Modelling Clay for Computers

J.Ogborn
Institute of Education
University of London

1. Abstract

How can students of all ages use the computer to model the real world? Modelling systems which iteratively solve difference equations are now common, and useful for older students. But they require that the world be imagined as composed of variables, not things. And they need some minimum mathematical sophistication. This paper discusses two new modelling tools suitable for quite young students, which could provide an introduction to modelling. One tool allows systems of variables to be constructed, without having to specify mathematical relations between them. The other provides for interacting objects whose behaviour can be specified, again without mathematics, through drawing 'before and after' pictures to express interactions of objects. It is argued that the different types of models fit naturally into a developmental sequence, matching modelling at various ages to student's intellectual growth. A radical re-sequencing of teaching about Mathematics in Science is proposed.

  To create a world, whether constituted of variables or of objects, and to watch it evolve is a remarkable experience. It can teach one what it means to have a model of reality, which is to say what it is to think. It can show both how good and how bad such models can be. And by becoming a game played for its own sake it can be a beginning of purely theoretical thinking about forms. The microcomputer brings something of this within the reach of most pupils and teachers.

2. Iterative modelling systems

We all know how to make simple iterative computational models (Roberts et al, 1983). Like many others I too have written modelling systems which use this idea (Ogborn, 1984; Ogborn and Holland 1986). An obvious example is a model of getting money from the administration for one's department - a matter of wide general interest. If the additional fractional appropriation in any year is proportional to how strongly one argues, but is also sensitive to how near an upper limit of funding one has already got, a model might look like Figure 1, which shows how it would appear on the screen in our Cell Modelling System CMS (Ogborn and Holland, 1986). As is well known, this

logistic model will show chaotic behaviour if the growth rate (strength of argument) is too large, which may be true of at least some institutions.
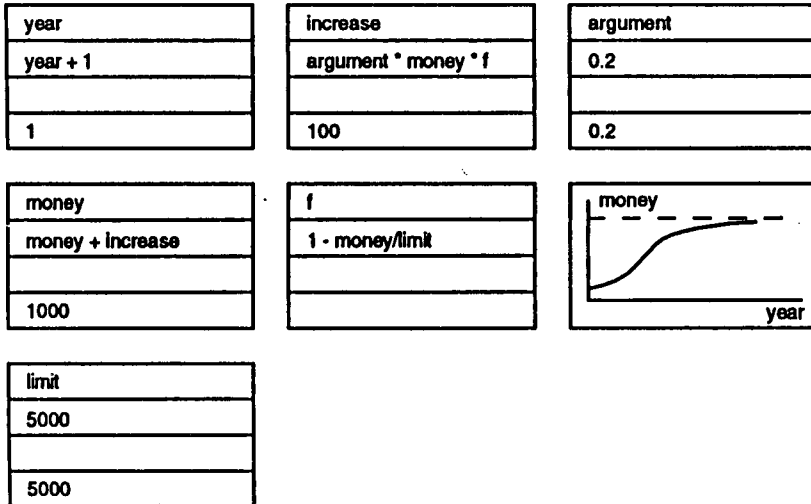
year
year + 1
1

increase
argument * money * f
100

argument
0.2
0.2

money
money + increase
1000

f
1 - money/limit

limit
5000
5000

Fig.1. Logistic growth model

Such models may be an excellent way to teach calculus. It is easy to build a variety of models, and they can be more realistic than models to be solved analytically. Plenty of suitable systems exist for doing this, from STELLA to one's favourite spreadsheet. The advantages and disadvantages of the computational and analytic approaches look something like:

Computational solutions	*Analytic solutions*
Steps close to physical reality	Formal methods of integration
Accessible early in learning	Needs previous mathematics
Adding complexity is easy	Adding complexity is difficult
Only particular solutions	General, manipulable solutions

Because the existence of analytic solutions is very sensitive to the detailed structure of the differential equations (in particular often requiring them to be linear) adding a small real life complexity to a problem may produce a very sharp rise in the mathematical difficulty of solving it. Figure 2 fancifully sketches a relation between the difficulty of getting a solution and the amount of reality the model includes.

