Annual meeting of the low countries 2022

Local linearity

Martin Kindt en Rogier Bos

References

Bos, R., Doorman, M., Cafuta, K., Praprotnik, S., Antoliš, S., & Bašić, M. (2019). Supporting the reinvention of the slope of a curve in a point. In U. T. Jankvist, M. van de Heuvel-Panhuizen, & M. Veldhuis (Eds.), *Proceedings of the Eleventh Congress of the European Society for Research in Mathematics Education*. Utrecht, the Netherlands: Freudenthal Group & Freudenthal Institute, Utrecht University and ERME

Bos, R. D., Doorman, L. M., & Piroi, M. (2020). Emergent models in a reinvention activity for learning the slope of a curve. Journal of Mathematical Behavior, 59, [100773]. https://doi.org/10.1016/j.jmathb.2020.100773

Plan

- What's the problem?
- Meaning making
- Research questions
- Mathematical approaches to the slope of a curve
- Results
- Discussion

What’s the problem?

- calculus students have difficulty seeing the tangent line as a limit of secant lines (Orton, 1977; Ferrini-Mundy and Geuther Graham, 1991).

- Students believe that the tangent is the same as a bounding line: a line that touches, but does not cross, the curve. (Vinner, 1982; Biza, Christou and Zachariades, 2008)

- the transition between geometric and analytic/algebraic representations of slope is problematic for students (Orton, 1983; Habre and Abboud, 2006).

- Once students learn to compute the slope symbolically and algorithmically, the geometric interpretation is lost -> originates in a daily educational practice with a quick shift from the conceptual introduction to calculation procedures (Thompson, 1994).

Hypothesis: Neither calculation algorithms nor formal definitions of slope and tangent line align with ideas that are meaningful for students.

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Meaning making

Build on ideas about slope that are meaningful to students

- Realistic mathematics education (RME): recursive approach to meaningfulness
- Embodied cognition -> grounded cognition (Barsalou, 2007):

"Grounded cognition reflects the assumption that cognition is typically grounded in multiple ways, including simulations, situated action, and, on occasion, bodily states"

Mathematical cognition on the slope of a curve needs to be grounded in situated action, simulations, and bodily states of experiences involving **steepness** and **smoothness**

Meaning making

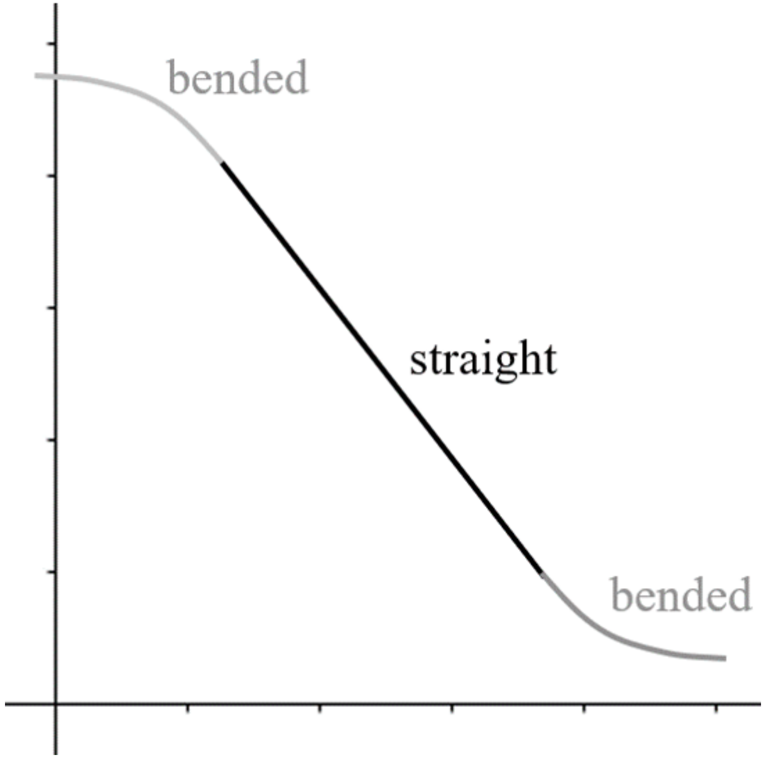
Task: design a slide with a straight and a bended bit joining smoothly, without bump. Give equations for both bits and coordinates of the point where both parts meet.

Minimal pre-knowledge: equations for lines and some curves.

