

## Tell me what you are doing - discussions with teachers and children<sup>\*)</sup>

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There are few places in the world where people are satisfied with the mathematical understanding displayed by their children, whatever the criteria of success dictated by either the needs or the educational philosophy of the country. The fault may lie in the nature of mathematics (or the nature of the mathematics we choose to teach) or in the methods we use to teach it. I suggest however that unless we can somehow match the mathematics to the child very soon, we are in grave danger of losing mathematics as a school subject, accepted by educators as a necessary part of every child's education. It may be replaced by social arithmetic and the use of the calculator for the majority of children with mathematics reserved for an elite few, as the study of Greek is today in Britain. Some would consider this a sensible step but if one's philosophy of education includes the desirability of giving all children access to their cultural heritage and the products of man's rational nature then the suggestion is to be deplored. The only alternative therefore is to reconsider the way we teach the subject and how we select material of a suitable level for the pupil. The suitability depends to a large extent on the level of knowledge already possessed by the child and a discussion of the methods we might use to discover this level form the major part of this paper.

### The child's level of knowledge

The means by which we ascertain what children understand seem to form a hierarchy of respectability in the minds of parents, employers and educators. Test papers which are published and therefore cost money are deemed superior to those written by the class teacher. Printed matter is in its very nature thought superior to oral communication although it is this last that adults rely on in their ordinary daily

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life. A growing body of research is now based on data obtained from interviews although this method of assessment is less popular with teachers. There are a number of drawbacks to the interview method, as we are reminded by Carpenter, Blume, Herbert, Anick and Pimm (1982) in their Review of Research on Addition and Subtraction.

Opper (1977) pointed out some of the procedural difficulties associated with the individual interview method. Among these were (a) the possibility that the child would not be at ease and perform naturally in the course of dialogue with the interviewer, (b) the problem of the interviewer maintaining neutrality and avoiding attempts to elicit 'correct' answers, (c) the misunderstanding of language not adjusted to the child's level, (d) insufficient time for the child to reflect on the problem and to develop his/her explanations, and (e) the interviewer's interpretation of the child's actions and responses on which subsequent questions are based.

One of the most serious problems with interview data is that children's explanations of how they solved a problem may not accurately reflect the processes that they actually used. The interview procedure may change how a child solves a problem, or children may have difficulty articulating the process that they really used and therefore describe another process that is easier to explain. Or they may try and second guess what they think the interviewer is looking for. Another serious problem is that the inferences drawn from an interview involve a great deal of subjective judgment on the part of the experimenter (pg.54).

Although the list is daunting, the objections can all be made equally against the method of assessment we have been using for many years - lists of computations to be completed in a fixed time in a fairly hostile atmosphere. The greatest disadvantage of a research methodology based on the acquisition of interview data is perhaps the amount of time that needs to be spent in order to truly listen to a child and then to transcribe the interview. It is this perhaps which leads researchers to limit the type of question discussed and the number of children interviewed or indeed to try to shorten the whole process by providing 'interview' booklets which can be completed by the child with a pencil and then 'marked' by the researcher. If we limit the means by which the child can answer we limit the richness of our data. To find out what children understand we must take into account what they 'get wrong' and why they fail to function adequately in certain areas. To do this we need to provide an opportunity for the child to convey his thoughts to us.

We would hope to influence teachers with our research so we must try to design research that they will believe. This means that our interviews should be so structured that the teacher can replicate the type of discussion and so verify the results we have obtained, besides possibly adding to his own repertoire of teaching skills by using interview techniques. Teachers are engaged in working with groups of children and a class is seen as needing something in common not as a set of individual needs. The identification of a learning difficulty which is common to many is of more interest to teachers than the pinpointing of a unique situation, so anecdotal data which might relate to only one or two special cases is of less value. To influence the practice of teaching, researchers must provide information to which teachers can relate and act upon.

