From Exotic Options to Exotic Underlyings: Electricity, Weather and Catastrophe Derivatives

Dr. Svetlana Borovkova

Vrije Universiteit Amsterdam

History of derivatives

Derivative: a financial contract whose value is derived from some other financial instrument (stock, index, commodity, exchange rate, bond, ...) $\longrightarrow$ *underlying*

THE KEY EVENT IN DERIVATIVES: Black-Scholes (and Merton) formula for the price of an option, discovered in 1970, published in 1973, Nobel prize for economics 1997 !

Trading in derivatives started May 16, 1972, on Chicago Board of Trade (CBoT)

- 1987: 1.1 trillion USD (10^{12})
- 1994: 20 trillion USD
- 1998: 33 trillion USD
- 2000: 98 trillion USD
- 2006: 270 (!) trillion USD

Financial Times, January 17, 2007: "... can (derivatives) market continue its monumental growth? Most (analysts) not only think it can, but believe it absolutely will".

Classic derivatives: plain vanilla options

European option: the right to buy (*call*) or sell (*put*) a financial instrument, e.g. a stock (*underlying asset*) on a specified maturity date T, at a specified strike price X.

Payoff of a call option:

$$c(T) = (S(T) - X)^+,$$

where $S(T)$ is the stock price on the date T.

An American option can be exercised anytime before the maturity date T.

These are the so-called *plain vanilla options*.

Black-Scholes option valuation

Main assumption: stock price $S(t)$ follows a Geometric Brownian motion: (picture)

$$\frac{dS(t)}{S(t)} = \mu dt + \sigma dW(t).$$

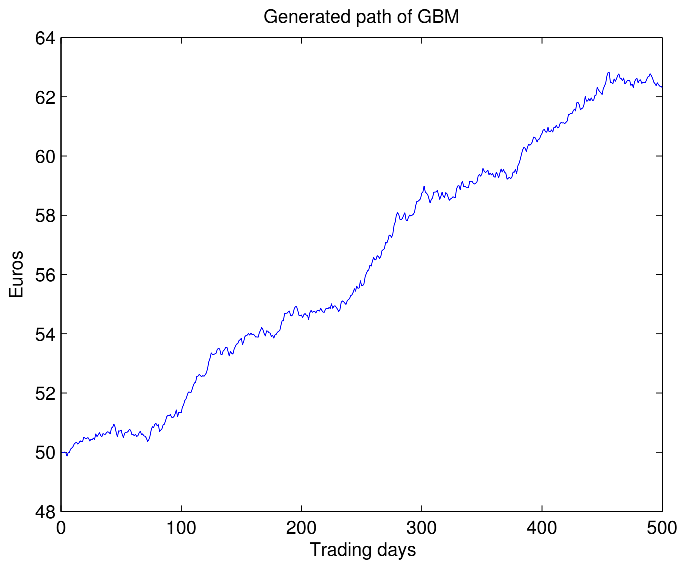
Discrete-time version:

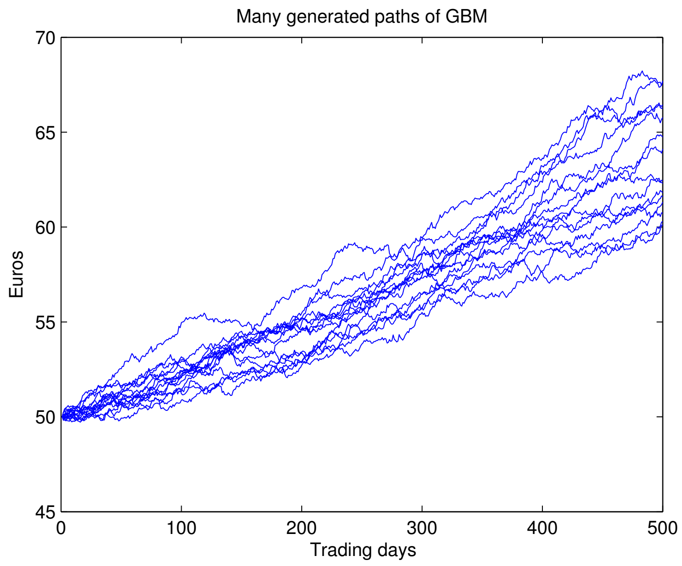
$$\frac{S(t + \Delta t) - S(t)}{S(t)} = \mu \Delta t + \sigma \times N(0, \Delta t),$$

so stock returns are normally distributed, and the price itself is lognormally distributed.