Reinvention principle: based on methods and ideas that are meaningful to students themselves

Emergent models design principle: from students' informal ideas and activities to more formal mathematical models for slope of a curve (Doorman & Gravemeijer, 2009; Gravemeijer, 1999)

Models of slope -> models for slope



Meaning making

The reinvention principle and emergent models principle in recent years:

- calculus (Doorman, 2005; Doorman & Gravemeijer, 1999; Herbert & Pierce, 2008; Oehrtman, Swinyard, & Martin, 2014)
- linear algebra (Andrews-Larson, Wawro, & Zandieh, 2017; Wawro, Rasmussen, Zandieh, Sweeney, & Larson, 2012),
- abstract algebra (Larsen, 2013)
- statistics (Schwartz & Martin, 2004)
- bifurcation diagrams (Rasmussen, Dunmyre, Fortune, & Keene, 2019)

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Research questions

How do RME-inspired task characteristics support students’ reinvention of the notion of slope of a curve?

Sub-questions:

- What strategies do students follow and how do these depend on their level?
- How do the student strategies relate to the four approaches as formulated in our a-priori analysis? What *models for* emerge from students’ *models of*? Are some approaches preferred over others?
- Is it possible for teachers to institutionalize a notion of slope based on the students’ informal models?

Issue: is the reinvention principle suitable and feasible at secondary/tertiary level?

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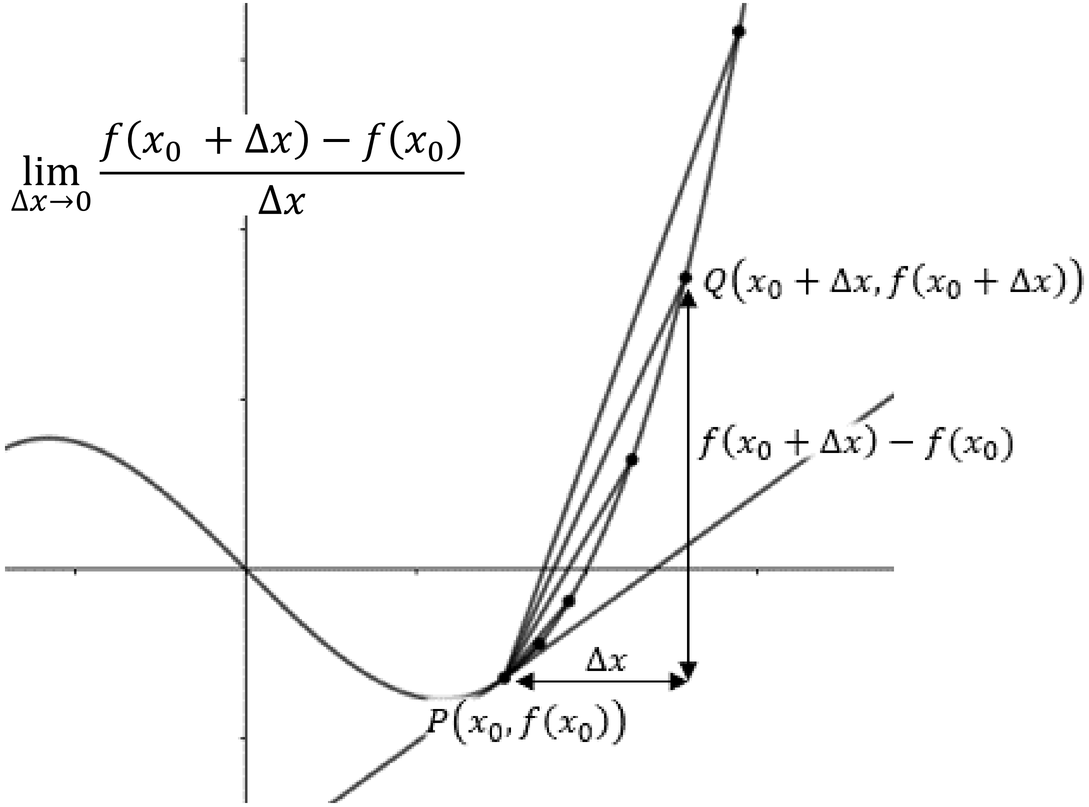
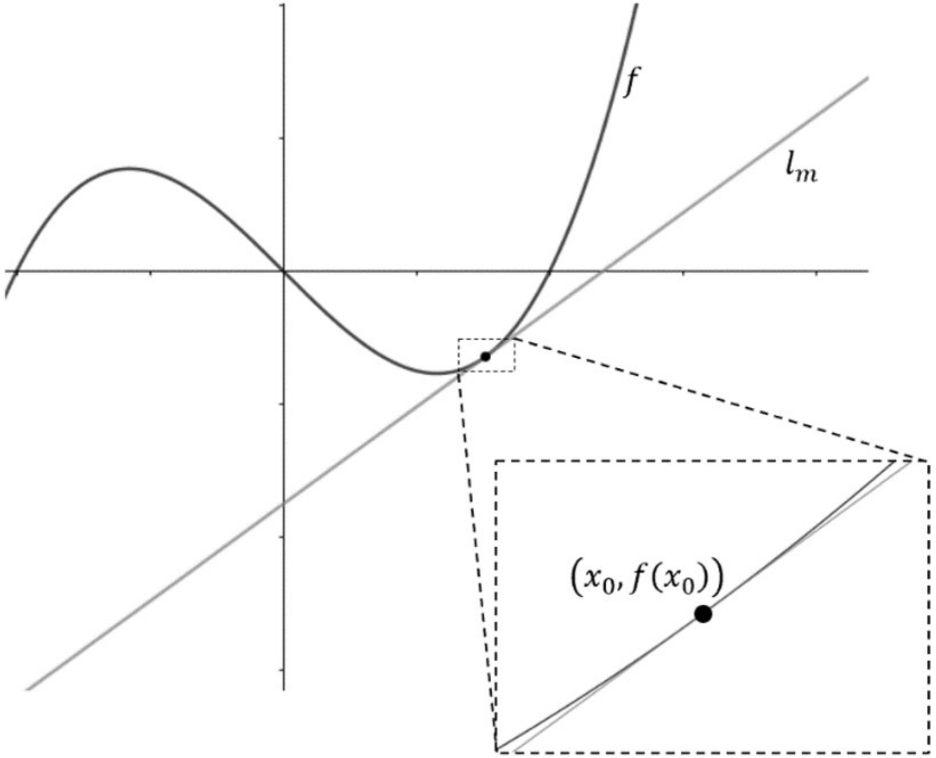
Mathematical approaches to slope of a curve

