(Early) Algebra in Primary School

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Overview

- Introduction project
- Working with a hanging mobile
- Student work
    - On paper
    - Movies
- Conclusions

Beyond Flatland Project

Aim: introduce more mathematical activities in primary school

→ Stimulate higher-order thinking (HOT)

Focus: grade 5 students

Three **mathematical domains**:

- Dynamic Data Modelling (graphing change)
- Probability
- Algebra

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(Early) Algebra

- The Netherlands teaching algebra starts in secondary school
- However, there is much evidence that it can successfully be implemented in the primary grades (e.g. Kaput et al., 2008)
  - E.g. 10-year olds solving linear equations with unknowns on both sides of the equal sign (Brizuela & Schliemann, 2004)

    e.g. 2T + 7 = T + 20
- EARLY algebra
- Does **not** mean: teach formal algebra only at younger ages

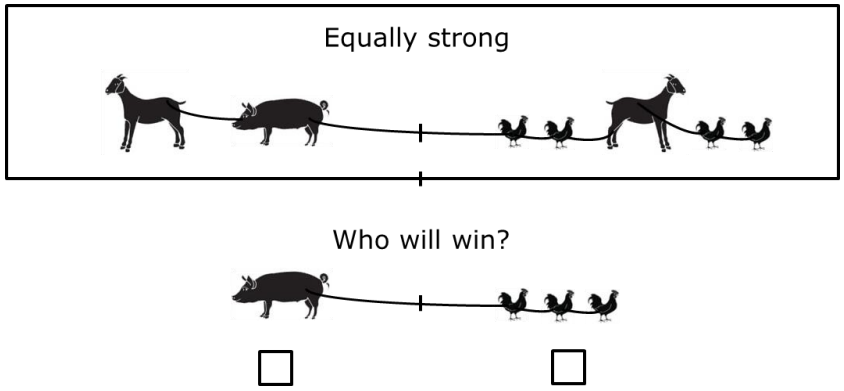
Early Algebra

- **Research question**: How can primary school students' algebraic reasoning be fostered?

To answer this question we have developed a teaching sequence for Grade 5 of six lessons on early algebra

Development of Teaching Sequence: Focus on equations

- Broad domain of algebra, chose to focus on *equations*
- Informal algebra
- More specifically: use context-based equation-like problems as starting point instead of formal equations



~~10Y = 2X~~
~~X = 2Y + Z~~
~~Z = ... Y?~~

Development of Teaching Sequence: Embodiment Theory

Offering students **bodily experiences** with a certain (mathematical) concept, can contribute to a deeper understanding of this concept.

E.g. offering perceptuo-motor experiences

Assignment in groups

- 5 groups

- **Assignment**: Discover what things can be done while keeping the hanging mobile straight

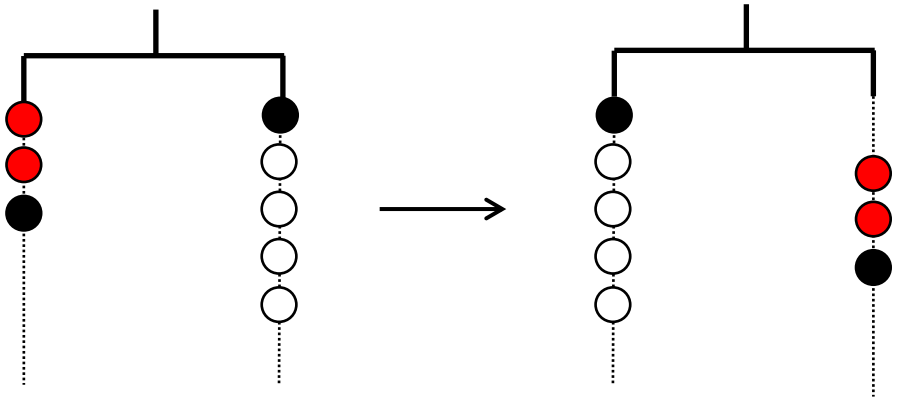
→ Make a poster of your findings (write/draw/…)

Discussion of findings

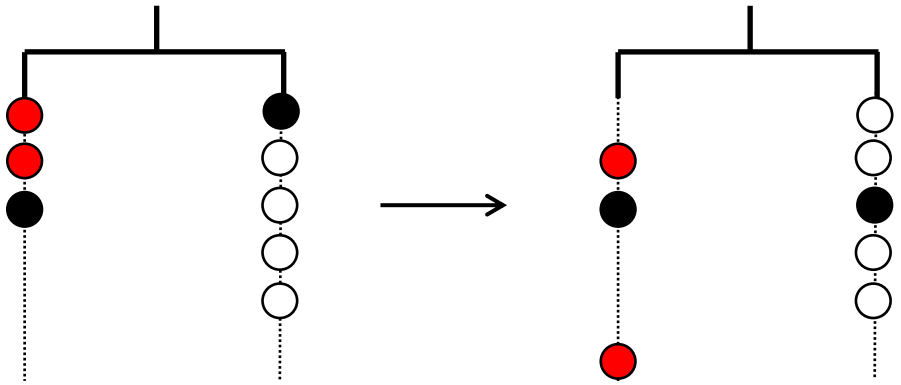
What things can be done while keeping the hanging mobile straight?