The key ingredient of Black-Scholes option valuation: the risk-neutrality argument, used for construction of a replicating portfolio:
A portfolio that, at any time, consists of a call option and an appropriate amount of stocks is, on expiry date, exactly equal in value to the option's payoff ! (example)

Stock price paths





Risk-neutral valuation

Call option price can be expressed as the expected (discounted) payoff under the risk-neutral probability measure Q:

$$c = e^{-rT} E_Q (S(T) - X)^+$$

Under such risk-adjusted probability measure, the rate of return on a stock is equal to the risk-free interest rate r:

$$\frac{dS(t)}{S(t)} = r dt + \sigma dW(t)$$

.

Mathematical tools: binomial trees, Itô calculus, martingale theory, change of measure, Radon-Nikodym derivative, Girsanov theorem.

Extensions of classical setup

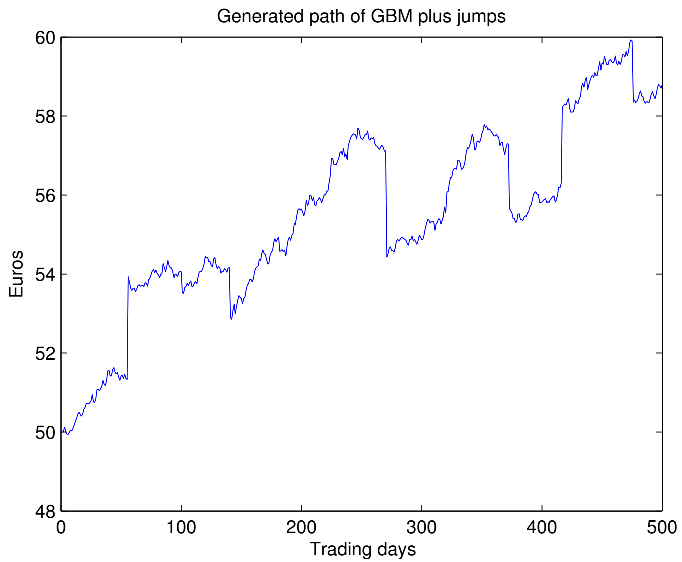
Extensions of the celebrated Black-Scholes formula:

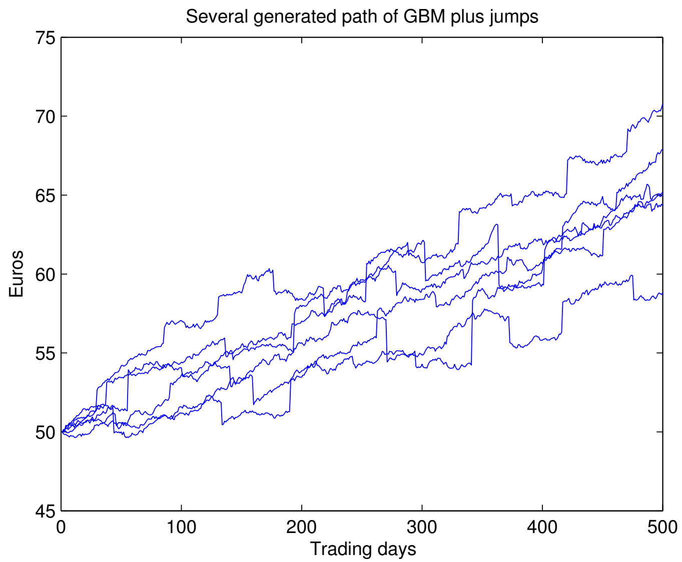
I. Replacing classic option payoff $(S(T) - X)^+$ by a more complicated ("exotic") payoff, which depends not only on the stock price at maturity date T but the entire stock price path during the lifetime of the option $[0, T]$.

