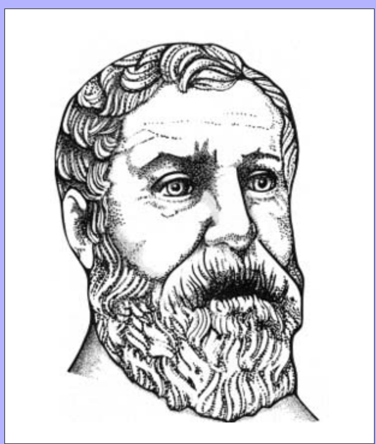
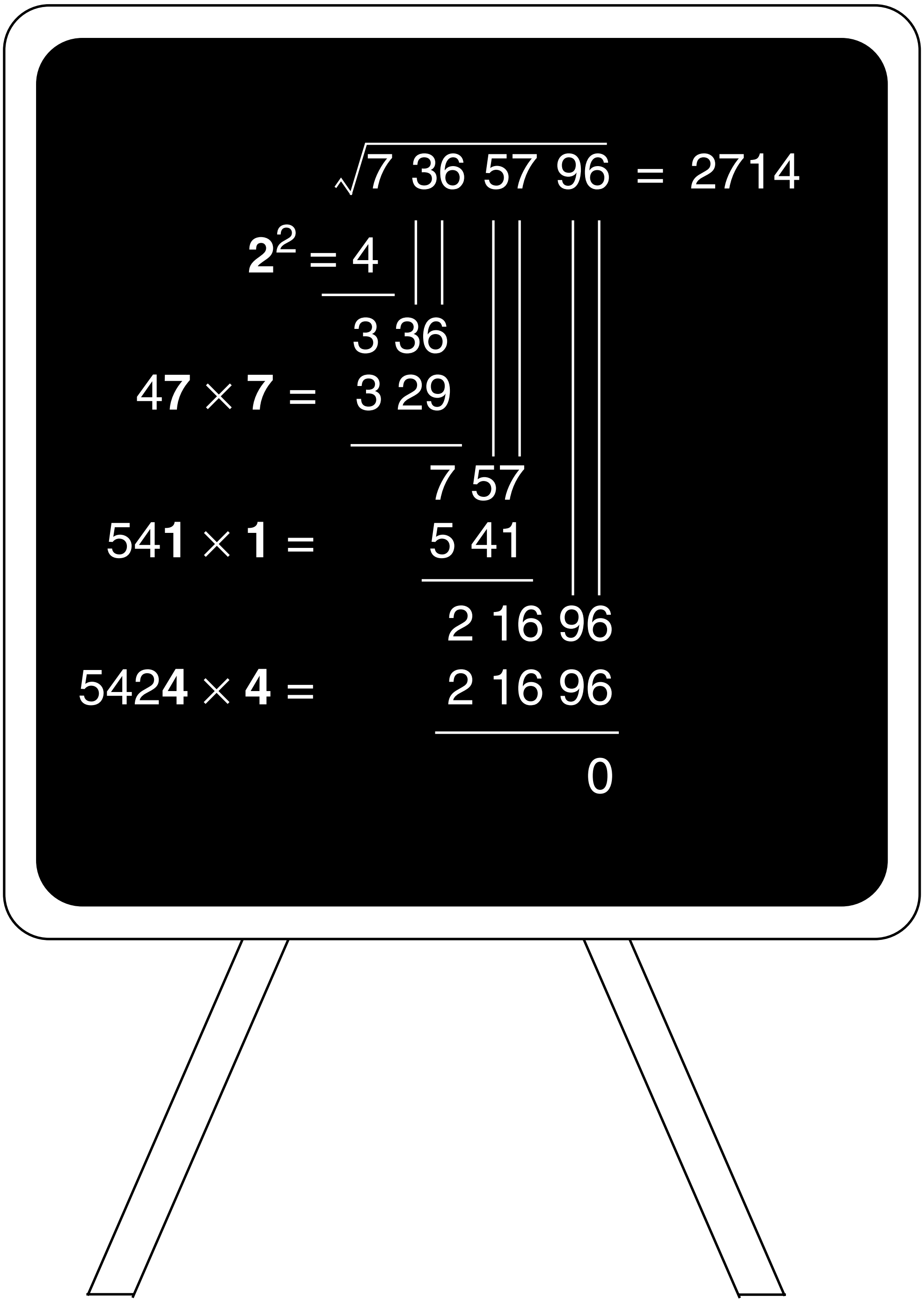
√ met een staart



