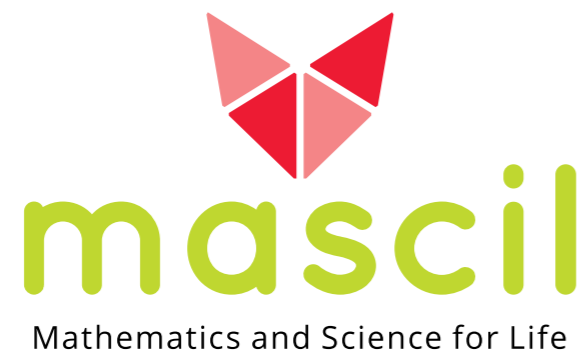


**Mathematics and Science
in Life: Inquiry Learning
and the World of Work**



Mathematics and Science in Life: Inquiry Learning and the World of Work

FOUR YEARS OF EUROPEAN COOPERATION
IN THE MASCIL PROJECT

Mathematics and Science in Life: Inquiry Learning and the World of Work

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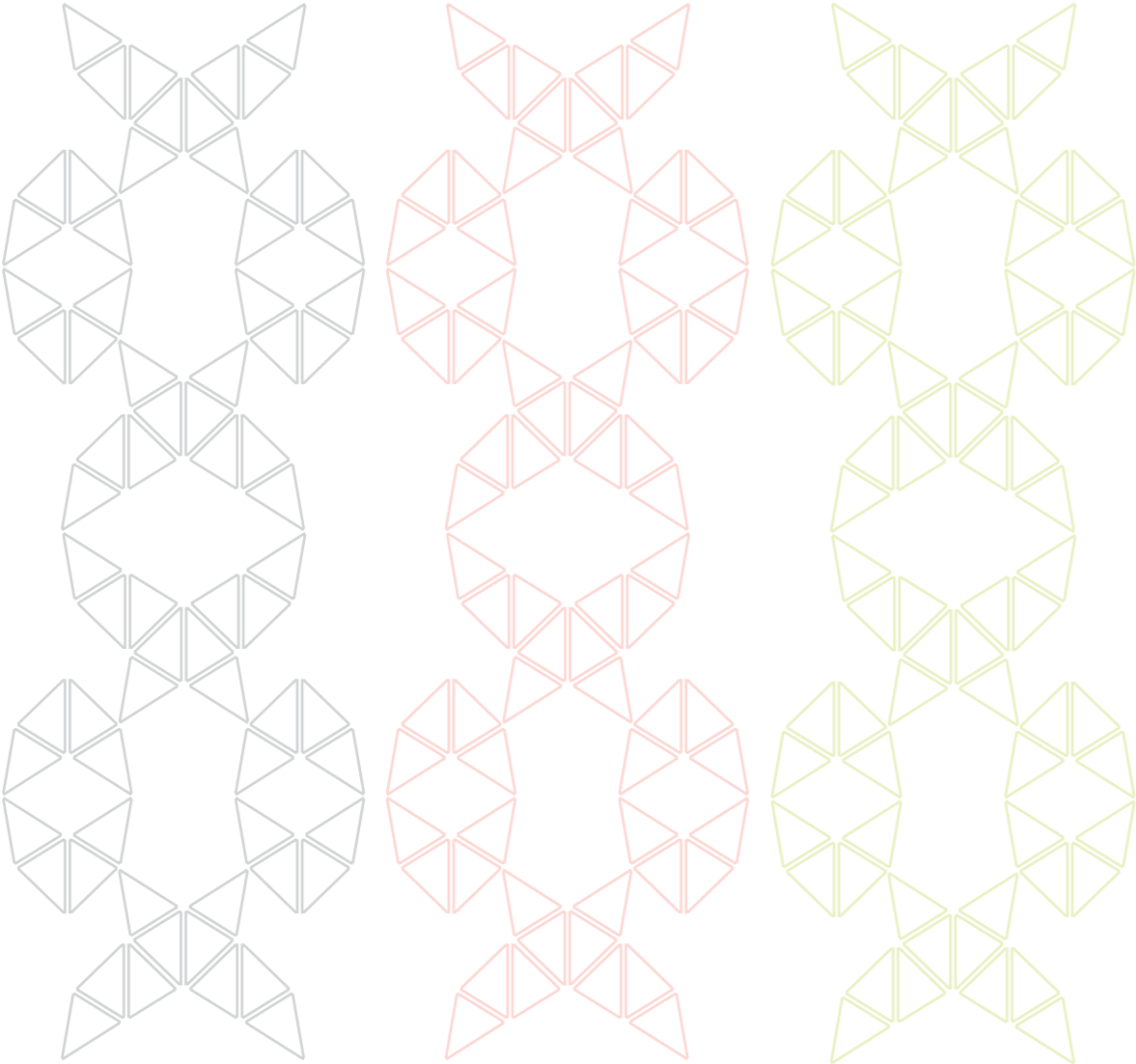
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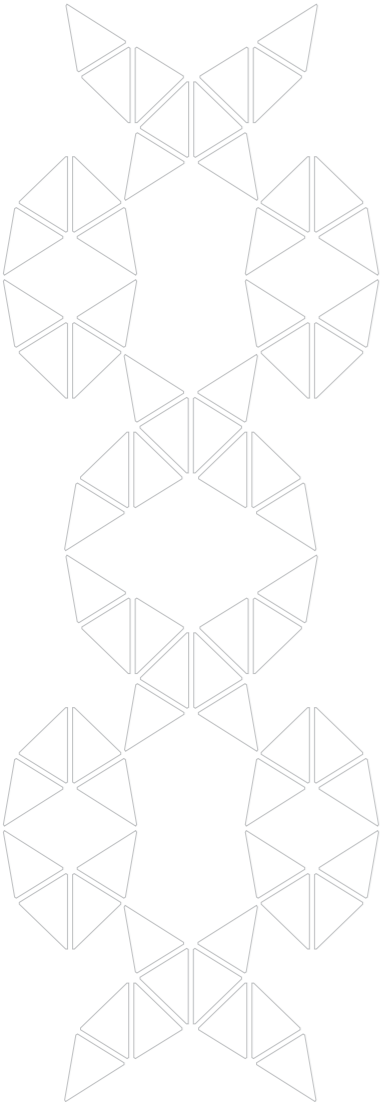
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Contents

- Chapter 1
Introduction..... **11**
- Chapter 2
Inquiry Based Learning
and the World of Work..... **15**
- Chapter 3
Contrasting examples..... **23**
- Chapter 4
(Re)designing tasks..... **33**
- Chapter 5
Mascil in the classroom..... **37**
- Chapter 6
Experiences with mascil tasks... **45**
 - Bicycle insurance **50**
 - Designing a parking garage..... **54**
 - Parachute food drop..... **60**
 - Counting people..... **64**
 - Holiday kennel for dogs..... **68**
 - Designing your non standard book shelf..... **72**
 - Chocolate chip mining..... **76**
 - GPS mathematics puzzle:
Holiday in Marocco..... **82**
 - Brine..... **86**
 - How do smartphone apps work?..... **92**



Foreword

Dear reader,

“Mathematics and science aren’t connected to real life.”
“You don’t need maths and science in your future job as long as you aren’t going to become a scientist or mathematician.”

