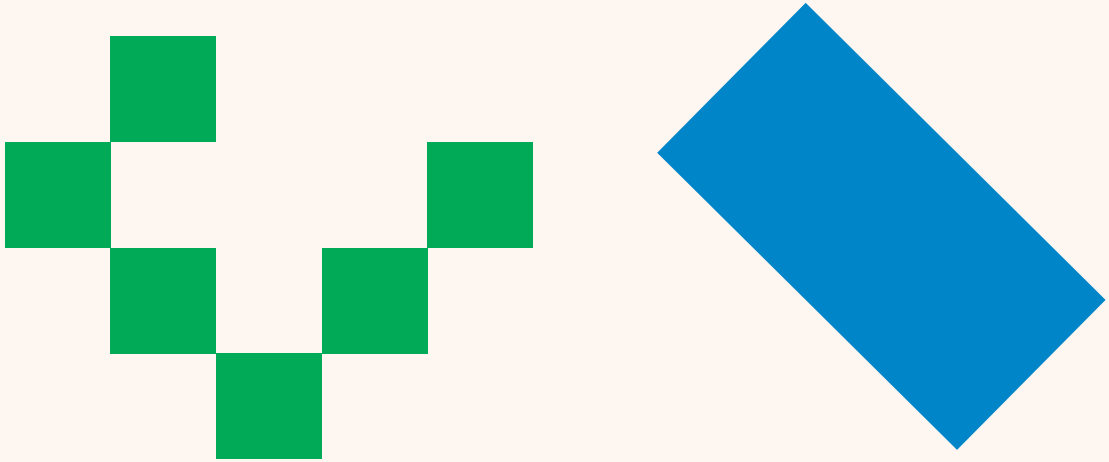


2p



1k

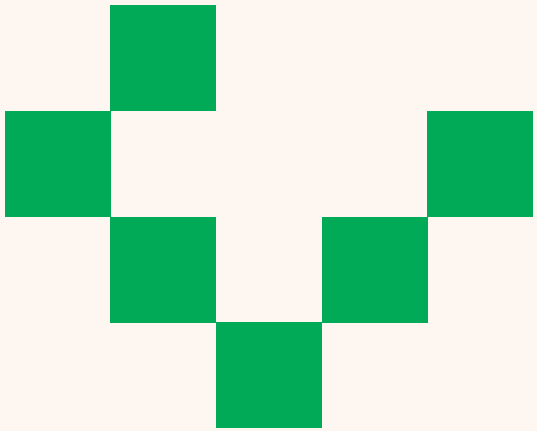
2. Representations of fractions without the unit.



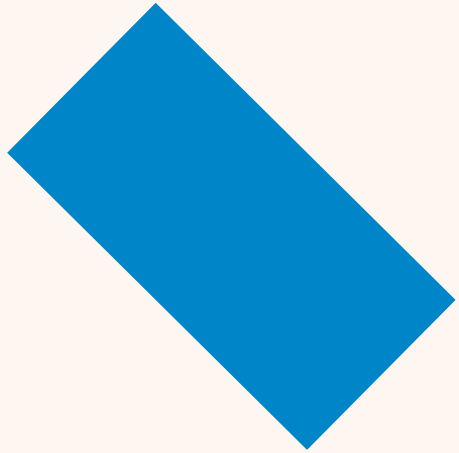
2p 1k

If you always show fractions with the unit...

2. Representations of fractions without the unit.



2p



1k

If you always show fractions with the unit...

$$\frac{1}{4} + \frac{1}{4} = ?$$

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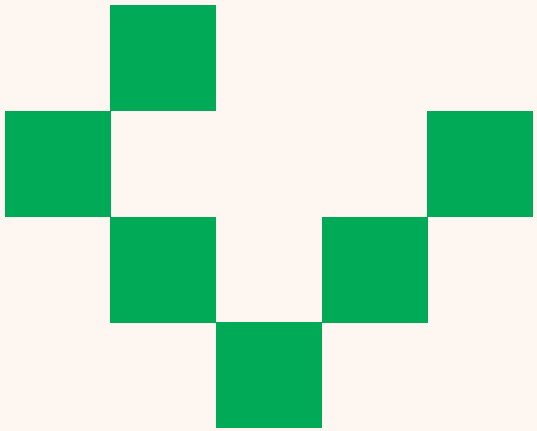
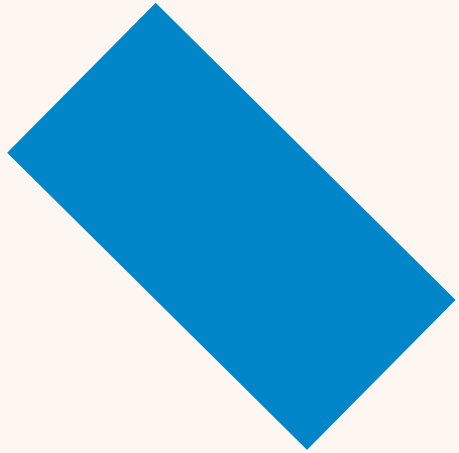


2p



1k

If you always show fractions with the unit...

$$\frac{1}{4} + \frac{1}{4} = ?$$

$$\frac{1}{4} + \frac{1}{4} = \frac{2}{8}$$

3. Several representations of the same fraction,

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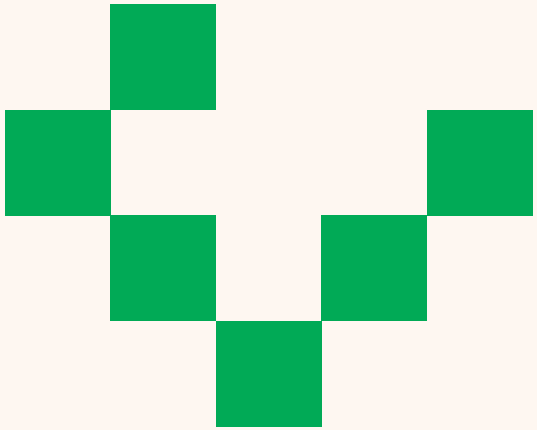
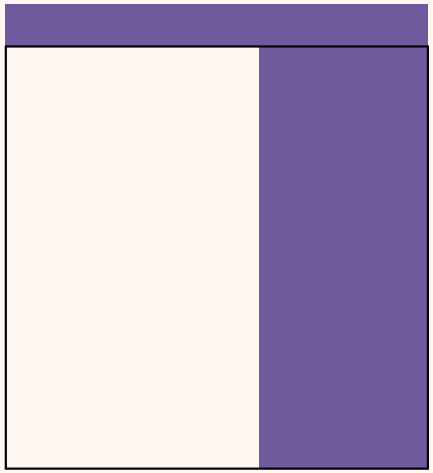
even representations that extend beyond the unit-square while the fraction is smaller than 1.



2p



2r

And some others interests of the material

4. With this material, you must estimate the number of parts in the unit ...

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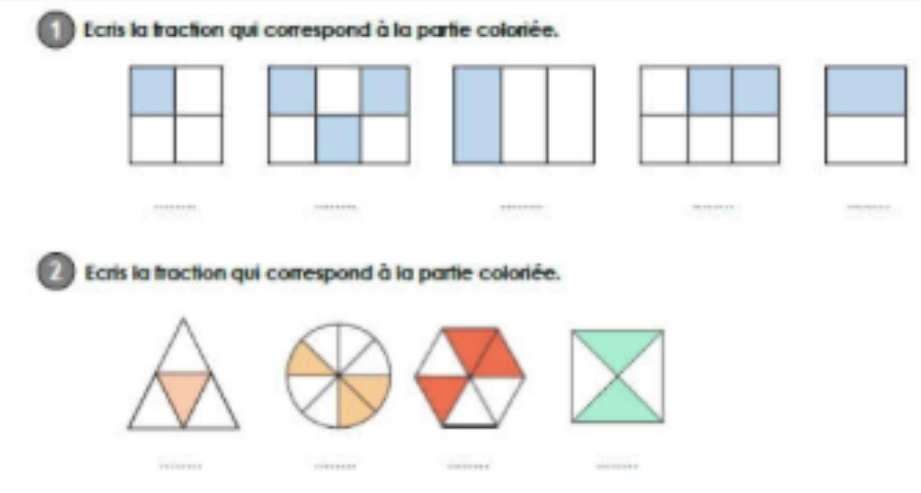
In standard student textbooks...



https://www.i-profs.fr/Fiches/cm2/numeration-calcul/cm2-exercices-fractions.php

In primary school

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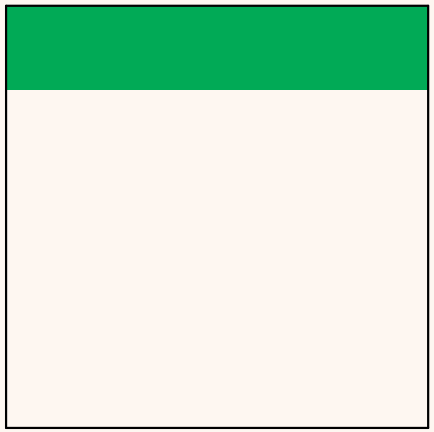
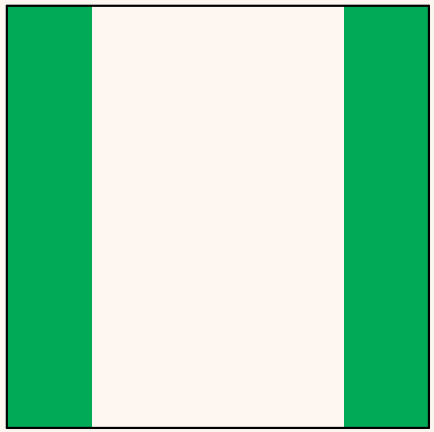
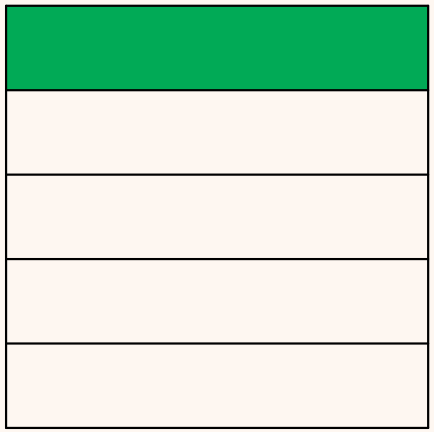


2a

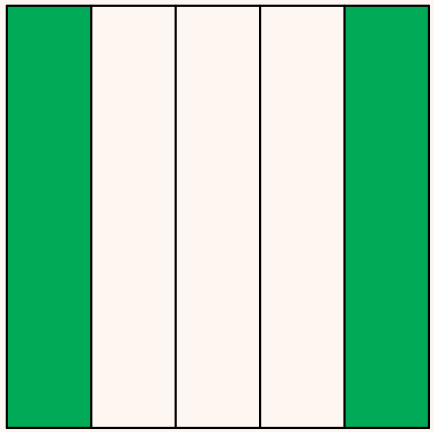


2b

In primary school



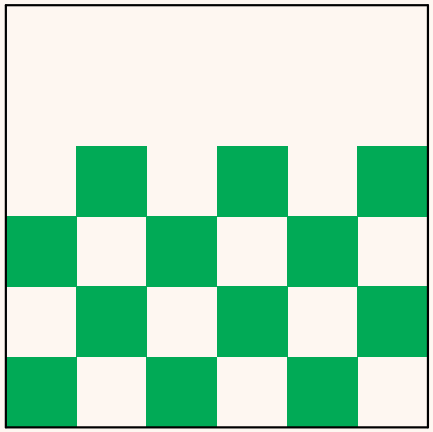
$\frac{1}{5}$



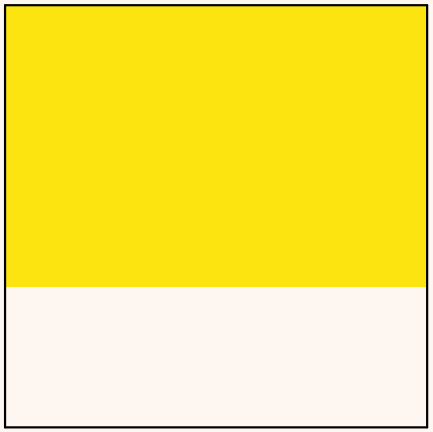
$\frac{1}{5}$

Equivalent fractions

Several answers arise naturally and we can discuss them.