Some things we thought of:

- Change L/R

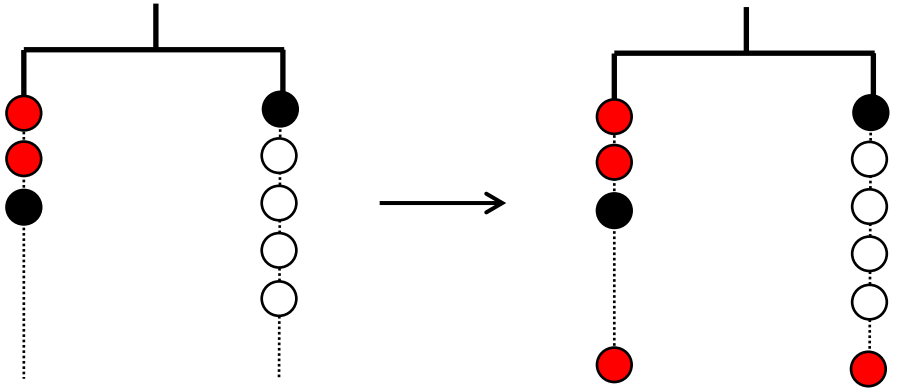


- Change order of bags on one side

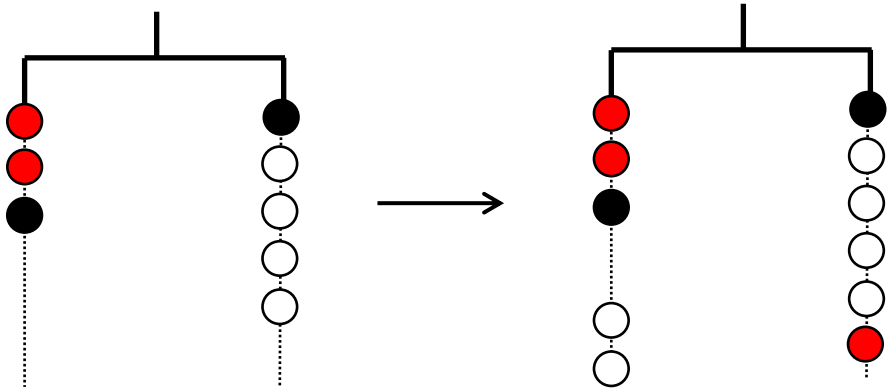


Some things we thought of:

- Add (similar) bags on both sides

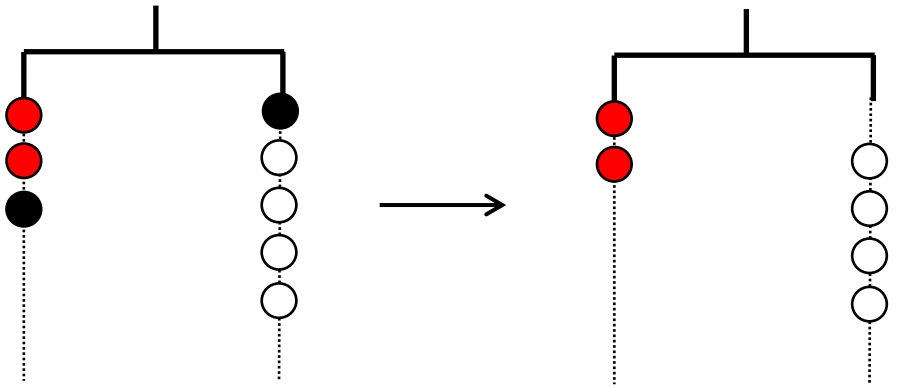


- Add bags (based on ratio) on both sides

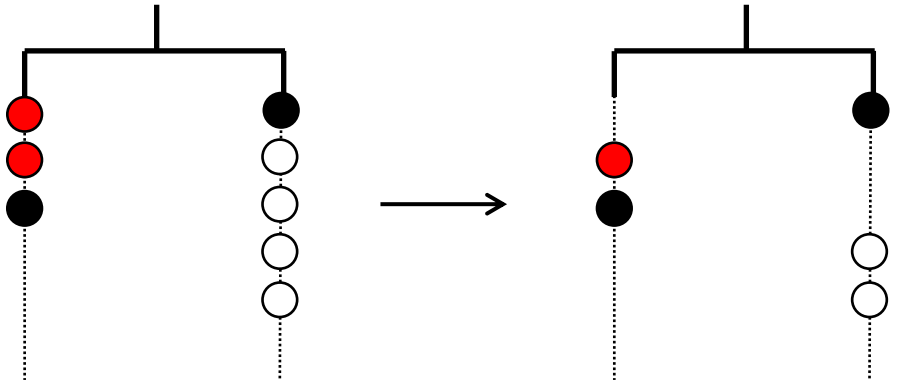


Some things we thought of:

- Take away (similar) bags on both sides



- Take away bags (based on ratio) on both sides



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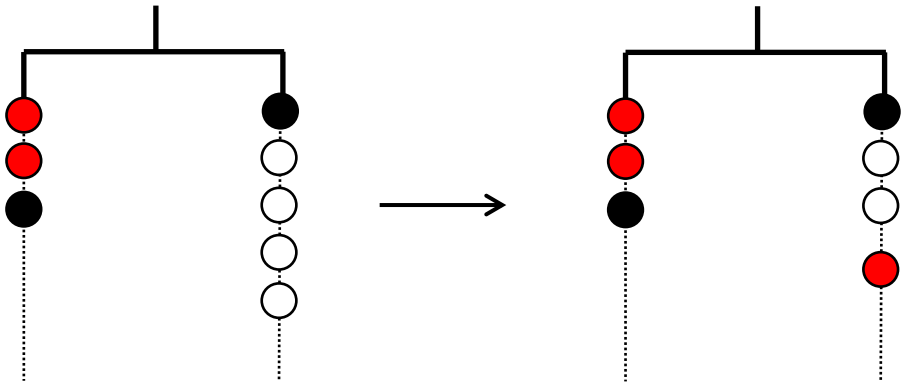
Some things we thought of:

- Replace bags of certain color by another color (based on the ratio)



- …

Discussion in groups

1. Which algebraic principles/concepts/strategies can be found in this task?

2. In what way does embodiment play a role? Can you think of a concept we are trying *to embody?*

Algebraic concepts/principles: what we thought of

- Equality / equivalence
- Restructuring principle:
  - Change L/R side
  - Change order of bags on one side
- Isolation principle:
  - Take away similar bags
  - Take away bags based on ratio
- Substitution principle:
  - Replace bags of certain color by another color
- Context-based notations
- …?

Embodiment theory?

Embodied cognition approach

- Offering students **bodily experiences** with the concept of **equality** can lead to a deeper understanding of the concept (cf. Núñez, Edwards, and Matos 1999).

- Constantly trying to keep the hanging mobile in balance.

- Builds on the **balancing** experiences most students already have in everyday life

- Understanding the concept of equality is crucial for solving equations (e.g. Greenes & Findell, 1999) working with the hanging mobile could provide a **foundation for developing algebraic reasoning**.

Student work

So far:

1. In what way can algebraic reasoning be elicited in primary school students?
2. Which algebraic strategies are elicited by letting students work with the hanging mobile?

From this point on: what algebraic reasoning do we see when students work with the hanging mobile?