### Research using interviews

The research carried out at Chelsea College over the last nine years has been focussed on classroom practice and designed to give information to teachers. We have employed interviews extensively besides collecting data in the more formal atmosphere of mathematical tests. The work of the Concepts in Secondary Mathematics and Science Project (Hart, 1981) gave us a crude picture of which aspects in eleven different mathematical topics were easy for secondary age children and which more difficult. The methodology involved the use of word problems firstly in an interview schedule and then in paper and pencil format. The latter enabled us to obtain a broad view of the mathematical performance of children in the 11-16 age range, whilst the 300 interviews enabled us to put forward tentative reasons for the different levels of difficulty.

From our testing ( $n = 10,000$  children aged 11-16) it seemed that at least half our secondary population was restricted to the ability to solve items which required at most two steps for solution, largely involved whole numbers and could be completed nearly always by the use of the operations of addition or counting. Counting is one of the most primitive methods for solving arithmetic problems although for young children it has the advantage of rhythmic naming encouraged by parents and grandparents and the satisfaction obtained from tapping with a finger. Easley (1982), impressed by the mathematical attainment in one Japanese school, suggests that one reason for the greater number dexterity exhibited by the children there was the absence of counting in Grade One. The children in Kitamaeno School did very little counting and concentrated on partitioning and regrouping.

With American teachers in third grade wondering, 'How can I get the children to stop counting?' it was impressive to see that counting was not necessary. Counting is a terribly inefficient method but

it's the foundation of our curriculum. So we wanted to see a curriculum in action where counting is not so central. Counting is one of those procedures which is very useful at times, but like any procedure, will not get you what you want, if you want mathematical thinking and greater confidence in tackling problems. As we have seen, a set of counting algorithms which children learn sets the tone, the pace, the attitude, from the first grade (pg.23).

The fact that adolescents of 15 years were using counting, was found from asking them how they attempted to solve the problems, we would not necessarily have discovered this except through interviews.

#### Alternative Frameworks

There is a growing body of research in both mathematics and science education which illustrates how children are employing strategies which are not teacher taught and which were unrecognised before children were asked to explain what they were doing. One recent statement on alternative methods was made by Collis and Romberg (1981), who reported on the rules used by children aged from four to eight years (of different 'Cognitive Processing Capabilities' - CPC) when faced with addition and subtraction problems:

Children at all CPC levels use the taught algorithm infrequently, between one-fifth and one-fourth of the number of times when it is appropriate. They appear to prefer to fall back on more 'primitive' strategies such as counting which they have used successfully previously ..... It is of interest to note that when the children cease to use inappropriate strategies they do not, in the main, turn to the algorithm which has been taught as the appropriate strategy. In fact, for this population, the use of the algorithm does not increase significantly with increasing CPC level (pg.140).

The CSMS interviews tended to show that secondary school children were employing strategies in mathematics which were adequate for some of the questions they were asked to solve but unlike the algorithms they had been taught, they were not generalisable. For example, a procedure of repeated halving is adequate for dealing with ratios 3:2, 5:2 but does not generalise to finding an enlargement in the ratio 5:3. Most secondary school mathematics is concerned with formalisation and generalisation and if the child is to succeed he must 'play this game'. Vergnaud (1983) states:

The formation of a concept, especially when you look at it through problem-solving behaviour, covers a long period of time, with many interactions and many decalages. One may not be able to understand

what a 15-year-old does, if one does not know the primitive conceptions shaped in his mind when he was 8 or 9, or even 4 or 5, and the different steps by which these conceptions have been transformed into a mixture of definitions and interpretations. It is a fact that students try to make new situations and new concepts meaningful to themselves by applying and adapting their former conceptions (pg.17).

To this I would add - sometimes the adaptation is a renewed allegiance to a naive version with which they feel comfortable and confident. The non taught mathematical methods used by children well into their secondary schooling may be 'naive' strategies they learned when young and which are assumed by teachers and textbooks to have been replaced, or they may be invented or 'common sense' techniques. As part of the research carried out for the British government report on mathematics education (Committee of Enquiry, 1982), Fitzgerald (University of Bath, 1981) and Sewell (1981) showed that adults seldom used taught algorithms in the mathematics they used in everyday life but they cope by using their own non-standard methods.