$$\lim_{\Delta x \to 0} \frac{f(x_0 \; + \Delta x) - f(x_0)}{\Delta x}$$



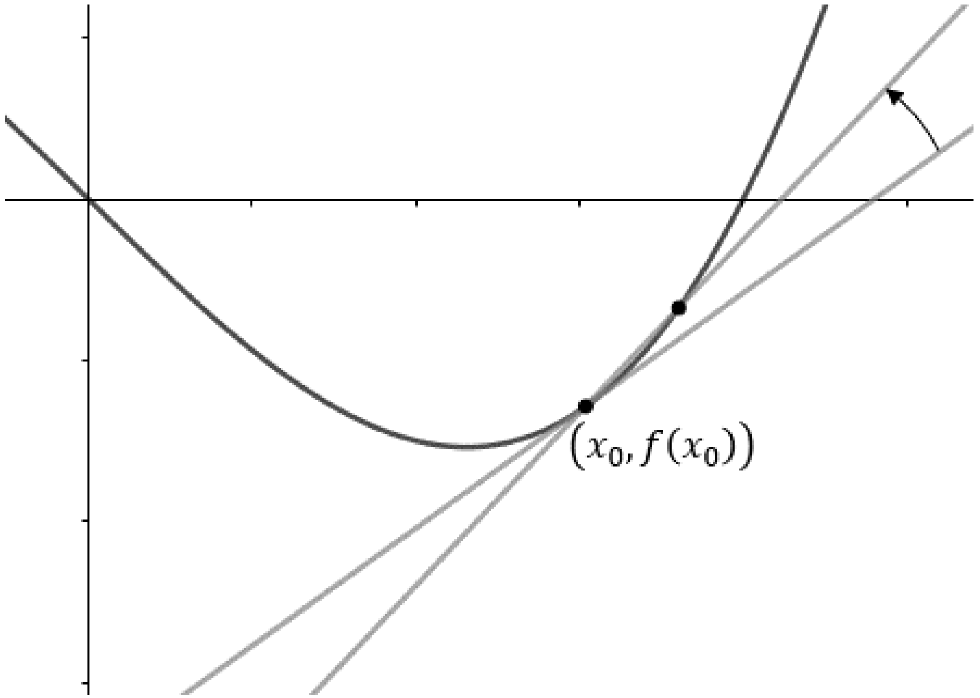
Secant line approach (S)