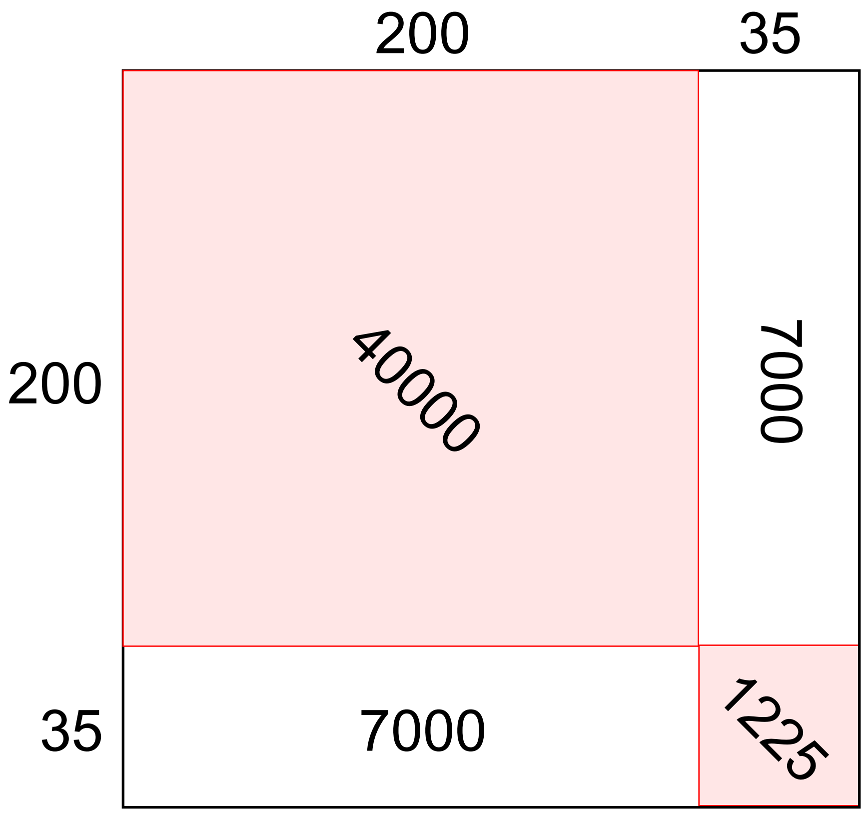
Martin Kindt

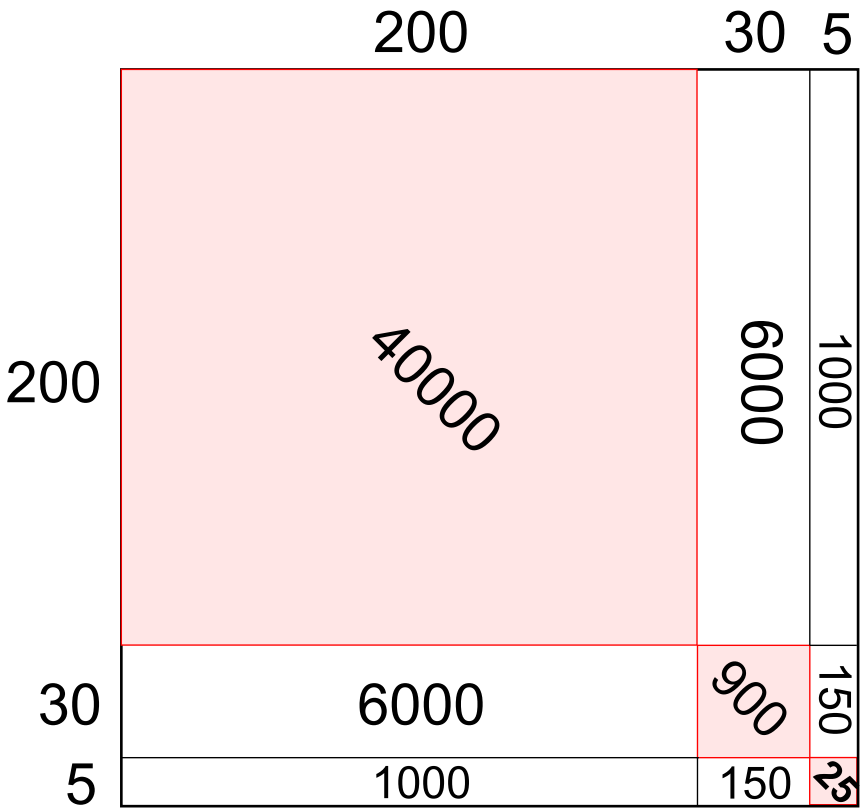
NWD 2007


√7 36 57 96 = 2714
2² = 4
3 36
47 × 7 = 3 29
7 57
541 × 1 = 5 41
2 16 96
5424 × 4 = 2 16 96
0

The Root of your first Period you
Must place in Quote, if you work true;
Whose Square from your said Period then,
You must subtract; and to th' Remain
Another Period being brought,
You must divide as here is taught;
By the Double of your Quote, but see
Your Unit's Place you do leave free;
Which Place will be supply'd by th' Square
Of your next quoted Figure there:
Next multiply, subtract and then
Repeat your work unto the End
And if your Number be irrational
Add Pairs of Cyphers for a Decimal.