The notation of fractions appears in some clerical and retail jobs, for instance  $4 \frac{3}{7}$  to represent 4 weeks and 3 days or  $2 \frac{5}{12}$  to represent 2 dozens and 5 singles. However, school-type manipulation is rarely found and then only in very simple cases; for instance, the calculation required to find the charge for 3 days based on a weekly rate is division by 7 followed by multiplication by 3 (pg.22).

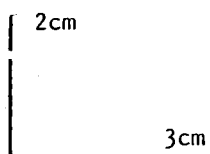
The needs of the working man and woman of 1982 should not be the guiding principles by which we decide on the mathematics to be taught to a child who will live his life in the 21st century, so although such common sense methods prove adequate for many mathematical exercises we require children to complete, they prove inadequate if the problems are complex or involve non integers and in these circumstances often lead to error.

### Errors

The CSMS data revealed that certain items on individual test papers produced the same wrong answer very often (40-50 per cent level). These errors were not restricted to particular schools or text book use and seemed worthy of further investigation. In 1980 the SSRC financed a further project at Chelsea called Strategies and Errors in Secondary Mathematics (SESM). The aims of this research were to investigate the identified errors more deeply and to try some remediation. We took errors in the topics of Ratio and Proportion,

Algebra, Fractions, Measurement and Graphs and based our initial work on interviews with children who had committed the errors in which we were interested. The CSMS word problems formed the first interview schedule and we attempted to find the methods children used correctly as well as the reasons for the later incorrect answers. The underlying rationale was that consistent errors in solving a particular type of problem were indicative of a mode of thinking and not just an example of a momentary lapse in concentration. About 60 children were interviewed in each topic investigation. The methodology is illustrated by examples from the Ratio and Proportion investigation. The error being investigated in this topic was the incorrect addition strategy (Piaget and Inhelder, 1956; Karplus, Karplus, Formisano and Paulsen, 1975) in which one enlarges a diagram by adding an amount as is shown in figure 1.

Enlargement of the figure on  
the left. New base of 5 cm



	5cm		
Age	13 (n=800)	14 (n=767)	15 (n=690)
	7.9	11.0	19.7
	47.6	39.4	39.7

per cent

Figure 1 The Incorrect Addition Strategy

The reasoning of children giving the answer '4' is that 3 cm in the smaller diagram became 5 cm in the larger by the addition of 2 cm, so the new upright must also be found by adding 2 cm. This same argument was put forward by those interviewed in the SESM research when the dimensions were changed so that (i) the base lines were 3 cm and 10 cm and (ii) the figures were triangles. The 'adders' who used this method on three-four of the hardest CSMS enlargement questions, could recognise when figures were not similar. Indeed when asked to draw the triangle resultant from the addition of 7 cm to the height, their comments showed their dissatisfaction with the new shape:

"It's too long"  
"It's too steep"  
"Looks more tilted"  
"Not as sloped"  
"They're not the same"  
"It's close"

The interview sample was taken from classes described as 'average' by their teachers; they had been taught some aspects of Ratio and Proportion and were not regarded as in need of remedial help. 'Adders' throughout the CSMS survey had proved to be children who could cope with a number of the CSMS items. They could for example enlarge a diagram in the ratio 2:1 and solve questions about a recipe in which the ingredients for eight people were given and those for four and six people were required (as long as fraction computation was not involved, i.e. 1/2 pint for 8 people. How much for 6?). The new SESM interview sample similarly gave answers of this type. In previous papers (Hart, 1981) I have referred to the correct but naive use of addition as 'building-up' to an answer. In the recipe question the ingredients for six people were found by adding the amount for four persons (the operation of halving) to the amount for two people (halving again). The 'adders' success then, was imbedded in these naive methods in which repeated addition replaced multiplication and fractions (other than '1/2') were avoided. It was a natural step to seek an additive method for solving the harder items in which the fraction element played a larger part. Fischbein (1983) describing some of the CSMS results comments

And this is not only because the notion of multiplication is, intuitively, related to a magnifying effect, but also because the operation of multiplying by a fraction has no intuitive meaning at all' Multiplying  $\frac{2}{3}$  (as a magnitude) by 6 (as a non-dimensional operator) means, intuitively,  $\frac{2}{3} + \frac{2}{3} + \frac{2}{3} \dots$ . What is the intuitive meaning of multiplying 6 (as a magnitude) by  $\frac{2}{3}$  (as a non-dimensional operator)? (pg.3)

### Information for teachers

#### SESM - Ratio and Proportion

The second part of the SESM work entailed intervention and some trials of materials that teachers could later use with children in their classes. Thus, having found a number of reasons why a child was giving the wrong answer we attempted to identify a series of constructs which matched the gaps in the reasoning of the 'adders', starting with a demonstration of the outcome of the method they were using and then stressing the operation of multiplication. The methodology was thus

tailored to the error and counter to the suggestions of Gagne (1983) when he referred to rules of computation.

