

## Students' reasoning in thermodynamics

S.Rozier & L.Viennot

L.D.P.E.S.

University of Paris 7

### Toelichting

*Dit artikel is een weergave van de voordracht die door Viennot gehouden is op het seminar "Relating macroscopic phenomena to microscopic particles: a central problem in secondary science education", dat plaats heeft gevonden in Conferentieoord Woudschoten te Zeist van 22-26 oktober 1989.*

### 1. Introduction

Thermodynamics is a subject which involves multivariable problems. The behaviour of a huge number of particles is described using a small number of variables, which are mean values or macroscopic quantities. These variables can be linked, at thermodynamic equilibrium, by certain relationships, for example  $PV=NRT$  for perfect gases. In any transformation, such relationships hold for initial and final equilibrium states. In *transformations* considered as "*quasistatic*", these relationships hold as well for any intermediate state, then also considered as equilibrium states. That is to say that we have to consider *several variables*, most of the time more than two, *changing simultaneously* under the constraint of one or several relationships.

Such a mental activity a priori raises obvious difficulties. Piaget and Inhelder (1941) have shown that children, dealing with three kinematic variables ( $s, v, t$ ), in fact consider one of these quantities as linked to a single other one: "the faster, the further". Other studies (Viennot, 1982; Maurines, 1986) show similar difficulties.

In this paper, we will illustrate, in the domain of thermodynamics, how students, and others, commonly reduce the intrinsic complexity of such problems. These tendencies towards "functional reduction" in common reasoning, will be shown to range from a simple reduction in the number of variables considered to a more elaborate procedure where all the variables are taken into account, but in a simplified way: the "linear causal reasoning".

The experimental facts supporting our analysis come from a study by S.Rozier (1987). The students in the study ( $N \approx 2000$ ) were drawn from three types of courses: one of the four first years at university of Paris 7, a selective course preparing french "grandes écoles d'ingénieurs" (two years after baccalaureat) and teachers ( $N=29$ ) in in-service training sessions. After undertaking exploratory interviews ( $N=9$ ), this study was conducted mainly on the basis of written questionnaires (14, only 4 of them are quoted here, many results being left aside for the sake of brevity). Because of the similarity of results for the different sub-samples we do not report the results for each separately. We will also quote excerpts from textbooks, popular science books and research papers in science education, as well as teachers' reactions in training sessions, in order to show to which extent and according to which modalities students' common ways of reasoning are shared by different categories of professionals in science.

The pedagogical implications finally discussed will relate mainly to our teaching goals.

## 2. Reducing the number of variables

### a) *Forgetting some of them*

A first question will illustrate students' most general and obvious tendency in coping with multivariable problems, which is to forget some relevant variables. Table I summarizes the question posed (a written test) and the most frequent response. Asked to explain in molecular terms why pressure increases in an adiabatic compression of a perfect gas, 43% students say, for instance:

"Volume decreases, therefore molecules are closer to each other, therefore there are more collisions, then pressure increases".

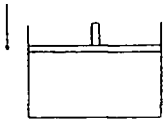
"Volume decreases, therefore there are more molecules per unit volume, then pressure increases".

These responses may be outlined in the following way:

$$" V \downarrow \rightarrow n \uparrow \rightarrow p \uparrow "$$

In these comments, an increase in pressure is ascribed only to an increase in the "number" (per unit volume, which is often implicit) or "density" of particles. Nothing is said about the

Table 1: Questions about an adiabatic compression (see Rozier, 1987), correct and typical responses