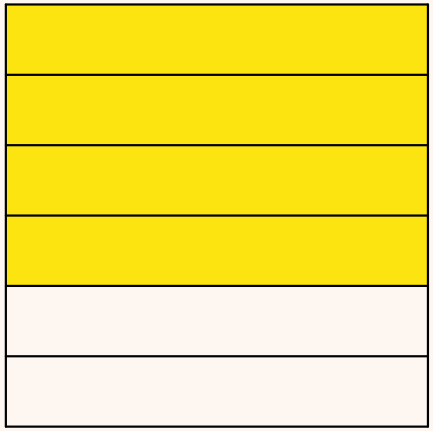
3a

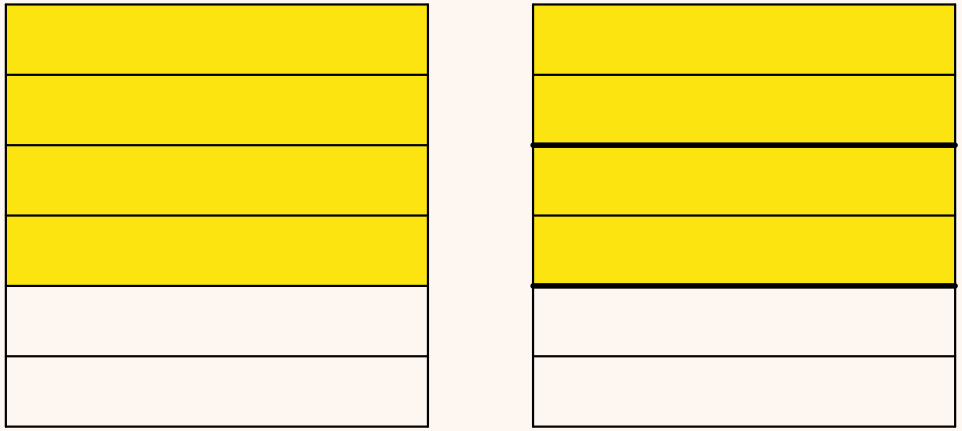


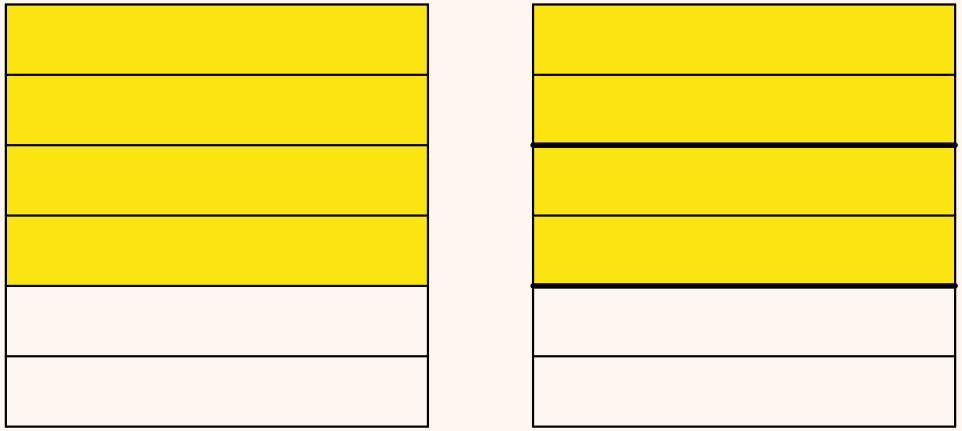
3b

3a. $\frac{12}{36}$? $\frac{1}{2}$ of $\frac{4}{6}$? $\frac{1}{3}$? ... Which one is correct ?

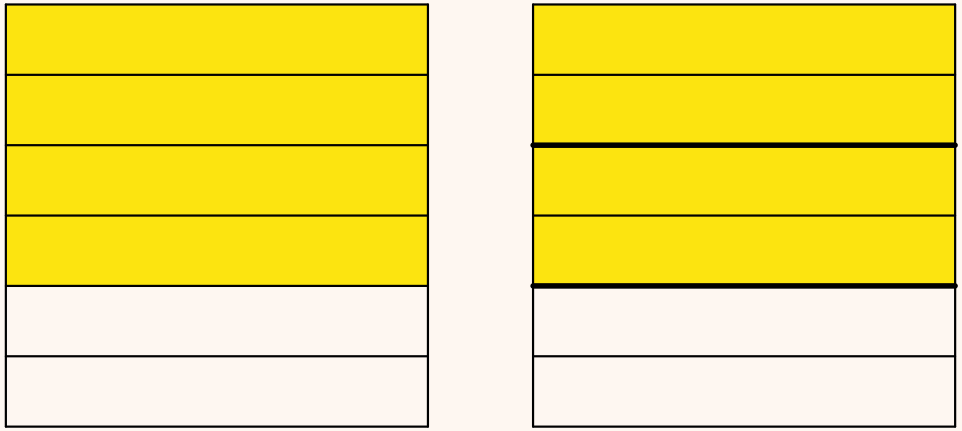
3b. $\frac{2}{3}$?$\frac{4}{6}$?







$$\frac{4}{6} = \frac{2}{3}$$



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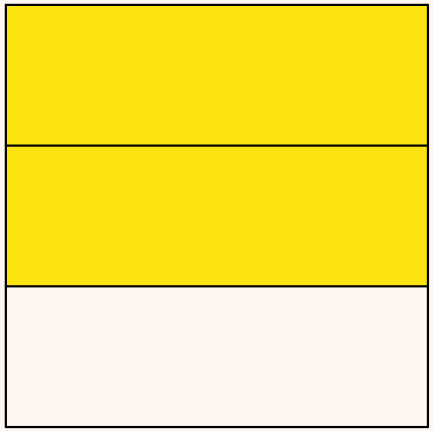

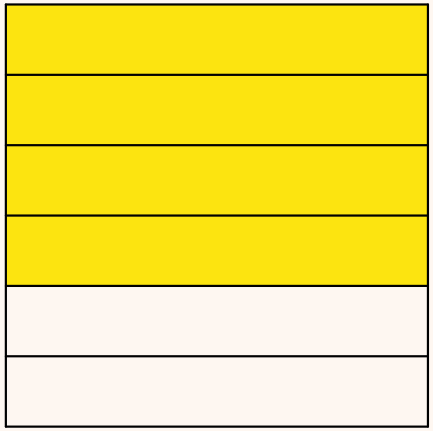

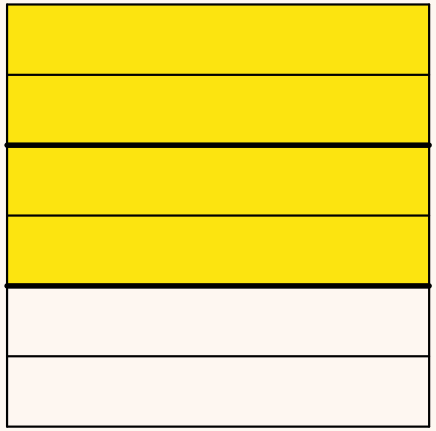

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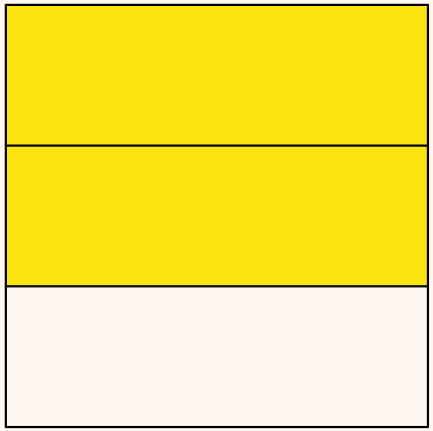

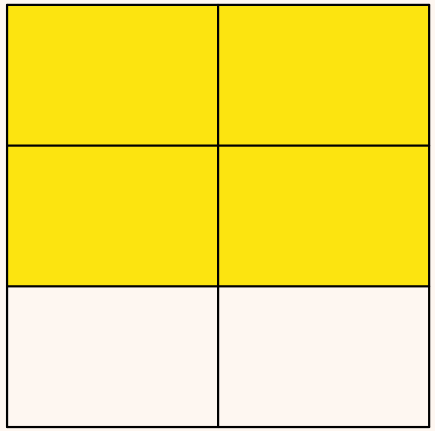

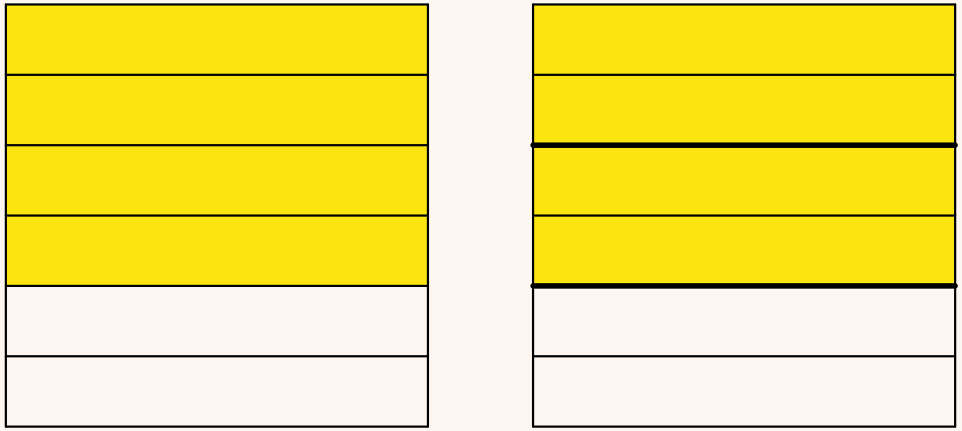

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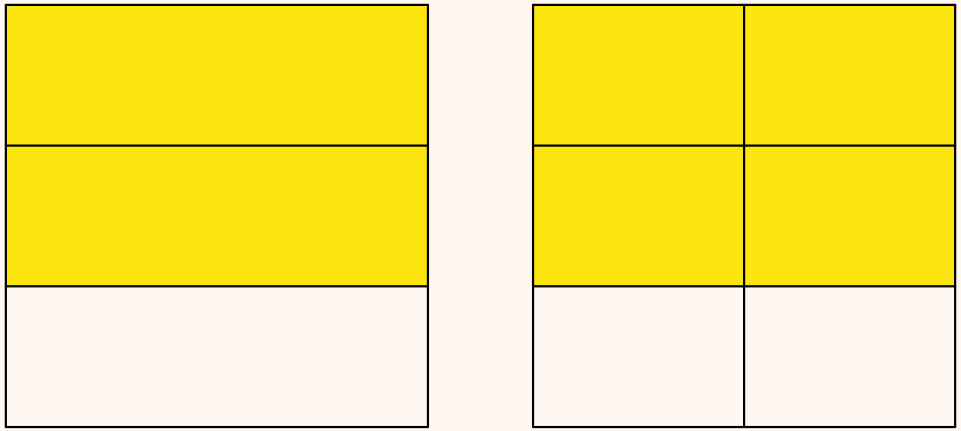


$$\frac{2}{3} = \frac{4}{6}$$

Other writings ?