Hypothesis 1 is this: The effects of incorrect rules of computation, as exhibited in faulty performance, can most readily be overcome by deliberate teaching of correct rules. My interpretation of previous psychological research on 'unlearning' is that it is a matter of extinction. This means that teachers would best ignore the incorrect performances and set about as directly as possible teaching the rules for correct ones. An unpreferable alternative is to make students fully aware of the nature of their incorrect rules before going on to teach the correct ones. It seems to me this is very likely a waste of time (pg.15).

In Ratio and Proportion I sought to distinguish between a schema and concepts and the skills needed for a successful demonstration of them within school mathematics. I took as a matter of educational belief that there were children within the group of 'adders' who were 'ready' to move their level of understanding if they were only given the right information at the time appropriate for their needs. An additional condition was that ideas for remediation should be in a form that teachers could and would use.

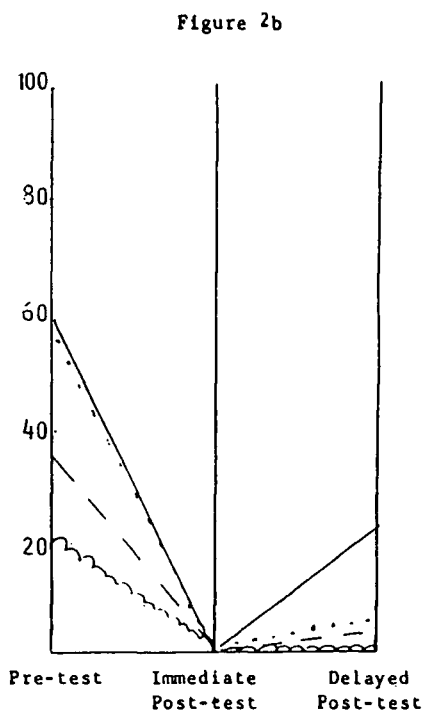
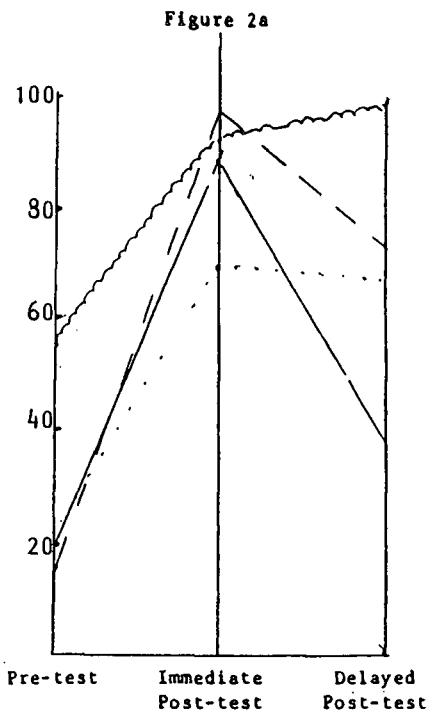
The module for Ration and Proportion addressed itself to four areas in which the adders appeared to be deficient:

- i) the recognition of the inaccuracy of the diagram resulting from the use of the incorrect addition strategy
- ii) the need to see that the crucial arithmetic operation involved was multiplication
- iii) the possession of a skill which would enable the child to multiply decimals (or fractions). This is distinguished as a separate entity from the understanding of what is needed to procure an enlargement
- iv) the possession of a method which would enable one to find a scale factor, given a dimension and its enlargement.