II. More sophisticated (and more realistic!) processes for the asset price, e.g. those incorporating price jumps. picture

III. Underlying asset is not a stock or index, but a commodity (gold, oil, agricultural products), electricity, credit, house, weather or insurance (against catastrophic events) $\longrightarrow$ *"exotic underlying"*.

Stock price paths with jumps





Exotic options: Asian options

The payoff is

$$(A(T) - X)^+,$$

where $A(T)$ is the arithmetic average of daily stock prices during the lifetime of the option $[0, T]$ - very widely used options, especially in commodity markets!

Main difficulty: the main assumption of Black-Scholes model is that the stock price has a lognormal distribution, but the sum of lognormal random variables is not lognormal!

<u>What to do?</u>

- Replace arithmetic average by geometric average - the product of lognormal random variables is again lognormal!
- Assume the arithmetic average is lognormal, and match first few moments.
- Run a Monte Carlo simulation, in the risk-neutral world!
- An exact solution involves sophisticated mathematical tools: Laplace transform of the call price with respect to maturity.

Other exotic options

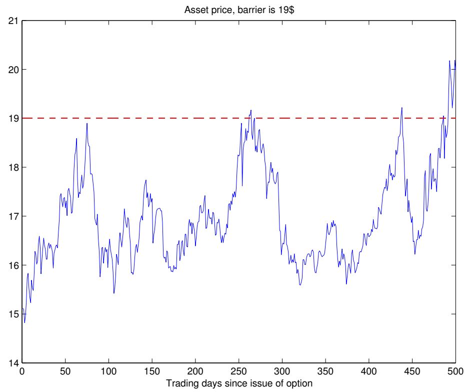
• **Barrier options**: provide the classical payoff $(S(T) - X)^+$ only if the asset price crossed (or not crossed) a pre-specified barrier B over the lifetime of the option.
Can be: "up-and-in", "down-and-in", "up-and-out", "down-and-out" $\longrightarrow$ clickfondsen.
(picture)

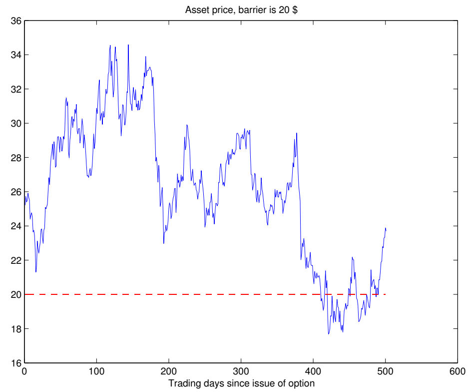
• **Bermudan options**: can be exercised at any of the N given dates $\longrightarrow$ *"between" American and European options*

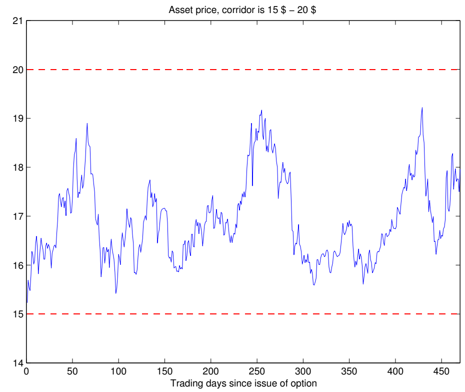
• More exotic options: Russian options, Parisian options, basket options, swaptions, quanto's, volumetric (swing) options, ...

Barrier options

Upper, lower and corridor barriers







Difficulties and mathematical tools for exotic options

• Non-lognormality of the underlying value (Asian, basket options and quanto's)
• Conditioning on some event(s) (barrier, double barrier)
• Optimization strategies and optimal stopping involved in American, Bermudan, swing and volumetric options

Tools available:

• **Risk-neutral valuation**: the option price = expected discounted payoff under the risk-neutral probability measure Q:

$$c(0) = e^{-rT} E_Q(\text{payoff})$$

- sometimes (rarely) the solution can be expressed in a closed form formula, most often it involves numerical evaluation of an integral.