<p>QUESTION:</p> 	<p>An adiabatic compression of a perfect gas: pressure and temperature both increase. Can you explain why in terms of particles?</p>
<p>notations used below: volume of gas: <math>V</math>, number of particles per unit volume: <math>n</math>, pressure of gas: <math>p</math>, temperature of gas: <math>T</math>, mean speed of particles: <math>v</math>, mean kinetic energy of particles: <math>e_c</math>, heat: <math>Q</math>; "increases", "\-": "decreases", "\_": "is produced", "\_": "therefore" (see text)</p>	
<p><b>p</b> outlines of ....</p> <p>...correct explanation:</p> $V \setminus \left[ \begin{array}{c} n \\ \text{and} \\ v \end{array} \right] \left[ \begin{array}{c} \text{number of collisions} \\ \text{per} \dots \\ \text{and } v \end{array} \right] \rightarrow p \setminus$ <p>...common explanation:</p> $V \setminus \rightarrow n \setminus \rightarrow \text{number of collisions} \setminus \rightarrow p \setminus$	
<p><b>T</b> outlines of ....</p> <p>...correct explanation:</p> $V \setminus \rightarrow v \setminus \rightarrow e_c \setminus \rightarrow T \setminus$ <p>common explanation:</p> $V \setminus \rightarrow \text{number of collisions} \setminus \rightarrow Q \setminus \rightarrow T \setminus$	

other relevant aspect, from a kinetic point of view, i.e. the mean speed of particles (see correct answer outlined in table 1). Other questions in this study confirm this preferential link between pressure and "number of particles". In what follows, we will refer to such links as "*preferential associations*" between two variables.

Such a tendency in reasoning is not limited to students. As an example, let us quote an excerpt from a book of popular science (Maury, 1989) considered as very good by many physics university teachers (informal evaluation, in France): " Planes fly very high, at an altitude where molecules of air are much less numerous, and therefore the pressure of the external air on the

window is much lower than at sea level." This explanation may be summed up in the following way:  $n \downarrow \rightarrow p \downarrow$ , nothing being said about temperature. The same single variable dependency as in students' comments is observed, despite the fact that at the altitude considered, ( $\approx 10$  km), the temperature is much lower than at sea level ( $\approx 70^\circ\text{C}$ , i.e. a decrease of about 25% in temperature) which also contributes to the lowering of pressure. Teachers in different training sessions ( $N=55$ )<sup>1</sup> have been invited to criticize this comment. In every session, more than 95% accepted it without any modification, and when the change in temperature was pointed out, the great majority of teachers said that it was "not the important phenomenon", so it was not necessary to specify what happened to this quantity. Five pages further in the same book, the hot air balloon is presented and "explained" using the fact that when the temperature increases, it contains "less and less air". So the "number of particles ..." decreases. Yet in the hot air balloon, the pressure inside is not lower than that outside, due to temperature. No connection is made with the explanation previously proposed for low pressure outside the aircrafts.

Such ad hoc variations on the equation of state for perfect gases,  $PV=NRT$ , are typical of the inconsistencies introduced by the common tendency towards "functional reduction" and a call on preferential associations with no mention of other relevant variables.

#### *b) Combining together two variables*

Reducing the number of variables may be obtained by another process also observed in other domains (Viennot, 1989a): two physical quantities seem to be "stuck together". This is the case, for instance, for mean distance between particles and mean kinetic energy of particles (Rozier, 1987). The name frequently used for this compound notion is "thermal motion", and its cement is the idea of disorder. In fact, only one of these quantities is determined only by temperature, namely the mean kinetic energy of particles. The other is also linked with other aspects: pressure, shape of potential of interaction between particles for solids and liquids. Students' reasoning and comments in this respect will be analysed in detail in what follows. Let us start, this time, with teachers' and researchers' quotations.

In the book previously mentioned, one may read: "particles need more room to move faster". In research reports, so called "accepted ideas" often give the impression of an adherence between these two - kinetic and geometrical - aspects. For instance, about thermal expansion (Lee, e.a., 1989):

"When a substance is heated, the molecules of the substance move faster and, therefore, move faster apart, which causes the substance to expand. In contrast, when the substance is cooled, the molecules move more slowly and move closer together, so the substance contracts."