Movie 1: working with the hanging mobile

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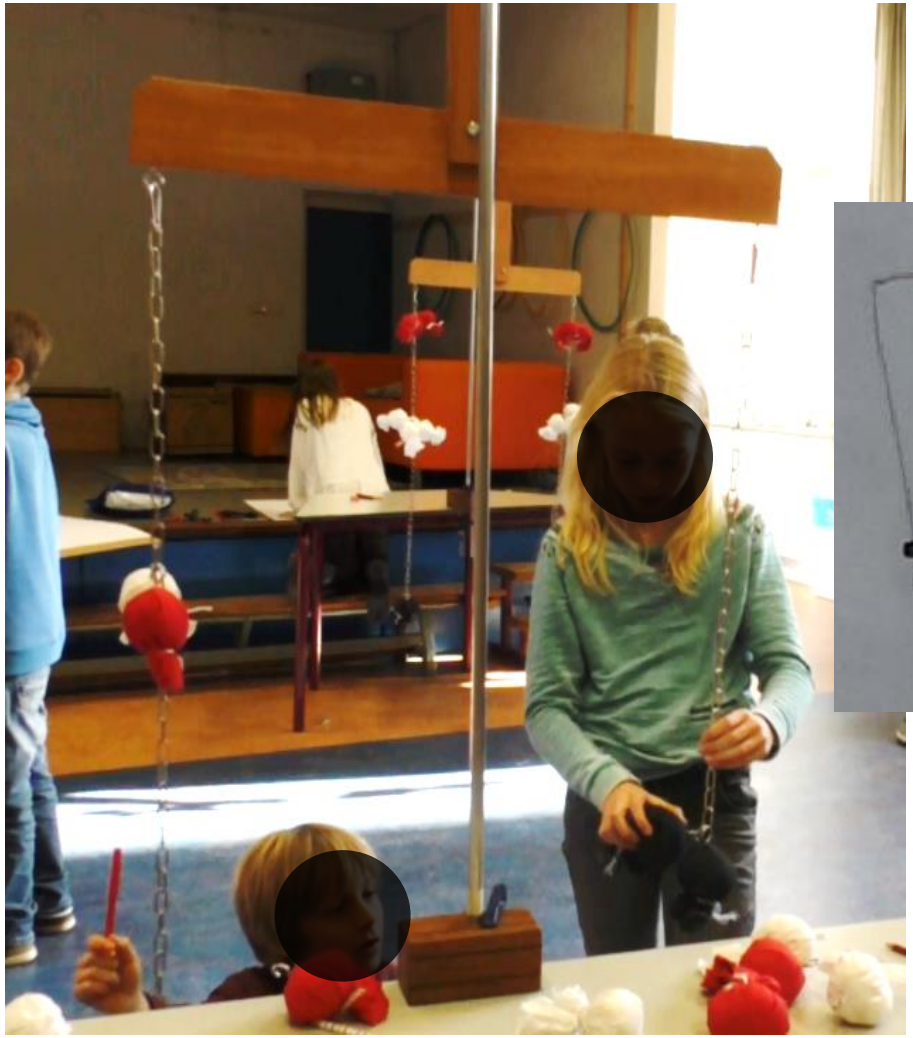
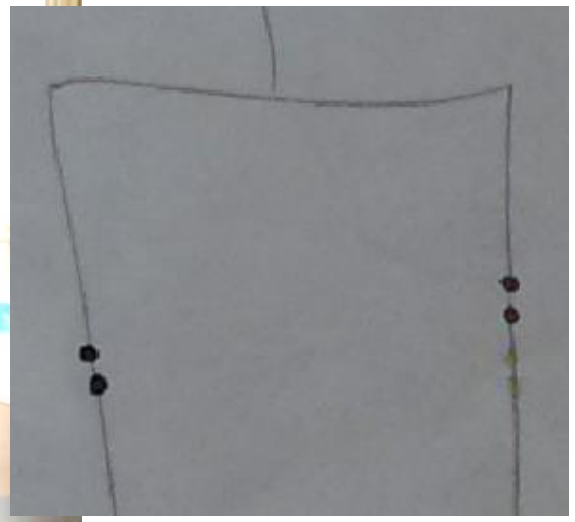
Movie 2: working with the hanging mobile





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Movie 3: classroom discussion

