Titles and Abstracts for 3rd International RME Conference[1]
September 23 – 25, 2011
Boulder, Colorado

Keynotes and Plenary Sessions

Helping students construct more formal mathematics

Koeno Gravemeijer, Eindhoven University of Technology, Eindhoven School of Education, the Netherlands

Formal mathematical knowledge is hard to transmit to students, for it typically concerns knowledge on a higher level of understanding than the students already possess. Therefore, instead of trying to help students in making connections with mathematical knowledge that is too abstract to them, Realistic Mathematics Education (RME) aims at helping students construct new mathematical knowledge by building on what they already know. The RME approach, however, does not offer an easy solution to the problem of how to teach more formal mathematics. Helping students (re)invent mathematics presents mathematics educators with the difficult task of building on the informal knowledge and ideas of the students, and at the same time making sure that processes of generalizing and formalizing debouch into the conventional formal mathematics one is aiming for. In this presentation we will address both the means of support RME offers and the pitfalls that threaten processes of generalizing and formalizing in everyday classrooms.

Learning Trajectories-- The Core of Standards, Teaching, and Learning

Doug Clements, SUNY Buffalo

Learning trajectories have been placed at the core of some standards, curricula, and approaches to teaching. Drawing on research abstracted from 6 recent publications that he co-authored, Clements will discuss different perspectives on learning trajectories and how they might form a solid core. One perspective, influenced by the RME approach, will be illustrated by a set of NSF- and IES-funded projects that produced and evaluated research-based mathematics curricula and assessments. The publications include (1) the report of President Bush's National Math Advisory Panel, (2) NCTM's new Curriculum Focal Points for early childhood, (3) a report on early childhood mathematics from the National Research Council, (4) Sarama and Clements' extensive research reviews, include RME work, (5) the TEAM (Tools for Early Assessment in Math), and (6) the Common Core State Standards.

Linking policy and practice through learning trajectories

Mary Pittman, Colorado Department of Education

National initiatives such as the Common Core State Standards and Response to Intervention impact state level policies for the learning and teaching of mathematics. The Common Core State Standards are based upon mathematics education research on student learning. While the standards provide guidance on landmarks of learning, teachers need learning trajectories with more detail to help students reach these landmarks. This is particularly true when teaching struggling learners within a Response to Intervention framework. This

[1] Additional information regarding sessions will be posted as soon as it becomes available.

presentation will focus on how learning trajectories are key to successful implementation of current educational policies.

Functions, Covariation, and the Cartesian Connection: Integrating Learning Trajectories for Pre-Service Secondary Teachers
Bill Jacob, UC Santa Barbara
Professional development for mathematics teachers often combines challenging inquiry with a close examination of student work. But how do these activities relate? Do they support each other? In this talk we examine hypothetical learning trajectories in a course for pre-service secondary teachers where it was believed certain investigations would enhance their abilities to make sense of K-12 students mathematical work on related topics. We will discuss on what worked and what didn't, what modifications in the contexts and problems helped, and what this tells us about preparing undergraduate math majors for secondary teaching. This research is in collaboration with Prof. Kyunghee Moon of the University of West Georgia.

Breakout Sessions: Elementary Grades (K – 6)

A Learning Trajectory for Fractions with Low-Achieving Students
Mieke Abels (Freudenthal Institute, Utrecht University, The Netherlands)
In a project for special-needs students in the Netherlands a learning trajectory is developed with many activities that support students' understanding of fractions. This session will show an overview of the activities as well as the learning trajectory itself, which goes from preformal via informal to formal understanding.

Make it Count - Improving mathematical outcomes for Australian Indigenous students
Steve Thornton (Charles Darwin University, Northern Territory, Australia) &
Joanne Statton (Alberton Primary School, South Australia)
This study looks at the value of mathematizing and contextualizing in promoting mathematical resilience and transfer of knowledge among Aboriginal students in an urban Australian setting. Teachers at Alberton Primary School, a large school located in the suburbs of Adelaide in which approximately one third of the students are Indigenous, teach mathematics through rich contexts, such as art, design, and science that are of interest to students. This is a unique way of organizing learning that capitalizes on the interests of both teachers and students and, in the process, helps to make mathematics relevant and engaging. The presentation will highlight findings to date, including video examples.