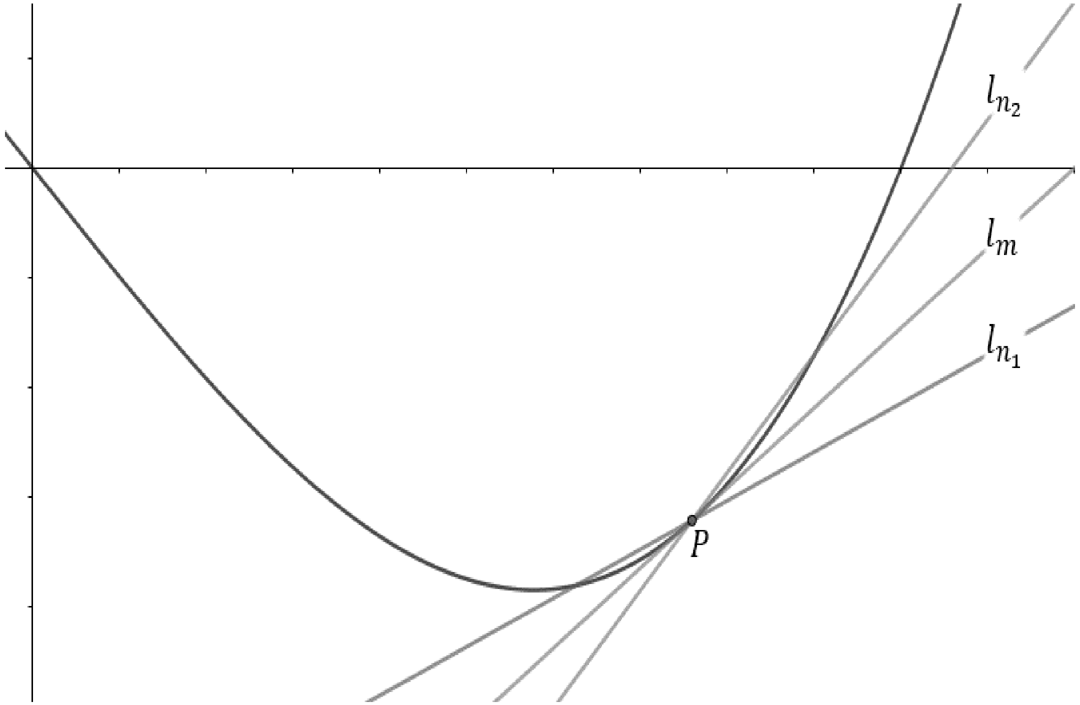
Locally linear approach (L)

Cf. David Tall

Mathematical approaches to slope of a curve



Algebraic multiplicities approach (A)
Cf. R. Michael Range



Transition point approach (T)
Cf. Marsden & Weinstein *Calculus Unlimited*

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Participants

Table 2. Overview participating classes, groups

Class label	School	Grade	Mathematics level	Length of action phase in minutes	Number of groups (n = 44)	% of groups with a (nearly) correct solution
1	A	9	Normal	25	8	12,5%
2	A	9	Normal	25	7	100%
3	B	9	Advanced	25	3	66,7%
4	C	10	Normal	25	7	42,9%
5	B	10	Advanced	35	9	55,6%
6	B	10	Advanced	50	10	80%

Analysis

Table 1. Classification labels for s

Label	Description
D	**D**raw a line and curve a top of a parabola) taken
PS	Choose a line and a cu
PT	Choose a line and a cu
PC	Choose a line and a cu
A	Use **A**lgebraic means t
HS	Use the tangent line pe
C	Use the tangent line pe
R	The **R**est: strategies no
O	**O**bscure, untraceable s
N	**N**o serious attempt regi