$$\sqrt{55225} = ?$$


200
35
200
40000
7000
35
7000
1225


200
30
5
200
40000
6000
1000
30
6000
900
150
5
1000
150
25

$$a^2 + b^2 + c^2 + 2ab + 2ac + 2bc$$

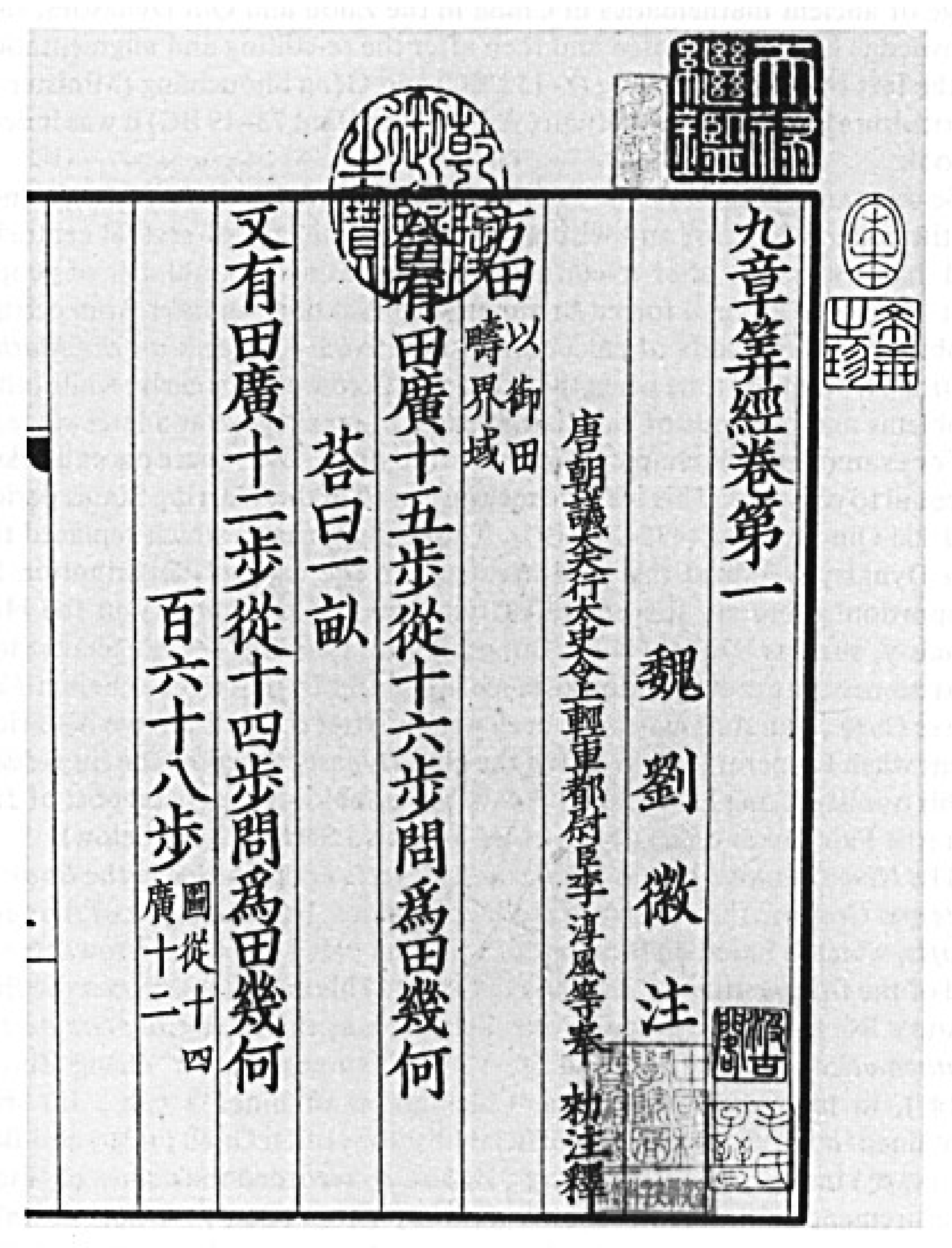
je schoonheid min je ogen noem ik a
de geest die in je dartelt b
je ogen
c

opgeteld en minstens een kwadraat gegeven:
$(a + b + c)^2$

K.Schippers

Uit: *Een leeuwerik boven een weiland*

Jiuzhang suanshu (*Han periode ca* –200,200)


九章筭經卷第一
魏 劉 徽 注
唐朝議大夫行太史令上輕車都尉臣李淳風等奉 勑注釋
方田 以御田疇界域
今有田廣十五步從十六步問爲田幾何
荅曰一畝
又有田廣十二步從十四步問爲田幾何
百六十八步 圖從十四廣十二

Negen hoofdstukken over de kunst der wiskunde

probleem 12 (hoofdstuk 4)
uit
‘Negen Hoofdstukken’

De oppervlakte van

een vierkant is 55225 pu

Hoeveel pu is de lengte?

Liu Hui (220 - 280)