Exploring Numeracy Throughout the Day
Cindy Pray (Adams 12 Five Star Schools, Colorado)
Having a strong sense of number is critical to future math success. In my first grade classroom, I give my students multiple opportunities to explore numeracy. Throughout the past school year, I incorporated numeracy explorations throughout the day in calendar, centers, transitions/seatwork time, game time, and intervention. My goal was for students to explore numbers in a variety of ways without impacting a lot of teaching time. This session will show a learning progression that began with subitizing up to 5 and knowing combinations of 5. New concepts such as anchoring to 5, doubles, combinations of 10, anchoring to 10, and combinations and partitions to 20 were added as the year progressed. Students recorded their explorations in a variety of ways on worksheets and journal pages.

Participants will be able to see student work from throughout the year in an interactive workshop setting. As the year progressed, students began moving away from using manipulatives and count-by-one strategies to strategies involving number sense and mental math. When assessing student numeracy, noticeable positive differences compared to the other first graders were found. This session will include presentation of end-of-year assessment results and further recommendations.

Designing a Learning Progression on Tangles
Meg Meyer (University of Wisconsin, Madison)
This interactive session will explore some of the mathematics contained in the toy known as a Tangle. A series of questions will be demonstrate a learning progression from middle grades to secondary. The last part of the session will be devoted to designing age appropriate tasks for elementary grades using Tangles.

Using Japanese Lesson Study as a Method to Investigate Students' Learning Trajectories
Michelle K. Reed, Susann M. Mathews, Aina Appova (Wright State University, Dayton, OH)
This presentation will provide the results of several groups of K-8 teachers' use of lesson study and how the teachers worked together to learn about their students' reactions and understandings of mathematical topics. Each example will show the evolution of the word problem or situation in reaction to their students' understanding of the content and the mathematics involved. For instance, our teachers revised a problem to lead students to envision perimeter and not area by changing their context to using a boarder rather than wallpapering a room.

The Ten Percent Who Never Get It: Number Sense Assessment of developmental math students at a community college and of Grades 3-8 at a charter school
Dorothea Steinke (Front Range Community College, Westminster, Colorado)
This study assessed students' number sense of whole number relationships. The assessment tool is based on the 3 Stages model of children's understanding of number relationships developed by Dr. Leslie Steffe and his colleagues at the University of Georgia (Steffe & Cobb, 1988). Children exhibited behaviors in interviews which identified a grasp (or a lack) of two basic number sense concepts: 1) equal distance of 1 between counting numbers; and 2) part-whole coexistence (i.e., if I have 9, I have 6 and 3 within and at the same time as 9). Participants will be invited to complete the number line assessment to use as a benchmark against the examples of student number lines that they will see during the presentation. Participants will also be invited to review several examples of student number line assessments and decide which stage of number sense each example best fits.

Breakout Sessions: Middle Grades (6 – 8)

A proposed instructional theory for introducing integer concepts and the operations
Michelle Stephan (University of North Carolina – Charlotte); George McManus, Ashley Dickey, Jennifer Smith, Kathleen Breitenbruck, Odyessa Moss & Tonya Fennell (Lawton Chiles MS)
This presentation explores best models for introducing integer concepts and operations (mainly addition and subtraction) by analyzing an instructional sequence that has been refined over five years. We used RME heuristics to create the sequence, using finance as an experientially real context for students. All activities were sequenced to move students from concrete to abstract reasoning. Finally, the sequence was designed to investigate the

emergence of students' models as they used a vertical number line for operations. In our presentation we will describe our process of designing the sequence, the hypothetical learning trajectory, student reasoning, and the teachers' means of supporting learning.