Mathematical tools for exotic options

- **Monte Carlo simulations**:

- a large number of price paths are generated under the risk-neutral probability measure Q
- these are used to compute the option's payoffs $c_i(T)$
- law of large numbers assures that the average payoff converges to the expected payoff under Q:

$$\bar{c}(T) = \frac{1}{M} \sum_{i=1}^{M} c_i(T) \longrightarrow E_Q(\text{payoff})$$

- discounted sample average gives the option price:

$$\hat{c}(0) = e^{-rT} \bar{c}(T).$$

Commodity derivatives

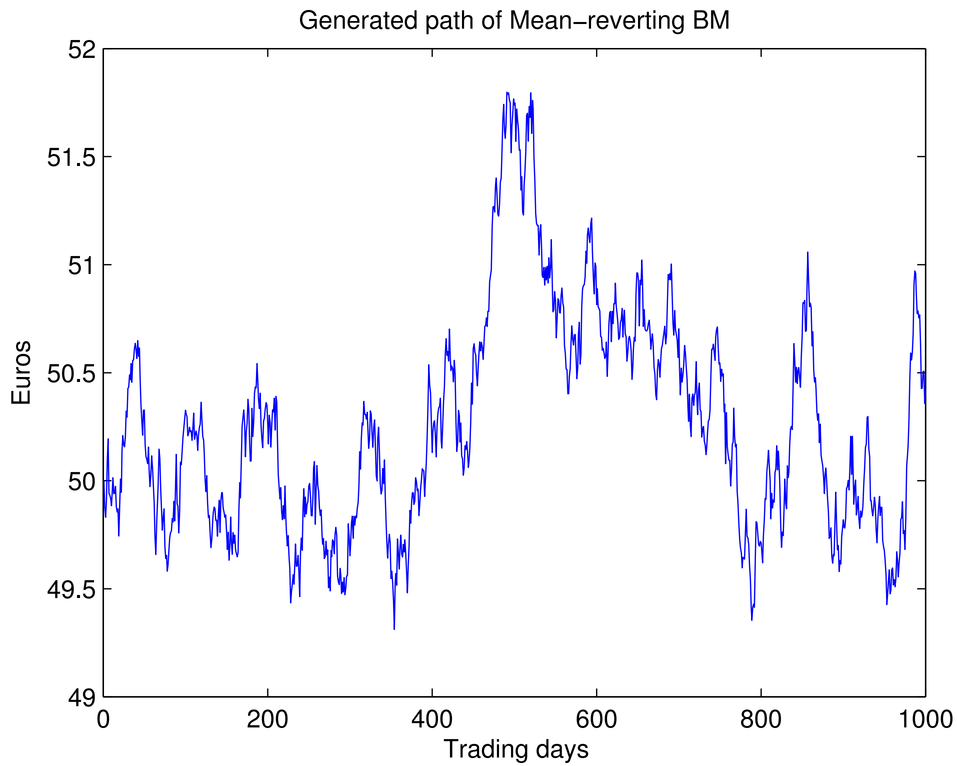
Underlying asset: not stock or index but **metals** (gold, aluminium), **energy** (oil, gas) or **agricultural product** (wheat, soya, coffee, orange juice, pork bellies).

Main differences:

• Underlying asset price is NO LONGER GBM, but can have (picture)
- seasonalities
- mean-reversion
- price jumps

• We cannot costlessly hold a commodity until option's maturity (either must pay storage costs or completely impossible (agricultural commodities)).