Have you ever heard your students make such remarks? Most likely, the answer is yes. That is because our students often just do not see how maths and science relate to real life. This in turn frustrates students, as learning these subjects seems to them to be “a waste of time” and useless. But mathematics and science are useful, in fact they form an important basis of our society, of professions and for decisions that have great impact on our future. Examples of this abound, whether we are trying to decide if vaccinations against measles should be

obligatory, or whether to build wind power stations in a certain place. As future decision makers, students need to learn how to apply sciences and see and appreciate the results and impact science has on their daily lives and the world they live in. By doing so, students develop competences in application, and gain a better understanding of mathematics and science by connecting it concretely to their realities. When students realise the relevance of maths and sciences, they develop a positive attitude towards these subjects.

This book aims to support you and other teachers in including applications of science and mathematics into day-to-day teaching, in particular the applications with connections to the world of work, and thus, demonstrate these subjects’ relevance to your students’ future professions. It shows you interesting examples of classroom materials, outlines their connections to the world of work and gives inspiring insights into real classroom experiences across 13 European countries.

We developed this book within the European Project mascil (mathmatics and science for life!) aimed at promoting a widespread use of inquiry-based science teaching in primary and secondary schools. In addition, we connected mathematics and science education to the world of work. In order to implement inquiry-based teaching and connect mathematics and science education to the world of work, mascil followed a holistic approach by carrying out a variety of activities, including the development of materials and running professional development courses. These courses for pre and in-service teachers ran from 2014 to 2016 and were supported by teachers from vocational education and representatives from industry.

We hope this book provides you not only with useful, practical information, but also with new ideas, approaches and inspiration for your invaluable work with today’s students and tomorrow’s working and contributing adults.

Katja Maaß
Coordinator, project mascil

chapter

01

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Introduction

This book aims to provide an introduction on how to connect inquiry based science and mathematics teaching in school to the world of work, and thus making mathematics and science more meaningful for students. The book describes tasks and their use in daily educational practice to show how teachers can enrich their practices with inquiry-based learning and the use of workplace contexts.

It is aimed at teachers and teacher educators and provides them with rich examples that show how inquiry-based learning and workplace contexts can be used in daily teaching practice.

In this book you will be introduced to the background of Inquiry Based Learning (IBL) and what it means to connect science and mathematics education to the world of work (WoW). We will highlight characteristics of tasks, (re)design guidelines and teaching methods that foster this connection.

How this can work in practice will be illustrated by inspiring classroom examples from mathematics and science and from various partner countries that connect IBL and WoW. The rich descriptions of classroom experiences to offer the information needed to adopt and use the classroom materials that are published on the mascil website: www.mascil-project.eu.

chapter

Unit 2

inquiry-based learning and the world of work

Inquiry Based Learning and the World of Work

Inquiry based learning (IBL) aims to develop and foster inquiring minds and attitudes that are vital in enabling students to face and manage uncertain and quickly changing futures. Fundamentally, IBL is based on students adopting an active, questioning approach and on teachers supporting students in taking this approach and using it for developing content knowledge and content related skills as well as for developing skills that are nowadays labelled as 21st century skills. In addition, to enforce the benefits of IBL and make science and mathematics more meaningful to students, the use of rich vocational contexts – from the World of Work (WoW) – is advocated.

When students learn by inquiry they explore situations, pose questions, plan investigations, experiment systematically, interpret and evaluate, collaborate and communicate results (processes of inquiry). In our interpretation of IBL, the teacher enriches his or her classroom practice with activities that are inspired by doing research and that involve students in one or more of the processes of inquiry.

We distinguish four criteria for tasks that support inquiry-based learning. Processes of Inquiry are supported by tasks that are cast in – for students – **meaningful situations**. This meaningfulness allows them to question the situation and to think of ways to tackle problems without using standard solution procedures or following a structured task sequence. Learning is driven by open questions that create opportunities for multiple-solution strategies. The openness of the task allows students to (initially) plan and

start the process of inquiry by themselves. The **multiple solution** strategies allow teachers to reflect on processes like the **planning** of inquiry, experimenting systematically and the evaluation of results. This does not imply that students have to go through all phases of a research cycle with each task. A task can also be specifically oriented on one of the processes of inquiry (e.g. what questions or problems are evoked by this situation, or what experimenting and organizing of data is needed for answering the main question). Finally, the tasks need to invite students to **collaborate and communicate**, for instance by providing information of how to distribute work, or by including the need for a presentation of results.



Four dimensions can be considered for connecting tasks to the World of Work: Context, Role, Activity and Product¹. First of all, the **context** in which the task is set relates to the World of Work. The **activities** students do should have a clear purpose, involve authentic problems and reveal how mathematics and science are used in the World of Work. The activities can be more or less similar to activities actually carried out by workers in the workplace with more or less use

of authentic tools or artefacts. Also, the ways of working are supposed to reflect characteristics of daily work, for example by creating an incentive for teamwork or division of labor. Within the task students are placed in a **professional role** fitting the context of the task. In some sense students step out of their role as a student. This is all the more clear if the outcome of the task is a **product** meant for an appropriate audience.

‘The owner of an apartment building wants to build a new parking lot. You are the architect who is given this assignment. Your task is to design a parking lot meeting the requirements ...’

‘The product you need to deliver is a technical drawing of your design as well as a letter to the owner of the building explaining your design and the decisions you made.’

Excerpt from the parking lot task

Not every task will have a similar emphasis on each of these four dimensions, but for a strong connection with the World of Work these dimensions all need to be taken into account.

Tasks that are designed for IBL and WoW will not automatically foster students’ inquiry and create a sense of purpose for them. Inquiry-based learning is not about using new tasks. Tasks offer students the opportunity to make decisions and to question situations, but the tasks do not in themselves guarantee inquiry-based learning. The role of the teacher is crucial here. Teachers need to scaffold the inquiry of students by being proactive: they support and encourage students who are struggling, and extend the skills of the ones that are succeeding through the use of carefully chosen strategic questions. They value students’ contributions – including mistakes – and scaffold learning using students’ reasoning and experiences (Crawford, 2000). The implementation of this support by teachers in daily practice asks for carefully planned lessons and valuing inquiry-related learning goals. The diagram below is an attempt to summarize the learning outcomes, the classroom culture, the learning environment and the roles of teachers and students in the inquiry-based classroom.



¹ A more detailed description can be found in Deliverable 1 and on the *mascil* website.

The mascil Framework

Valued Outcomes

- Inquiring minds
- Applying science in real life
- Preparing for active citizenship and lifelong learning
- Understanding the nature of science
- Understanding how mathematics and science are used in the World of Work

Teacher Guidance

- Values and builds upon student’s reasoning and reflections
- Connects to student’s experience
- Motivates students by connecting school and work

IBL Tasks

- The context is **meaningful**
- The situation evokes **multiple solution** strategies
- The students **plan** inquiry
- The task supports **collaboration and communication**

What Students Do

- Inquire, pose questions
- Explore problems, engage in solving them, use their knowledge to find solutions
- Explain situations and phenomena
- Reflect on the results and processes
- Make sense for themselves
- Explore the World of Work

Classroom Culture

- Shared sense of purpose/justification
- Value mistakes, contributions (open-minded)
- Dialogic
- Shared ownership
- Collaborative

World of Work

- The **context** of the task relates to the WoW
- Students have to take a professional **role**
- Students’ **activities** reflect workplace practices
- The task asks for a **product**

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Contrasting examples

Tasks that teachers give to students have a major influence on the learning that takes place. Many textbooks structure tasks in such a way that students do not encounter too many problems and are able to solve the tasks independently. In general, such tasks are not very helpful for creating opportunities for students to learn to inquire problem situations, to plan solution strategies and to think of ways to collaborate with peers or to communicate results. In this section we present different versions of the same problem situation to show how changes in the setting of the tasks might promote different approaches to the problem and allow for different learning goals. These contrasting examples are used to illustrate how rather small changes in the task might increase the promotion of inquiry-based learning and the use of workplace contexts. The first problem situation considers how the concentration level of medication in blood evolves over time. The second situation involves students in separating salt from brines. In the description of the differences between the tasks we explicitly refer to the task characteristics from the mascil framework that is presented in section 2.