Local Instructional Theories and Routines of Practice: Supporting Teachers to Engage Students in Fraction-Based Algorithmic Thinking

Debra I. Johanning & Kimberly S. Shockey (The University of Toledo)

This research session shares first-year findings from a larger study examining teacher practice to identify key routines of practice, and related local instructional theories, that exemplary teachers draw upon as they engage students in fraction operation algorithmic thinking using a guided-reinvention approach.

Division of Fractions: A Learning Trajectory Designed to Promote Conceptual Understanding

Amanda Geist (Centennial Middle School, Boulder, CO)

Many students are presented with formal algorithms to divide fractions but lack the understanding behind them. In this session, I will share the development and results for a learning trajectory designed to promote students' deeper conceptual understanding of division with fractions.

Reform trials of mathematics education and professional development in Fukui, Japan

Saburi, Y., Irei, M., Matsumoto, C., Nishimura Y. (Univ. of Fukui);
Nagamizu, T., Tsubokawa, T., (Fukui Nat. Coll. of Tech.); Makida, H. (Fukui Pref. Inst. for Edu. Res.); Murai, S. (Sabae Jr. High.), Takama, Y. (Shimin Jr. High.), Takegawa, K. (Meido Jr. High.), Kusaoke, H., Taisho, H. (Attached Jr. High. of Univ. of Fukui), Miyagoshi, T. (Keishin Sr. High.), Yamauchi, Y. (Fujishima Sr. High.)

The purpose of our study is to advance our reform of mathematics education and professional development. In Fukui, Japan, there are some trials to reform mathematics education, accomplished mainly by classroom teachers within the current condition of Japanese mathematics education. These trials give teachers opportunities for their professional development as well. We are going to report those trials in this talk. Since our trials are in the early stages of reform, we expect to hear helpful feedback or advice from conference participants. We expect as well to exchange ideas together with participants on what kinds of reform approaches are possible and appropriate under different environmental conditions surrounding mathematics education.

A teacher's local theory of instruction: The development of teacher knowledge through enacting challenging tasks

Jeffrey Choppin (University of Rochester)

Teachers can develop local theories that help them utilize resources in curriculum materials in ways that help students make connections between informal and formal ways of thinking. A notable feature of local theories according to the RME tradition is that instruction should elicit and build from students' pre-formal thinking, which is progressively refined toward more formal mathematical representations and terminology, in a process of progressive formalization. This session focuses on how a teacher's knowledge informed how she revised and enacted challenging tasks in ways that elicited and refined student thinking around integer addition and subtraction.

Surface area without length times width

Mieke Abels (Freudenthal Institute, Utrecht University, The Netherlands)

The concept of surface area is not very difficult. However, when students have to solve problems it seems they haven't really understood the concept. From the moment on when students apply the rule area = length x width, the concept of area is gone: they apply the rule for triangles, and they cannot find the area of a circle, because a circle has no length and width. In a project for special need students in the Netherlands, a learning trajectory is developed with many activities that show how the concept of surface area can be developed. Classroom observations showed that the students were very engaged and creative in doing the activities and solving problems.

Breakout Sessions: Secondary/Post-Secondary

Hypothesis Testing in Statistics: A Learning Trajectory Designed to Promote Theoretical Understanding

Monica Geist, Ph.D. (Front Range Community College, Westminster, Colorado)

Introductory Statistics students often struggle with understanding the theory behind hypothesis testing. Many students rely on their ability to follow procedural, cookbook methods to conducting hypothesis tests with little understanding of the theory. Using the principle that "richly connected bodies of knowledge are well retained," this session will illustrate a learning trajectory designed for deep understanding of hypothesis testing.