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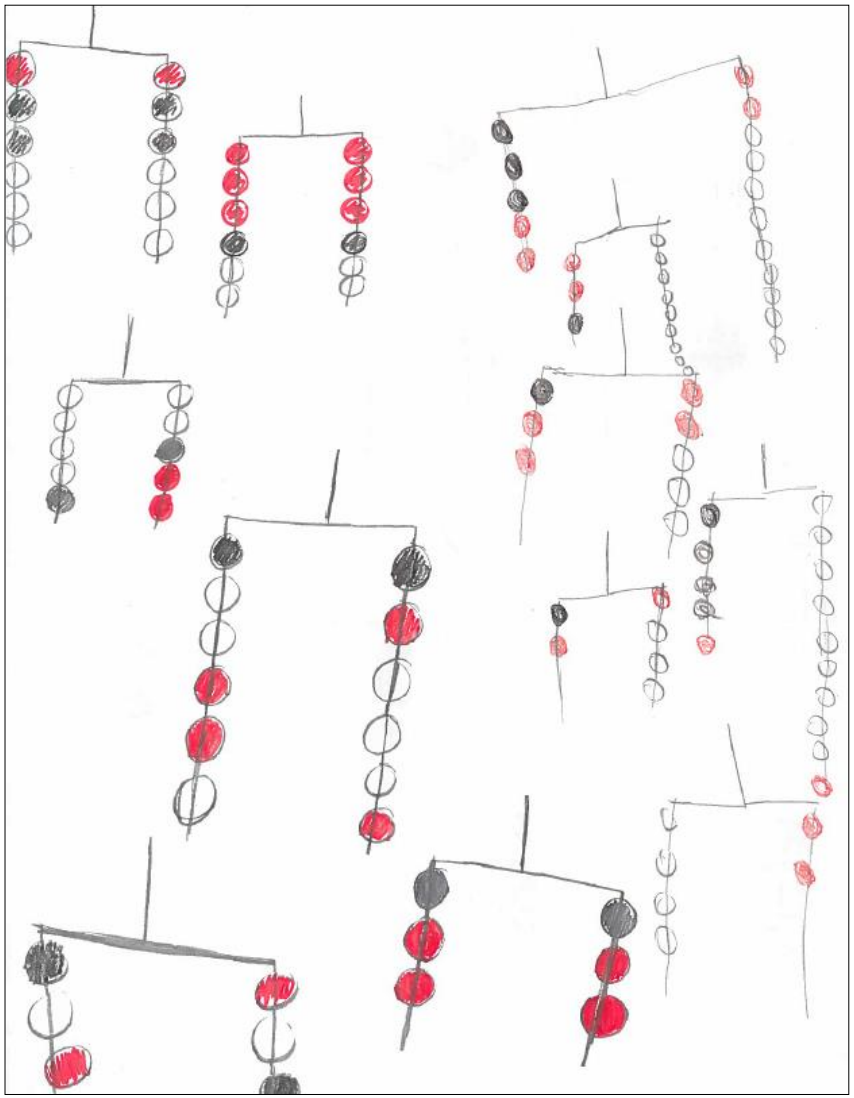
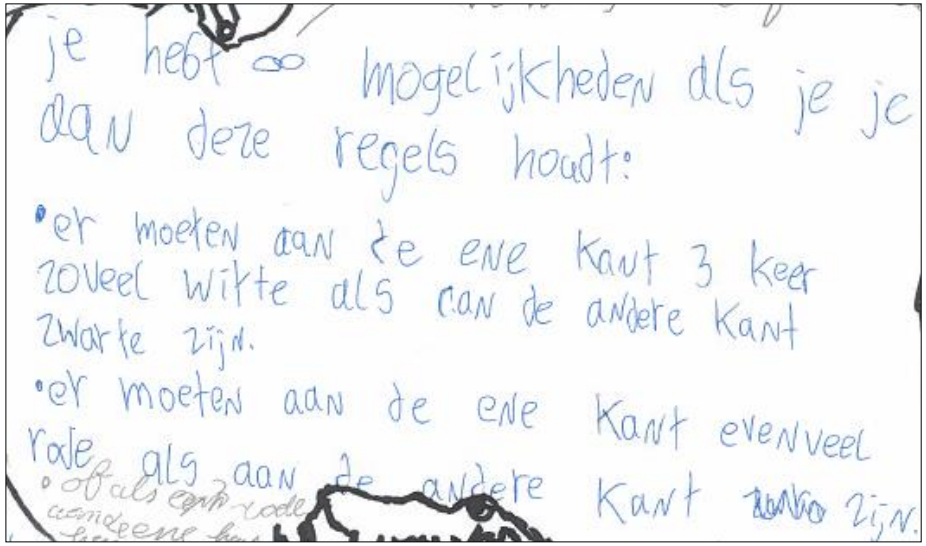
Posters 1 en 2



There are unlimited possibilities when you stick to these rules:

- *There must be three times as many whites on the one side, as blacks on the other side*
- *There must be the same number of reds on the one side, as reds on the other*