Traditionally, we teach Science and some calculus alongside one another, so as later to be able to develop analytic solutions for differential equations.
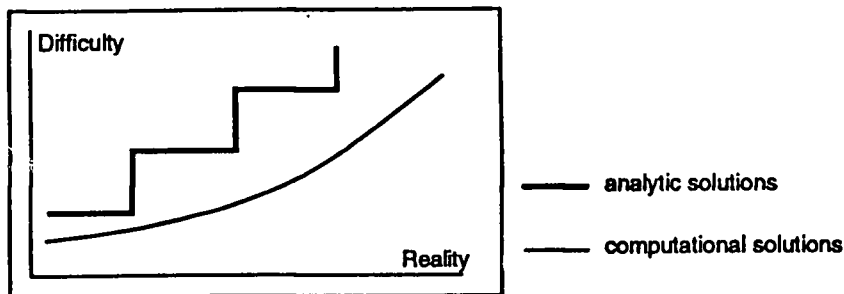
Fig.2. Difficulty and reality

Much later, perhaps only in graduate school, is the student introduced to numerical methods. The alternative is to teach Science by means of some very elementary numerical methods, and to use this to develop the ideas of the calculus so as later to develop analytic methods and numerical methods in parallel.

3. Modelling without mathematics

Up to now, what has been suggested is hardly revolutionary. The next suggestion is more shocking: it is that we need to begin modelling without mathematics. Consider what is needed if one is to make models of the kind discussed so far:

1. Imagining the world constituted of *variables*
2. Conceiving physical relations as *mathematical relations* between variables
3. Giving appropriate *values* to variables
4. Seeing a model as a structure with *possibilities*.

Of these, the first is perhaps the hardest. As scientists we have become so used to imagining the world as analysable as the interaction of quantitative variables that we forget what a huge step in imagination this is. There is good evidence, supported by commonsense observation, that young students see the world as built of objects and events, not as built of variables.

We have built, and tested with students in the age range 12-14 years, a modelling programme which focuses just on imagining variables and the connections between them, without having to specify the form of mathematical relations. It was developed in the project Tools for Exploratory Learning, in association with Joan Bliss, Rob Miller, Jonathan Briggs, Derek Brough, John Turner, Harvey Mellar, Dick Boohan, Tim Brosnan, Babis Sakonidis, Caroline Nash and Cathy Rodgers. The background to this project is given in Bliss and Ogborn (1988, 1989). The design of the modelling programme

is in Miller et al (1990) and results are discussed in Bliss, Ogborn at al (1992) and Bliss and Ogborn (1992). The modelling system is called IQON (Interacting Quantities Omitting Numbers). In IQON one creates and names variables, and links them together graphically. The best introduction is by example: figure 3 shows what an oscillator looks like when expressed in IQON.
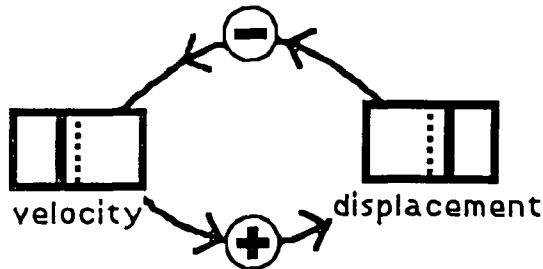

Fig.3. An oscillator in IQON

A positive velocity progressively increases the displacement, through the 'plus' link. But a positive displacement progressively decreases the velocity, through the action of a spring, represented via the 'minus' link. The outcome is that the system oscillates, an example of the principle mentioned before, that negative feedback plus delay gives oscillation. What is shown in Figure 3 is all that the user has to do: to create and name two variables and to link them as shown. No equations are written at all.

However, IQON is also intended for thinking about systems where we have much vaguer ideas about quantities and their relationships. Consider the quality of a conference. We may imagine that much depends on the quality of the workshops. If that is high, the participants become happier and happier as the week goes by. But if they are happy they may perhaps participate more actively in workshops, so that the quality of workshops itself increases. Figure 4 shows this idea expressed in IQON.

This model is overly optimistic. It contains positive feedback, so that if as in figure 4(a) the quality of workshops is somehow increased by a small amount, then after some time all the variables are driven to their positive limits. It does not matter whether the model is correct; what matters is that such effects are possible and will certainly arise in some cases, whatever the details of the system. An increase in global temperature causing melting of polar ice, which by reducing reflectivity increases the energy absorbed from the Sun and so leads to a further increase of global temperature is an example.