Geometrical reasoning and proving in small groups: the role of the tasks

Sonia Palha, Rijkje Dekker, Bernadette van Hout-Wolters (University of Amsterdam, Research Institute of Child Development and Education, Faculty of Social and Behavioral Sciences, the Netherlands); Koeno Gravemeijer (Eindhoven University of Technology, Eindhoven School of Education, the Netherlands)

Learning geometrical proof at upper high school is not easy for students and it remains a problem in mathematics education. In our research we developed and studied problem tasks that, embedded in an interactive learning setting, aims at supporting students when learning geometrical proof. Drawing on the idea of mathematics as activity and following a design research approach we developed and experimented sequences of problem tasks ('shift problems') with 16/17-year-old students in the Netherlands. An interesting finding was that these problem tasks together with the interactive setting (students were solving these problems in small heterogeneous groups) can create a micro-learning context in which students handle other socio-mathematical norms and mathematical activity than the ones usually associated with normal classroom practice (Lithner, 2008). In this workshop we will present and discuss a selection of "shift problems" and students work. The discussion about the role and function of the tasks in student learning will be approached from two complementary perspectives:

1) The designer (and teacher) perspective, on one side restricted by the written curriculum and, on the other side aiming at engaging students in meaningful activities. What criteria can be considered to select and adapt tasks from the textbook and how are they connected? How is it possible to elaborate on existing students materials, as they come in the textbook?

2) The learner (and student) perspective, usually having difficulties with this topic. Recent research in this field provides us with more insight about cognitive and social aspects that

can be behind some of these difficulties. For instance, lack of awareness of theoretical perspectives (Laborde, 2004) and difficulty in connecting private and public aspects of proof (Raman, 2003). How can we elaborate on these insights when developing tasks?

What mathematics is important for (future) work?
Henk van der Kooij (Freudenthal Institute, Utrecht University, the Netherlands)
The way mathematics is used in most work settings is quite different from the mathematics that we teach (and our students learn) in school. Since the 70-80's, when Schliemann, Carraher and Lave observed workers using mathematics in simple work settings, a number of math educators (like Steen and Forman in the USA, Hoyles and Noss in the UK and Zevenbergen in Australia) have investigated mathematics at work. Their findings are in line with the ones we experienced with a Dutch project in vocational education in upper secondary schools. Economic demands for math educations (as reported in SCANS, USA, 1991 and in the key competences for work, EU, 2007) are quite different from the curricula we are used to. How can we integrate work-related math in our school practice? That is the central question to be discussed in this workshop. After an introduction of the problem, illustrated with some hands-on activities, the floor is open for discussion.

The Great Balancing Activity: A Learning Progression for Descriptive Statistics Which Supports Learning With Understanding and Encourages Sense Making
Kevin J. Reins (The University of South Dakota)
The featured activity of this session, The Great Balancing Activity, makes use of a pre-formal structure, simply, a physical model-for the development, teaching, and vertical mathematizing of three concepts from descriptive statistics, mean, mean absolute deviation, and standard deviation.

Length times width equals area, and line times line equals parabola: Incorporating two RME models into a cohesive learning trajectory for quadratic functions
Frederick Peck (University of Colorado and Boulder Valley School District) and Jennifer Moeller (Boulder Valley School District)
RME researchers have discussed two alternatives to the projectile motion model for quadratic functions: (1) "line times line equals parabola" (Kooij, 2000), and (2) the area model (Drijvers et al., 2010). Our learning trajectory incorporates and connects these models in a cohesive unit that begins with a motivating contextual problem, guides students to construct both of the above models, and concludes with formal algebraic representations. We have used this unit for the past two years in our Algebra I courses. During this time, we have collected a body of evidence that supports the approach: 1) Assessment results suggest that students learn quadratic functions at or above the level expected in a typical algebra course; 2) Student work suggests that learning multiple models leads to deeper understanding and flexible problem solving; 3) Student feedback suggests that the models help students solve problems and understand formal mathematics. Participants will receive our complete unit in electronic form. Furthermore, we will discuss our lessons learned, and avenues for extensions and future research.

Contexts and Models for Multiplying, Dividing, and Factoring Polynomials
Raymond Johnson (University of Colorado at Boulder)
While multiplying binomials, factoring, completing the square, and polynomial division are important skills, belaboring these skills can interfere with students' broader vision of algebra. In this session, we will explore a learning trajectory involving the area model and its derivatives in ways that promote student proficiency and understanding.