The module was tried with small groups of adders, then half classes and finally by teachers with classes of children whom they considered to be of 'average' ability. The results for the four teachers who used the materials for two weeks teaching are shown in figure 2. In figure 2b we can see that the incorrect addition strategy has completely disappeared at the immediate post test but some children have reverted to it by the time of the delayed post test, 11 weeks later. Of the 24 children in the four schools who would have been designated 'adders'



on the CSMS test because they used the incorrect addition strategy on three/four of the four hardest questions, 23 were no longer in this category on the delayed post test. Solving the items correctly is more difficult and as can be seen from figure 2a, although the performance of every class improved, school four's results show a sharp decline between immediate and delayed post tests.



Correct Answers  
 .... School 1  
 ---- School 3  
 ——— School 4  
 ~~~~~ School 6

Addition Strategy Answers

The percentage is obtained:  $100 \times \frac{\text{Number of answers given}}{\text{Number of answers possible}}$

Figure 2 Results for the school trial

Positive intervention by the teacher, designed to meet a specific and identified need has been shown to be effective. It would seem very often that intervention which is too general or too far removed from the misconception fails to effect any improvement.

### Implications

I have been told by various teachers that the CSMS results mean that half our population is incapable of doing secondary school mathematics; that the subject is for an elite; that primary school teachers have failed or that secondary school teachers have failed; that practical work and concrete aids/manipulatives need to be used to a much later age than is currently the practice etc. All of these are of course matters of opinion and even politics and their generality hides the subtlety of the problem of teaching mathematics to children. The statement 'he is not ready because he is at the concrete stage' can be as crude a diagnosis in its own way as 'he is 14 years of age and so should be able to do ....'. Neither necessarily describe the child and its needs. Our evidence of the existence of child-methods and alternative frameworks suggests a picture of a far from smooth transition from the state of reliance on concrete aids to 'formal' mathematics. We would agree with the statement by Driver and Easley (1978).

It has been suggested that in designing instructional sequences attention needs to be given not only to the sequence of operations or ideas to be taught but to the operations or ideas pupils tend to apply spontaneously to tasks.

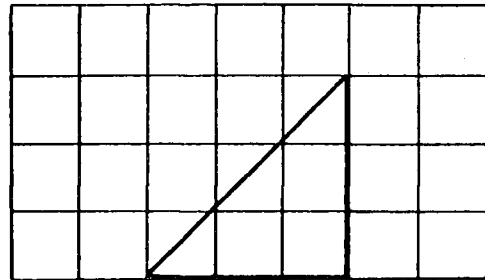
'..... until one understands what students do spontaneously one will not be able to demonstrate the limits of this approach to them' (Case 1976).

Knowledge of pupils' alternate frameworks has been used in constructing learning tasks in science (pg.78).

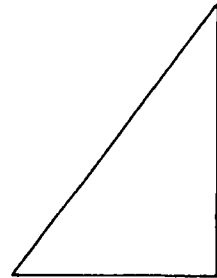
The path is littered with the jagged rocks of methods found reliable by the child when he was young and in which he still places his confidence, the methods he has invented and which cause him to be talking on a different level to his teachers and finally, the belief that it is all magic anyway. The teacher must find out the location of these rocks and without destroying them lead the child to a higher plane. The division between the higher plane or 'formal' mathematics and the level of the child may, to continue the allegory be rather a large plateau, a huge chasm or a small step, we have very little information on this. We do for example know from our CSMS data that the two questions in figure 3, given to the same children on the same day, gave very different results.

There seems to be a large gap between the difficulty of question (i) which can be solved by counting and adding and the formalisation needed for question (ii).

Find the area



|     |                 |      |        |
|-----|-----------------|------|--------|
| Age | 12              | 13   | 14 yrs |
|     | 77.5            | 86.9 | 91.4   |
|     | percent correct |      |        |



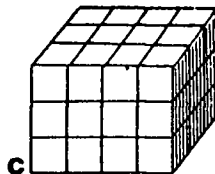
|     |                 |      |        |
|-----|-----------------|------|--------|
| Age | 12              | 13   | 14 yrs |
|     | 31.4            | 38.7 | 47.5   |
|     | percent correct |      |        |

Figure 3 CSMS area questions

### The Path of Formalisation

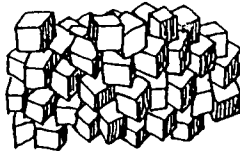