Or, still more simply, a very commonly accepted idea is that thermal motion is much higher in gases (larger mean distance between particles) than in liquids (smaller mean distance between particles), and larger in liquids than in solids. See for instance these excerpts from french textbooks or written materials at university:

"In some solids, such as glass, and many plastics, molecules are squashed against each other and cannot move" (Sciences Physiques, 1980).

"when, cooling down a liquid, particles become motionless without any order, it is an amorphous solid" (DEUG SSM, 1985). However, as said before, thermal motion, if meant as mean kinetic energy of particles, is only a matter of temperature. It is therefore the same for the water in the sea, the air just above, and a stone on the beach, in as much as they are at same temperature.

### *c) Lack of symmetry in implications*

A striking feature in the way single-variable dependencies are commonly handled is a lack of symmetry in implications. Indeed, in the accepted theory of quasistatic transformations, variations are simultaneous and therefore, the implications are symmetrical (provided that the variables which are kept constant are specified).




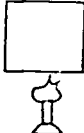
A typical example is the following: the commonly accepted implication  $V \downarrow \rightarrow p \uparrow$ , which was discussed above, seldom appears to be applied in reverse:  $p \uparrow \rightarrow V \downarrow$  (see below section 3).

This lack of symmetry may even occur in implications concerning some variables which are, most of the time, simply stuck together and therefore interchangeable in a symmetric relationship. This is the case for two variables evoked about the com-

pound notion of "thermal motion": temperature and volume. As shown further in the paper, students are familiar with the  $T \uparrow \rightarrow V \uparrow$  implication for a heated gas. But it is not so frequent at all, as classroom practice shows, to say that expanding a gas results in an increase in temperature.

Another result also suggests, although indirectly, that students would not unconditionally reverse the preceding implication. A question proposed to students in Rozier's inquiry (see table 2) presents the following situation: an equal amount of heat is transferred to two systems consisting of same numbers of particles of perfect gases at same temperature, but in vessels of different volumes. 22% of students ( $N=255$ ) or teachers ( $N=28$ ) give responses equivalent to this one: "the amount of heat is more diluted in the larger vessel, so the temperature does not increase as much as in the smaller vessel", which can be summarised by "larger volume  $\rightarrow$  smaller increase in temperature"

Table 2: A question from (Rozier, 1987) and corresponding rates of response

$(N_1, V_1, T_1)$ (1) 	$(N_2, 2V_1, T_1)$ (2) 	<p>Two rigid vessels (1) and (2) are filled with a perfect gas, in respective states <math>(N, V, T)</math> and <math>(N, 2V, T)</math></p> <p><math>N</math> = number of moles in each vessel  <math>V</math> = volume of vessel 1  <math>T</math> = temperature of each vessel</p>
$(1)$ 	$(2)$ 	<p>The two vessels are heated up for the same time with identical heat sources, then one measures their respective temperature.</p>
Do you think that		<i>Rate of response (N=283)</i>
$T_1 > T_2$		<b>37%</b> <i>22% : "because <math>V_1 &lt; V_2</math>"</i>
$T_1 = T_2$		<b>48%</b> <i>30% correct justification</i>
$T_1 < T_2$		<b>5%</b>
I don't know		<b>8%</b>
<u>Why?</u>		_____↑

In conclusion to this first section we suggest that common types of reasoning observed in students and teachers are characterised in the following way:

In the implications used,  $\Phi_1 \rightarrow \Phi_2$ , " $\Phi$ " refers to a *phenomenon specified with only one variable*, for instance: " $p$  increases", or "input of heat". When several variables are mentioned (see table 1), this is done through an argument which links the variables in a linear chain:

$$\Phi_1 \rightarrow \Phi_2 \rightarrow \Phi_n \rightarrow \dots$$

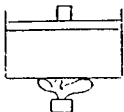
Each specific implication  $\Phi_n \Phi_{n+1}$  does not imply that the reverse implication would be accepted by the same person.