(a) initial setting

quality of workshops   happiness of participants

activity of participants
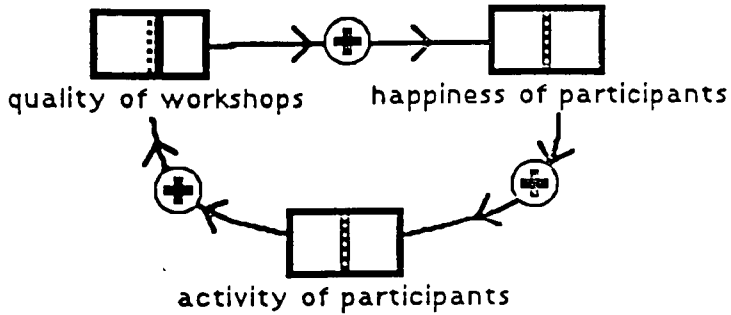
(b) positive feedback causes runaway

quality of workshops   happiness of participants
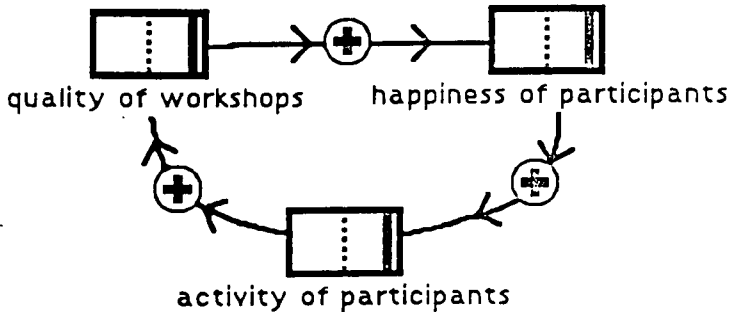
activity of participants

Fig.4. An IQON model for success of workshops

In its present implementation, all IQON variables are alike. Any input from other variables simply modifies the rate of increase or decrease of a variable. Each has a central 'neutral' position at which its output has no effect. Figure 5 shows this schematically.
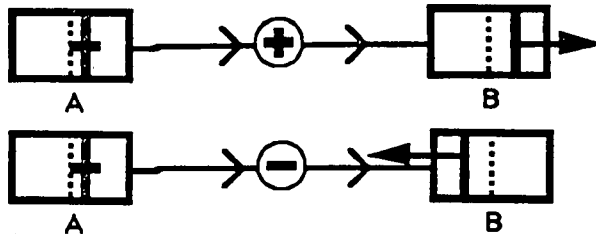

A   B

A   B

Fig.5. Behaviour of linked variables in IQON

If variable 'A' is above 'neutral', a positive link from it to variable 'B' drives 'B' up progressively until it reaches the limit of its box. Similarly, a negative link to 'B' drives 'B' progressively down. Thus 'A' determines the rate of change of 'B'. Multiple inputs to a variable are simply averaged, taking account of sign, to determine the rate of change, though some inputs can be given greater weight than others. The response of each variable is made non-linear, through a 'squashing function' which restricts its values to the range minus one to plus one. A variable also has some (adjustable) internal damping. In fact, the behaviour is similar to that of some forms of artificial neuron (McClelland and Rumelhart, 1987). One may of course also regard a variable as a (non-linear) integrator of its inputs.

These features mean that any system of inter-linked variables a user designs will have a smooth behaviour, with no tendency for variables to go to infinity or to produce large step function outputs, and that any system will have a unique stable condition from a given starting point.

Figures 6 and 7 show two examples of models created by pupils aged about 13 (Bliss and Ogborn, 1992). Nancy (figure 6) sees fitness depending both on general health and on whether one is getting plenty of sleep, and additionally on attitude. Jokingly, she says that if the school gives her a lot of work to do at home she gets less sleep. Health she sees as affected positively by sensible diet and negatively by disease, in both cases sliding a little away from quantitative variables towards events. Disease has a direct negative effect on fitness, and also an indirect effect via attitude. The point is not whether Nancy is right, but that she has produced a model which is discussable, and whose results when run may surprise her and lead her to think some more.
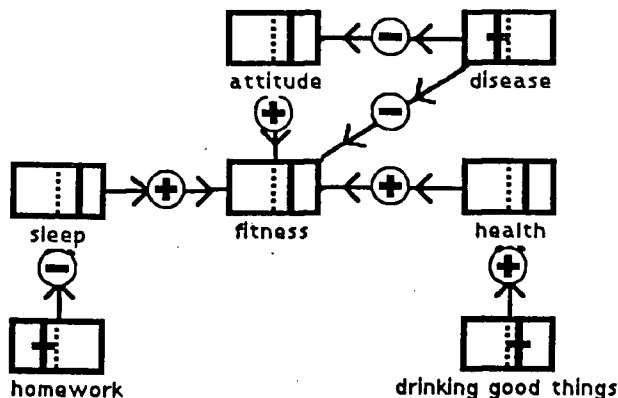