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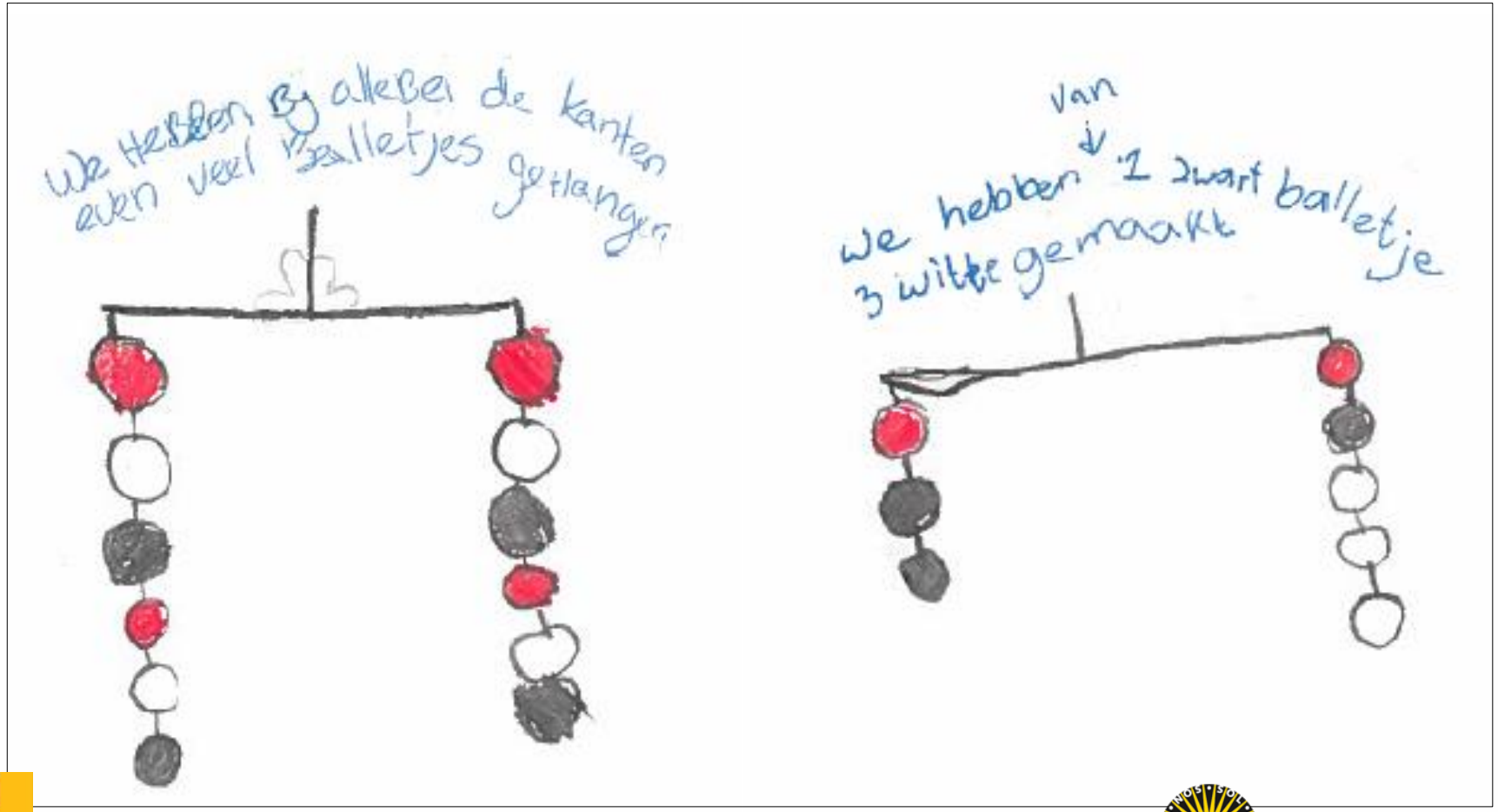
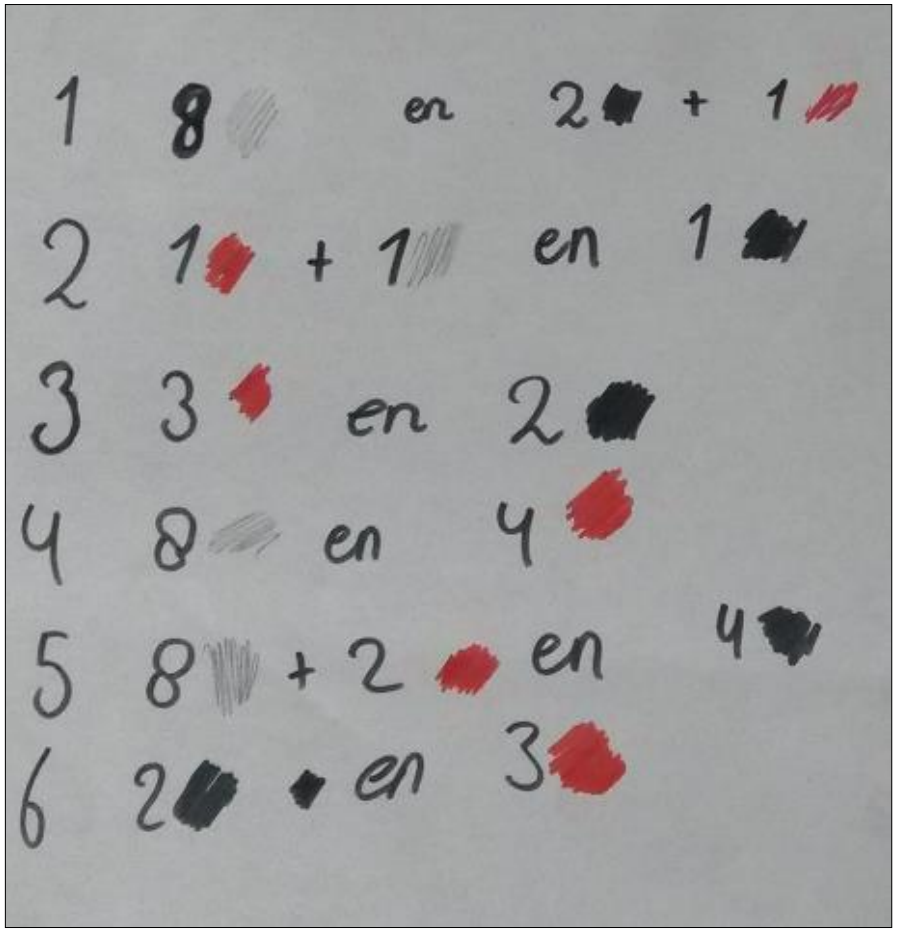
Poster 3