Drug concentration

These two versions of a similar task show how a task can be redesigned to support IBL and to connect to the World of Work. The second open version of the task does not provide the sub-questions that guide the students along the solution process. In addition, it asks for a clear product that provides a purpose and is connected to a workplace practice.

A structured version of the task

A patient is ill. A doctor prescribes a medicine for this patient and advises to take a daily dose of 1500 mg. After taking the dose an average of 25% of the drug leaves the body by secretion during a day. The rest of the drug stays in the blood of the patient.

- 1. How much mg of the drug is in the blood of the patient after one day?
- 2. Finish the table.

Day	Mg of drug in blood
0	0
1	1125
2	
3	

- 3. Explain why you can calculate the amount of drug for the next day with the formula:
$$\text{new_amount} = (\text{old_amount} + 1500) * 0,75$$
- 4. After how many days has the patient more than 4 g medicine in the blood? And after how many days 5 g?
- 5. What is the maximum of amount of the drug that can be reached? Explain your answer.

A version for IBL and connecting the task to the WoW ²

A doctor presents the following details about the use of a specific drug:

- An average of 25% of the drug leaves your body by secretion during a day.
- The drug is effective after a certain level is reached.
- Therefore it takes a few days before the drug that you take every day is effective.
- Do not skip a day.
- It can be unwise to compensate a day when you forgot the drug with a double dose the next day.



N.B. These details are a simplification of reality.

Investigation

- Use calculations to investigate how the level of the drug changes when someone starts taking the drug in a daily dose of 1500 mg with for instance three times 500 mg.
- Are the consequences of skipping a day and/or of taking a double dose really so dramatic?
- Can each drug level be reached? Explain your answer.

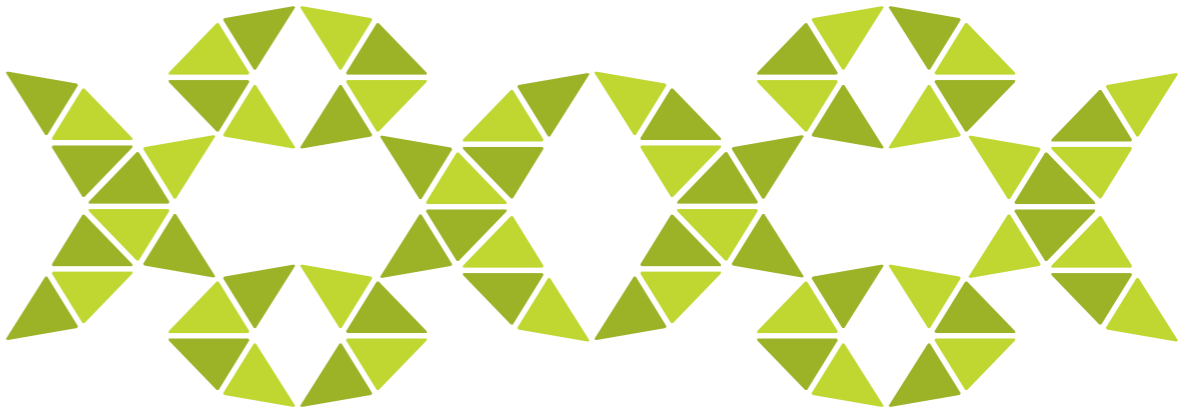
Product

Design a flyer for patients with answers to the above questions. Include graphs and/or tables to illustrate the progress of the drug level over several days.

The IBL version of the task allows students to plan the inquiry themselves. They have to consider the presented information and decide whether more information needs to be collected, investigate what can be calculated and how calculations can be organized to explore and answer the main issues in the task. Furthermore,

the connection to the world of work can be recognized in the structure of the task and the final product. The request to design a flyer invites students to think of ways to communicate their mathematical results to a broader audience as if they act in an environment out-of-school.

² Drawn from mascil task ‘Drug Concentration’
www.fi.uu.nl/toepassing/22038



Brine ³

This example provides three versions of a task that show how a structured version of a task can be redesigned into a task supporting IBL by deleting the sub-tasks and having the students themselves think of the equipment that can be used. Finally, the alternative introduction of the task shows how it can be connected to the World of Work by including a workplace practice, providing a practitioner's role and asking for a clear product.

³ Drawn from mascil task 'Brines':
www.fi.uu.nl/toepassingen/28121

Structured version of the Brines task

Worksheet: Purifying a brine

To separate different substances and obtain pure sodium chloride, you have to conduct a two-step experiment. In the first step, the separation technique of filtration is used to separate sand and stones from the brine. In the second step, the brine is heated (technique: evaporation) to separate water from the salt (sodium chloride).

Substance: Brine consisting of dissolved sodium chloride unpurified with sand and stones

Material: Erlenmeyer flask, beakers, Bunsen burner, tripod, wire gauze, funnel, filter paper, eye protection

Safety measures: Wear your eye protection!

**Task 1**

Insert the funnel into the Erlenmeyer flask. Fold a filter paper, place it into the funnel and wet it with some water. The solution is poured into the funnel. Remove the funnel and discard the filter paper.

**Task 2**

Place the Erlenmeyer flask with the remaining solution onto the wire gauze on top of a tripod and heat it with a Bunsen burner. Wait until all water is evaporated and observe the substance that is left over.

**Task 3**

Name the properties of the substances that make separation possible.

Task 4

Explain the process involved in the two steps of the experiment on the molecular level.

A version for IBL and connecting the task to the WoW ²

Your task is to purify a sample brine and obtain the pure substance sodium chloride. You only know that the brine is a mixture of sodium chloride, sand and stones in water.

In your group you have to develop an experiment to purify the brine. You will get a sample of the brine and a selection of equipment that you may use to conduct your experiment.

For your experiment, you are allowed to use the following equipment:

- Beakers
- Erlenmeyer flask
- Bunsen burner
- tripod
- wire gauze
- funnel
- filter paper
- ...



Wear your eye protection!

If you think a different device would be useful, discuss your idea with the teacher!

Follow-up questions:

1. Explain the processes involved in the steps of your experiment on the molecular level.
2. Explain how you would separate the substances if different ionic compounds were involved.

Student task:



You are an engineer at the salt production company and get the instruction to develop a process that will purify your impure salt brine.

You will get a sample of the brine and a selection of equipment. Your task is to develop a process to come up with salt that might be used in the kitchen.

(re) des ign ing tas ks

(Re)designing tasks

Mathematics and science teachers can use ready-made tasks for inquiry based learning set in rich vocational contexts – like the mascil tasks. Although this type of tasks is rarely found in regular textbooks, existing textbook tasks can often be adapted to provoke IBL and/or reflect a connection to the World of Work. This is easier if the textbook problems are already situated in a context. Below are guidelines for redesign⁴ that a teacher can apply to adapt suitable textbook problems.