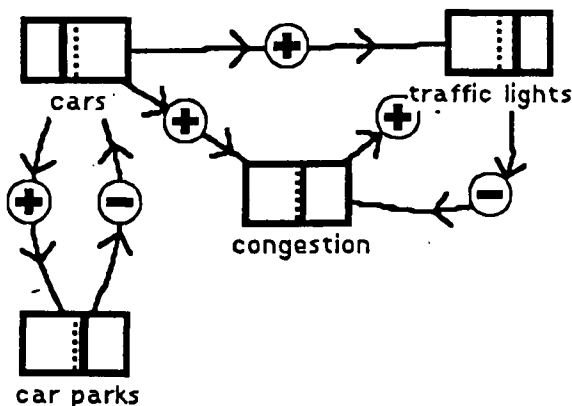Fig.6. Nancy's IQON model for keeping fit

Fig.7. Burgess' IQON model for traffic congestion

Burgess (figure 7) was modelling traffic congestion. His 'variables' are more like objects than like amounts of something. Because of the feedbacks in the model, when it is run it can give surprising results. Increasing 'car parks' can at first decrease 'congestion' but, because of the loops between 'cars' and 'car parks' and between 'traffic lights' and 'congestion', the model is liable to oscillate. Again, what matters is that this is likely to lead the pupil to reconsider ideas.

Overall, the results of our studies with IQON (Bliss and Ogborn 1992) can be stated as follows:
- all pupils could make *some* model;
- half or more made models with fairly sophisticated interconnections;
- those who *made their own* models were more radical in criticising or reformulating them than were those who were given previously prepared models;
- many had difficulty creating amount-like variables. The tendency was to create *objects* and *events*.
- some pupils could argue about feedback effects
- most pupils' work produced *discussable ideas*, capable of leading to progress in modelling.

In summary, we have a simple graphic modelling facility, for pupils to build such models out of just a few building bricks, and for them to be able to see some of the basic qualitative interactions at work, without yet having to consider exact functional relations between variables. The significant information is in the *qualitative pattern of relationship and change* amongst variables. In Physics, one might *begin* with such qualitative models. Later,

it would be time to see how well defined relationships in similar models can give more precise answers, in numerical simulations.

4. Modelling with objects and events

If one wants to make computational models with even younger pupils - say 8 to 12 years - then it would seem to be a good idea to model not variables but objects and events. WorldMaker (Boohan, Ogborn and Wright, forthcoming) is a system of this kind, largely designed and written by Dick Boohan and Simon Wright. A WorldMaker model of sharks preying on fish might look like figure 8.
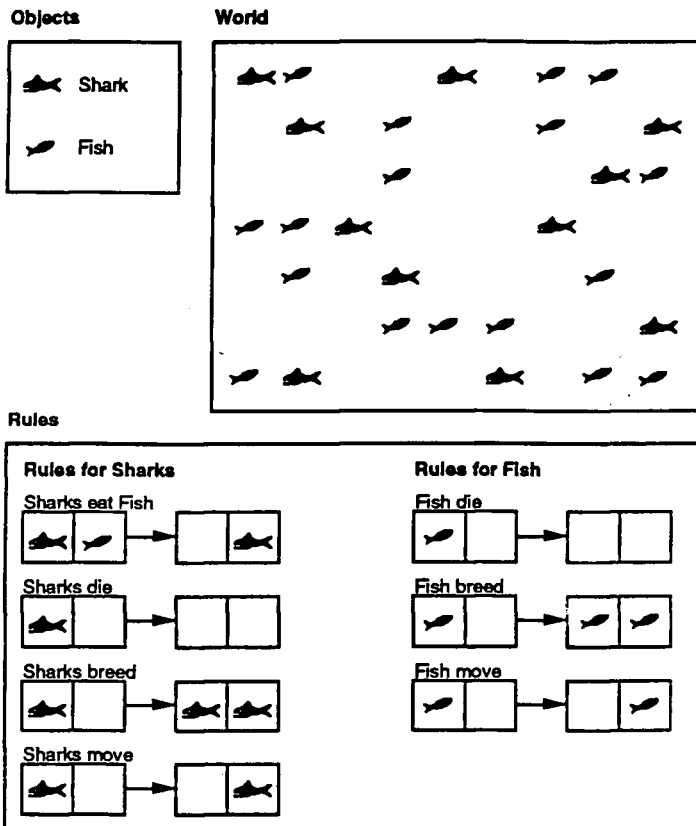
Fig.8. Predator and prey in WorldMaker