Teachers in Britain who follow the 'official line' in teaching would approach a formal mathematical statement through a wealth of practical experience for the children but there would come a time when they expected the class to deal with symbols and abstraction without the concrete aids. At Chelsea we are currently looking at the transition stage between practical experience and formal mathematics for children aged 8-13. The step from one to the other may be too great and experiences which provide a bridge between them may be called for. I give as an anecdotal illustration a conversation with a ten year old, Paul, who has just spent a month doing practical work relating to volume prior to learning that the volume of a cuboid is given by ' $V = l \times b \times h$ '. The problem used is the well known Piagetian example of decomposition of a cuboid as shown in figure 4.

Block 'C' is made by putting some small cubes together:

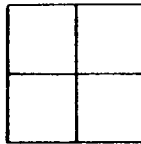


How many cubes make this block 'C' if there are no gaps inside?....

c) All the cubes from block 'C' are put in a pile:



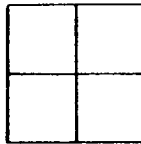
I am now going to use all these cubes from block 'C' to build a 'sky-scraper' so that the bottom floor is 4 cubes



How many cubes high would this 'sky-scraper' be from the ground?..

Figure 4 Volume questions used on interview

(Interviewer: I - Paul: P)

I: I have a picture here. That is made like that (showing cuboid) - that is just the picture. Then I tear it all to pieces, to get these bricks like that. Then I build it up again. This time I build it up on top of that (pointing to ). How high would it go? (Repeats briefly).

P: Can I do it any way, but just find the answer?

I: I would prefer you not to break it up but if you need to break it up, well go ahead. Can you do it without breaking it up?

P: It's 3 fours in each layer.

I: Good

P: So, 1, 2, ... that makes 4 there. Another ... (long pause)

I: Can I tell you what you said when you worked that out? You said three fours. Then you wrote that

|   |   |
|---|---|
| 3 | 4 |
| 3 | 4 |
| 3 | 4 |

I: Then you said three fours and you wrote that. What are you going to do ?

P: Just ... it's 12 isn't it ?

I: Twelve Fours ?

P: Twelve into ...

- I: Twelve is what ... ? Is it all the lot. Or is it a layer? Or is it three fours ? Or what ?
- P: It's ... kind of how many fours is three times 12 fours.
- I: Could you show me 12 fours, on that, you see that block there. Where are the fours.
- P: Do you want me to break it up ?
- I: No, you didn't break it up to get to that answer. Did you?
- P: There's a four here. You said 3 up, to make it how high.
- I: Are you giving me the answer then, 12.
- P: Yes
- I: So, is it 12 high ?
- P: Yes
- I: I don't understand. That is the drawback. Do you think you could explain to me very slowly, what you were doing.
- P: Alright.
- I: I found that bit I thought you said three fours and that was a layer, was it not?
- P: This is four, four and four, so it's three. Three ...
- I: I see, three little bits of four
- P: Yes ... three time it - sounds a little bit like a table
- I: That is what I thought you were doing, you see. But you are not. There are 3 fours in that top layer, I agree.
- P: Then there's another four and another four. That's what I've written here and that all comes up to 12.
- I: Twelve fours ?
- P: Yes.
- I: So I see
- P: And that's the height.
- I: Splendid. Now I understand. So that is the height of our skyscraper?
- P: Yes ... Do you understand now ?
- I: Yes ... I'm not sure I agree but I understand.

Paul has not moved from counting cubes to multiplication but to a collection of fours ( $\begin{array}{|c|c|} \hline \square & \square \\ \hline \square & \square \\ \hline \end{array}$ ), he is not at a formalisation stage but not completely tied to the concrete either.

The research design of 'Children's Mathematical Frameworks' involves the teacher in writing a scheme of work and justifying it to us. Then we interview six children in the class just prior to the lesson(s) involving the formalisation, to try to ascertain the nature of the pre-requisite knowledge (identified in the scheme of work) that they possess. We listen to and tape record the lesson(s) when the teacher formalises the idea and then we talk to the six children again, immediately and after three months have elapsed in order to see whether they utilise the formalisation and then how the nature of

their understanding has changed in the intervening period. From this we hope to obtain evidence which sheds light on the source of a number of misconceptions and child methods shown by secondary school children.

### Children and teachers

The case has been made that interviews with children enable us to discover considerably more about the nature of the child's understanding than we could from his performance on a pencil and paper test. Let us extend this idea to include other types of verbal interchange.