From a structured textbook task to a task supporting IBL

- Look for the ‘real problem’ within the context. Take this as the focal point for redesign.
- Create opportunities for students to become owner of the problem and a solution strategy.
- Skip sub-questions and pose the real problem or the main question. In some cases it is even sufficient to only sketch the situation and ask the students what questions it evokes.

- Scaffold students’ inquiry process with a lesson plan (the introduction, process support and a final goal need more attention than compared to a structured task).
- Provide guidelines about the final evaluation of the result.

Connect a task to the World of Work (WoW)

The starting point for a mascil task may be an existing IBL task for mathematics or science that is not yet related to the World of Work. In this case it is often possible to add contextual information from the WoW, to formulate activities for the students related to similar authentic practices from the WoW, to give students a professional role, and to define an appropriate product.

- Explore the context of the task and try to relate it to the WoW.
- Think of a workplace practitioner and a workplace activity fitting the context and/or the task.
- Determine a product that fits the task, is connected to the WoW and is useful or meaningful for a specific audience.

⁴ See Guidelines for redesign on the mascil website (www.mascil-project.eu)

Stimulate cooperation and communication

- Ask for products that can be presented and discussed.
- Make sure the task asks for cooperative work (e.g. sharing of responsibilities, partial division of labour, etc.).
- Organise peer feedback.

When a task has been redesigned be aware of the changing role of the task in the learning process of the students. In addition to content-related goals, the new task aims at developing process skills. In some cases this might be at the cost of attention for content knowledge. In other cases it might offer opportunities to deepen content knowledge, or to better assess students’ abilities.

The questions below can be used to review the task resulting from the redesign process, and decide whether it now supports IBL and/or is connected to the WoW and what are the potential benefits for students working with this task:

- Does the task require students to adopt a role or ways of working that are different to usual?
- What knowledge, skills, or understanding might be gained, either mathematical or science-related or otherwise, through using this task?
- In what ways might this learning be different from that stimulated by a more conventional school task?
- Would the task help motivate students or stimulate student inquiry in any way?

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Mascil in the classroom

When working with students on mascil tasks – open tasks set in rich vocational contexts that ask for inquiry – there are several ways to give students some insights into how mathematics and science might be used in a range of different workplaces. We can organize students to work in ways that reflect the sorts of roles and responsibilities that occur in a workplace where science and/or mathematics play an important role. We could also have students work on a task or activity that draws on data and information or artefacts from a specific workplace or vocational area. Another option is to use vocational contexts in mathematics and science lessons that provide students with insights into the usefulness (purpose and utility) of mathematics and science in the World of Work.

“At a certain moment we understood how everything worked out and from that moment on we ‘raced’ through the tasks. Because we had divided the work efficiently we could finish the tasks fast [...] we were better at math than we thought and that is worth something as well.” (Celia, 17 years old)

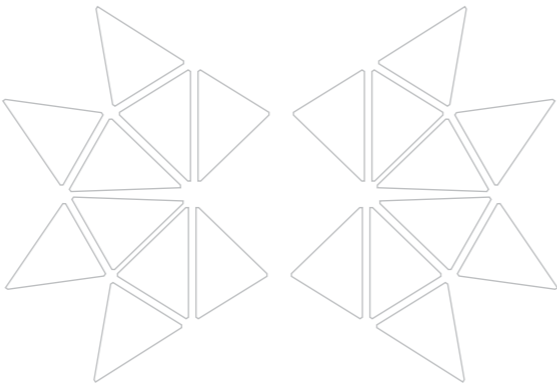
In all cases it is important to provide students with the opportunity to explore the professional context of the task: what are typical activities, tools, data, language outcomes, products, problems of this workplace? This may be done by including video or photos or artefacts from the workplace, by inviting professionals to the classroom, by having students visit a workplace or a website of a company. In this first exploration of the context it is important that students think about what they already know and can do to solve the problem and what other information, knowledge, actions or tools are needed.

In order to facilitate the inquiry and problem solving processes a teacher might insert a brief whole class discussion after the first exploratory phase. In this discussion students can share their first ideas and exchange any problems or questions they encountered.

“In the introductory tasks we were confronted with mathematics we didn’t fully understand. We kept to our initial problem solving strategies throughout the tasks and we believe this led to a very good outcome. By wrestling through the introductory tasks we got more and more familiar with the context and the mathematics.” (Dahoud, 16 years old)

“We learned things and it was fun. We were free to plan the work ourselves. [...] After ‘hard thinking’ we understood what was going on. In the beginning we were frustrated, but after we found a way to approach the problem and to divide the work, it became much more fun.” (Geertje, 14 years old)

Teachers can use a lesson plan that facilitates inquiry based teaching and scaffolds students inquiry process. The exemplary lesson plan below is written for the open, unstructured version of the drug concentration task.



A sample lesson plan

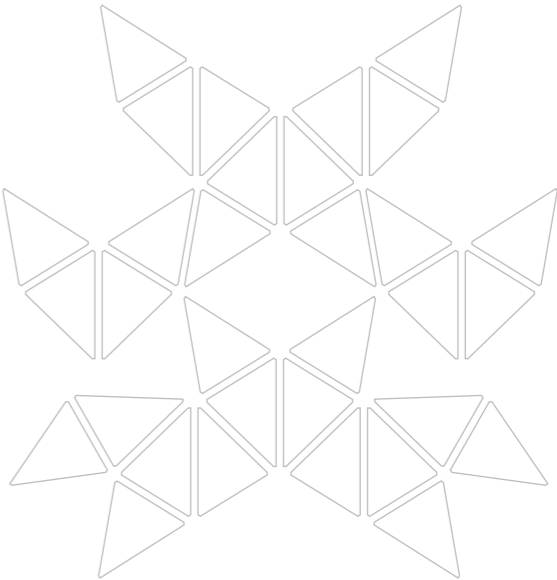
- Lesson 1**
10 minutes: create groups & introduce the problem and the working plan and distribute the task
10 minutes: students work in groups on the task
10 minutes: discuss with the whole class whether all groups have an idea how to start and how to proceed. Exchange strategies and make sure that everybody has an idea what is expected.
15 minutes: students work on the task, finish calculations and prepare the building blocks for their flyer.
- Lesson 2**
20 minutes: students finish their flyer
20 minutes: presentations of a few examples
10 minutes: reflection on the task (and positioning it in further work)

To provide students with optimal opportunities for exploration the tasks should be open and have the potential to evoke multiple solution strategies. If that is the case the students themselves need to think about how to structure the task and divide it into smaller parts. This fosters inquiry by students and stimulates ownership of the problems that need to be solved to fulfill the task. The teacher provides (limited) time for exploration, supports questioning and sharing ideas, takes care of the distribution of work (often in small groups),