Students' responses to other questions will now introduce a new feature in the interpretation of such chains, which gives some coherence to these preliminary conclusions.

### 3. Causality and chronology: linear causal reasoning

A very common (43%, N=120 students) "explanation" of the increase in volume resulting from the heating of a perfect gas at constant pressure is of the following type (see question in table 3):

Table 3: A question about an isobaric heating of a gas (see Rozier, 1987), correct and typical responses

QUESTION	
	A perfect gas is heated at constant pressure...Its volume and temperature both increase. Can you explain why?
Notations used below: see table 8, and: $c_p$ : molar specific heat at constant pressure, R: constant, N: total number of moles, $\Delta$ : algebraic increment of...	
Outlines of ... correct explanation: $Q$ (supplied to gas) = $c_p \Delta T$ and $Q > 0$ and $c_p > 0$	$\left. \begin{array}{l} \Delta T > 0 \\ \text{and} \\ PV = NRT \\ \text{and} \\ p, N, R, \text{ all constant} \end{array} \right\} \rightarrow \Delta V > 0$
common explanation: supply of $Q \rightarrow T \nearrow \rightarrow p \nearrow \rightarrow V \nearrow$	

"The temperature of the gas increases. Knowing that in a perfect gas  $PV=NRT$ , therefore at constant volume, pressure increases: the piston is free to slide, therefore it moves and volume increases".

This response can be outlined in the following way: supply of heat  $\rightarrow T \uparrow \rightarrow p \uparrow \rightarrow V \uparrow$  (with obvious notations).

In such comments, one of the evoked events,  $p \uparrow$ , contradicts data presented in the problem, namely that  $p$  is kept constant.

*Such a contradiction, and others as we will see, disappears if one admits that this form of argument is interpreted temporarily. An arrow, then, does not mean only "therefore", but also "later". Table 4 shows how, in three and probably many other languages, these logical and chronological levels melt into a single word, totally ambivalent, in english: "then".*

Table 4 Shift in meanings from logical to chronological levels

level ↓	french	english	spanish
logical	donc	therefore	por eso
intermediate	alors	then	entonces
chronological	ensuite	later	despues

From this point of view, the previous chain subdivides into two steps:

- *first step*: "Supply of heat  $\rightarrow T \uparrow \rightarrow p \uparrow$ ", volume being implicitly or explicitly kept constant. Notice that such a constancy of volume is a sufficient condition for the two first implications to be straightforward. At constant volume, an input of heat, in the accepted theory, necessarily results in an increase of temperature (no work being transferred to the exterior of the gas). The same condition also allows the otherwise not obvious conclusion that if temperature increases, then pressure increases.

- *second step*: " $p \uparrow \rightarrow V \uparrow$ ". The piston is now released (this is said explicitly by some students) and moves until the internal pressure equals the external one. In such a chronological view, the seemingly contradictory argument " $p \uparrow$  (during isobaric heating)" becomes acceptable, as well as the statement "at constant  $V$ ", followed by this other: "Volume increases". These events indeed are understood as *successive*, and therefore as *temporary*. So they seem no longer contradictory.



To sum up: this kind of response supports the hypothesis (see Rozier, 1987) that a linear type of reasoning is used:

$$\Phi_1 \rightarrow \Phi_2 \rightarrow \Phi_n \rightarrow \dots,$$

in which, as said earlier, each phenomenon  $\Phi$  is specified with only one physical quantity, and where the causality referred to by the arrow has a both logical and chronological content. The temporal connotation of such an implication accounts for the lack of symmetry described in section Ic. This way of reasoning contradicts the accepted theory of quasistatic phenomena, in which all the changing physical quantities are supposed to change *simultaneously* under the *permanent* constraint of one or several relationships. But this enables variables to be coped with two by two, and to say different things about one of them at different stages of the argument.

Other inconsistencies become acceptable in this linear causal reasoning, as we will see now.