becommentarieerde
de negen hoofdstukken

$$\sqrt{5\ 52\ 25} = 235$$

$$2^2 = 4$$

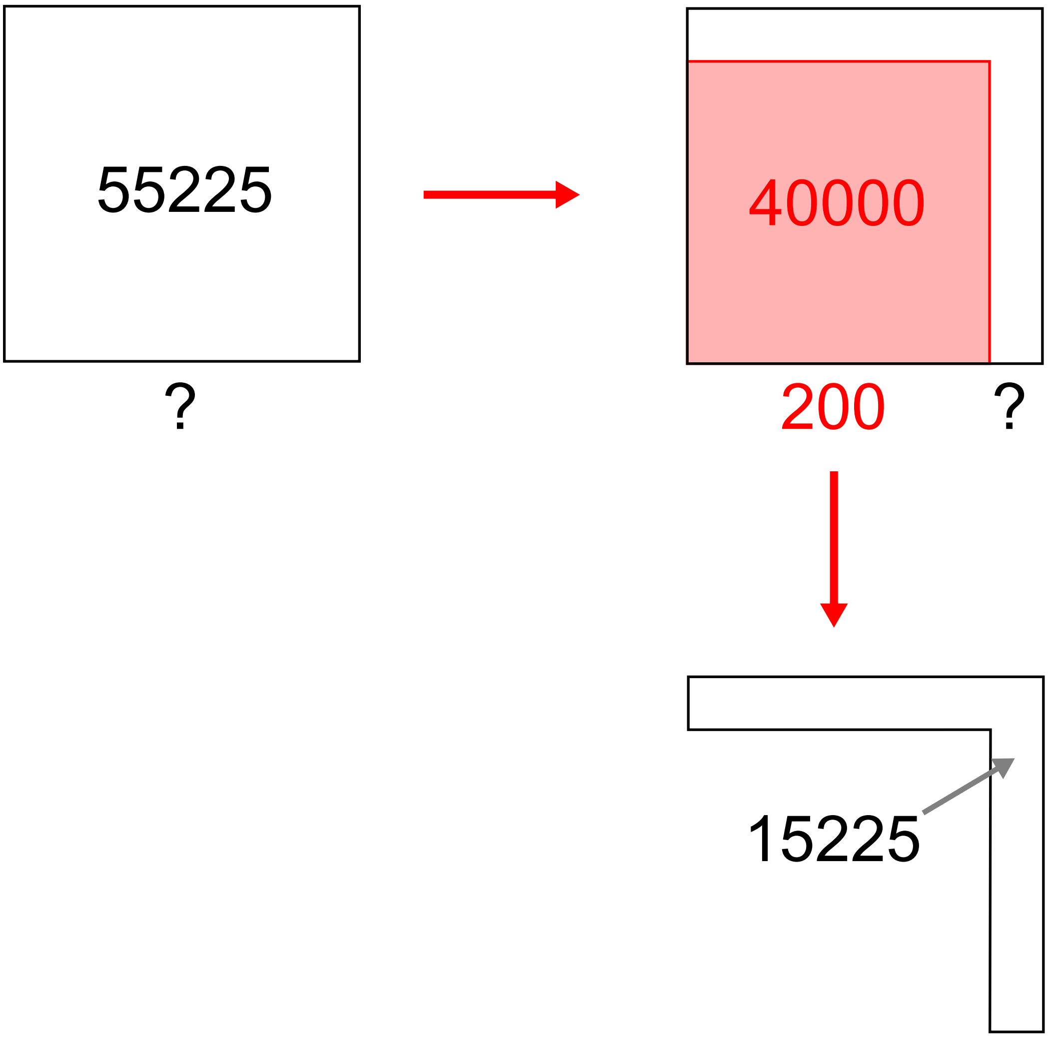
$$1\ 52$$

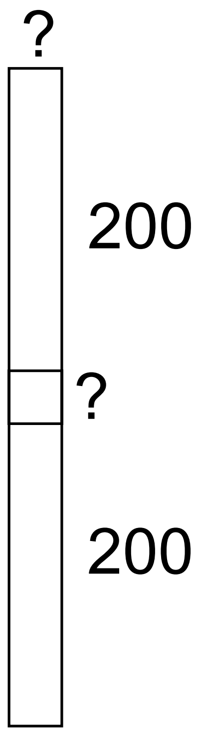
$$4\overset{3}{.} \times \overset{3}{.} = 1\ 29$$

$$23\ 25$$

$$46\overset{5}{.} \times \overset{5}{.} = 23\ 25$$

$$0$$


55225
?
40000
200
?
15225

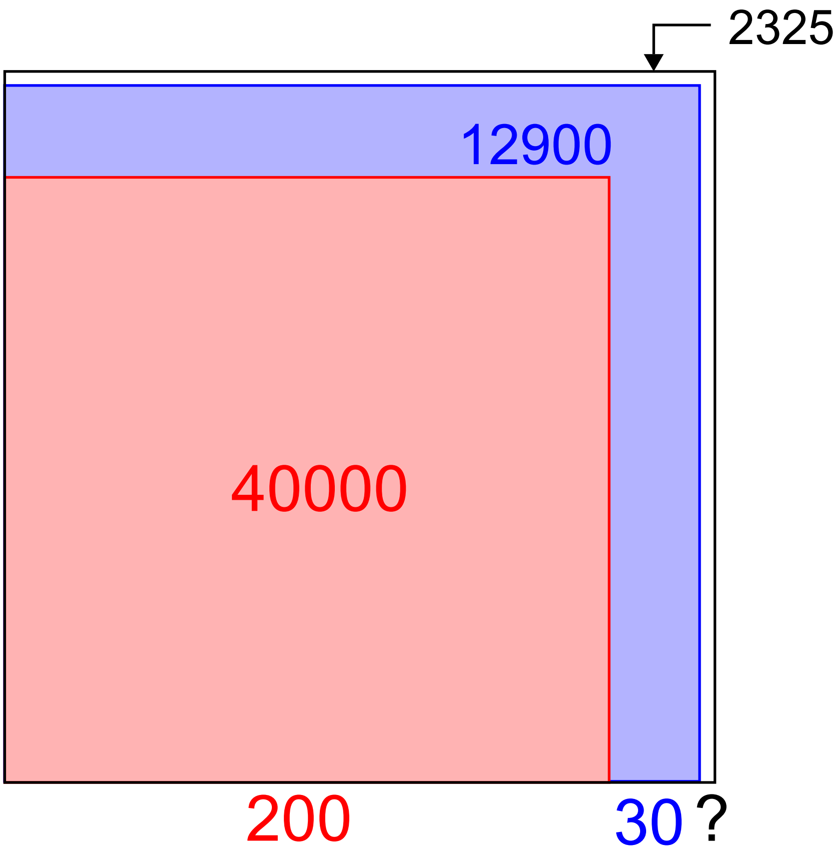


$$(400 + ?) \times ? \leq 15225$$

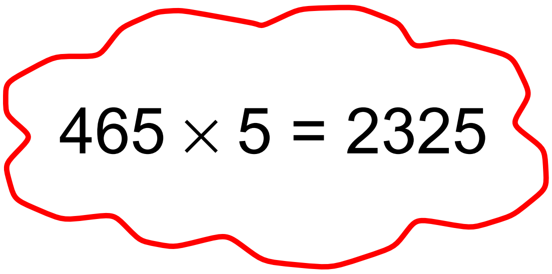
10-tal

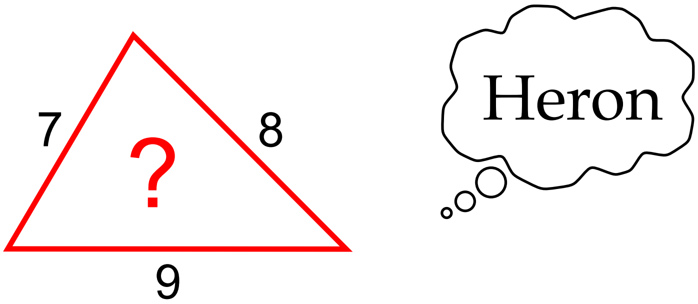
$$430 \times 30 = 12900$$

restant = 2325



$$(460 + ?) \times ? \leq 2325$$



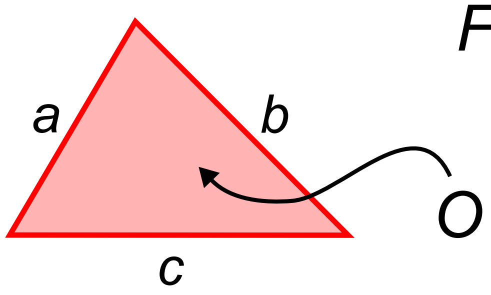


Er is een algemene methode om, zonder een hoogtelijn te trekken, de oppervlakte te vinden van een driehoek waarvan drie zijden gegeven zijn ....