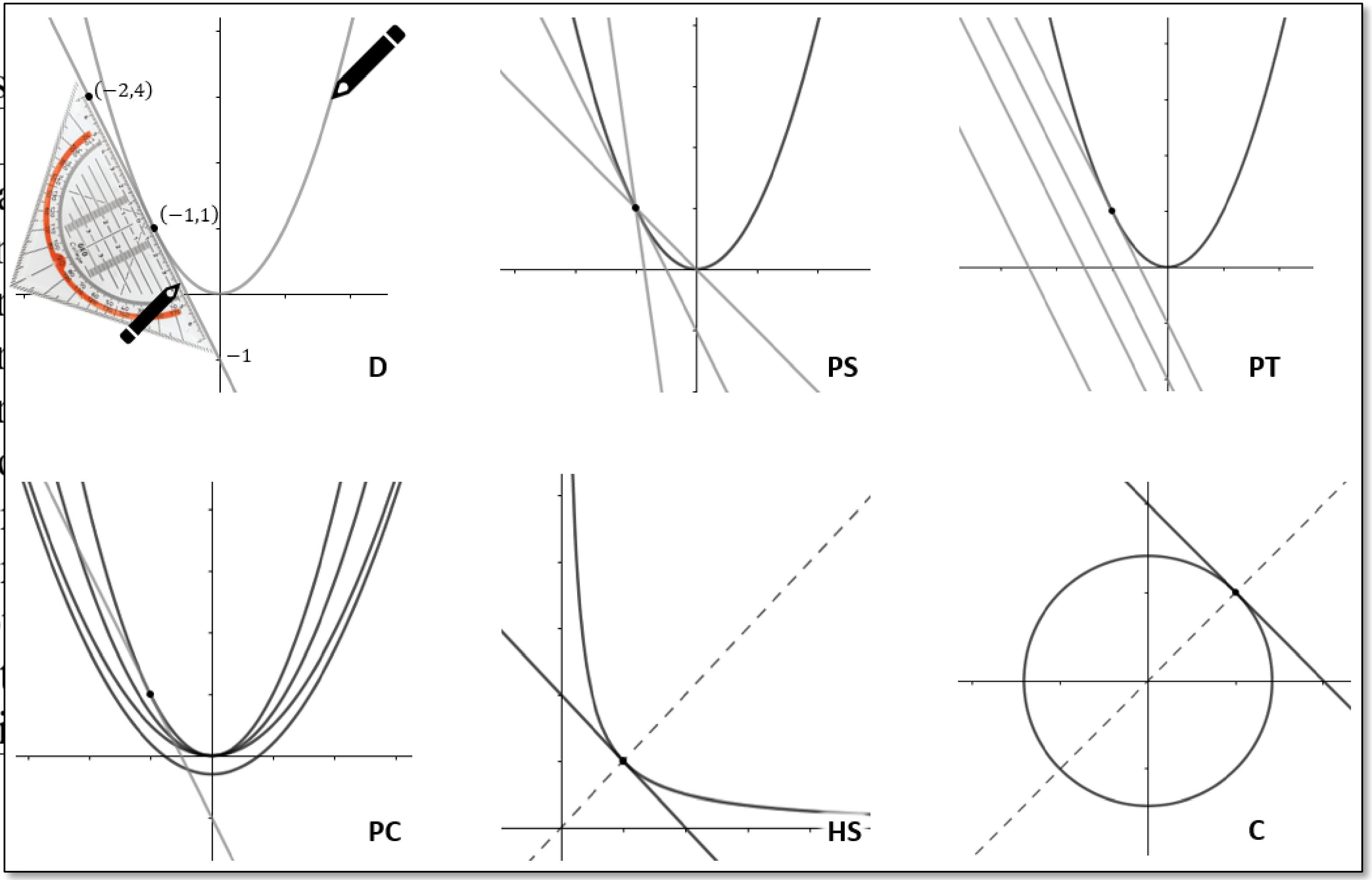
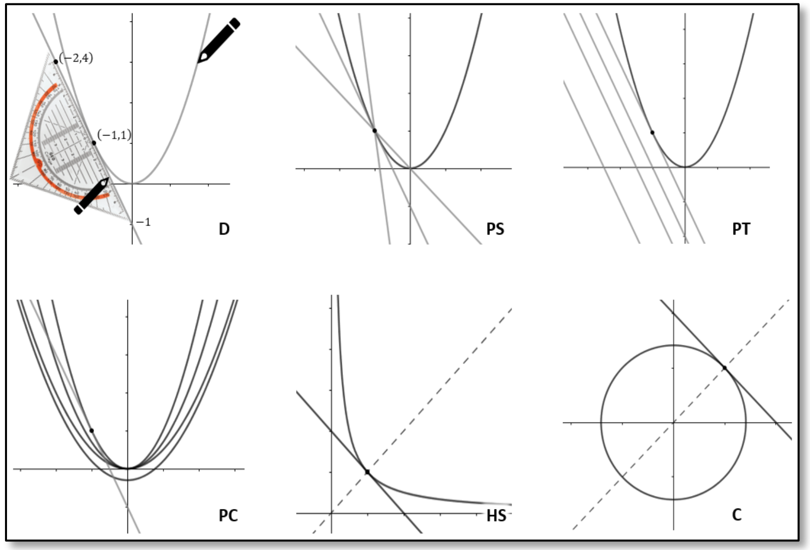