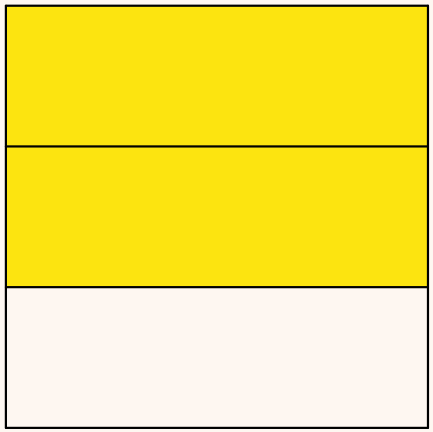
$$\frac{2}{3} =$$

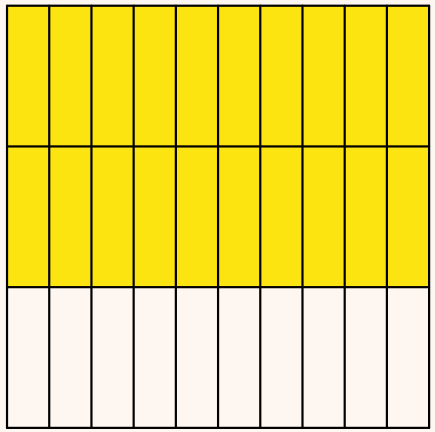
Other writings ?

$$\frac{2}{3} = \frac{20}{30}$$

Other writings ?

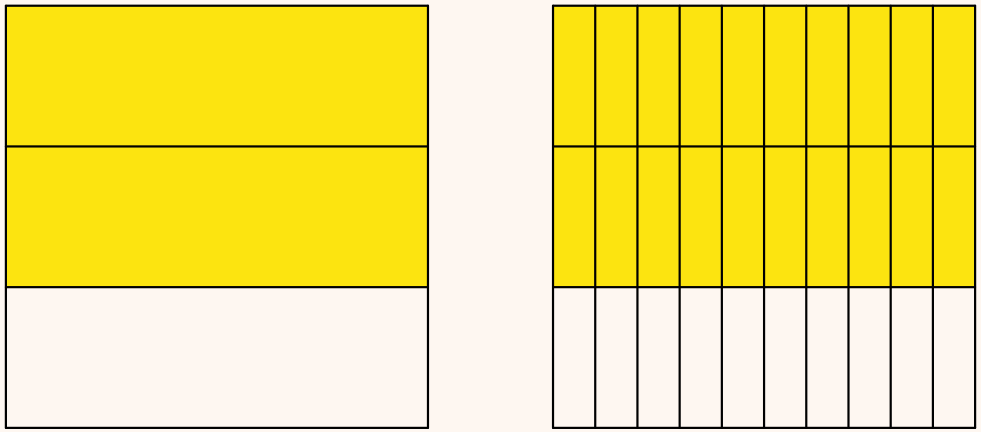
$$\frac{2}{3} = \frac{20}{30}$$





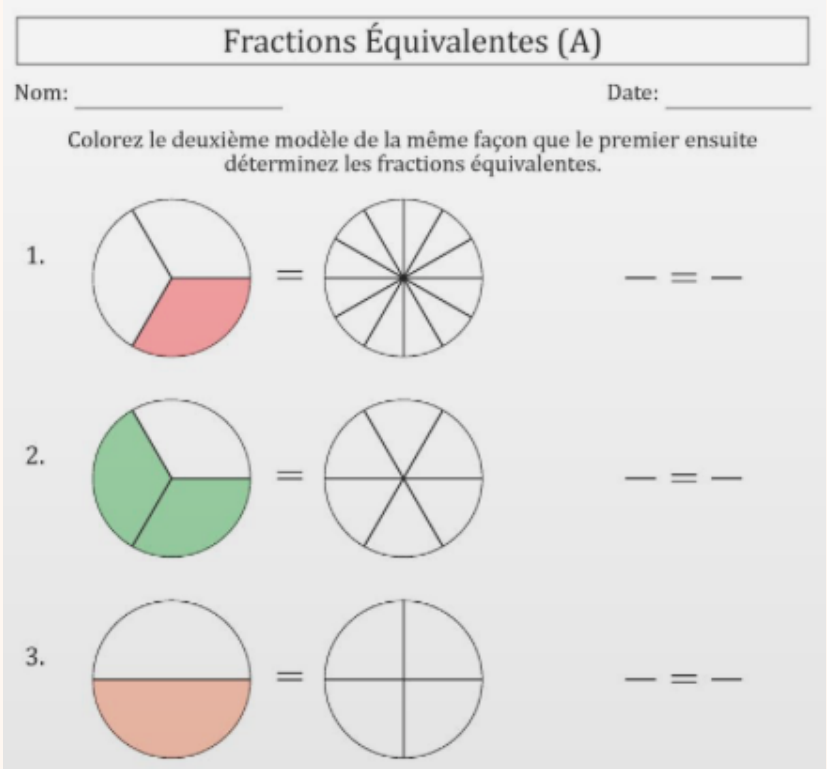
Other writings ?

$$\frac{2}{3} = \frac{20}{30}$$



$$\frac{2}{3} = \frac{20}{30} = \frac{2000}{3000}$$

A classical exercice.



Fractions Équivalentes (A)

Nom: _____ Date: _____

Colorez le deuxième modèle de la même façon que le premier ensuite déterminez les fractions équivalentes.

1. = — = —

2. = — = —

3. = — = —

https://www.mathslibres.com/fractions/frac_equiv_modeles_simplifie_premier_001.php

Representation of fractions

Activity 3. Fraction representations.

a) Use the window to represent a half. You can be original !

b) Using the window, compare $\frac{2}{3}$ to $\frac{3}{5}$.

Representation of fractions

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b) We can take a length as a unit !
The representation of fractions by length is essential

- to allow the representation of fractions by points on a graduated line,
- to apprehend the product of fractions.

Adding and subtracting fractions

Activity 4. Fiche 6.
a) 6a, 6c, 6d, 6g. Find the fraction represented by the coloured part (total).
b) 6h, 6d. Find the fractions represented by each coloured part then find the fraction represented by the (total) coloured part, then the difference between the two parts.

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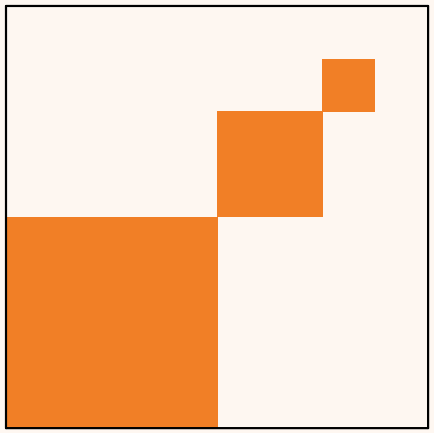
6a



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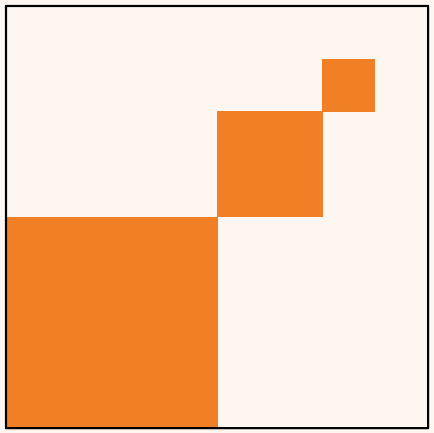

$$\frac{1}{4} + \frac{1}{16} + \frac{1}{64}$$

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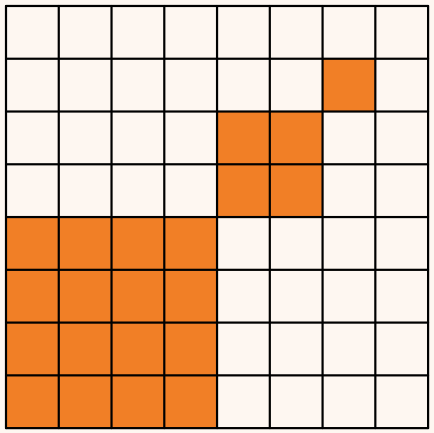

$$\frac{1}{4} + \frac{1}{16} + \frac{1}{64}$$

$$\frac{21}{64}$$

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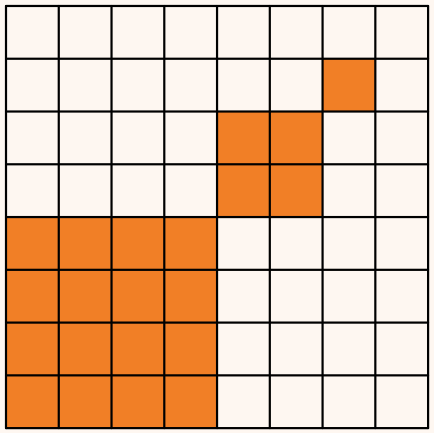

$$\frac{1}{4} + \frac{1}{16} + \frac{1}{64}$$

$$\frac{21}{64}$$

$$\frac{1}{4} + \frac{1}{16} + \frac{1}{64} = \frac{16}{64} + \frac{4}{16} + \frac{1}{64}$$

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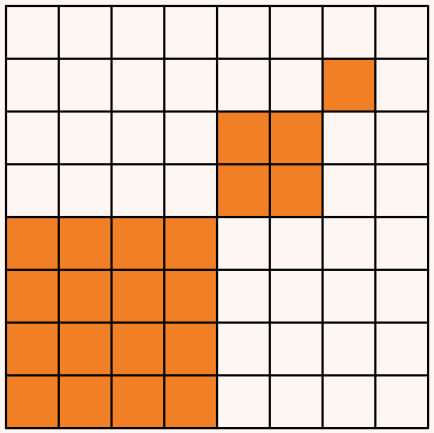


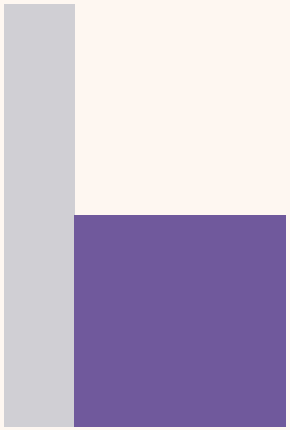
$\frac{1}{4} + \frac{1}{16} + \frac{1}{64}$

$\frac{21}{64}$

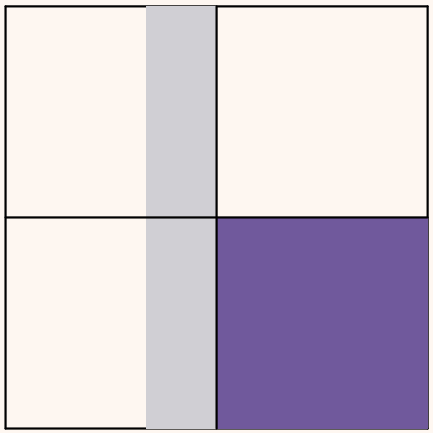
$\frac{1}{4} + \frac{1}{16} + \frac{1}{64} = \frac{16}{64} + \frac{4}{16} + \frac{1}{64}$

Here the common denominator is present.

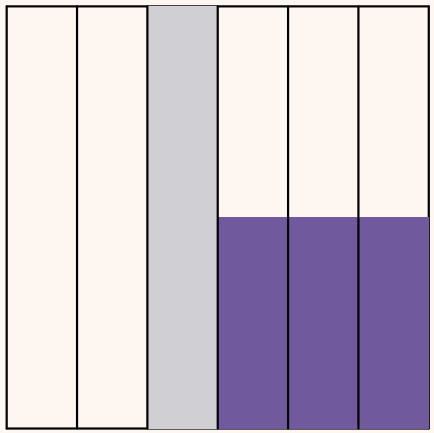
For the others, the common measure is not present. But when we superimpose the transparencies that identify the two fractions, it appears.



