Software for Mathematical Explorations: Attempting to Make a Curricular Agenda Visible[1]

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Since I developed with Judah Schwartz the Geometric Supposer in the mid 1980's we collected solid evidence that it is possible to organize and teach school mathematics in such a way that math students can learn it as they adopt a mathematician's habits of mind. Inspired by this cycle of research and development, my more recent work has been directed towards inventing, designing, teaching and studying the "VisualMath" curriculum (1995/2003) which offers new forms of learning algebra and calculus with technology. Visual Math" was developed at the Centre for Educational technology (CET) Tel-Aviv, Israel in cooperation with teams at the University of Haifa. Written material and software can be viewed, download and used in http://www.cet.ac.il/math-international/first.htm. Newer developments in the format of electronic book and applets are at http://www.cet.ac.il/math/function/english . Mobile Learning environment that is based on similar educational agenda can be viewed and downloaded at www.math4mobile.com A major goal of VisualMath is to help students develop strong modelling and algebraic abilities, learn a variety of standard techniques, develop meaning to signs used and an understanding of the graphical meanings of these techniques, as well as a sense of the purposes for which such techniques are useful. The art and craft of this technology-intensive guided-inquiry curriculum was based on the assumption that any curriculum represents a point of view and that this view could be amplified by especially designed software tools.

An important purpose of the tools was in-line with what Goldenberg (1999) defined as "habits-of-mind orientation" where "a primary purpose of technology will be to help students formulate, express and reason about mathematical ideas;" (p.212). The orientation of the learning tools was neither to make complex algorithms easy nor to make it serve to reduce the knowledge one needs for manipulating using sophisticated procedures. It does not make solution the central feature and it often provides tools that are explicitly trivial for professionals. The design of the software tools meant to support the design of long-term sequence of learning activities in which the learner jump into what Schwartz (1995) calls the “interesting middle.” To do that, the building blocks (the major options offered by the tool) are mathematical objects and processes that are primitive enough to allow

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construction of new objects by the given processes, but interesting enough to promote uses of higher-order mathematical language, argumentation, and proof.

Clearing the confusion regarding the role of technology

Many misunderstand curriculum reform with respect to the use of symbol manipulators in teaching algebra and eventually there is still confusion about the role of the four-operation calculator in teaching arithmetic. While CAS just as numerical calculator can support explorations, the design of CAS often delivers the opposite message. Clearing the confusion regarding the role of technology and proficiency in manipulations of equations and inequalities was my major concern.

A central decision of function-based algebra curricula designers is to view any equation and inequality as a comparison of two functions. Numerical [almost-correct] solutions can be read on a graphing screen by reading intersecting points of two graphs or by reading values of zeros of the difference function of the two expressions. However, the major strength of 2D graphing of the two sides of the equation as two functions is its support in viewing the processes involved rather than viewing the solution (Yerushalmy 1999). In the VisualMath curriculum resources and occurrences are designed so that students come to understand what operations on equations are legal ones, and which operations on equations are not mathematically sound and do that while performing manipulations themselves as a way to conjecture and understand "on screen" results. Students are asked to explain how the effect of algebraic operations on the solutions of a comparison depends on the type of comparison (equation, inequality), the type of operation applied to the side/s of the comparison. Software that provides a vertical “rulers” at intersection points, that graphically trace the change of the x-values of the solutions and restricts the free input to well defined algebraic and graphical (e.g.; translations and stretch of each or both graphs) operations on the comparison is used to enable explorations of operations on both functions. This design could well be interpreted as awkward and restrictive in comparison to the slick, transparent, and quick operation favored by "solution tools". But this design is meant to support the construction of a visible map of the point of view of the curriculum.

Several studies of the VisualMath students have been carried out. They suggest that the use of multiple representation technology does not at all omit the structural ideas of expressions from the study of algebra but rather introduces a new style of activities that have a chance at introducing important ideas. A special attention was given to the ways that the tools for explorations may support less successful mathematics students learn inquiry-based curriculum that demand creativity and flexibility. We found differences between the work of these less successful students and the traditional problem-solving patterns of less successful students. The less successful students used the graphing software to obtain a broader view, to confirm conjectures, and to complete difficult operations. However, their process of reaching a solution was found to be relatively long and they delayed using symbolic formalism, and most of their solution attempts focused on numeric and graphic representations. Comparing VisualMath students and equations' based algebra students solving algebra problems in context, it was found that the students who were successful students of a traditional algebra sequence which focus on unknowns and stress paper & pencil manipulations' procedures were substantially less capable than the function-based students to solve these problems.

From bodily actions to symbolizing and meaning production

Going beyond the work in multiple representation systems, technology has proved to be a powerful tool for physical interaction. Thus, another important goal of the design was to emphasize relation between bodily actions, artifact mediated activities and the processes of symbolizing and meaning production. The capability of artifacts to be part of semiotic mediations, support experimentation with temporal processes by means of embodied actions, and turning these processes to produce mathematical symbols of space and motion has played a major role in various stages of learning in technology-intensive reform curricula. Using MBL software, students study the graphs of the process as it changes in time and develop narrative to connect the actions of the situation with the features of the graphs. A planar movement of the hand motion, a motion of an object or an operation of pre-designed simulation provides the input that appears on the screen as a graph of one or two-dimensional path. In order to abstract the data plotted by the software but to still reflect the essential physical actions of the learner, a set of graphical icons is designed in the Function Sketcher environment. The different components of the lexical system (a set of seven icons and a limited verbal list of function properties is designed) are eventually adopted as manipulable objects that support students when solving problems that are too complicated for them to describe symbolically. This intermediate bridging language helped to form a mathematical construction with language that developed from acquaintance with physical scenarios. It supported the abstraction of everyday phenomena into a smaller set of mathematical signs that are manipulated with software tools as "semi-concrete" objects. The tools were designed based on Vygotskian's notion of semiotic mediation, according to which cognitive functioning is intimately linked to the use of signs and tools, and affected by it. Algebra beginners, advanced algebra students and calculus students (Botzer &Yerushlamy 2006) were all benefit from the perceptual and visual resources and from the direct and sensual manipulations of graphs.

References

Botzer, G. & Yerushalmy, M. (2006) Interpreting motion graphs through metaphorical projection of embodied experience. International Journal for Technology in Mathematics Education. 13(3).

Goldenberg, P. E. (1999). "Principles, Arts, and Craft in Curriculum Design: The Case of Connected Geometry ".International Journal of Computers for Mathematical Learning 191-224 (4) .

Schwartz, J. L. (1995). The Right Size Byte: Reflections of an Educational Software Designer . In Perkins et al. (Eds.) *Software Goes to School.* New York, Oxford University Press. Pp. 172-181

Yerushalmy, M. (1999) Making Exploration Visible: On Software Design and School Algebra Curriculum. International Journal for Computers in Mathematical Learning, 4 (2-3), 169-189.