Mean-reverting diffusion process (model for e.g. oil price)



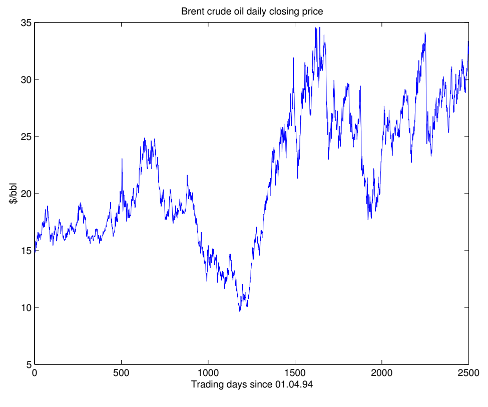
Commodity prices

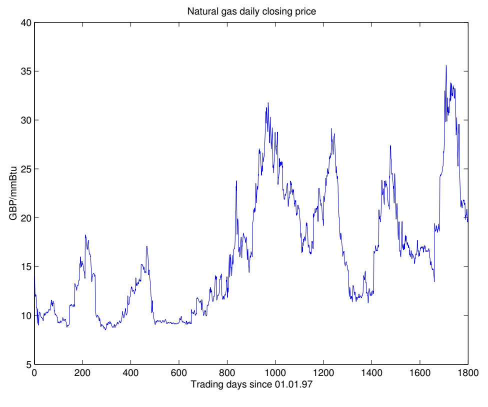
Crude oil, Natural Gas and Soybean

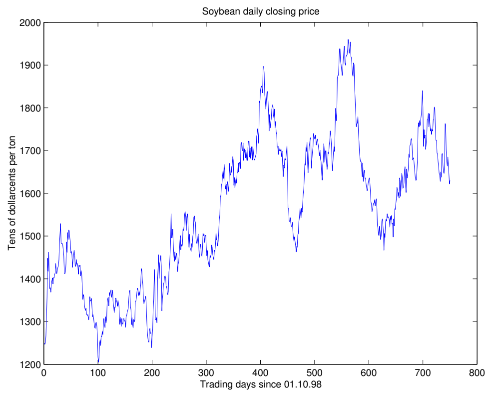
Crude oil: April 1994 - May 2004

Natural Gas: January 1997 - April 2004

Soybean: October 1998 - October 2001







Exotic underlyings: Electricity

Leap in difficulty: totally different CLASS of markets and derivatives: *exotic underlyings*.

Newly liberalized electricity markets, where electricity is traded as any other commodity.

US: PJM (Pennsylvania-New Jersey-Maryland), COB (California-Oregon Border)
Europe: Nordpool (Scandinavia), EEX (Germany), APX (Netherlands), UKPX (UK)
in the next few years also Italy, France, Belgium, ...

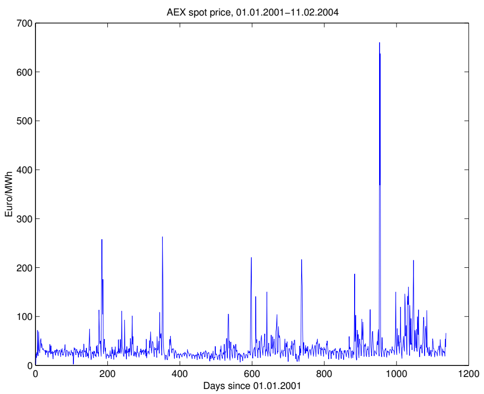
BUT: Electricity is a totally new type of commodity! (picture)

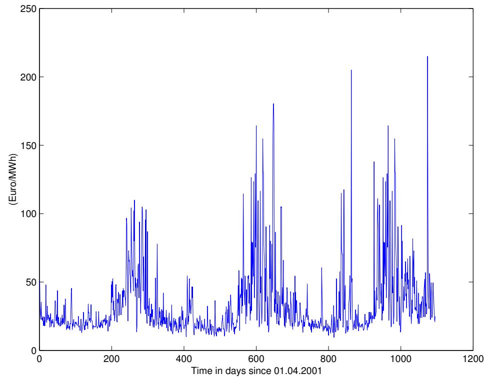
- seasonality
- high volatility
- non-elasticity of demand $\Longrightarrow$ **price spikes**
- limited transportability
- **non-storability!**