6d

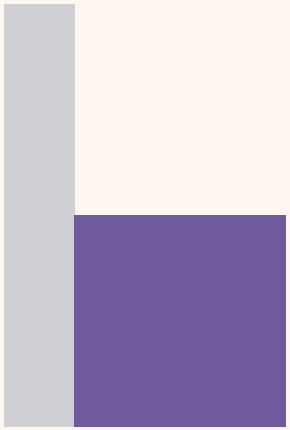


$\frac{1}{4}$

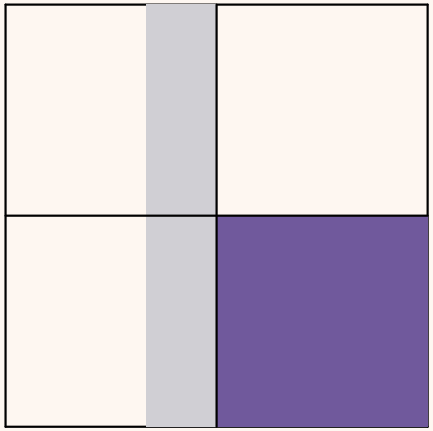


$\frac{1}{6}$

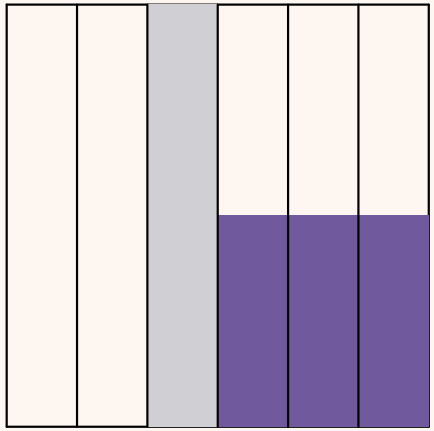
For the others, the common measure is not present. But when we superimpose the transparencies that identify the two fractions, it appears.



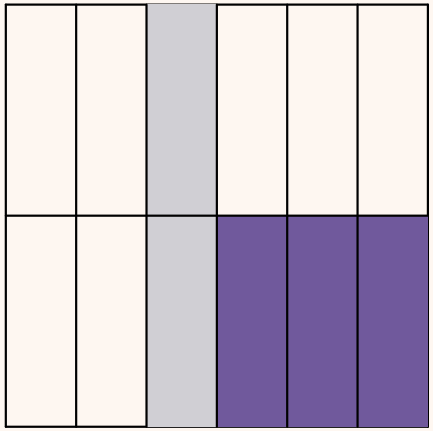
6d



$\frac{1}{4}$

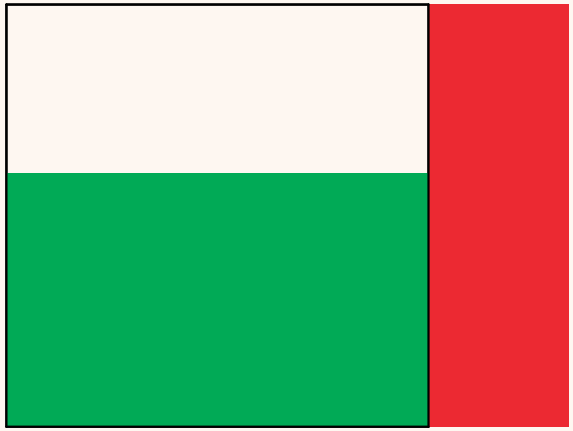


$\frac{1}{6}$

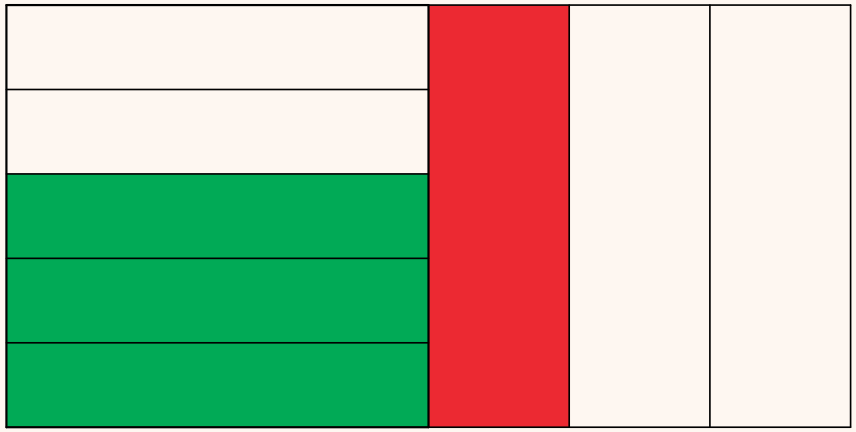


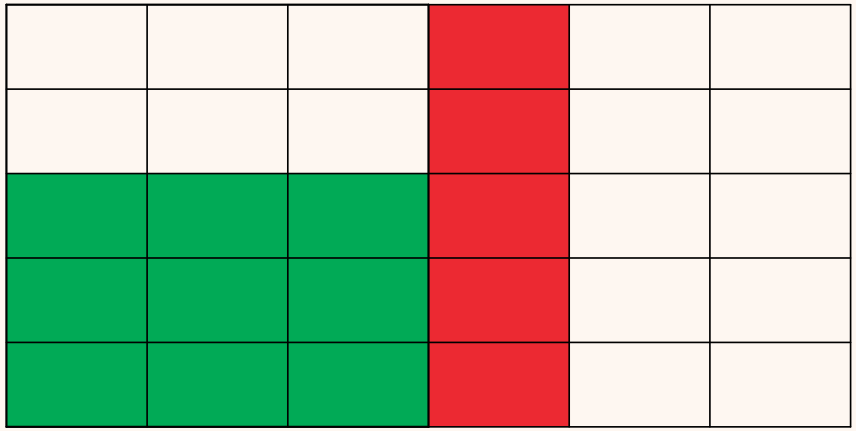
$\frac{1}{12}$ appears

The arrangement of the two parts of 6h and 6j can be used to represent all the sums of fractions.

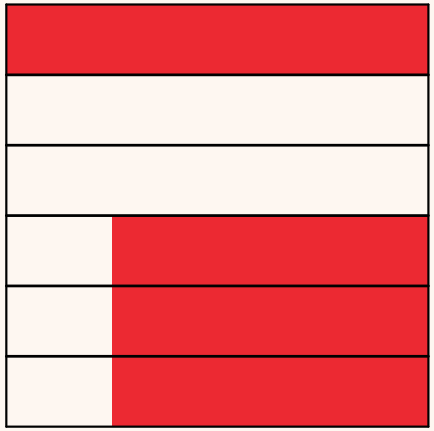


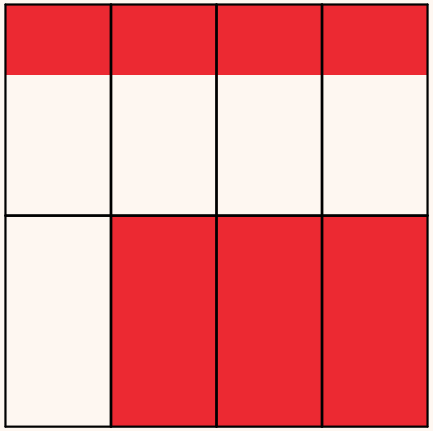
6j

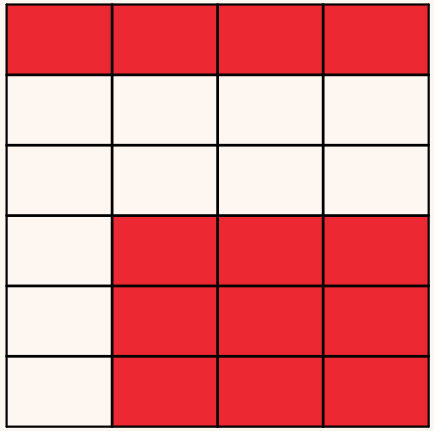




The common denominator is not always de product.

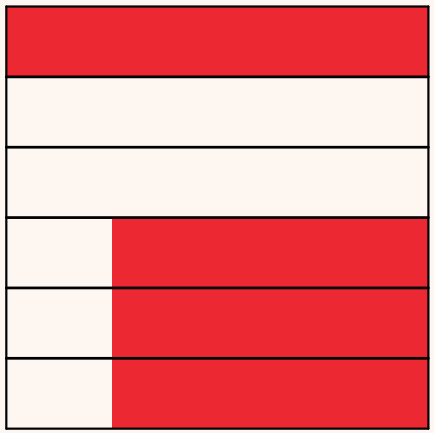




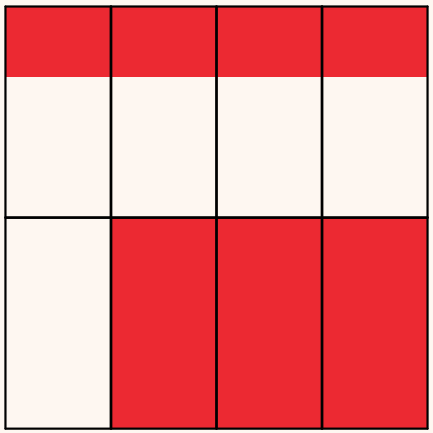


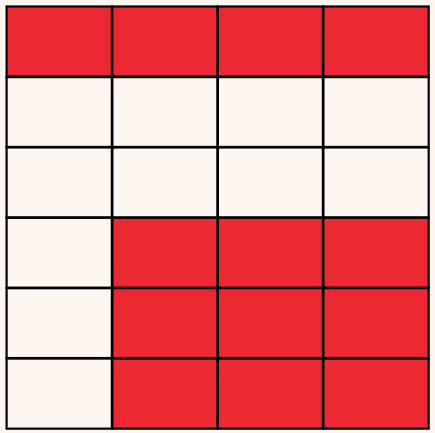
6c

The common denominator is not always de product.



6c



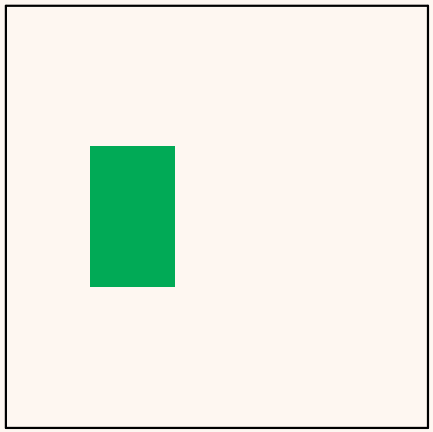


$$\frac{1}{6} + \frac{3}{8} = \frac{4}{24} + \frac{9}{24} = \frac{13}{24}$$

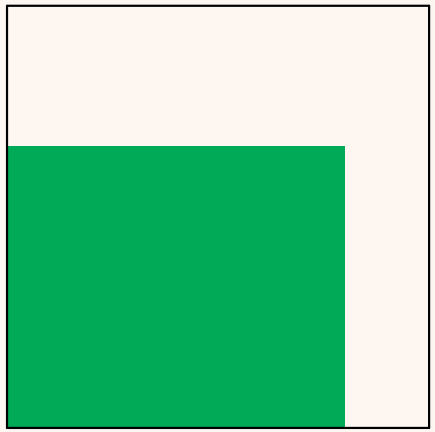
Activity 5.
Represent $\frac{4}{3} + \frac{1}{2}$ and find the "answer".

An activity that allows you to write the class sumary.