A WorldMaker world consists of objects on a grid. Rules telling the objects what to do are defined graphically. Thus in Figure 8, the two kinds of object,

sharks and fish, swim around the grid, being placed on it using drawing tools. Rules are specified by drawings, too. A shark next to an fish eats the fish. A shark on its own may die. A shark next to an empty space may breed or may move. The three rules for fish are similar to the last three rules for sharks. All rules have the form 'condition - effect'. Any rule can be set to 'fire' with a probability selected by a slider bar, so that for example relative breeding rates can be altered, or sharks can be made very long-lived. In this model, if sharks breed too fast, they can destroy the fish population and then themselves die out. As is well known, such predator-prey systems can oscillate.

The concept of WorldMaker derives from that of Von Neumann's cellular automaton (one of the best known instances being Conway's Game of Life), with the addition of moving objects each of which retains its identity, and of the possibility of random choices of allowed changes. A cell automaton consists of an array of cells, each of which has a small finite number of states. The state of a cell changes in relation to its own present state and those of its immediate neighbours. Thus the rules for evolution of the system are local rules, the same everywhere. A useful general account is given by Toffoli and Margolus (1987).
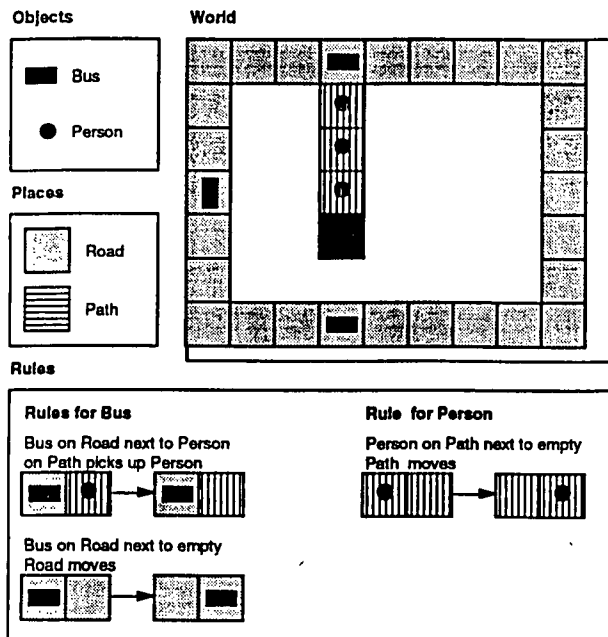

Fig.9. WorldMaker model for buses travelling in groups

The system as a whole is not represented explicitly at all, but is visible to a person watching the model evolve, as some pattern of behaviour of the assembly of objects. A simple model suitable for young pupils addresses the question why buses in town always seem to come in groups. Figure 9 shows the idea.

If buses stop to pick up people when they are there, the buses soon become clustered on the road around which they travel. WorldMaker allows directions of movement to be given to an object by the background it is on, making it simple to construct paths or tracks for objects. The example illustrates one of the several ways in which backgrounds and objects can interact, which include either changing the other into a different one. An example of such changes is a 'farmer' who moves around the grid 'planting crops' (i.e. changing bare earth to plants) and one or more 'pests' who move around destroying the crops. Another is shown in figure 10, in which a creature moves purely at random, but moves more frequently in the 'light' than in the 'dark'. The result is that any initial distribution of creatures ends up with most of them in the 'dark' region.
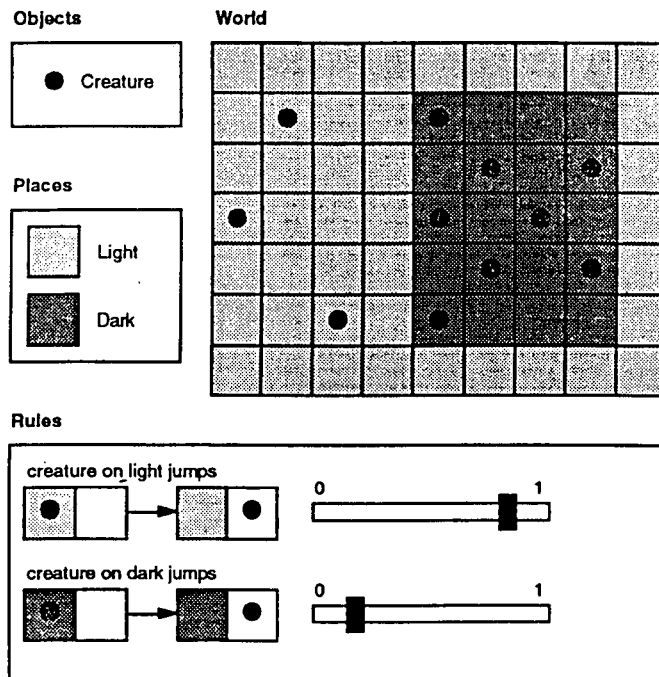
Objects