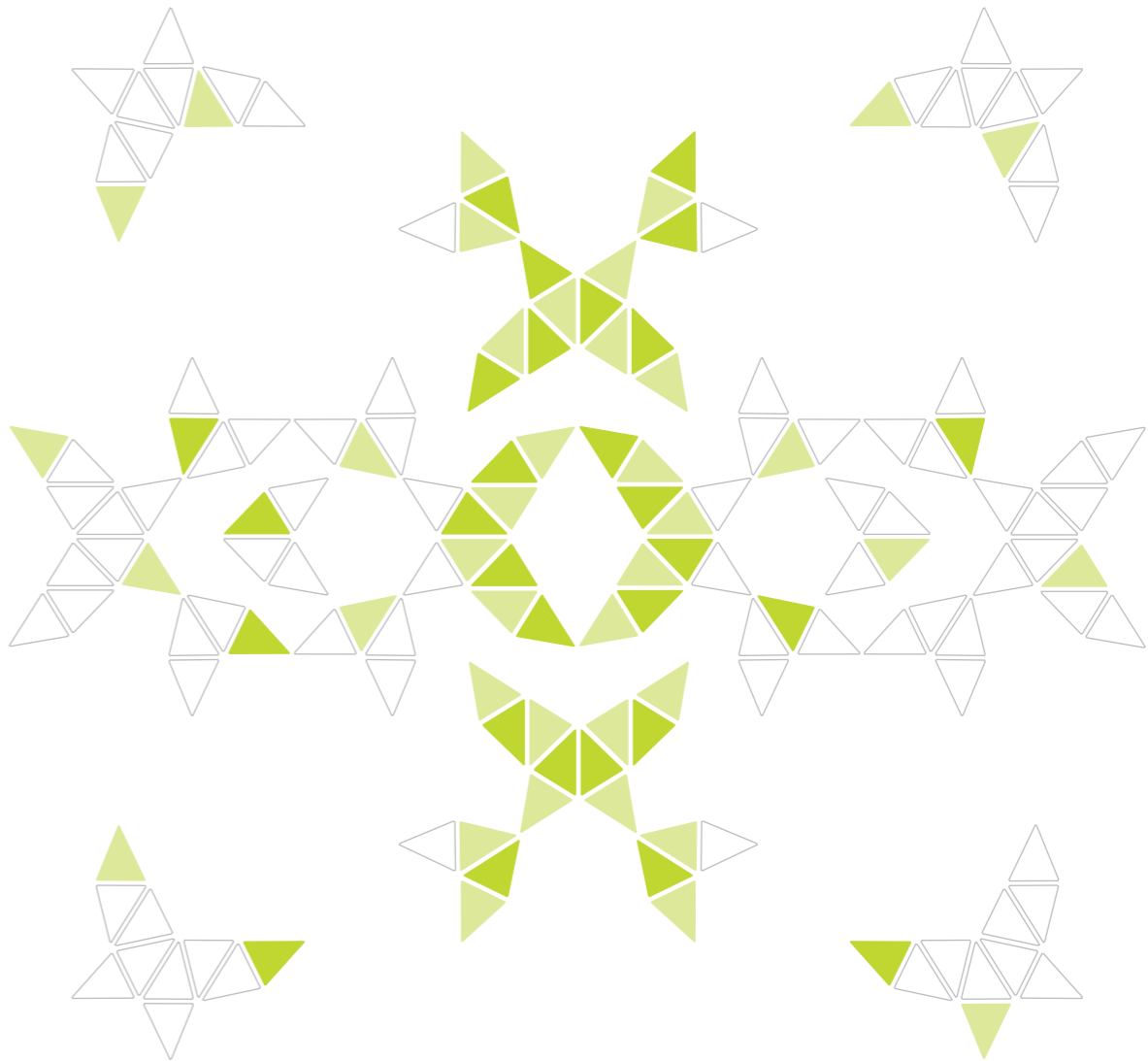
creates time for whole class process feedback (to prevent running from table to table), lets the students present and comment their work (e.g. peer-feedback), and finally organizes a reflection on the inquiry processes and the science and mathematics learning.

Below is advice for teachers on how to deal with unstructured problems and scaffold students' inquiry processes by posing appropriate questions⁵.

⁵ For further information see the mascil professional development toolkit: www.mascil-project.eu/professional-development/teacher-pd-toolkit



IBL TEACHING STRATEGIES	SUGGESTED QUESTIONS
Allow students time to understand the problem and engage with it Discourage students from rushing in too quickly or from asking you to help too soon.	<ul style="list-style-type: none">• Take your time, don't rush.• What do you know?• What are you trying to do?• What is fixed? What can be changed?• Don't ask for help too quickly – try to think it out between you.
Offer strategic rather than technical hints Avoid simplifying problems for students by breaking it down into steps.	<ul style="list-style-type: none">• How could you get started on this problem?• What have you tried so far?• Can you try a specific example?• How can you be systematic here?• Can you think of a helpful representation?
Encourage students to consider alternative methods and approaches Encourage students to compare their own methods.	<ul style="list-style-type: none">• Is there another way of doing this?• Describe your method to the rest of the group• Which of these two methods do you prefer and why?
Encourage explanation Make students do the reasoning, and encourage them to explain to one another.	<ul style="list-style-type: none">• Can you explain your method?• Can you explain that again differently?• Can you put what Sarah just said into your own words?• Can you write that down?
Model thinking and powerful methods When students have done all they can, they will learn from being shown a powerful, elegant approach. If this is done at the beginning, however, they will simply imitate the method and not appreciate why it was needed.	<ul style="list-style-type: none">• Now I'm going to try this problem myself, thinking aloud.• I might make some mistakes here – try to spot them for me.• This is one way of improving the solution.



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Experiences with mascil tasks

In the following, you will find several experiences with mascil tasks. There are experiences with tasks in the primary and secondary levels, in mathematics and science lessons, in various European countries. You can see a complete description of the tasks and their relevant worksheets and teacher guides on the mascil website: www.mascil-project.eu.

Each of the experiences starts with a description of the task, its potential for inquiry-based learning and its connection to a workplace context. Next, the classroom practice is outlined by describing the country, the level and the lessons that were used for the implementation of the task. Finally, the experiences with the task are described including copies of student work showing their products.

This collection of experiences illustrates the rich potential of implementing IBL and WoW in daily practice. It shows what students in very different European countries are capable of as soon as they are given the opportunity to be creative and to collaboratively work on open tasks. The collection is supposed to provide ideas, support and, hopefully, it will stimulate and motivate teachers to develop and implement their own IBL and WoW tasks for use in their classrooms. Allow yourself to be inspired – and you will be surprised by your students' capacities, enthusiasm and learning potential.





Experiences



Bicycle insurance

The task is to create a manual for an insurance company on how to calculate a 'fair' insurance rate for a bicycle and a 'fair' compensation for a stolen bicycle. The students have to consider different variables, such as:

- The likeliness for a bicycle to be stolen (in this area)
- The value of different bicycles
- The age of the bicycle and how this affects their value

The product is a written document or manual for the insurance company where the suggested calculations are described and reasoned for, which may or may not (depending on the students' age) include tables, graphs, diagrams, formulas etc. The task involves making choices, justifying and defending decisions and comparing different solutions.