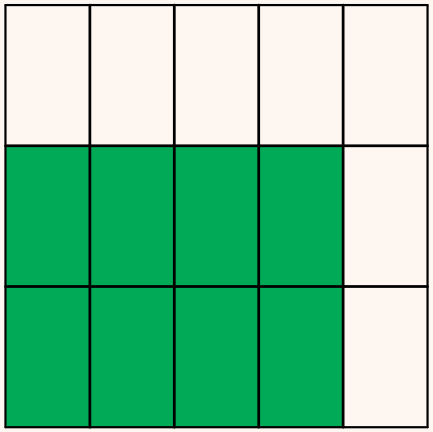
Multiplying fractions



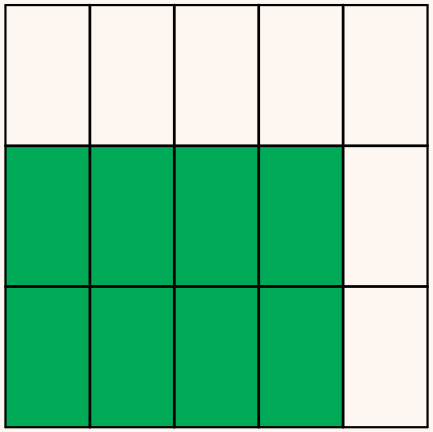
7b



7c



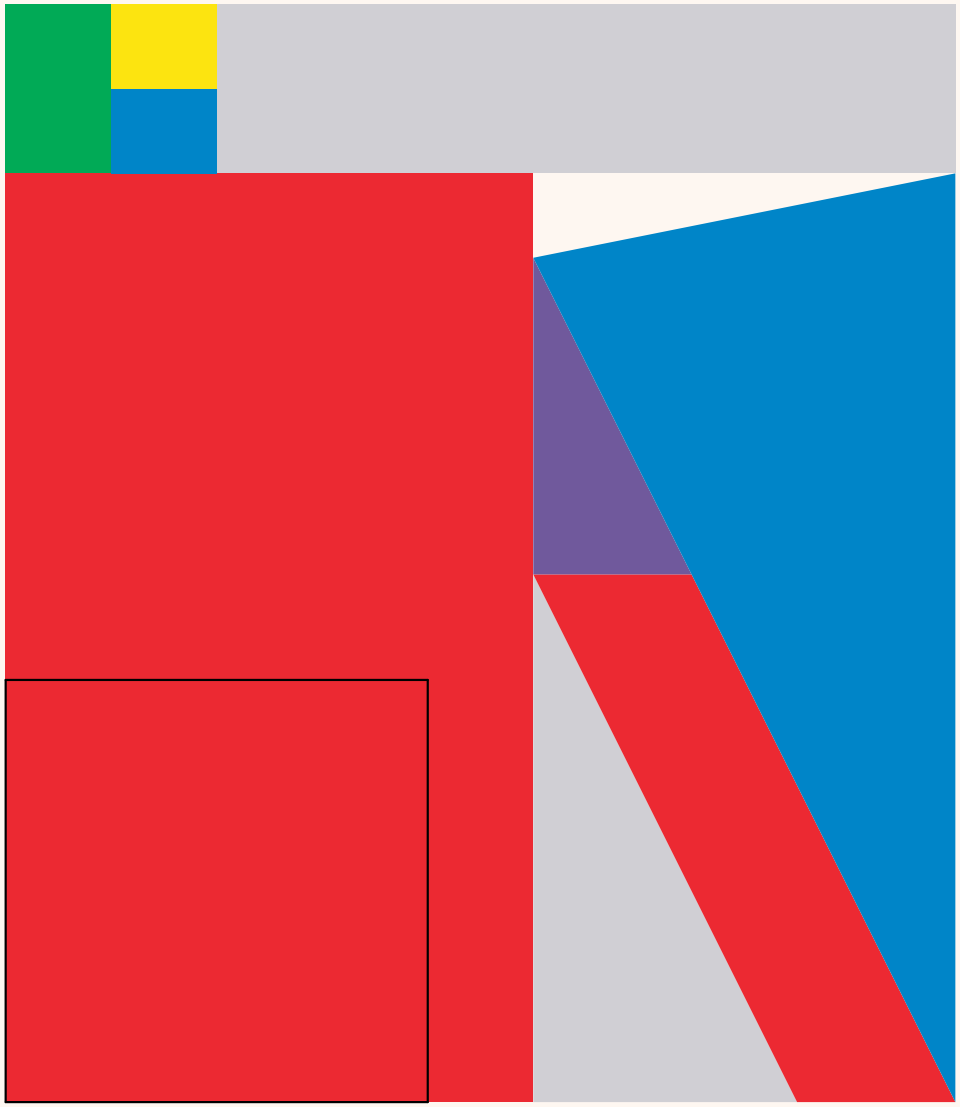
$$\frac{2\times 4}{3\times 5}$$

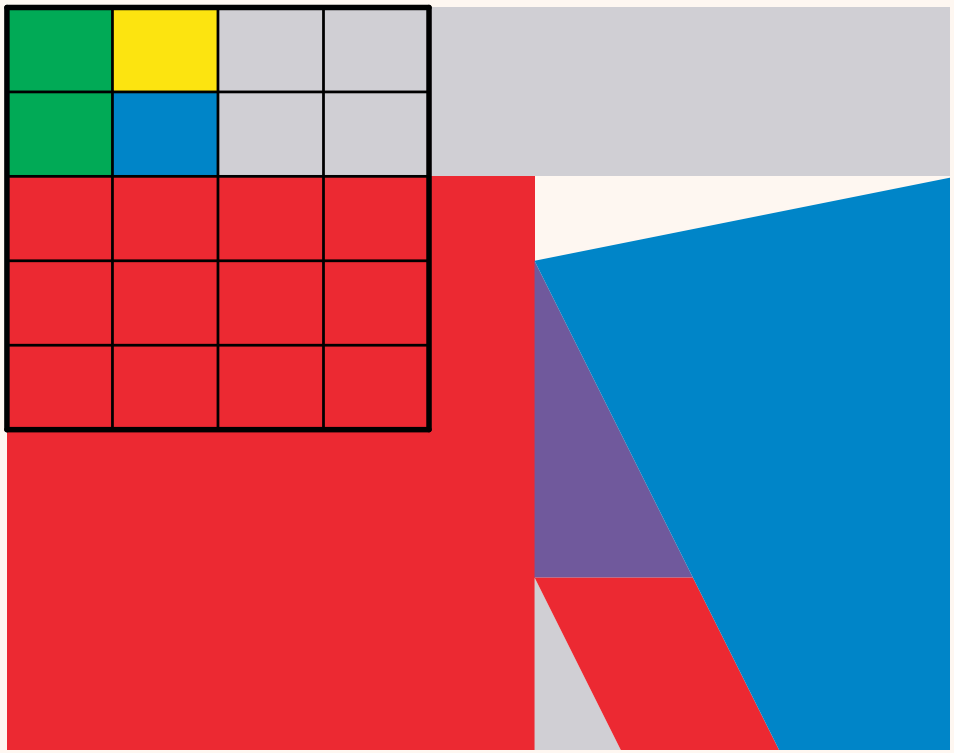


$$\frac{2\times4}{3\times5}$$

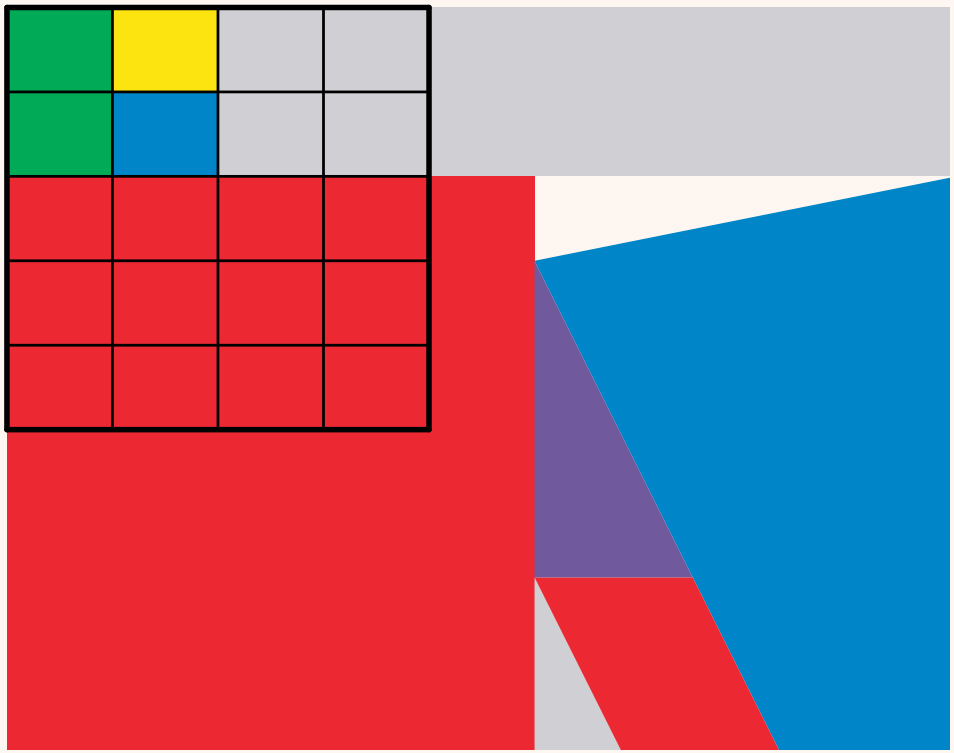
but not

$$\frac{2}{3}\times\frac{4}{5}$$





$$\frac{2}{4\times 5}$$

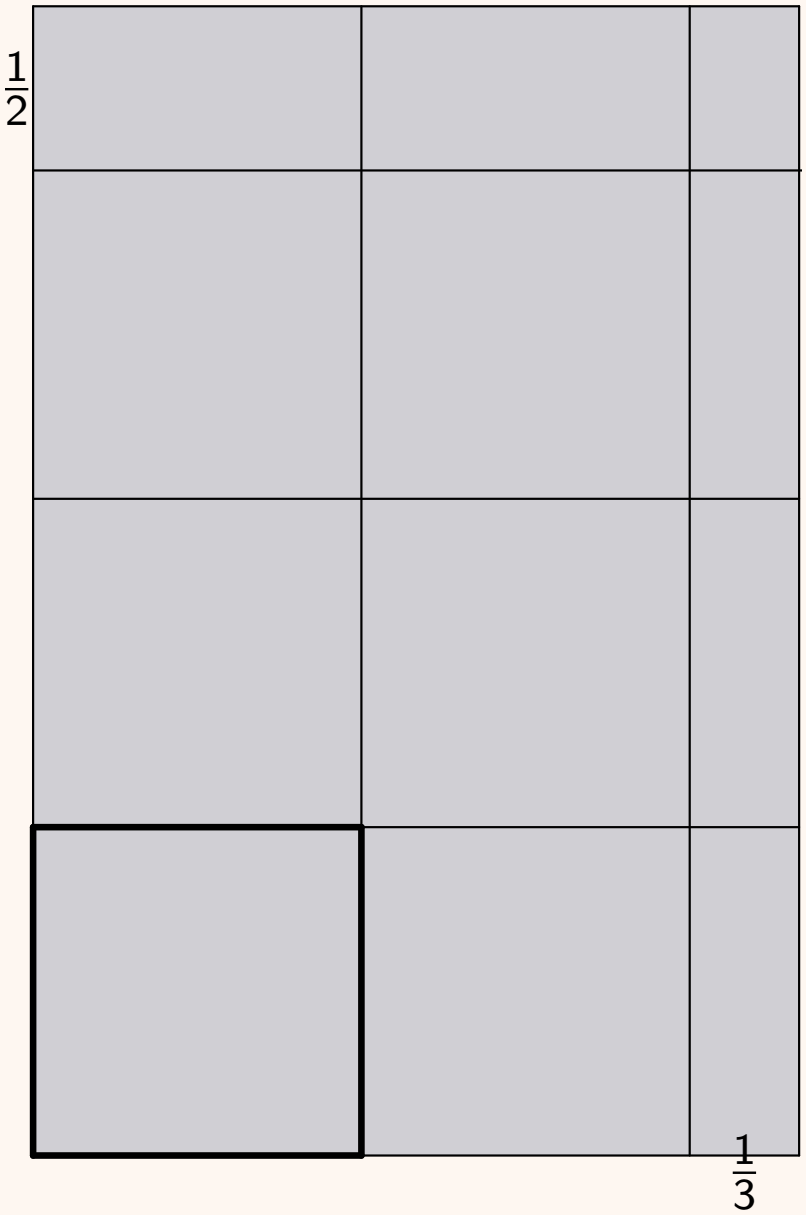


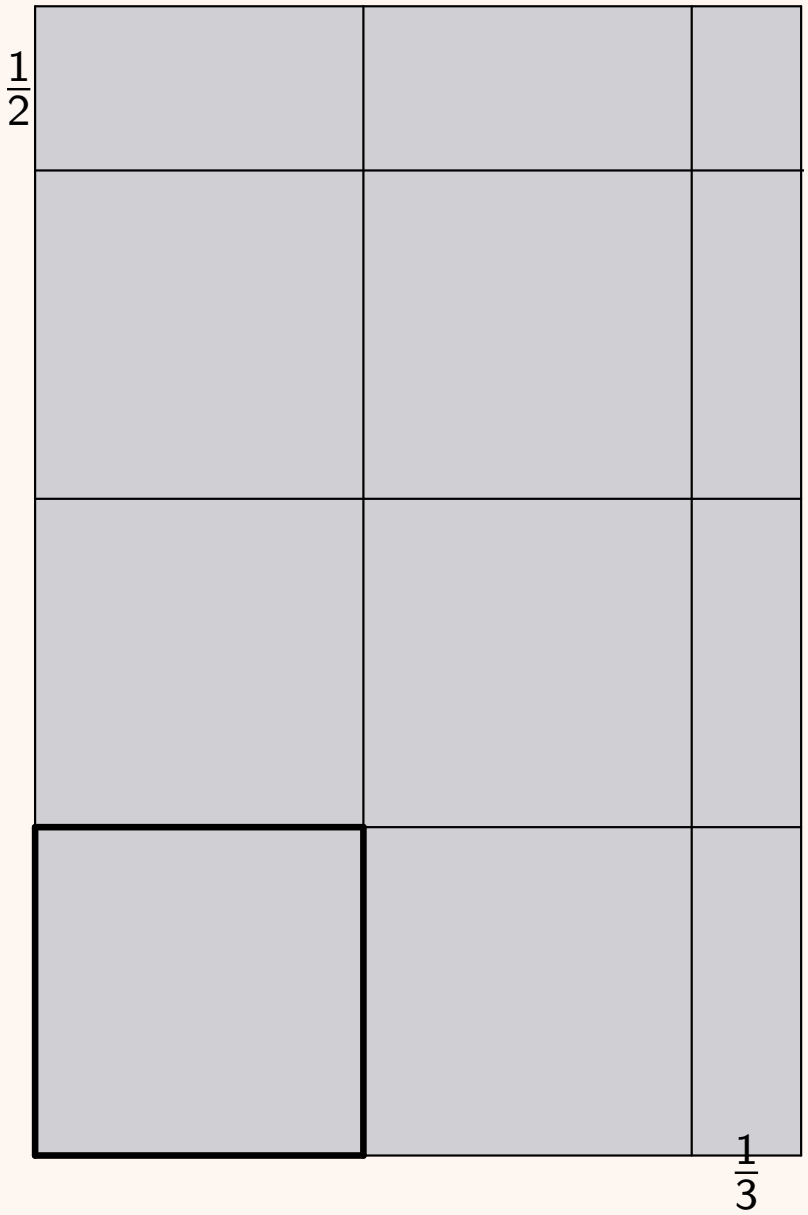
$$\frac{2}{4\times 5}$$