Methode

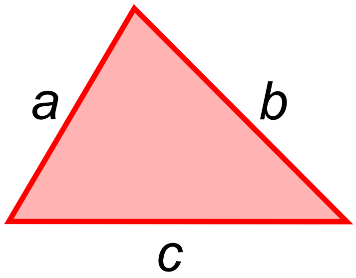
Tel bij elkaar **7**, **8** en **9**. Uitkomst **24**.
Neem hiervan de helft : **12**
Trek hiervan **7** af; de rest is **5**
Opnieuw: trek **8** af; de rest is **4**
En nu ook met **9**; de rest is **3**
Vermenigvuldig **12** met **5**, dat is **60**
Vermenigvuldig dit met **4**, dat is **240**
Vermenigvuldig dit met **3**, dat is **720**
Trek hier de vierkantswortel uit, dit is de oppervlakte van de driehoek!

Formule van Heron



$$\frac{a+b+c}{2} = s$$

$$O = \sqrt{s(s-a)(s-b)(s-c)}$$



$$O = \tfrac{1}{2}ab\sin\gamma$$

$$c^2 = a^2 + b^2 - 2ab\cos\gamma$$

Elimineer γ

↓

Formule van Heron

$$\sqrt{720,000000} = 26,832\ldots$$

$$2^2 = 4$$

$$320$$

$$46 \times 6 = 276$$

$$4400$$

$$528 \times 8 = 4224$$

$$17600$$

$$5363 \times 3 = 16089$$

$$151100$$

$$53662 \times 2 = 107324$$

$$43776$$

....

enzovoort

Heron:

720 heeft geen rationale wortel

Dus: benaderen!

Meest naburig kwadraat: $729 = 27^2$

$$720 \div 27 = 26\frac{2}{3}$$

$$26\frac{2}{3} + 27 = 53\frac{2}{3}$$

$$53\frac{2}{3} \div 2 = 26\frac{1}{2} + \frac{1}{3}$$

$$\sqrt{720} \approx 26\frac{5}{6}$$

$$27 \quad \& \quad 26\tfrac{2}{3}$$

meetkundig gemiddelde

rekenkundig gemiddelde

$$\sqrt{720} \quad < \quad 26\tfrac{5}{6}$$

$$26\frac{5}{6} \times 26\frac{5}{6} = 720\frac{1}{36}$$

dus de afwijking is $\frac{1}{36}$

Wie een kleinere afwijking wil,

kan dezelfde methode toepassen

met $720\frac{1}{36}$ in plaats van 729

d.w.z. met $26\frac{5}{6}$ i.p.v. 27

Volgende ‘Heron-stap’ zou geven:

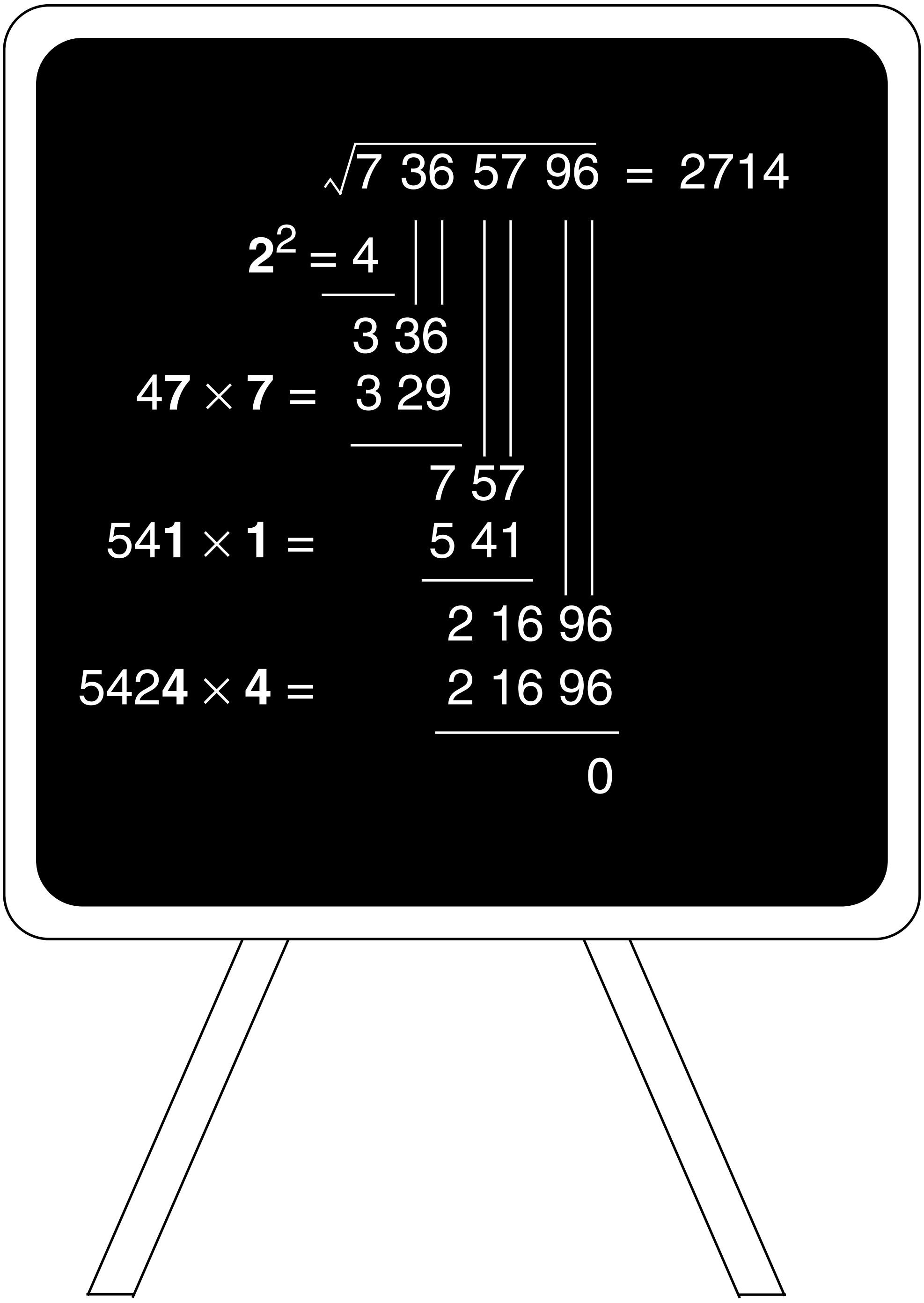
$$26\,\frac{1609}{1932}$$

26 + 1609/1932

26.83281573

$\sqrt{(720)}$

26.83281573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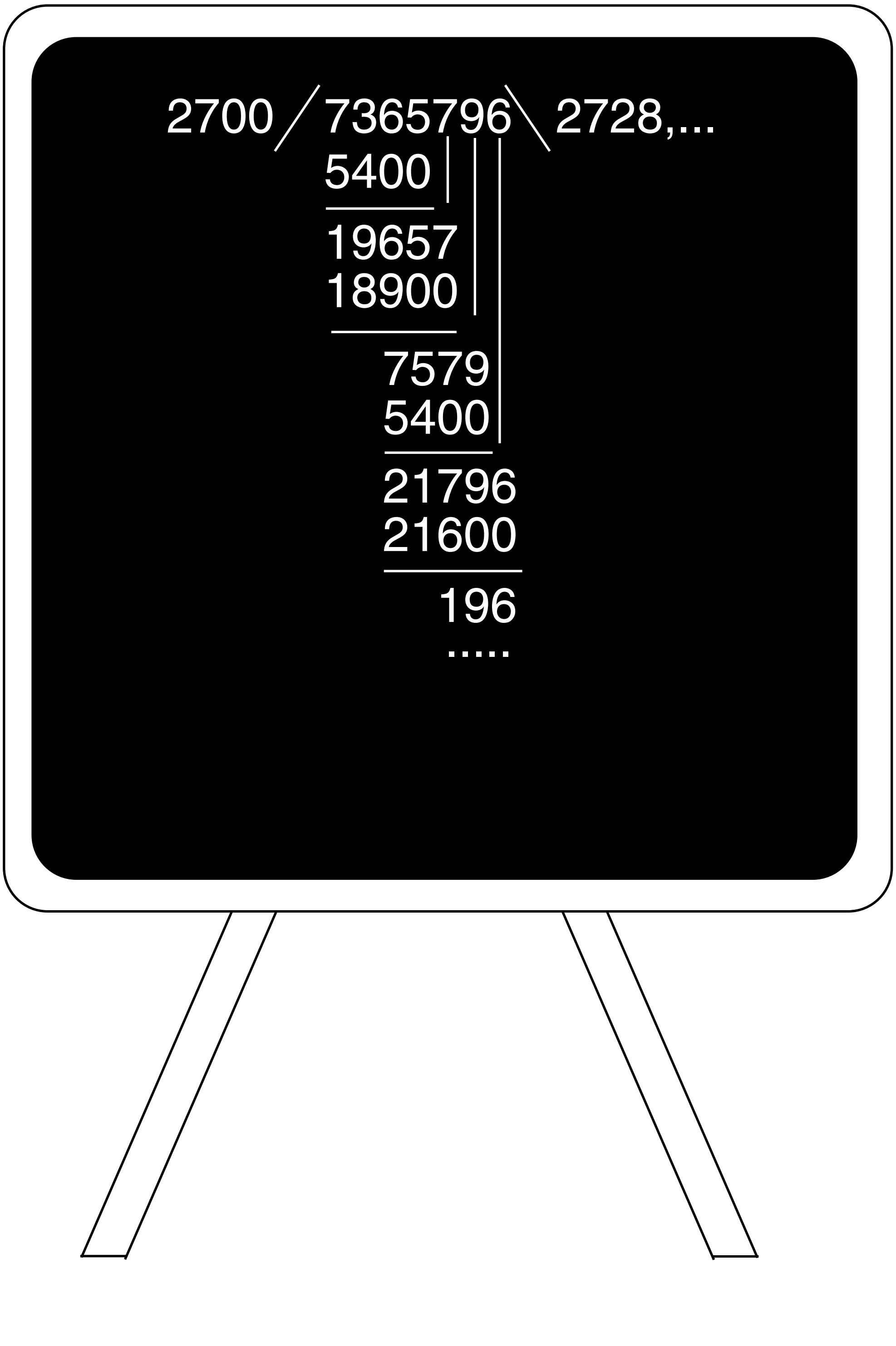

2700 7365796 2728,...
5400
19657
18900
7579
5400
21796
21600
196
.....

$$\sqrt{2,0000000000} = 1,41421\ldots$$

$$1^2 = 1$$

$$100$$

$$24 \times 4 = 96$$

$$400$$

$$281 \times 1 = 281$$

$$11900$$

$$2824 \times 4 = 11296$$

$$60400$$

$$28282 \times 2 = 56564$$

$$383600$$

$$282841 \times 1 = 282841$$

$$10075900$$

....