#### 4. Linear causal reasoning and the problem of steady states

Another question from this study (Rozier, 1987) puts in evidence how the features of linear causal reasoning just described fit in with students' most common responses and allow comments which in the accepted theory lead to contradictions. Asked to explain in molecular terms why an adiabatic compression of a perfect gas results in an increase of temperature (see question in table 1), 42% of students (N=140) give comments of this type:

"When the piston is pushed down, volume decreases, therefore particles are closer to each other, whence more collisions occur between them.. and there is an increase in temperature"

"Same number of particles in smaller volume, then particles more squashed, more collisions, more heat produced"

"More collisions between particles, more energy produced due to friction"

These responses can be outlined as follows:  $V \downarrow \rightarrow n \uparrow \rightarrow$  number of collisions  $\uparrow \rightarrow Q$  is produced  $\rightarrow T \uparrow$ , the fourth statement being justified by the fact that "collisions produce heat".

Again a linear form is observed. Let us see now how the hypothesis of a temporal content is supported by this last comment: "collisions produce heat".

In such a comment, one can see an emergence of the well known preferential association between temperature and heat, an increase in temperature being necessarily ascribed, in common reasoning, to a supply of heat. One can also say that macroscopic properties of bodies colliding inelastically are ascribed to microscopic particles.

Valid as they may be, these interpretations do not explain how it is that none of these students realise the incompatibility between this statement: "collisions produce heat" and the idea of steady state. Indeed, if in an adiabatic vessel, collisions between particles were continuously producing heat, an explosion would soon occur. But if the statement: "collisions produce heat", or "there is some heat produced", refer to a temporary phenomenon, as in the "chronological" interpretation of students' reasoning, then there is no longer any incompatibility with the idea of steady state. Interestingly, some students in this inquiry, and others informally questioned in a class room, said that more collisions produced more heat, *during the transformation*, but that at the end of the transformation, the heat production stopped: the end of the argument is also the end of the story...

So, it seems that seeing the evoked phenomenon as temporary avoids the difficulties inherent to the analysis of steady states. Such states are not envisaged for themselves, but as the result of transitory phases, themselves analysed as step by step - variable by variable processes. All this is done, in common reasoning, without saying it, and probably without being aware of it.

Most probably, teachers share to a large extent this tolerance towards explanations incompatible (according to accepted logic) with steady states. Some teachers were asked, during training sessions ( $N=45$ )<sup>2</sup>, to consider what answer they would give to a student who says "collisions between particles produce heat". *None* of them proposed a counter argument in terms of steady states.....Other examples of this teachers' tolerance are given in Rozier's study (1987, see also Viennot, 1989b).

##### **5. Interpreting a common idea in terms of linear causal reasoning: changes of states and thermal motion**

As said before, an idea widely spread among students and teachers, is that thermal motion is more intense following the order: solid, liquid, gas. At first sight, this might be simply a

manifestation of the adherence between mean kinetic energy and mean distance between particles commonly referred to by the expression "thermal motion" and cemented by the idea of disorder.

An experiment (Rozier, 1987) has been done with students at university to refine this point of view and to see if the linear causal reasoning was an help in interpreting common ideas in this field.

An excerpt from a textbook (Valentin, 1983) was first given to students, who were asked to read it carefully:

"Thermal energy possessed by each molecule is large enough to prevent the molecules of the gas from being bound: in a gas, molecules are continuously hitting each other and bouncing. But if temperature is lowered, the system will be able to become liquid and even solid. Such physical phenomena occur when, with decreasing temperature, molecules have so low a mean kinetic energy that they cannot any longer resist the electromagnetic interaction. They first gather in liquid state and finally get bound in solid states"

The subsequent questions are:

I. Do you think that this text suggests the following statements:

*Statement 1:* At a given time during the liquefaction, mean kinetic energy of a molecule of gas is larger than mean kinetic energy of a molecule of liquid (liquid and vapor are in thermal equilibrium at the time considered).