Inquiry Based Learning and the World of Work

This task has a clear starting point and a specified goal, yet there are many solutions to reach this goal and the task can therefore be characterised as an inquiry based task at the level of a guided inquiry. The task is situated in the field of consulting for start-up companies where the students adopt the role of the consultant. First of all, the students have to understand how insurance works, what the different revenues and expenditures are, and what the average monthly or annual salary of an insurance employee is. Then, they collect data of bicycle thefts and estimate the amount on bicycles in their region. Based on these estimations, they develop a simplified business plan for bicycle insurance. The students collaborate in small teams, discuss their different approaches and report their findings. The teacher provides some additional information from the insurance industry and supports the students in collecting their data, in developing their ideas and in overcoming possible obstacles. Depending on the students' age the task can be adapted, focusing on different aspects: data

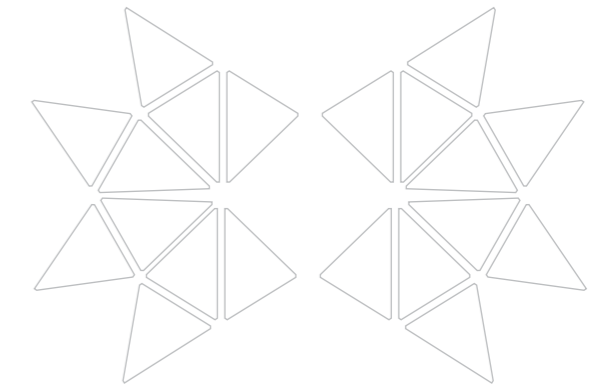
collection and estimation of an average bicycle theft rate; describing the different revenues and expenditures with simple functions; simulation of the success of the insurance company under the assumption of randomly distributed bicycle thefts and bicycle prices (this is already very close to real-life modelling practices of an insurance company). The final product (written document or manual) can then be presented to the other teams, for example in a competitive setting.

Classroom practice from Austria in cooperation with Bulgaria

This task was tried out with year 9 students (14 years old) in an engineering-focused secondary college in Austria. The class consisted of 15 students who worked together in four teams. In total, they spent three mathematics lessons (50 min. each) with the task.

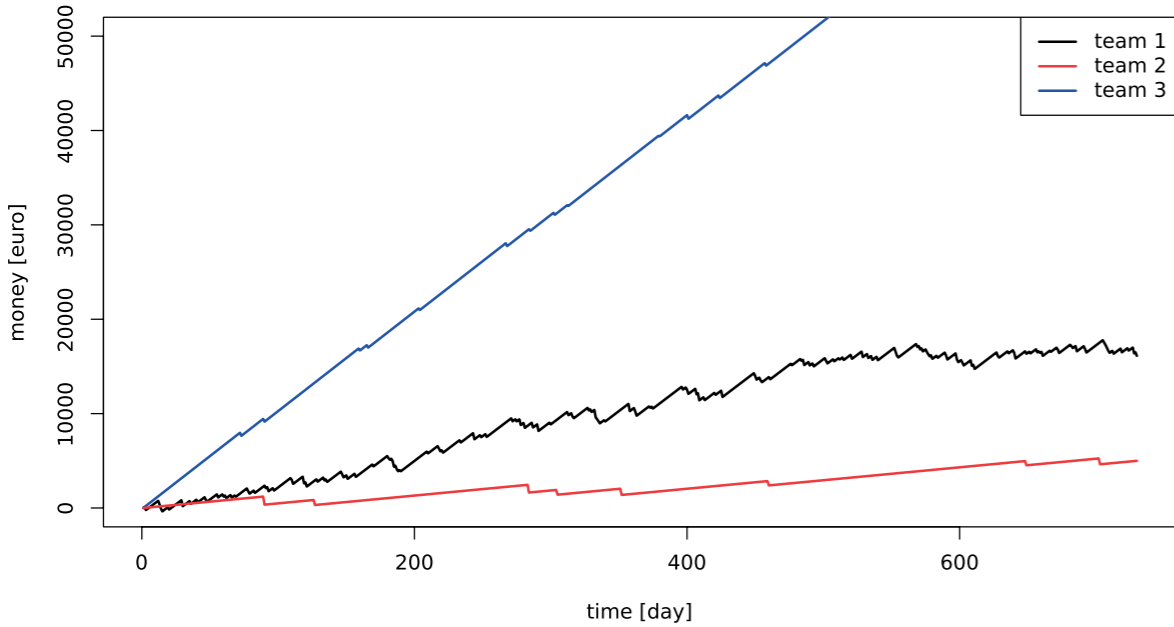
As homework for the first lesson, the students had to collect data on the number of bicycles and the number of bicycles stolen annually in each federal state of Austria. During the first lesson, the teacher presented the task and explained how an insurance company works. The students had many questions, especially regarding the fact that one might receive more money from the insurance in case of a theft than they had actually paid in. The teacher then discussed the different revenues and expenditures of

insurance companies. Due to insufficient data collection in each team, the teacher gathered all the data and combined it in a spreadsheet, which was then sent to all teams (each team had at least one laptop). In addition to the original task, he encouraged the students to describe the revenues and expenditures with simple functions, to sketch the graphs of these functions and to think about the development of their insurance company in the next few years. Then the students started working in their teams, but since they were not used to working on inquiry based tasks, the teacher had to support them on several occasions. At the end of the second lesson, most teams had found a strategy to estimate an average bicycle theft rate, calculated one or two average bicycle prices and had an idea about average annual salaries. Based on these data, they calculated an average insurance premium and looked at the profit of the company.



Between the second and the third lesson, the teacher programmed a simulation for randomly distributed events (number of bicycle thefts and bicycle prices). This allowed for the testing of the students' solutions with randomly generated data. The graph below shows the development of some the students' companies. Here, the money of the company in each month is the

sum of the different revenues and expenditures, which was then used to introduce the sum of functions. This simulation was quite fruitful due to the visualisation of the propositions' outcomes and how these are influenced by the contributing factors (e.g. salary, annual premium and indemnifications).



Development of the companies from different teams

During the third lesson, the students finalised their reports and after a feedback loop the reports were handed in. It turned out, that the students had difficulties to understand the basic principles of an insurance company, thus we suggest the context be introduced carefully. The notion of function was quite new for the students, as a consequence only one team modelled the revenues and expenditures with stepwise linear functions.

The same task was also tried out in Bulgaria at Baba Tonka High School of Mathematics in the town of Ruse. Instead of a manual, the Bulgarian students designed slides to advertise their own bicycle insurance company and provided some additional slides with their assumptions and calculations for a profitable company.

The Austrian reports and the Bulgarian slides were translated into English and then exchanged between the two classes. Each class discussed

the results of the other class and prepared some questions. During a virtual meeting the Austrian and Bulgarian students could ask each other questions and discuss the different assumptions and solutions. The Austrian teacher included the Bulgarian approach into the simulation and presented it to the Bulgarian students (and a revised version of the simulation program was then sent to the Bulgarian teacher). This collaboration culminated in a face-to-face meeting of the supervising teachers in Sofia, at a special Poster Session within the National conference on IBME.