Table 3. Strategies and connecting approach per group

Class label	School	Strategies per group	Connecting approach
1	A	PC, PC, O, PT, HS, D/PS, O, N	V, V, N, T, N, T, N
2	A	PC/PT, O/PT, O, O, O, PS/PC, O	V, V, V, V, V, V, V
3	B	A, C, C	A, T, N
4	C	PT/PC, D, O, C/R, O, R, N	V, V, V, L, N, L, N
5	B	D, R, PC, O , R, PC/PS/PT, PC, HS, O	V, V, V, N, A, L, A, T, N
6	B	A, C, D, PT/PC/C, D, R, PC/PT/PS/A, C, D, PC	A, N, L, V, N, V, A, V, V, A



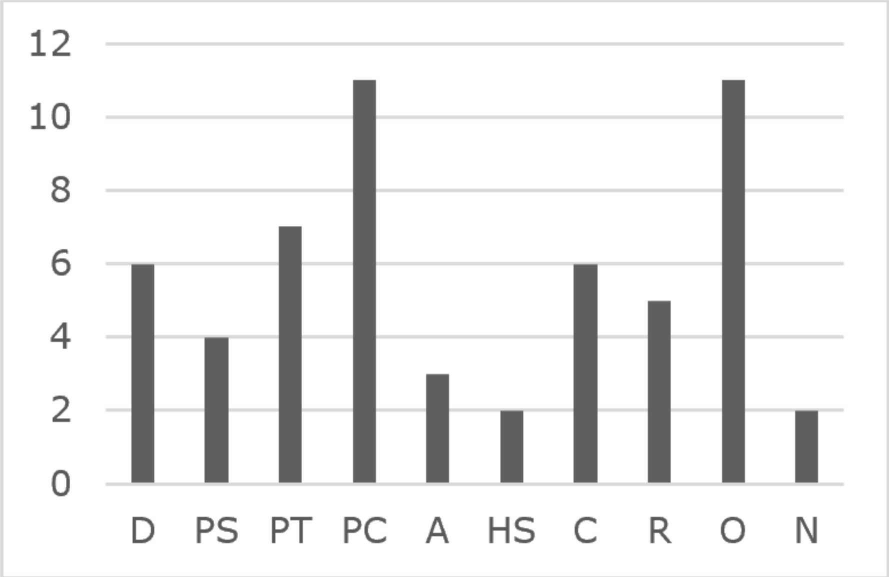


Figure 13. Frequency of student strategies

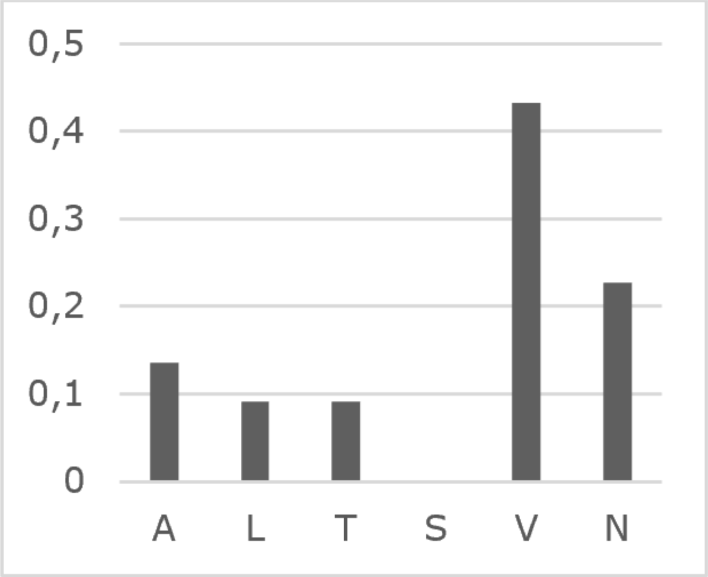


Figure 14. Relative frequency of approaches

A: algebraic multipl.
L: Locally linear
T: Transition point
S: Secant lines
V: Various options
N: None apply

Results

Case 1 (Class 4, classified as C/R→L).