enzovoort

$$\sqrt{2} = ?$$

eerste benadering: $1\frac{1}{2}$

$$2 \div 1\frac{1}{2} = 1\frac{1}{3}$$

$$1\frac{1}{2} + 1\frac{1}{3} = 2\frac{5}{6}$$

$$2\frac{5}{6} \div 2 = 1\frac{5}{12}$$

→ $\sqrt{2} \approx 1\frac{5}{12}$

$$\sqrt{2} = ?$$

eerste benadering: $1\frac{1}{2}$

$$2 \div 1\frac{1}{2} = 1\frac{1}{3}$$

$$1\frac{1}{2} + 1\frac{1}{3} = 2\frac{5}{6}$$

$$2\frac{5}{6} \div 2 = 1\frac{5}{12}$$

$$\rightarrow \sqrt{2} \approx 1\frac{5}{12}$$

tweede benadering: $1\frac{5}{12}$

$$2 \div 1\frac{5}{12} = 1\frac{7}{17}$$

$$1\frac{5}{12} + 1\frac{7}{17} = 2\frac{169}{204}$$

$$2\frac{169}{204} \div 2 = 1\frac{169}{408}$$

$$\rightarrow \sqrt{2} \approx 1\frac{169}{408}$$

Sulvasutras

(Vedische Wiskunde, –600)

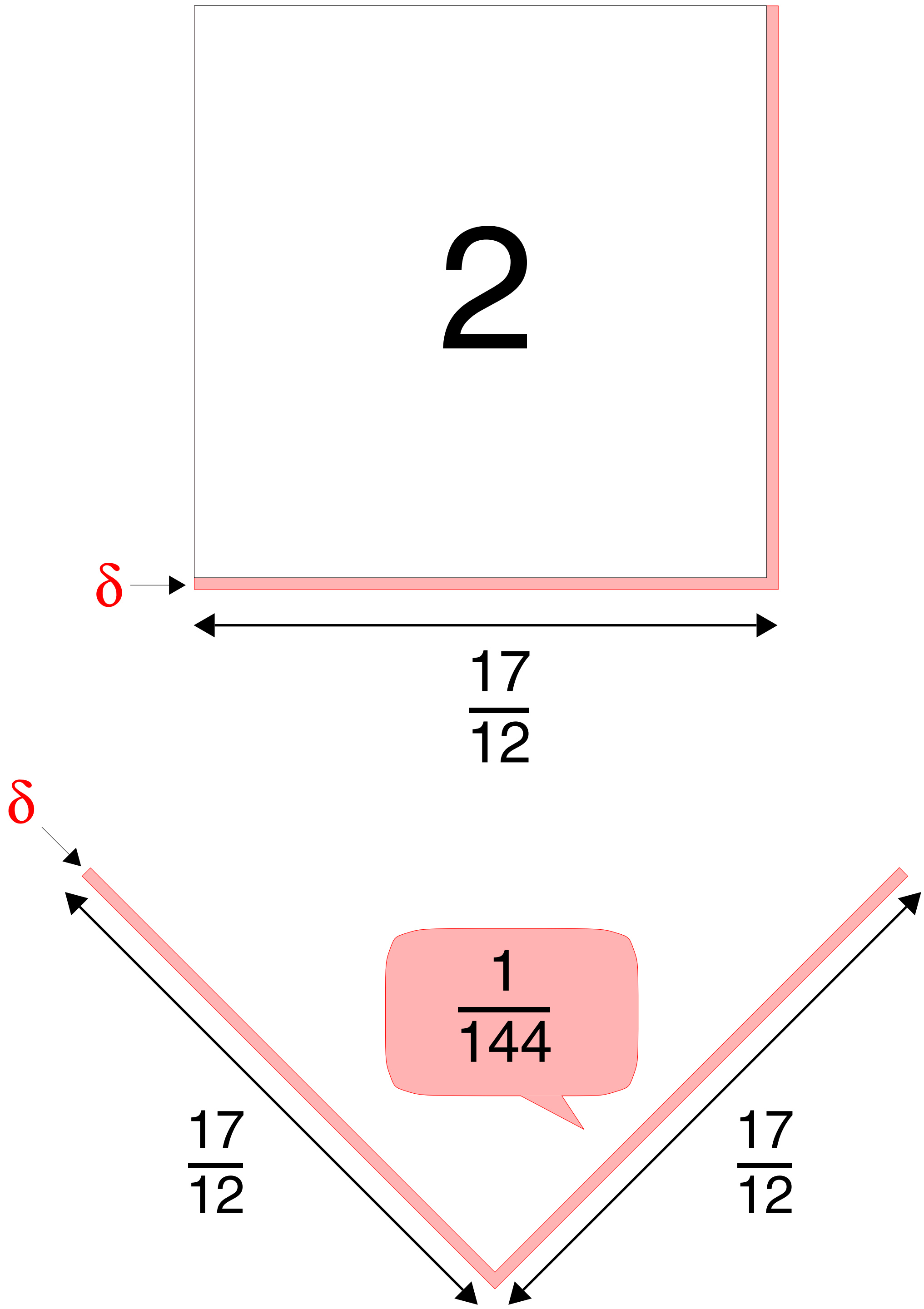
$$\sqrt{2}$$

$$\approx$$

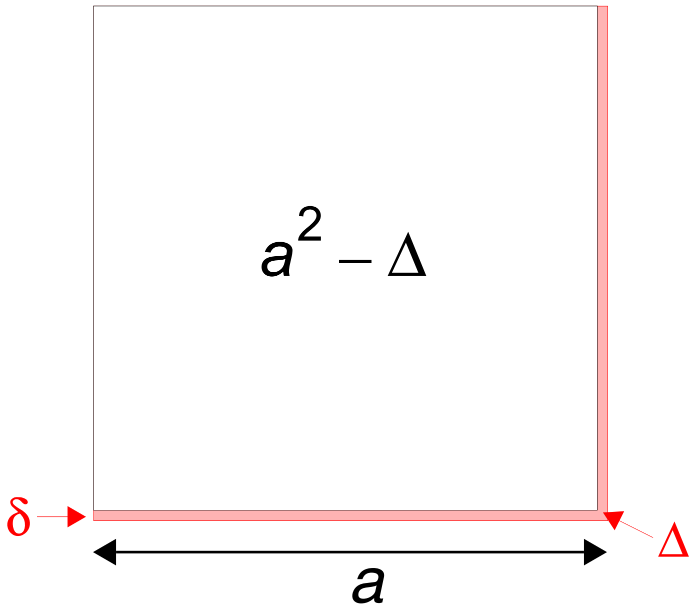
$$1 + \frac{1}{3} + \frac{1}{3 \times 4} - \frac{1}{3 \times 4 \times 34}$$