but not

$$\frac{1}{4}\times\frac{2}{5}$$

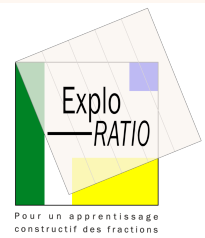
On a white paper, represent a rectangle of $2 + \frac{1}{3}$ on $3 + \frac{1}{2}$.
Write the calculation that determines its area.
Then determine this one.


$\frac{1}{2}$
$\frac{1}{3}$

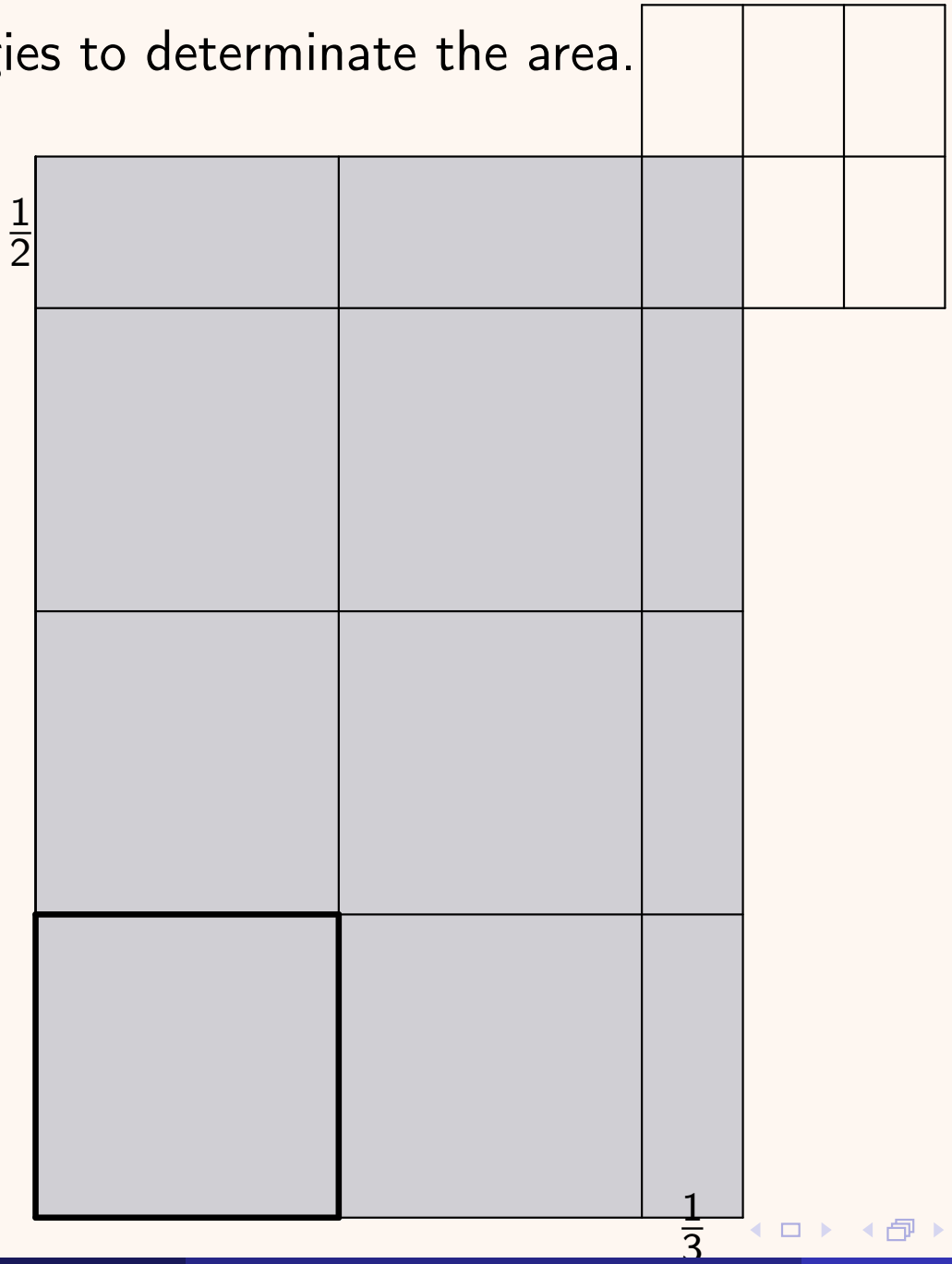


$$(2+\tfrac{1}{3})\times(3+\tfrac{1}{2})$$

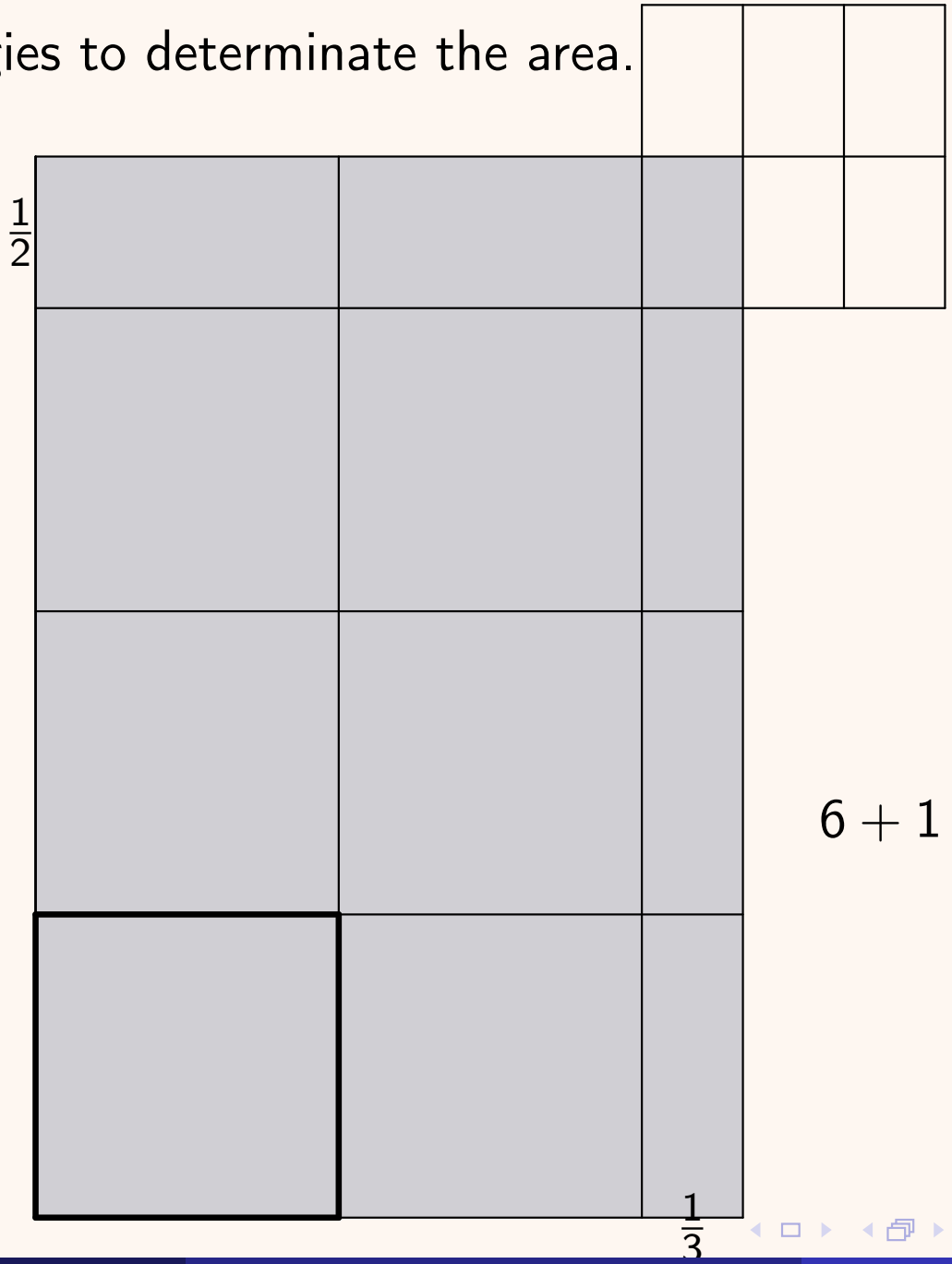
Different strategies to determinate the area.



Different strategies to determinate the area.