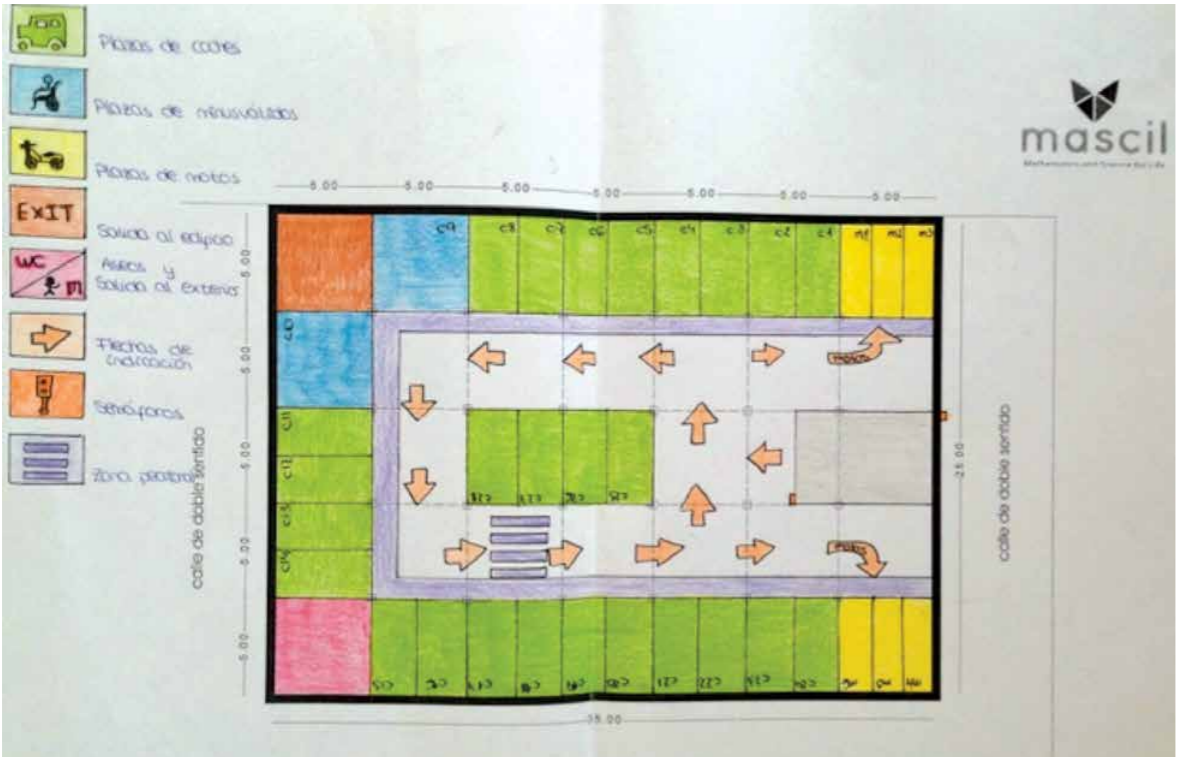
Author: Florian Stampfer

Link to mascil website:
www.fi.uu.nl/toepassing/28226



Meeting of the supervising teachers in Sofia

Designing a parking garage

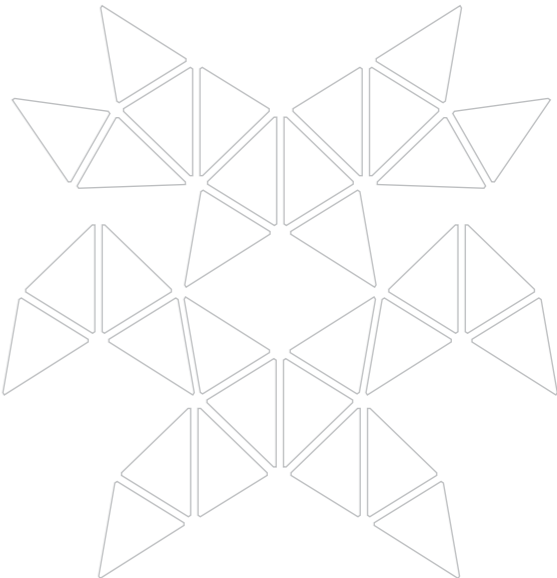


The design of a building is a complex task involving many variables. Architects have to think about the structure, installations (electricity, water, heating...), the distribution of the space (staircase, corridors, rooms, entrance hall...), orientation of the building, etc. Often, decisions taken in prior steps affect what it is possible to do in the next ones. In this task students work as architects on the design of a car park (garage). The structure of the building and the distribution of the pillars have already been decided and cannot be changed. Students design the lay-out of the car park, the parking spaces and the entrance ramp.

Inquiry Based Learning and the World of Work

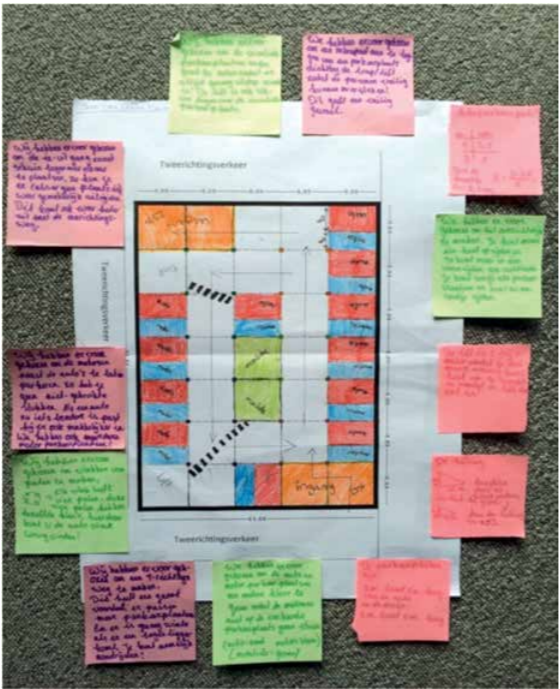
The task has been designed by an architect, mirroring her work in the professional context. Constraints have been taken from existing regulations. The task reflects how mathematics is used in the world of work, where the problems are ill-structured and no unique solution is expected. The evaluation and improvement of the possible solutions is part of the solution process. Professionals need to make use of a variety of mathematical knowledge and tools to come up with a good solution.

The task does not provide all information that students might want to use for their design. During a first inquiry they list missing information (like the size of cars) and create plans of how to collect and share this information. Students adopt the role of an architect designing a car park with a building. A scale drawing of the outline is available on a worksheet. Students use mathematics to decide on the lay-out and measurements of the parking spaces and other elements like ramp and staircase. The product is a scale drawing of the car park with an explanation. The product can also include a report or letter for the owner or the users of the car park in the building.



Classroom practices from Spain, Greece, the Netherlands and Romania

In Spain, the task was used with twenty-five grade 9 students. Initially, three 50 minutes sessions were planned: session 1 to introduce the task and initiate students' exploration of the situation, session 2 in which students should work in groups, designing their parking lot, and session 3 where students will present their work and discuss their designs. Students were used to working collaboratively, but not with an open-ended task like this. According to the description of the task, students' work was mainly oriented on two issues: (1) optimising the distribution of the parking places and other parts of the parking, and (2) calculating the dimension of the ramp so that the gradient is less than 25%. Students had difficulties to interpret ramp's gradient in terms of percentages. They had some previous knowledge about gradients, but interpreted as the tangent of the angle. Instead of a problem, this was perceived by the teacher as an opportunity to (re)visit the mathematics of trigonometry and triangles. Finally, all the groups could translate the ramp's problem into a math problem, solve it, and interpret the solution in terms of the real situation. At the end, students were asked about their experience with the task. According to their feedback, they especially liked the open-ended nature of the task, and doing mathematics like professionals do outside school.



In Greece, a group of three prospective teachers chose the parking problem task to investigate the implementation of IBL in an 8th grade mathematics classroom. The students worked in small groups and by adopting the role of an architect they constructed their own plans and argued about their appropriateness. The prospective teachers identified students' solution strategies, their arguments and realized student's difficulties with the mathematisation process and the understanding of the underlying concepts. They also reflected on their own teaching and realized the potential of the tasks in promoting students' motivation and mathematical meaning but also the difficulties to manage these tasks in the classroom. They reported: "The students were really involved in the tasks and they were keen to express their opinions and to argue for them ... We could have managed better the time allowing students to provide more arguments for supporting their claims"; "I was surprised that students who were not considered as good students by the classroom teacher were really engaged in the task."; "The students seemed to link the problem with reality. They were checking whether their calculations provided realistic results. In one case they discovered an error in calculating the covering area, as their result was larger than the parking area. Visualization seemed also very important in tackling the task."