$$1\frac{5}{12} = 1\frac{170}{408}$$

$$\frac{1}{408}$$



$$\delta \times 2 \times \frac{17}{12} \approx \frac{1}{144}$$



$$\delta = \frac{\Delta}{2a + \delta} \approx \frac{\Delta}{2a}$$

$$\sqrt{a^2 - \Delta} \approx a - \frac{\Delta}{2a}$$

Net zo: $$\sqrt{a^2 + \Delta} \approx a + \frac{\Delta}{2a}$$

$$a + \frac{\Delta}{2a} = \frac{1}{2}\left(a + \frac{a^2 + \Delta}{a}\right)$$

$$\sqrt{2\frac{1}{4} - \frac{1}{4}} \approx 1\frac{1}{2} - \frac{1/4}{3} = 1\frac{5}{12}$$

$$\sqrt{2\frac{1}{144} - \frac{1}{144}} \approx 1\frac{5}{12} - \frac{1/144}{34/12} = 1\frac{5}{12} - \frac{1}{12 \cdot 34}$$

$$\sqrt{4a^4 + 12a^3 + 29a^2 + 30a + 25} = 2a^2 + 3a + 5$$

$$(2a^2)^2 = 4a^4$$

$$12a^3 + 29a^2$$

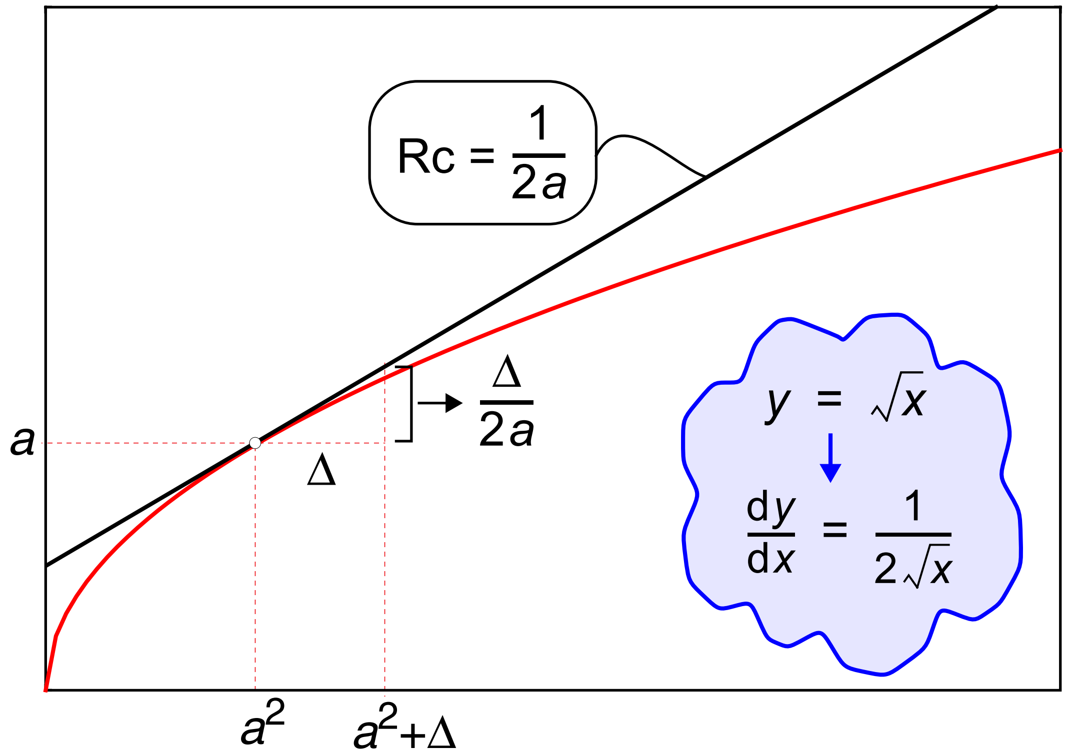
$$(4a^2 + \underset{.....}{3a}) \times \underset{.....}{3a} = 12a^3 + 9a^2$$

$$20a^2 + 30a + 25$$

$$(4a^2 + 6a + \underset{.....}{5}) \times \underset{.....}{5} = 20a^2 + 30a + 25$$

$$0$$

Eerste-orde benadering van $\sqrt{a^2 + \Delta}$



Aryabhata (499): *derde-machtswortel trekken*

	1	**2**	**9**	**7**	**7**	**8**	**7**	**5**	]	**2**	$\mathbf{2}^3 = 8$
		8									
1 2		**4**	**9**						]	**3**	$3 \times \mathbf{2}^2 = 12$ quotiënt **3**
		3	**6**								
		1	**3**	**7**							
			5	**4**							$3 \times \mathbf{2} \times \mathbf{3}^2$
			8	**3**	**7**						
				2	**7**						$\mathbf{3}^3$
1 5 8 7			**8**	**1**	**0**	**8**			]	**5**	$3 \times \mathbf{23}^2 = 1587$ quotiënt **5**
			7	**9**	**3**	**5**					
				1	**7**	**3**	**7**				
				1	**7**	**2**	**5**				$3 \times \mathbf{23} \times \mathbf{5}^2$
						1	**2**	**5**			
						1	**2**	**5**			$\mathbf{5}^3$
								0			

$$\sqrt[3]{12977875} = 235$$