*Statement 2:* At a given time during the liquefaction, the mean distance between particles is larger in the gas than in the liquid.

II. Do you think that

*Statement 1* is    true            false            why?

*Statement 2* is    true            false            why?

Among 181 students in the three first years at University, 77% think that the text suggests statement 1 and 69% think that this statement is true. The corresponding percentages for statement 2 are 80% ("the text suggests statement 2") and 85% ("statement 2 is true").

As recalled earlier in the paper, mean kinetic energy depends only, in classical thermodynamics, on temperature and is therefore the same for systems at same temperature, for instance two

phases of a substance at thermal equilibrium. This is recalled by the author of this text one page further (not reproduced in the test).

In interpreting these facts, one may first notice the strong input of temporal connotations in the text: "if ... the system will be, .... they cannot resist any longer, ... first .... finally ...".

This suggested chronology superimposes on the logical chain, as follows:  $T \searrow \rightarrow e_c \searrow \rightarrow$  electromagnetic interactions win  $\rightarrow$  liquid state  $\rightarrow \dots \rightarrow$  solid state.

Linear and chronological, this text seems in perfect resonance with the features characterising the "linear causal reasoning". The idea subtly induced by such a chronology is that the story begins with high temperature and gaseous state and finishes with low kinetic energy and liquid state, no room being left to envisage simultaneously gaseous and liquid states at same temperature. All these students, however, know that at thermal equilibrium the two phases are at same temperature.

The very high percentage of students who accept statement 1 as true supports the hypothesis that they share the type of reasoning described earlier (linear causal reasoning), and seemingly encouraged by the text.

## 6. Discussing our teaching goals: some remarks in conclusion

There are various points which can be discussed at length about the greater or lesser correctness of some of the excerpts quoted above. One might then ask whether comments such as : "at high altitude, there is less molecules, so pressure is lower", or "thermal motion is higher in gases than in solids", or "molecules have so low a kinetic energy that they cannot resist any longer the electromagnetic interactions..." should be banished or not.

This is not the point of interest here. Rather than discussing the correctness of these statements, let us just note that such "*soft qualitative reasonings*" gloss over the difficulties of multi-variable reasoning, that this is, most of the time, not pointed out, and that the contradictions which may arise from a careless extension of these simple and evocative explanations are not confronted. These facts deserve attention and bring us back to the crucial question: what are our teaching goals,

- to make students familiar with particulate, or atomic structure of matter, or with other ideas or phenomena
- or to teach them how to reason in a coherent way (in par-

ticular with several variables), and to show them the limits of each level of explanation?

This alternative is put in a provocative way. In fact, in the constructivist view so widely shared now among researchers in science education, familiarity with ideas is of no real value if a personal construction of concepts by children has not occurred. In other words, there cannot be any conceptual learning without any reasoning. So we can drop our first alternative and replace it by this question:

- which kind of reasoning do we aim at for our pupils or students when introducing such and such ideas or phenomena?

This question is double faced:

- which (available) kind of reasoning will help them to grasp new concepts (for example in an inductive progression)

- which kind of reasoning will they learn?

It seems to us that it is important to be extremely careful in such a specification. For instance, inductive procedures aimed at introducing particulate ideas raise the following questions:

which experiments in physics, and *according to which logic*, support a particulate model rather than a continuous one? A classical theory, hydrodynamics, accounts for changes of volume and flows with a continuous model which, of course, respects all the necessary conservations, dynamical ones included. Not to speak of quantum mechanics which is also continuous with respect to space. Many teachers are not aware of this lack of evidence. In a workshop in a recent international conference<sup>3</sup>, participants were asked which experiment(s), among the following, were the most appropriate to introduce particulate ideas:

- change of state

- dissolution

- difference in color for different concentrations

- expansion and compression of a gas

- diffusion

- non additivity of volume in the mixing of water and alcohol, about a third of participants chose expansion and compression of a gas. So, there is a danger of pseudo demonstrations.