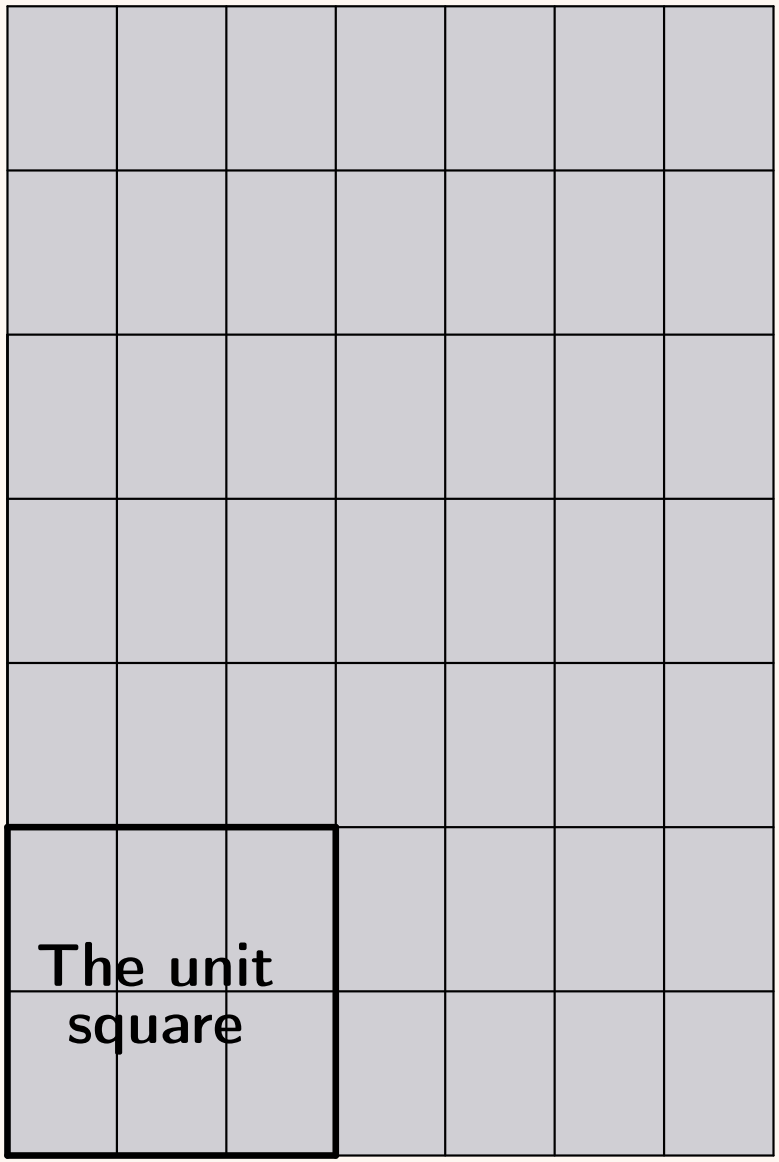


Different strategies to determinate the area.

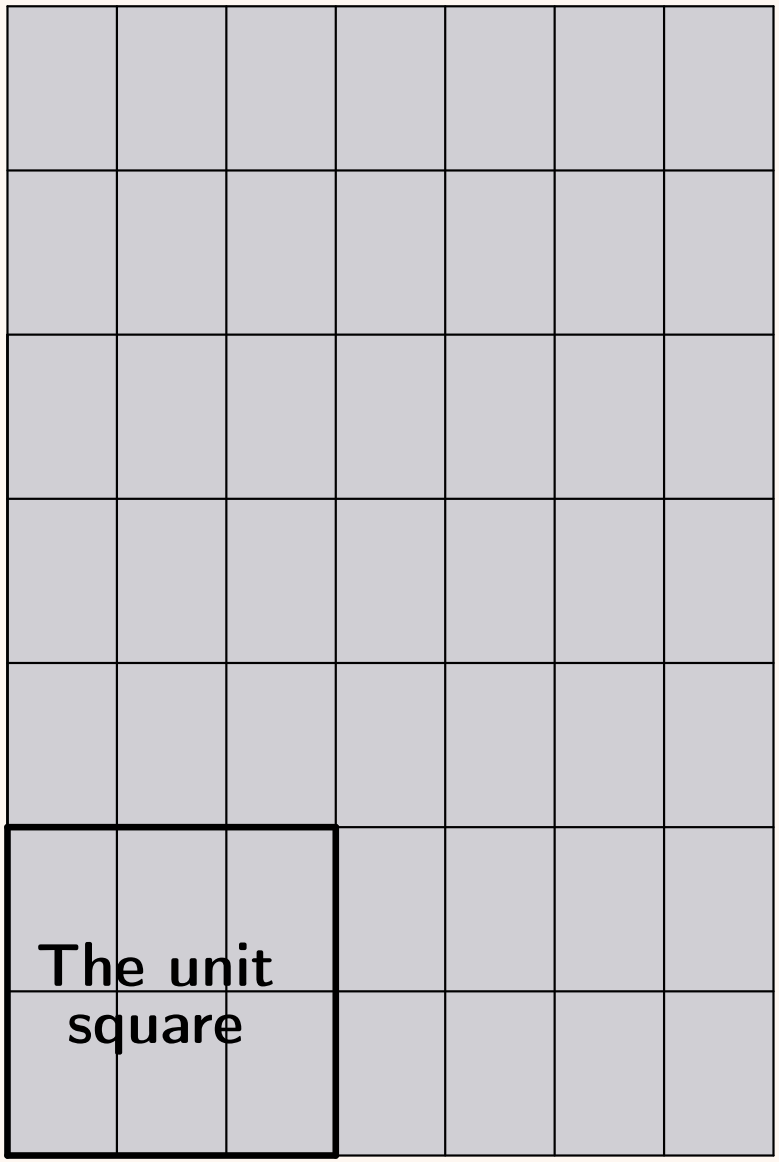


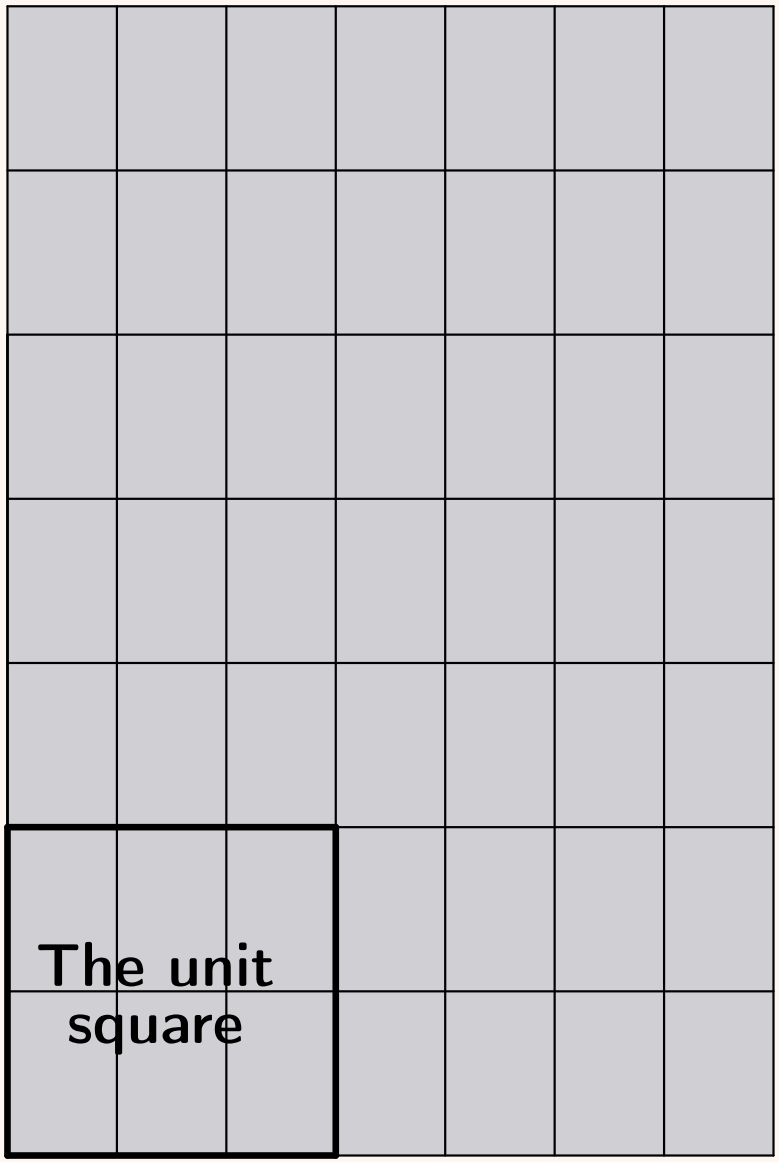
$$6 + 1 + 1 + \frac{1}{6}$$

$2 + \frac{1}{3} = \frac{7}{2}$



$3 + \frac{1}{2} = \frac{7}{3}$


$\frac{49}{6}$
$2 + \frac{1}{3} = \frac{7}{2}$
The unit square
$3 + \frac{1}{2} = \frac{7}{3}$