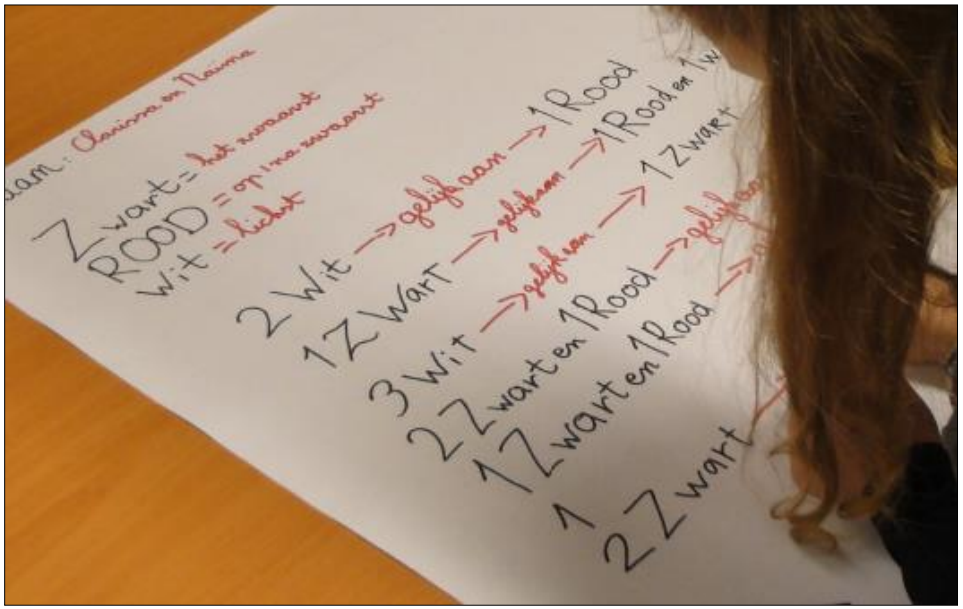
We put the same number of bags on both sides

We turned one black bag into three white ones



Posters 4 en 5





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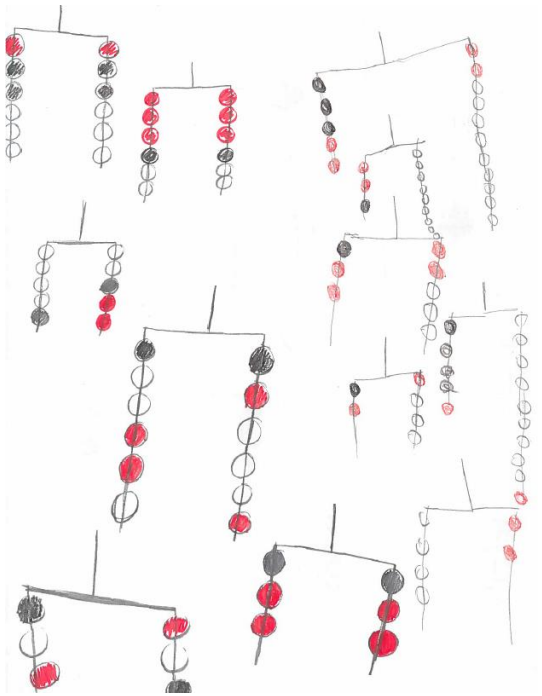

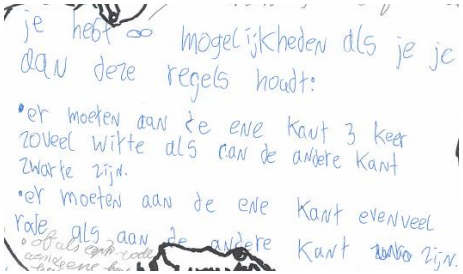

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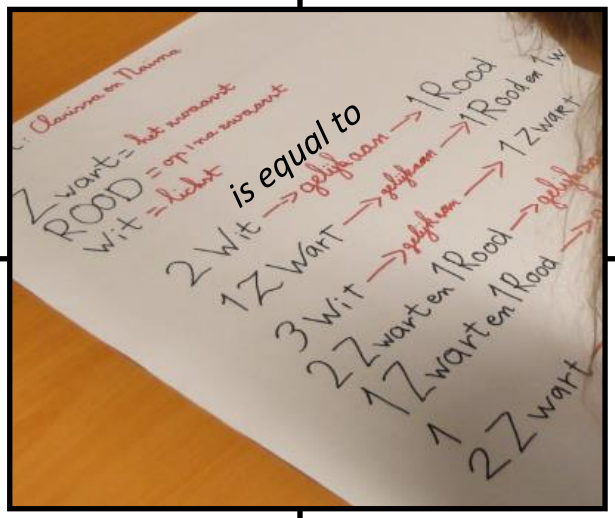