- GeoGebra to visualize their designs
- Equations: $y = 2x + 8$ and $y = \frac{1}{2}x^2 + 2x + 8$.
- *"b in the formula of the parabola and a in the formula of the line must be the same."*
  (they had the formulas $y = a\,x^2 + bx + c$ and $y = a\,x + b$ in mind).
  "the directional coefficients of the line and the curve must be the same"

Results

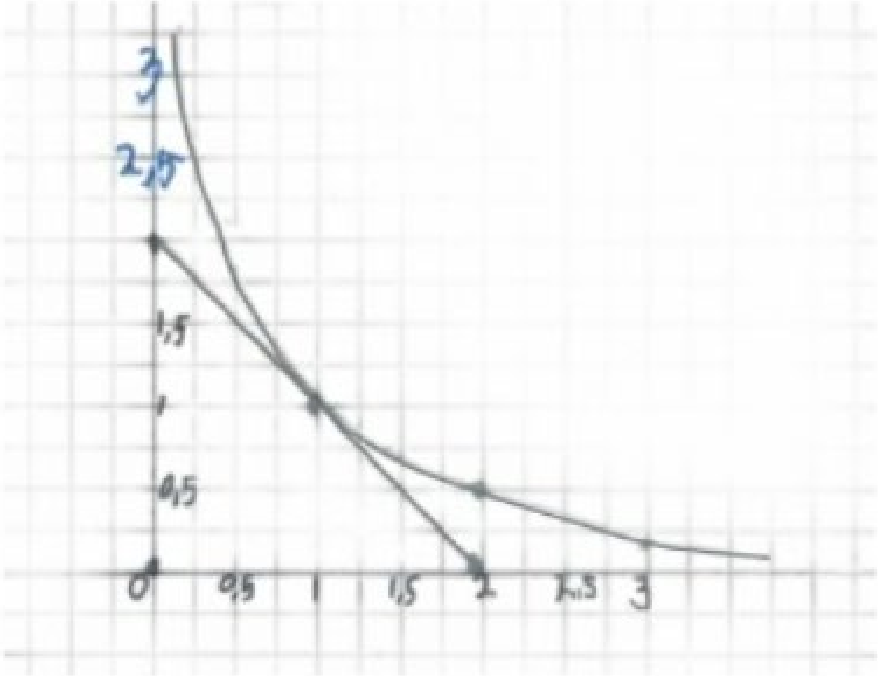
Case 2 (Class 4, classified as HS→ T).

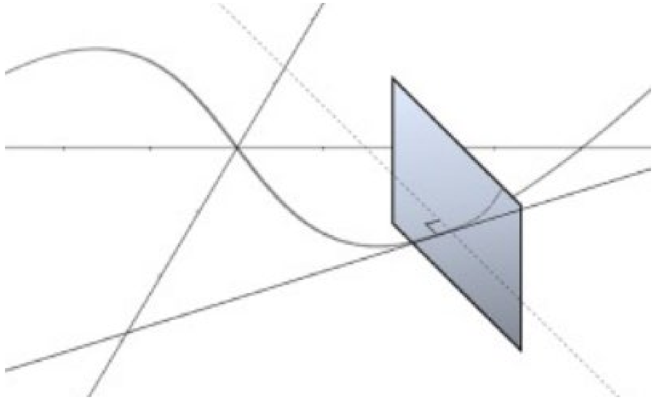
- Equations: $y = \frac{1}{x}$ and $y = -x + 2$.

- Students mentioned symmetry as a justification

- They mentioned that they would like to zoom in on the intersection point… that to have absolute certainty, they would like to "zoom in forever"

- Symmetry, rotating mirrors and transition points





Results

Case 3 (Class 5, classified as PC/PS/PT L)

- used a graphical calculator

- First the students fixed a seemingly random line and tried to adjust the parameters in the equation for the parabola.

- Then they changed strategy and fixed the parabola to $y = \left(\frac{3}{10}x\right)^2$ and fixed a point on it $(10,9)$. Next, they wanted to adjust the parameters of the line.