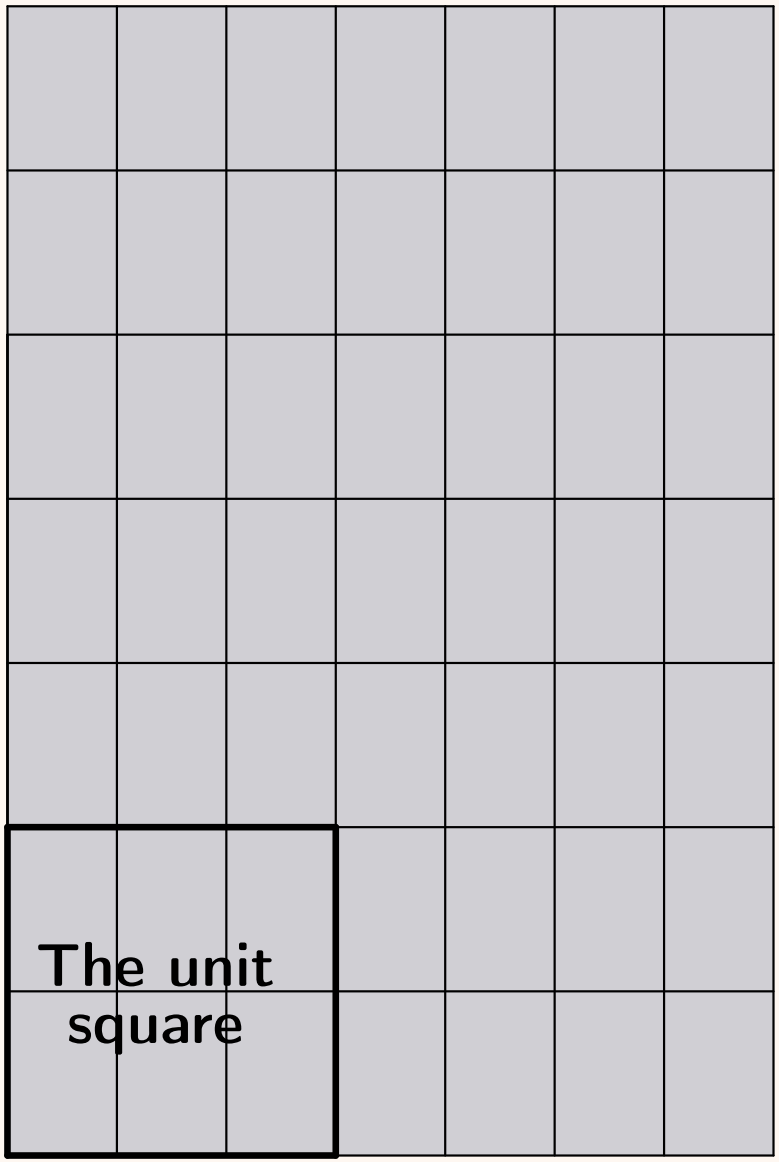


$2 + \frac{1}{3} = \frac{7}{2}$

$3 + \frac{1}{2} = \frac{7}{3}$

$\frac{49}{6}$

$(3 + \frac{1}{2}) \times (2 + \frac{1}{3})$



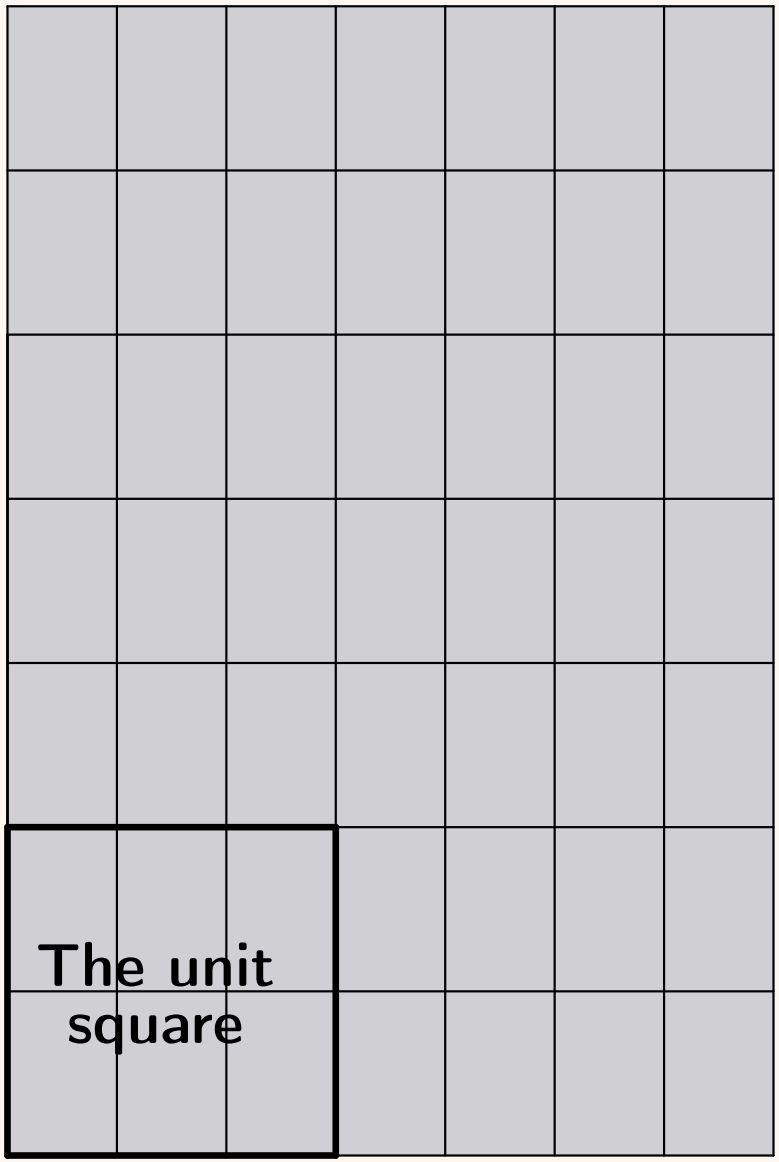
$2 + \frac{1}{3} = \frac{7}{2}$

$3 + \frac{1}{2} = \frac{7}{3}$

$\frac{49}{6}$

$(3 + \frac{1}{2}) \times (2 + \frac{1}{3})$

$= (\frac{6}{2} + \frac{1}{2}) \times (\frac{6}{3} + \frac{1}{3})$

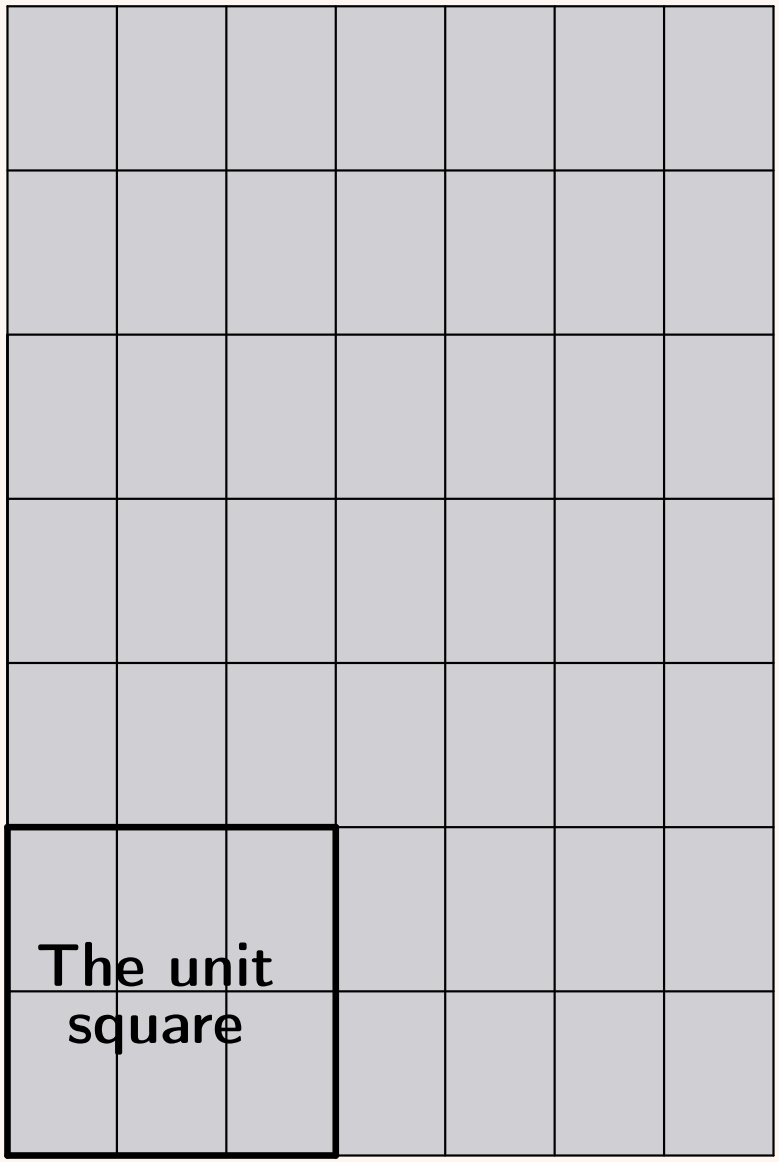


$2 + \frac{1}{3} = \frac{7}{2}$

$3 + \frac{1}{2} = \frac{7}{3}$

$\frac{49}{6}$

$$(3 + \tfrac{1}{2}) \times (2 + \tfrac{1}{3})$$
$$= (\tfrac{6}{2} + \tfrac{1}{2}) \times (\tfrac{6}{3} + \tfrac{1}{3})$$
$$= \tfrac{7}{2} \times \tfrac{7}{3}$$

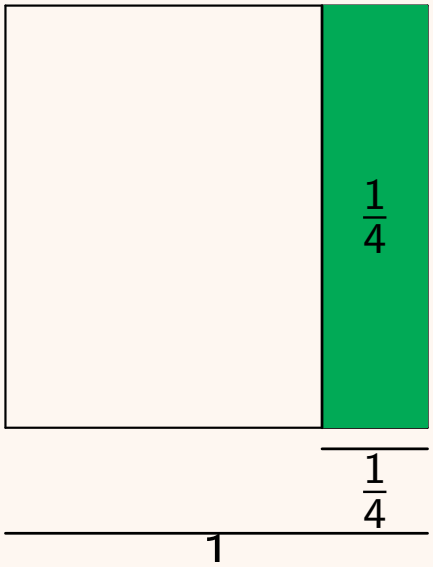


$2 + \frac{1}{3} = \frac{7}{2}$

$3 + \frac{1}{2} = \frac{7}{3}$

$\frac{49}{6}$

$$(3 + \tfrac{1}{2}) \times (2 + \tfrac{1}{3})$$
$$= (\tfrac{6}{2} + \tfrac{1}{2}) \times (\tfrac{6}{3} + \tfrac{1}{3})$$
$$= \tfrac{7}{2} \times \tfrac{7}{3} = \tfrac{49}{6}$$


1/4
1/4
1

Comparison of fractions

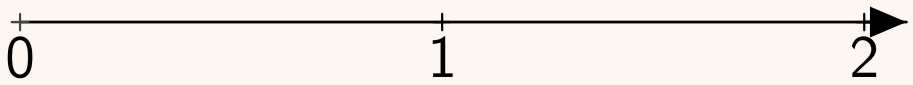
a) Represent 3/4 and 4/5 (segments below). Color the requested parts. Compare these two fractions.

_____

_____

b) (With the windows) Represent these fractions on the graduated line. Order the fractions.

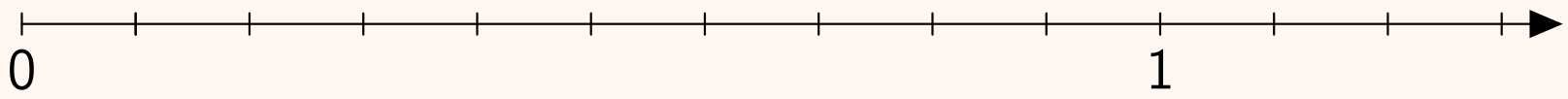
$$\frac{3}{4}, \frac{1}{2}, \frac{3}{5}, \frac{11}{9}, \frac{7}{9}, \frac{13}{10}, \frac{11}{8}, \frac{5}{9}.$$



Decimal writing of fractions

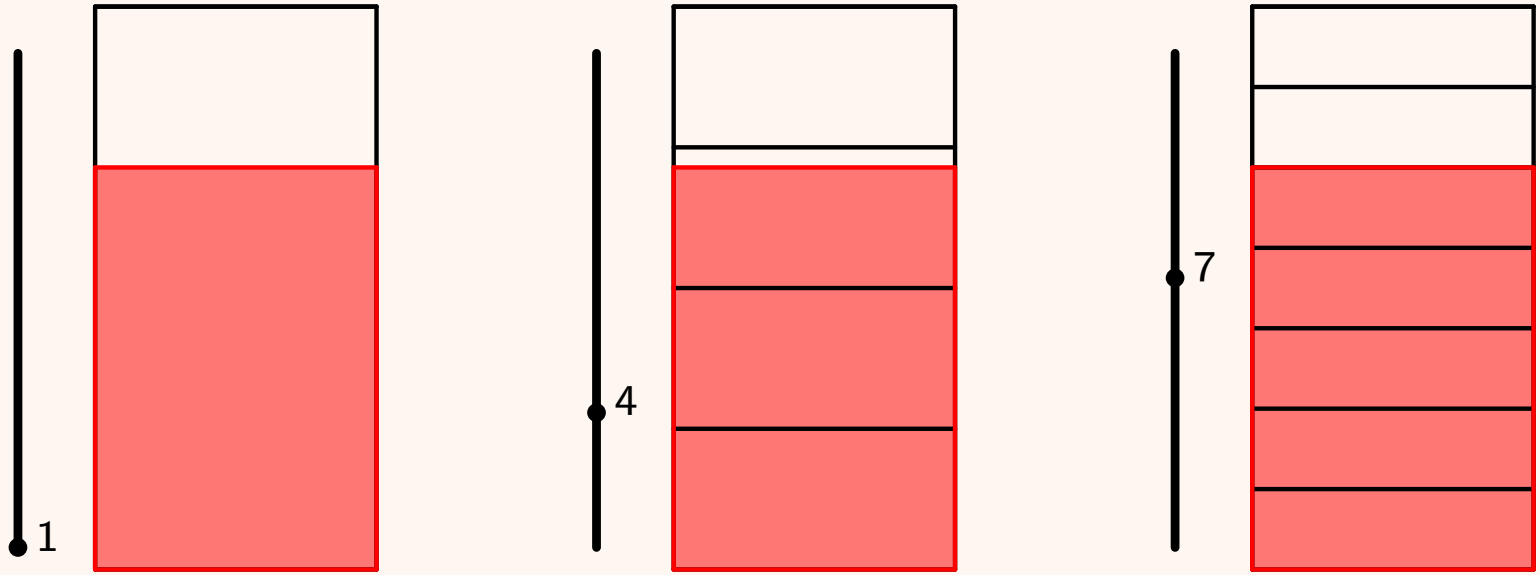
Represent these fractions on the graduated line (unit = 10 cm) with the calculator.

$$\frac{3}{4};\ \frac{1}{2};\ \frac{3}{5};\ \frac{11}{9};\ \frac{7}{9};\ \frac{13}{10};\ \frac{11}{8};\ \frac{5}{9}.$$



The origin of the device

Régionale des Bas Pays, 2014.
Mieke Abels and Martin Kindt presented an activity about fractions.
The fraction had to be evaluated and checked with a cursor.



Interest of this device

Content that can be built with this device :

- notion of fraction (in its operator and measurement aspects) (Fiches 1 and 2) ;
- fraction of a fraction (Fiche 2) ;
- equivalence and comparison of fractions (Fiche 3) ;
- the transition from division to fraction bar (Fiche 4) ;
- the fractional framing of measurements (Fiche 5) ;
- the sum and difference of fractions (Fiche 6) ;
- the product of fractions and the transition from "of" to "times" (Fiche 7) ;
- the division of a fraction by a natural (Fiche 8) ;
- the ratio of fractions, the reversed fraction, the division by a fraction and the composition of operators (Fiche 9).

Thank you !


Explo
RATIO
Pour un apprentissage
constructif des fractions