### *Discussion and Debate*

Adults place particular regard on oral communication; we all attend a conference because although we can read the papers we value the opportunity to hear people and to ask them questions.

We seem not to think children might benefit from the same type of interchange. I suggest that children are expected to ask only the questions the teacher has planned for and to which the answer is known.

We tend to think that we are protecting children when we omit the awkward example or slide over the less than satisfactory explanation. It is thought children are less confused if many of the complexities of a situation are disguised. For example in  $lxbxh$ , is  $l$  sometimes  $b$  and sometimes  $h$  or is the label permanently attached to a side? We essentially leave the child to discover these inconsistencies when he is alone. We tend to think that children's opinions are not worth heeding and although we may sometimes listen to a child explanation, we quickly interject with our interpretation of the situation. I suggest in fact we should make a positive and penetrating effort to find out what the child is doing, as Bauersfeld (1978) said.

Erlwanger's case studies are related to programs from individual Prescribed Instruction. His documentation of students' mathematical misconceptions and deficiencies demonstrate how mathematics learning can be damaged by restricted teacher-student communication - a restriction which leads to the nearly total absence of negotiations over meanings. (pg.5)

Teaching and learning mathematics is realised through human interaction. It is a kind of mutual influencing, an interdependence of the actions of both teacher and student on many levels. It is not a unilateral sender-receiver relation. Inevitably the student's initial meeting with mathematics is mediated through parents, playmates, teachers. The student's reconstruction of mathematical

meaning is a construction via social negotiation about what is meant and about which performance of meaning gets the teacher's (or the peer's) sanction (pg.19).

The effectiveness of teachers is closely linked to amount of developmental work (as opposed to practice) they are prepared to introduce into their lessons, Grouws (1982).

Development is a part of most mathematics lessons in which the teacher actively interacts with pupils. In very general terms, development can be thought of as that part of the lesson devoted to the meaningful acquisition of mathematical ideas, in contrast to other parts of the lesson such as practice or review (pg.5).

If development is viewed in a global way, then teachers' attention to the meaning of material presented is vitally important, and development must be understood in terms of promoting student thought. Future classroom research must study more extensively the content that is presented to students rather than instructional time per se (pg.17).

Both of these aspects can be achieved, I suggest, if one involves the children in a discussion of the topic under consideration. Easley (1982) after his research in Japan has been trying to encourage some Illinois teachers to build-in debate and discussion as part of their normal teaching.

In Kitamaeno School, we had seen teachers urging students to find as many methods as possible, and urging students to learn several methods for each kind of problem. This approach contrasts with that of the teachers we were working with, who would feel happy if they could teach one method well for each kind of problem. (pg.137)

Some further research carried out by Malvern and Bentley (1982) adds weight to the argument that verbal communication between teacher and child could be a rewarding experience for both.

The aim of the Project was to augment the routine class teaching of six to eight years old with some extra work in mathematics, increasing the scope for discussion, and paying particular attention to the use of appropriate, simple, unambiguous language. The purpose of the evaluation was to ascertain if any measureable improvement in mathematics was achieved. To provide the extra teaching six primary teachers were seconded to the Project for two years. All six were experienced teachers but none was particularly specialised in mathematics before the Project began (pg.1).

On average an extra six hundred minutes mathematics teaching was provided by the Project teachers to each infant pupil and six hundred and twenty-five minutes to each junior pupil. Again on average this means each pupil received 17 or 18 minutes per week extra mathematics taught in a small group of between 3 and 5 pupils for the most part. Some pupils received much more, some much less but the bulk of the pupils received about the average. Fifteen to twenty minutes a week is a modest amount of time compared to the normal class time given over to mathematics teaching estimated at between 5 and 6 hours per week (pg.2).