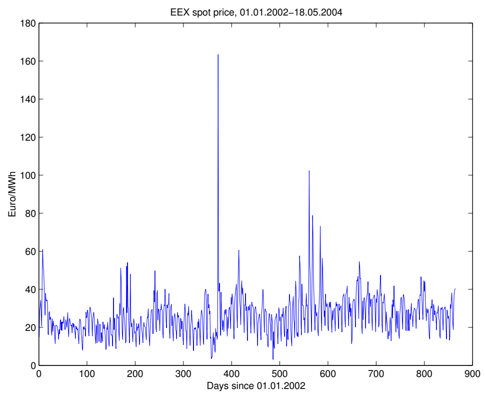
Three major European power exchanges:
APX, UKPX and EEX

APX: Amsterdam Power Exchange
UKPX: UK Power Exchange, London
EEX: European Energy Exchange, Leipzig, Germany

All prices for 2001-2004:

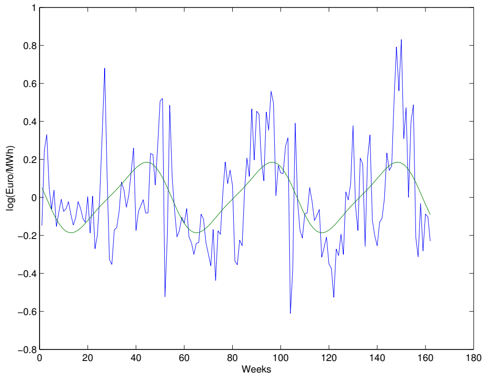


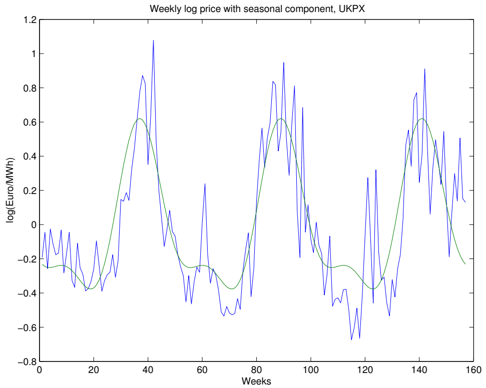


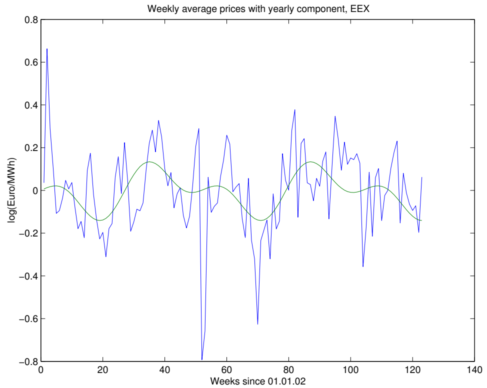


Yearly seasonalities

The yearly seasonal component:
$f(t) = \sum_{k=1}^{2}(A_k \sin(2\pi kt) + B_k \cos(2\pi kt))$