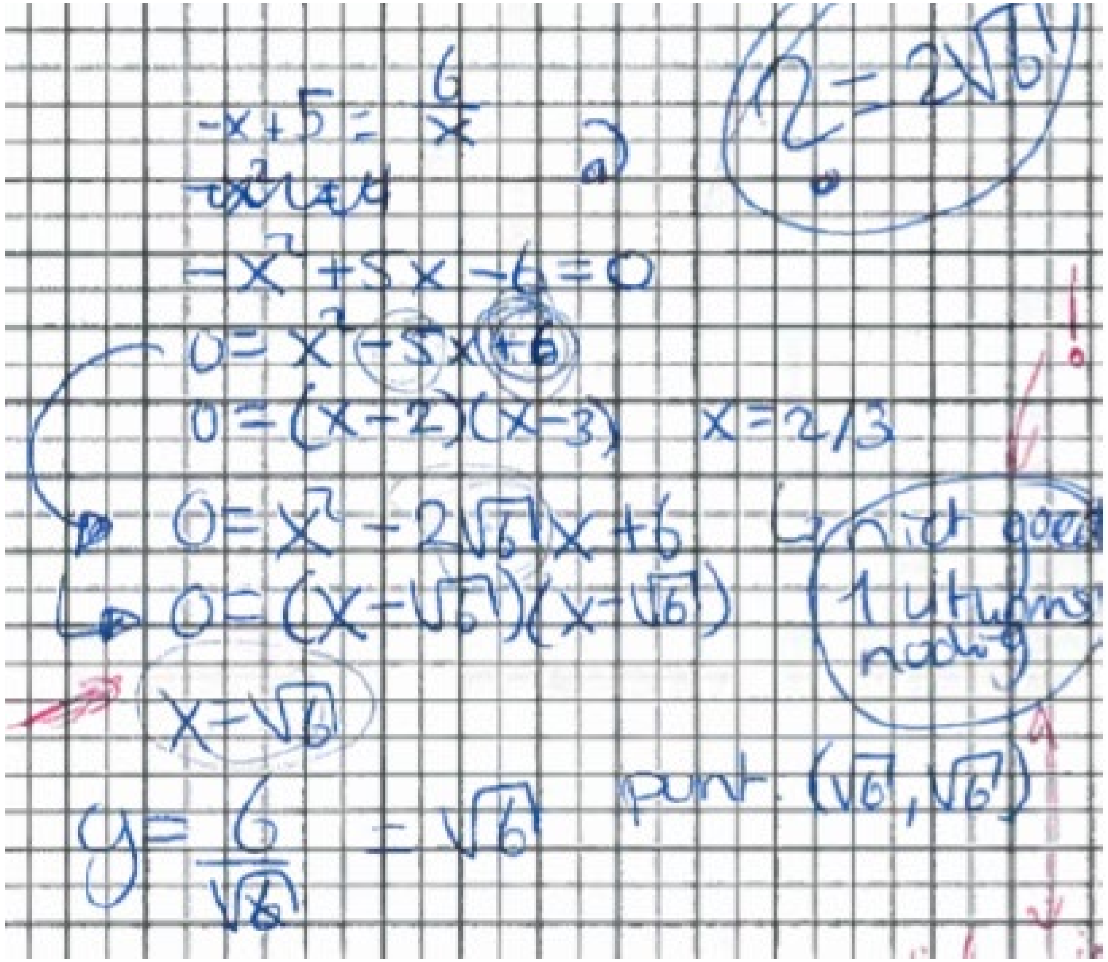
- In the end, they settled for $y = \frac{15}{10}x - 6$, which is not "correct". However, graphing it on a GC on a "standard" scale shows a convincing picture

- one students said: "I think that when the line touches the curve, in that small part the equation of the parabola must be the same as the line".

Results

Case 5 (Class 6, classified as A→A)

- First: hyperbola, $y = \frac{6}{x}$, and a line, $y = -x + 5$
- They compute the intersection points at $x = 2$ and $x = 3$.
- “not good, we need 1 outcome”.
- in the next line the “5” is replaced by “$2\sqrt{6}$”, which allows a unique solution, with multiplicity 2.



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Discussion

- Why fix line before curve?

- Use of GeoGebra

- Aspect of time for action phase

- Why does the secant lines approach not occur?

- Why does the idea of overtaking lines (as a step towards the transition point approach) not occur?

(1) Algebraic multiplicities approach and the locally linear approach are most often/easy connected to.

(2) In 77% of student groups and 100% of classes, teachers had opportunities for institutionalizing the lesson goal

(3) Only 4,5% of the student groups were not engaged

No specifics for participating groups -> evidence that reinvention principle is feasible at secondary level.