This would support the choice made, for example, by Meheut and al. (1987), i.e. introduce *ex cathedra* the basis of a particulate model, then ask children to work on it.

This however leads us to ask the question: what kind of work, should the students be involved in the learning activity?

A work about conservation of mass and number of particles through changes of volume or changes of state has been proposed by several authors (for instance Meheut e.a.), a goal very appropriate to pave the way for learning the basis of chemistry. Then the difficulty is again to specify what kind of work it is possible to do in a consistent way. One may envisage activities of a descriptive type: children or students have to describe in terms of a particulate model changes of volume or changes of state. This may also be consistent with goals which emphasise explanations. The difficulties stressed in this paper suggest that, at any level of teaching, only two attitudes are self-consistent.

- One is to be extremely careful about the degree of "explanation" actually expected, and to specify what cannot be accounted for in the frame of the proposed description. Thus, for instance, the following levels of understanding may be envisaged: "Gases can change their volume to a large extent but (without the beginning of a kinetic theory) we cannot explain why they resist a compression before molecules are in contact"

"Solids expand when heated (contract when cooled), we cannot (yet) explain why. Knowing that thermal motion increases (decreases) in such a case is not enough to explain why this makes the solid expand. Indeed, the particles might vibrate more intensely, and stay around the same place without drifting (a matter of anharmonicity of the potential of interaction between particles!)."4

"At equilibrium between, say, liquid and gas, thermal motion (mean kinetic energy) is the same in the two phases, and we cannot (yet) explain this surprising thing. In other words, we cannot explain why, with the same thermal motion, some molecules are linked to each other and others are free. We cannot explain why thermal motion keeps the same during the change of state. We know indeed that an input of heat is used to break the links between particles in the liquid. But we do not know why it is used only for this and not also to increase thermal motion."

- Another possible teaching strategy is to work with some "soft" explanations, but without hiding the dangers of a careless extension of such explanations to other cases. For instance, to work with the following ideas:

"At an altitude, there are fewer molecules, therefore pressure

is lower"... adding: "this reasoning works only if the molecules have (more or less, admittedly) the same velocity in the two compared cases.

"When a tyre is heated up, it becomes harder because the molecules have a larger mean speed"... adding "this reasoning works only if the same number of molecules is still in the same volume" (obviously not the case since the tyre is harder, but an approximate constancy of volume may be invoked).

This kind of *harder qualitative reasoning* may be considered too demanding, but it is the price to pay for consistency in dealing with such phenomena.

Of course, if one is interested in fostering the multivariable reasoning for itself, rather than illustrate phenomena connected with particulate structure of matter, one may choose simpler examples first. The area of a rectangle is a function of two variables: hard qualitative reasoning may be trained on similar simple examples.

However such teaching goals, linked with general features of reasoning, are not much in favour at the moment, overshadowed as they are by more content-specific objectives. However, one point at least must be made clearly: in our students, linear causal reasoning will be the most likely outcome of teaching which never confronts it.

It seems therefore that we cannot avoid a debate about our teaching goals, which should more explicitly consider the kinds of reasoning we expect our pupils or students to learn.

### Acknowledgement

The help received from Rosalind Driver in the preparation of the english version of this paper is gratefully acknowledged.

### Notes

1. L.Viennot, Paris 1986-7, first cycle in secondary education (grades 6 to 9), N=30, training in physics; Milan 1989, all levels of teaching, N=25, training in didactics
2. L.Viennot, 1989, all levels of teaching, Paris N=20, Milan N=25, training in didactics
3. IIIrd International Conference on Research in Science and Mathematic Education, Santiago de Compostela, 1989, Workshop by Enciso, E. Llorens, J.A., Sebadra, F.

4. It happens even that they vibrate more intensely being closer to each other, for instance when ice melts and the resulting liquid water is subsequently heated.

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