We can say without equivocation that the measured achievement in mathematics was enhanced by the Project. The enhancement was large in scale, and the results overall showed a major improvement had been achieved through this work (pg.2).

The enhanced performance of the pupils is accounted for by a number of features of the project, the authors mention four, one of which is:

Small Groups and Discussion. The Project teachers worked with small groups. This allowed not only a lot more individual attention to be given to each child, but also the encouragement of discussion of mathematics. It was the Project teachers' intention to promote talk about the work among the children both to improve linguistic skills generally and to make mathematics active and lively. The small size of the groups was important in creating the circumstances where each pupil was able to contribute, as well as resulting in the teachers' guidance being quickly available immediately it was required (pg.3).

### Conclusion

In this paper I have attempted to provide an argument for research which is based on interviews by illustrating the advantages with results and insights obtained in the CSMS and SESM research at Chelsea. Children often use methods for solving mathematical problems that are far removed from the algorithms they are assumed to be utilising. The process of teaching children is very complex and we do a disservice to both teachers and children if we pretend it is straightforward. The subtlety of approach needed might best be accomplished by greater verbal interchange in the classroom, including discussion and debate. We must surely learn to change so that the future is not as David Page (1983) sadly reflected on the introduction of 'New Math' into American schools:



Teachers treated the new ideas in mathematics as they had learned to treat other educational innovations: put in the required time at the workshops and 'wear wide ties while they are in style'. Then go back to your classroom, close the door, and do what you have been doing (pg.1).

### References

- Bauersfeld, H. *Hidden dimensions in the so-called reality of a mathematics classroom*. Paper given at the Psychology of Mathematics Education Workshop, December, 1978.
- Carpenter, T.P., Blume, G., Hiebert, J., Anick, C.M. and Pimm, D. *A review of research on addition and subtraction*, Wisconsin Center for Education Research, September, 1982.
- Committee of Inquiry into the Teaching of Mathematics in Schools. *Mathematics Counts*, London: Her Majesty's Stationery Office, 1982.
- Driver, R. and Easley, J. Pupils and paradigms: A review of literature related to concept development in adolescent science students. *Studies in Science Education*, 5, p.61, 1978.
- Easley, J. and Easley, E. *Math can be natural: Kitamaeno priorities introduced to American teachers*, Bureau of Educational Research, Collene of Education, University of Illinois, At Urbana-Champaign, October, 1982.
- Fischbein, E. *Intuition and logic in mathematical education*. Paper given at Trento, Italy, January, 1983.
- Gagne, R.M. Some issues in the psychology of mathematics instruction, *Journal for Research in Mathematics Education*, 14, 1983.
- Hart, K. *Children's understanding of mathematics: 11-16*, London: Murray, 1981.
- Hart, K. Strategies and errors in secondary mathematics - the addition strategy in ration, *Proceedings of the 5th International Conference for the Psychology of Mathematics Education*, Grenoble, 1981.
- Karplus, R., Karplus, E., Formisano, M. and Paulsen, A.C. Proportional reasoning and control of variables in seven countries. *Advancing Education Through Science Oriented Programs*, Report 10-65, June, 1975.
- Malvern, D. and Bentley, C. A report on the evaluation of mathematics achievement in the primary mathematics/language project, 1982.
- Page, D.A. Some problems and some proposed solutions or Here is one horse to ride on the merry go round's next trip. Paper given at AERA, Montreal, 1983.
- Piaget, J. and Inhelder, B. *The child's conception of space*, London: Routledge and Kegan Paul, 1967.
- Romberg, T.A. and Collis, K.F. Cognitive functioning and performance on addition and subtraction tasks. *Proceedings of the 3rd Annual Meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education*, Minneapolis, Minnesota, September, 1981.

- Sewell, B. Use of mathematics by adults in daily life. Advisory Council for Adult and Continuing Education, Leicester, England, 1981.
- University of Bath, School of Mathematics. *Mathematics in Employment*, p.16, Bath: 1981.
- Vergnaud, G. *Cognitive and developmental psychology and research in mathematics education: Some theoretical and methodological issues*, Paper given at Trento, Italy, January, 1983.