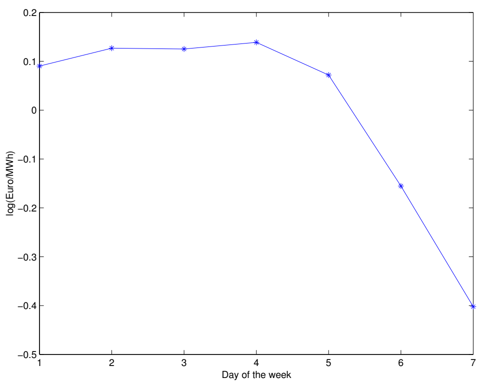


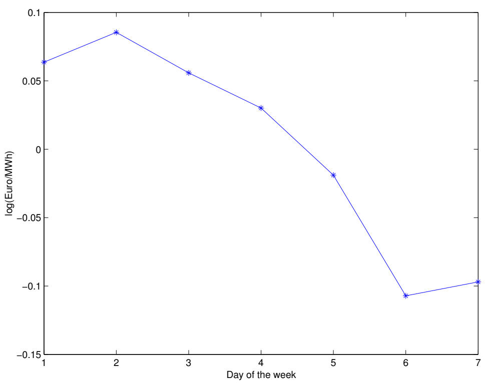


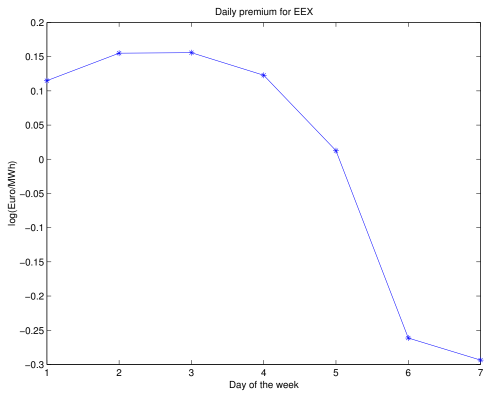


Weekly pattern

Here we plotted price premia corresponding to a particular weekday, starting on Monday







Electricity derivatives (cont'd)

Main problems:

- Realistic models for electricity price is needed.
- Option replication is impossible because electricity cannot be stored!
- Other, new types of options: volumetric options, swing options, flexible supply contracts...

New tools:

- Levy processes (pure jump processes), regime switching models, jump diffusions;
- Risk management with natural gas and weather derivatives;
- Power plants as real options.

Catastrophe (insurance) derivatives

Before 1993: reinsurance.
December 1993: introduction of catastrophe insurance futures and options (CAT) on Chicago Board of Trade.

- The payoff of a CAT derivative is paid if a there was a large amount in insurance claims in a certain area, over a certain period.

- This happens in case of a catastrophic event, such as a hurricane, tornado or an earthquake.

- The payoff is based on the **PCS (Property Claim Service) Index**.

Use of CAT derivatives

- Insurer will buy CAT futures or CAT call options.

- Sellers of CAT derivatives: construction companies, reinsurance companies, speculators willing to take risk for profits.

CAT derivatives = perfect diversification instrument, the so-called **zero-beta assets**: low correlation to financial markets, investors willing to diversify their portfolios will buy/sell CAT futures and options.

Mathematical difficulties: seasonalities, spikes in case of a catastrophic event, no tradable underlying value, insurance versus financial valuation ...

Weather derivatives

Underlying value - any measurable weather factor: temperature, precipitation, snowfall, ...

Most popular: measures of temperature closely reflecting energy demand:
HDD (heating degree days) and CDD (cooling degree days):

$$HDD(\text{day } t) = \max(18^o C - AVT(t), 0); \quad CDD(\text{day } t) = \max(AVT(t) - 18^o C, 0),$$

$AVT(t)$ is the average temperature on the day t.

• HDDs/CDDs are summed over a period
• The term of a contract may be a full year of a season:
- "Heating": November-March
- "Cooling": May-September
• The payoff depends on "strike" and the number of HDD's or CDD's exceeding the strike times a nominal amount.

Using and valuing weather derivatives

Users: Energy, Agriculture, Construction, Tourism, Leisure, Transport, Retail, ...

Fundamental difficulty: the underlying asset (e.g. temperature) is NOT TRADED $\Longrightarrow$ options cannot be hedged, i.e. replicated with the underlying asset.

Two existing approaches:

• Actuarial, or insurance method: uses historical statistical distributions of the weather variable $\longrightarrow$ requires a large diversified weather derivatives portfolio, plus extensive historical weather databases are needed.

• Financial option theory: more in line with financial markets, but the underlying asset is not traded, so option replication does not hold!

Weather insurance: low probability, high risk events (e.g. avalanche destroying a skiing resort).

Weather derivatives: high probability, lower risk events (e.g. no snow, such as this winter (2006-2007) $\Longrightarrow$ low or no profits for a skiing resort).

Conclusions

• A single mathematical development (Black-Scholes option pricing theory) solely gave rise to an entire multi-trillion finance industry of derivatives!

• Sophisticated mathematical tools are needed to deal with exotic derivatives, realistic asset price models, exotic underlyings.

• New classes of derivatives are growing and establishing their importance in enterprize-wide risk management and in the financial marketplace.

• "Bermuda triangle" is formed by the energy, weather and insurance derivatives.