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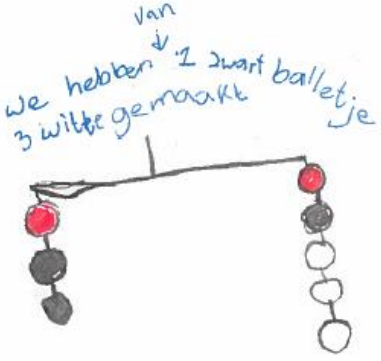
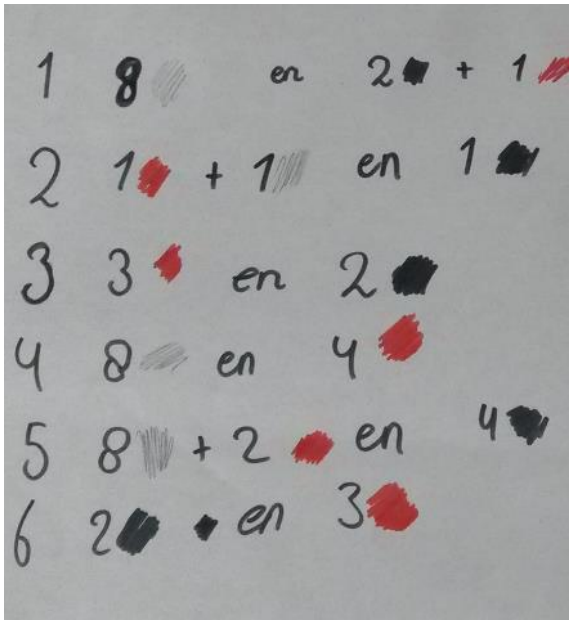
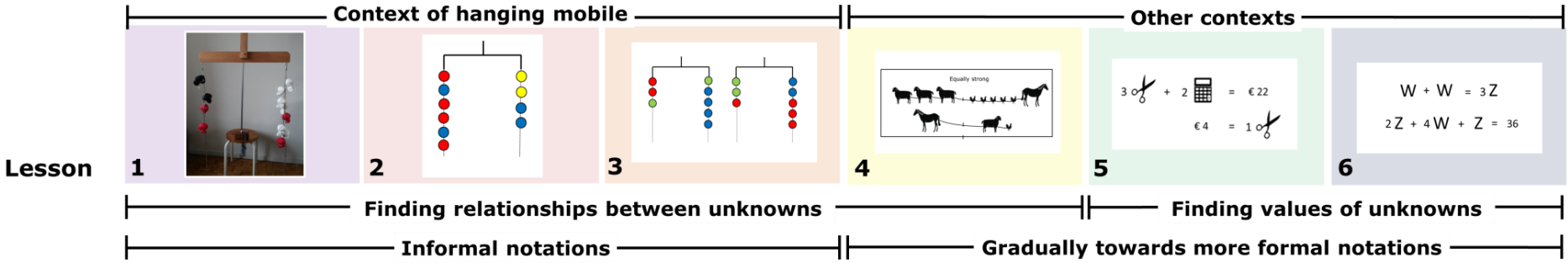
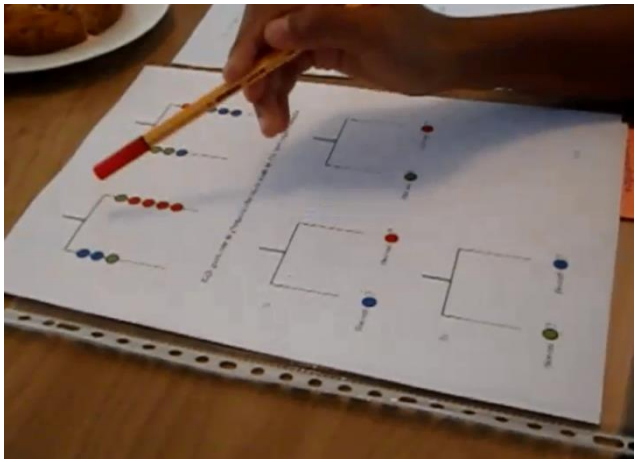




From this point on…?





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Thanks a lot for your attention!
Do you have any questions??

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