Mathematizing:
A human activity in arts and sciences

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The beauty and utility of mathematics

Euler's identity:

\[ e^{i\pi} + 1 = 0 \]

- \( e \): Euler's constant \( (\approx 2.71828) \)
- \( i \): imaginary unit of complex numbers
- \( \pi \): pi \( (\approx 3.14159) \)
Island problem

SEE? MATH ISN'T SO BAD!

DAD, WILL I EVER USE THIS STUFF IN REAL LIFE?

ABSOLUTELY!
The beauty and utility of mathematics

What do we do wrong?
Realistic Mathematics Education [RME]

- Freudenthal (1905 – 1990): anti-didactical inversion = endpoint of the work of mathematicians is used as a starting point for instruction

- Alternative: learning mathematics as a human activity
Realistic Mathematics Education [RME]

- Freudenthal (1905 – 1990): anti-didactical inversion = endpoint of the work of mathematicians is used as a starting point for instruction

- Alternative: learning mathematics as a human activity
  - organizing subject matter from reality
  - organizing mathematical subject matter
Content

1. Three examples
2. Reflection on RME
3. Mathematics and Arts
4. Conclusion
Example 1: How many?
Example 2: How long?

- A rope of 30 meter is divided in 5 short and 3 long parts. A short and a long part together are 9 meter.
- How long is a short part?
Example 2: How long?

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- How long is a short part?
Example 2: How long?

Geometrical models for Algebra

- [http://www.fisme.science.uu.nl/toepassingen/02020/toepassing_wisweb.en.html](http://www.fisme.science.uu.nl/toepassingen/02020/toepassing_wisweb.en.html)
Example 3: sending a text message?


How many text messages are sent if four people all send a message to each other?

How many text messages are sent with different numbers of people?

Have a look at the students’ work and put them in some kind of order.
All three examples have the potential of supporting mathematizing towards …

1. Place value
2. Algebra
3. Functions

What are common characteristics between these examples?
Mathematizing?
Freudenthal on mathematics

“It is an activity of solving problems, of looking for problems, but it is also an activity of organizing subject matter. This can be matter from reality which has to be organized according to mathematical patterns (...). It can also be a mathematical matter, new or old results, of your own or of others, which have to be organized according to new ideas, to be better understood, in a broader context, or by an axiomatic approach.”

(HF100 special, p. 107)
Freudenthal on learning mathematics

• Starting point: mathematics is a human activity

• Mathematics should start and stay within the students’ reality

• Students should be given the opportunity to reinvent mathematics
Freudenthal on learning mathematics

- Students reinvent mathematics by mathematizing
Mathematizing

Mathematical objects, structures, methods

Abstract

Vertical mathematizing

Translate

Mathematical model

Horizontal mathematizing

‘Realistic’ context

Mathematizing

Mathematical model
Mathematizing

Mathematical objects, structures, methods

Abstract

Vertical mathematizing

Rope problem

‘Realistic’ context

Mathematical model

Translate

Horizontal mathematizing
Mathematizing

Mathematical model

Total = n \cdot (n - 1)

Mathematical objects, structures, methods

Translate

Vertical mathematizing

Abstract

‘Realistic’ context

Translate

Horizontal mathematizing

Translate

Mathematical model
Mathematizing

• Related to modeling:

  learning to apply
  (use an available mathematical toolkit: modeling as a goal)

  <>

  learning new mathematics
  (new symbolizations emerge: modeling as a vehicle)
Mathematizing

How to organize/design it?
Realistic Mathematics Education (RME)

Organizing mathematizing by

1. Starting in ‘realistic’ contexts with non-routine problems to evoke tentative representations
2. Change focus from informal strategies to formal mathematics (mathematizing)
3. Use models to support students’ learning
4. Interactive whole-class teaching

Can you recall an example of each principle from this summer school?
Mathematizing

A process of mathematizing is needed to learn mathematics and to prevent the Island Problem.

Is this specific for learning mathematics?
Mathematics and art
A few points on a circle knew they must be more – at least a few for each of their two neighbors had one of their own – or so they said.

Then, with improved communication, they felt they were a Whole Creation.

They wondered if a common bond came from some Point that lay beyond.

They felt this Source that kept them ’round must be thanked and must be found.

One point worked itself free and zigzagged across their inland sea.

The others followed and roamed on till circle and centre both were gone.
Mathematics and poetry

• Mathematics as a topic (circle and centre)
• Mathematics as a metaphor (circle and centre)
• Mathematics determining the product ($\pi$)
  
  *E.g. Pi Based on Edgar Allan Poe’s "The Raven":*  
  Poe, E.: Near a Raven. Midnights so dreary, tired and weary. Silently pondering volumes extolling all by-now obsolete lore.

• Mathematics for the process (algorithmic)
  
  *E.g. http://rhymeless.hamiltonulmer.com/
And architecture?
Painting
Mathematics and arts?

• Mathematics as a topic
• Mathematics as a metaphor
• Mathematics determining the product
• Mathematics for the process / construction
Mathematizing and arts

Organizing subject matter
Creating and developing (mathematical) patterns and structures

Are these processes specific for mathematics?
Mondrian (1872 – 1944)
Developing a visual language for …
DE OASE VAN MATISSE
Developing a visual language
Developing a visual language

Still Life, Interior in Venetian Red of 1946 is illustrative of Matisse’s depictions of his studio in Vence. This new visual language is made up of contrasts: between interior and exterior, between still life and landscape, between straight and curved lines, and, more than anything, between flatness and spatiality. His gradual development towards a flat, decorative style now attains its final phase, in which the scene is transformed into a composition of areas of dazzling color and dramatically simplified contour lines.
Creative processes

• Abstraction is deleting reality?
• No, it is about developing, constructing a language
• In arts:
  – Renaissance painters: a language for 3D illusions, perspective
  – Arabic artists: a language for pattern, mosaics and symmetry
  – Mondrian: a language for pattern, line, area and colour, …
  – Matisse: a language for visualizing shape, colour, straight and curved lines, flatness and spatiality, …
• In mathematics:
  – a language for grasping structure, patterns, symmetry, shape, …

• Involvement in and knowledge about these processes helps to understand and to appreciate (and to apply)
Mathematizing & RME

Challenge for mathematics educators is:

1. To support the development of mathematical symbol systems

2. To prevent isolated pieces of knowledge → to help students bridge the gap between
   – informal, situated knowledge
   – abstract, formal mathematical knowledge
Realistic Mathematics Education (RME)

Prevent the Island Problem, organize a process of mathematizing by:

• Starting in meaningful contexts with non-routine problems that ‘beg’ to organized (‘ask’ for representations; e.g. the rope problem)
• Change focus from informal strategies to formal mathematics (progressive schematization)
• Use models to support students´ learning
• Interactive whole-class teaching
Bridging the gap

• Being able to trace concepts & procedures
  – prevent isolated knowledge
• Being able to act mathematically in daily-life, work and further study
  – Have confidence in creating representations
  – Flexible user in non-routine situations
  – Ability to deal with missing or superfluous information
as a conclusion