World

Places

Rules

creature on light jumps

0   1

creature on dark jumps

0   1

Fig.10. WorldMaker model of preferential random distribution

An even simpler system, is able to illustrate molecular diffusion, as in figure 11. The walls can be drawn anywhere one likes, and the initial distribution can be varied. The educational lesson here is important. A large scale, macroscopic appearance of systematic change can be generated by what is at the microscopic level random. Exactly the same rule will produce the outward diffusion of particles placed in a cluster at the centre of an otherwise empty screen.
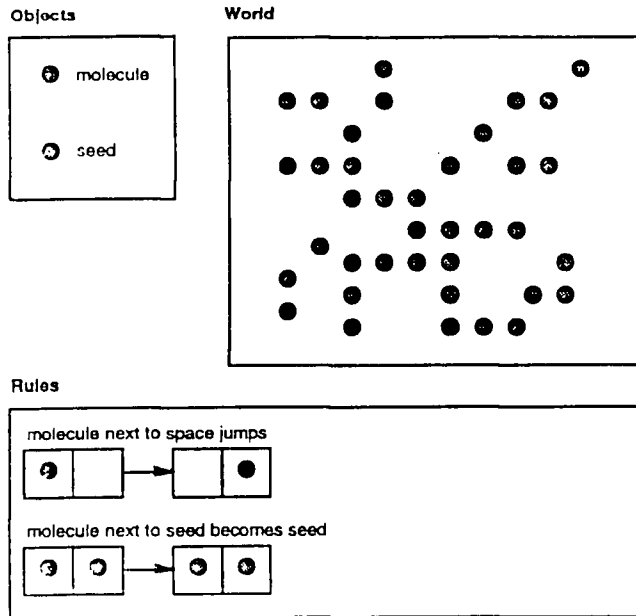

Fig.11. WorldMaker model of molecular diffusion

An adaptation of the model in figure 11 leads to a model of diffusion limited aggregation. One just adds another object, a seed, which does not move, and the additional rule that a molecule alongside a seed is captured and turns into a seed. Figure 12 illustrates the kind of fractal structure which can result.

Let us mention some other models, simple and more advanced, which WorldMaker makes possible. One is radioactive decay, in which the rule is simply that an object representing a nucleus has a finite probability of changing to a stable nuclide. Such a model is' readily extended to a decay chain.

Marx (1984) gives the example of a forest fire, which belongs to the large class of percolation problems. A cell can be empty, or can contain a tree

which is alive or is burnt. Trees are placed at random with a certain density over the screen, and one of them is 'set on fire' (fig.13).
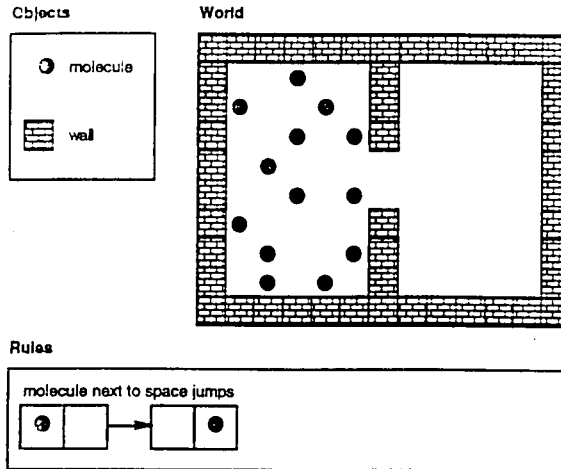

Fig.12. WorldMaker model of diffusion limited aggregate

A tree burns if one or more of its neighbours burns. Will the fire travel all through the forest? It turns out that there is a critical density of trees for this to be likely. An equivalent problem is that of whether a mixture of conducting and insulating grains will be conducting, or of whether there are continuous percolation paths for oil through cracked rock strata. Marx (1984, 1987) gives many other interesting similar ideas.
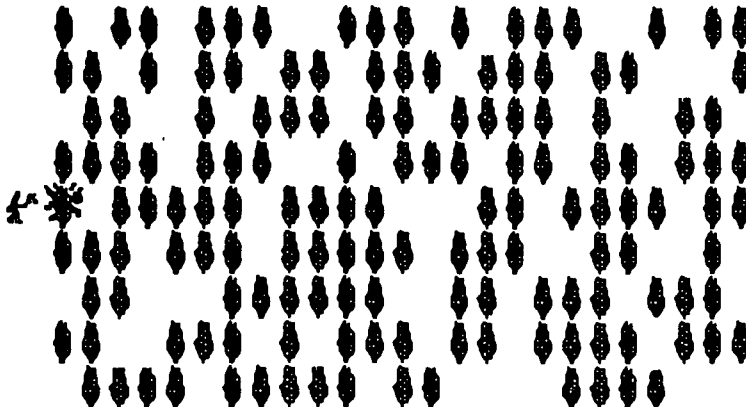

Fig.13. Forest fire: one tree is set on fire - will all the forest burn?

Simple examples of chemical reactions can be modelled by having cells filled with two or more species of 'molecule'. Molecules may move to empty cells or may combine with others nearby to make product molecules, which themselves may react in the reverse direction.

All these models have the great advantage that the objects one is talking about are directly represented on the computer screen. If the work concerns sharks eating fish, there are icons of sharks and fish to look at. If the problem is about molecules, one looks at an array of entities representing molecules, not at a display of variables such as temperature and pressure (though the system might in addition calculate these). The behaviour of the whole system is represented to the student by the visible pattern of behaviour of the objects, not as values of system variables. In general, the rules for the behaviour of entities are simple and intuitive, usually relating directly to their behaviour in the real world. Despite this simplicity, quite complex and analytically intractable systems can be studied.

5. Conclusions

I have in this paper suggested three things:

a. that there is an important role in science teaching for quantitative system modelling;
b. that there is scope for qualitative computational modelling of systems of variables;
c. that use can be made of models which manipulate the objects in a system rather than the variables; and that cell automata provide a useful formalism for this concept.

Systems to provide for (a) already exist, and are in use in some schools, mainly in the upper age range. Those who cannot get or afford such a system, or who prefer an alternative already known to many pupils, can do a great deal with a spreadsheet program. The possibility is opened up of teaching Science through modelling without having to wait until students know the calculus, and indeed of teaching the calculus in this way.

Suggestion (b) is more radical. We have built and tested a prototype, and can say that with it quite young pupils can produce interesting models. There are good psychological reasons for thinking that qualitative reasoning about variables is important, because of its pervasiveness in all human thought. The opportunity offers for teaching quite young students about systems of variables and effects of feedback, before they are ready to deal with quantitative formalised relations between variables.

Plenty of simulations which belong within the concept of (c) already exist, and are not difficult to program, though speed may be a problem. What I have suggested is the value of a generalized facility for building such models,

and I have described one such system. Here we can see how the idea of modelling could be extended to pupils even in the Primary School. Let me finally try to put these thoughts in a more general perspective. The normal order in which people come to appreciate the role of computational models, is far from ideal. One is first supposed to learn functional relations between quantities (Ohm's law, Newton's laws etc.), then some differential calculus, then integration, then numerical methods, and finally one is expected to see the unity in all this. This path is followed hardly any distance by most pupils, and the whole distance by almost none except the best doctoral students.

This leads me to propose in a sense to reverse the normal order. We should perhaps concentrate from the beginning on form, defined at first loosely and then more precisely. At present we leave form until last, if we ever reach it at all. If it is true that children would find computational representations of objects easier to deal with than representations of system variables, then this suggests one kind of beginning with modelling in which the child tells the objects what to do, not the variables. Form is then represented by patterns of behaviour of collections of objects.

A second beginning, directed towards analysing systems into related variables, might be with modelling systems supporting qualitative reasoning, or patterns of cause and effect, involving variables. Here one has the possibility of looking at form as the typical kind of behaviour of systems with a given structure. The reason why oscillators oscillate is fundamentally the same. The reasons why stable systems are stable are often basically similar.

I want to emphasize the very real importance, equally for young pupils and for the best experts, of qualitative reasoning about form. The young child can often guess how things may go, and can look at a model on the computer to see if it 'goes right' or not. The expert is an expert just by virtue of having passed beyond the essential stage of being able to do detailed calculations, to have reached the even more essential stage of knowing what kind of calculation to do, and what kind of result it will give.

To create a world, whether constituted of variables or of objects, and to watch it evolve is a remarkable experience. It can teach one what it means to have a model of reality, which is to say what it is to think. It can show both how good and how bad such models can be. And by becoming a game played for its own sake it can be a beginning of purely theoretical thinking about forms. The microcomputer brings something of this within the reach of most pupils and teachers.

References

Bliss, J., J.Ogborn (1988). *Tools for exploratory Learning Occasional Paper* InTER/5/88/: InTER, University of Lancaster.

Bliss, J., J.Ogborn (1989). 'Tools for Exploratory Learning', *Journal of Computer Assisted Learning, 5*, 37-50.

Bliss, J, J.Ogborn, R.Boohan, J.Briggs, T.Brosnan, D.Brough, H.Mellar, R.Miller, C.Nash, C.Rodgers, H.Sakonidis (1992). 'Reasoning supported by computational tools', *Computers in Education, 18*, 1-9.

Boohan, R, J.Ogborn, S.Wright (forthcoming) *WorldMaker Software, Teachers' Guide and Manual*: ESM Cambridge.

Marx, G. (1984). *Games Nature Plays* (with associated software): Lorand Eötvös University, Budapest and National Centre for Educational Technology, Veszprém, Hungary.

Marx, G. (1987)(ed.). *Welcome to our non-linear Universe*, Proceedings of the International Workshop on teaching non-linear phenomena in schools and universities; National Centre for Educational Technology, Veszprém, Hungary.

McClelland, J.L., D.E.Rumelhart (1987). *Parallel Distributed Processing*, Cambridge Mass.: MIT Press.

Miller, R, J.Ogborn, J.Turner, J.H.Briggs, D.R.Brough (1990). *'Towards a tool to support semi-quantitative modelling'* Proceedings of International Conference on Advanced Research on Computers in Education, July 1990, Gakashuin University, Tokyo.

Ogborn, J. (1984). *Dynamic Modelling System, Microcomputer software*, London: Longmans.

Ogborn, J., D.Holland (1986). *Cellular Modelling System, Microcomputer software*, London: Longmans.

Ogborn, J. (1986). Computational Modelling in Science, In: R.Lewis, E. Tagg, *Trends in Computer Assisted Learning*, Oxford: Blackwell.

Ogborn, J. (1989). Computational modelling: a link between Mathematics and other subjects. In: W.Blum e.a., *Modelling, Applications and applied problem solving*, Ellis Horwood.

Ogborn, J. (1990). 'Modellizzazione con l'elaboratore: possibilità e prospettive' *La Fisica nella Scuola , Anno XXIII*, 2, 32-43.

Ogborn, J. (1991). Modelaço com o computador: Possibilidades e perspectivas. In: V.D.Teodoro, J.C. de Freitas (eds.) *Educaço e computadores* Ministério da Educaço: Lisbon, Portugal.

Roberts, N., D.Anderson, R.Deal, M.Garet, W.Shaffer (1983). *Introduction to Computer Simulation*, New York: Addison Wesley.

STELLA (1987). High Performance Systems, 13 Dartmouth College Highway, Lyme, New Hampshire, 03768, USA.

Toffoli, T., N.Margolus (1987). *Cellular Automata Machines: a new environment for modelling*, Cambridge Mass.: MIT Press.

Tools for Exploratory Learning Programme (1992). Final Report in four volumes:

1 *Executive Summary* plus Summary Report
2 Technical Report 1 *Tools plus Examples of Tasks*
3 Technical Reports 2 and 3 *Semi-Quantitative Reasoning, Expressive and Exploratory*
4 Technical Report 4 *Quantitative Reasoning* plus Technical Report 5 *Qualitative Reasoning*

Available from Dr. Joan Bliss, Centre for Educational Studies, King's College London, Cornwall House Annexe, Waterloo Bridge, London SE1