In the Netherlands, the task was used by an experienced mathematics teacher in two 8th grade classes with about 30 students. Initially, the teacher was a bit sceptical about the possibility of using the task as it is rather open and students need to deal with quite some missing information. After discussing the lesson plan that scaffolds the students' activities, she was willing to try out the task. She started the lesson with introducing the task, watching a video of the construction of a parking lot, and a short discussion about the task situation and what further information the students would need to start designing. Some students, for instance, asked for the size of an average car. She rephrased the question and asked how you could solve such a question. Students suggested to go outside and measure cars or to browse the internet. Both possibilities were rewarded and

after dealing with all the questions the students started working on the task. Also in her case, they worked in small groups, adopted the role of an architect seriously and designed their own plans for a parking lot. Afterwards, the teacher was amazed by the enthusiasm of her students and of their work. In a quite natural way they worked with scale and ratio and created various products with accompanying information to ‘sell’ their design (e.g. colourful drawings, 3D models and websites).

In Romania, the mascil team was convinced that an IBL task – especially when connected to the World of Work – will have the most success in being implemented if it is solved first by the teachers themselves. This is why the task was introduced at a professional development course, where 23 teachers had to solve the problem and then discuss about the different solutions. Teachers worked in small groups and were left to work entirely by themselves within a time limit. Due to the openness of the problem and all missing data, teachers spent almost an hour documenting and calculating. When the time was up and a tentative solution was presented by each group, every group had five minutes to optimize their solution. The presentation of different solutions was followed by a brief discussion and all teachers were convinced that this task should be implemented in classroom practice.

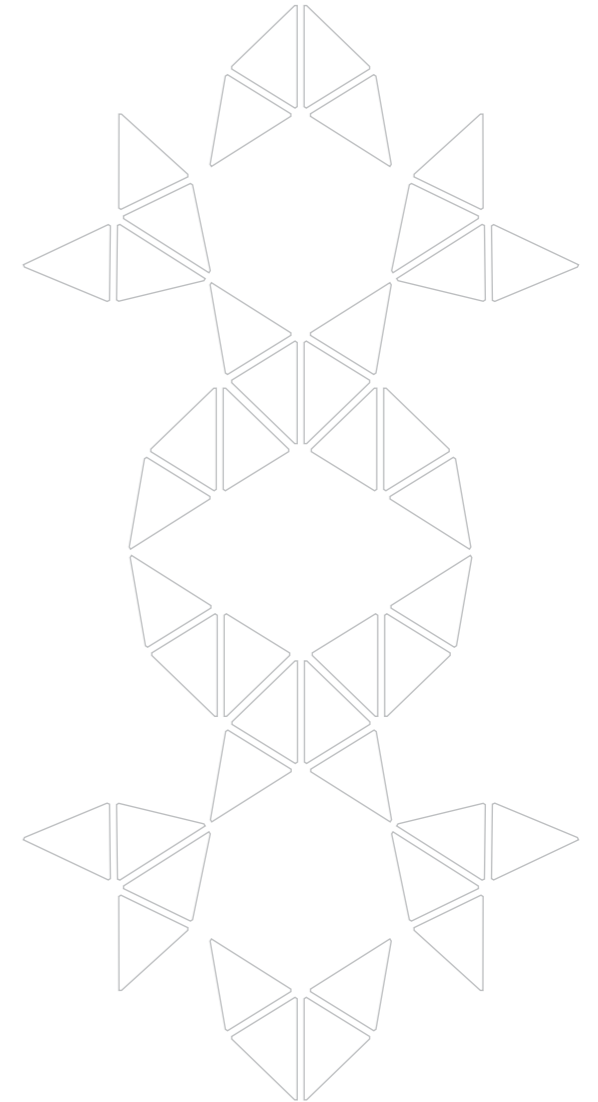
The task was used by primary school students (grade 3), secondary school students (grade 8), mathematics education master’s course students, and by a high school class (grade 10). At all different levels the quality of solutions was almost the same while the main focus of the discussions, the main difficulties and the possible lessons learned were quite different. Since the task was initially proposed for secondary school and high school students, implementing it in primary school was a great challenge. Parking lot plans were considerably enlarged, paper cars were printed out, already made stairwells were used and also some tools (like measuring tapes) were brought in. Working with the same task at several levels offered a unique perspective on the ongoing processes, the typical reactions that came up in the activities. A striking difference was in the time management and in the use of tools. While teachers understood the task quite quickly, they spent a lot of time on gathering the missing information and in creating their first design, the younger students spent more time on understanding the task and only a small amount

of time with the first design. A central question for the Romanian team was: at what level could this task be implemented? Especially in Romania, with a strongly content focused curriculum and a tight time frame for teaching mathematics? During the activities it became clear that the task can be used at all levels. In primary school, to help understand the concept of areas, scaling, multiple solutions, optimizations; in secondary school as application of trigonometry to real-world problems like the length of the ramp, the maximum slope of the ramp or in high school as an optimization problem. However, the success of the implementation highly depends on the preparation of the teacher.

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Link to mascil website:

www.fi.uu.nl/toepassing/28343



Parachute food drop



A military transport airplane drops humanitarian aid load, dealing with the aftermath of the 2010 Haiti earthquake. Attribution: By TSgt. James L. Harper Jr Derivative work: Durova [Public domain], via Wikimedia Commons

Historically, airdrops have been used for different purposes, from resupplying troops, to deliver food and medicines as humanitarian aid in areas, otherwise inaccessible. The design of parachutes for airdrop is a complex task, which has to take into account different variables. Among other things, it is necessary to consider the material, size, shape and structure of the parachute required to deliver a particular load at adequate velocity. In this activity, students take the role of an aeronautical engineer, specialised in parachute engineering and design. They will have to decide which variables they want to investigate, make systematic experiments and try out different prototypes, in order to design the best possible parachute. Finally, they are asked to present their work to an audience backing up their final product in the results obtained through the research and piloting phase.

Inquiry Based Learning and the World of Work

The introductory scenario places students in the role of aeronautical engineers, specialised in parachute engineering and design, who have to make an adequate parachute for the airdrop of humanitarian aid. They are asked to present a final product to an audience, explaining the foundations of their parachute. This is an open and unstructured task suitable

for fostering inquiry minds and process skills in the classroom. In order to successfully address the challenge, students will have to raise questions, make assumptions, design and conduct experiments and discuss their findings with others, in order to better understand the factors affecting a falling object. The task offers opportunities for the understanding and meaningful application of key science concepts such as weight, speed and acceleration, while investigating the effects of gravity and air force resistance. Shape, size and effective area will be other key concepts to be used when designing an appropriate parachute, along with the investigation of how the properties and behaviour of different materials affect a parachute efficacy.

In sum, the use of an open task in a 'world of work' scenario provides a sense of purpose and meaning to students' activity, raising their engagement and motivation. It also offers opportunities for the development of inquiry skills and the application of key science and mathematics concepts.

Classroom practice from Spain

This activity has been tried out with two different groups of teacher students (N = 160, average age 21), in a pre-service teacher education course on science education, but it can be also used with secondary school students (age 12-18).

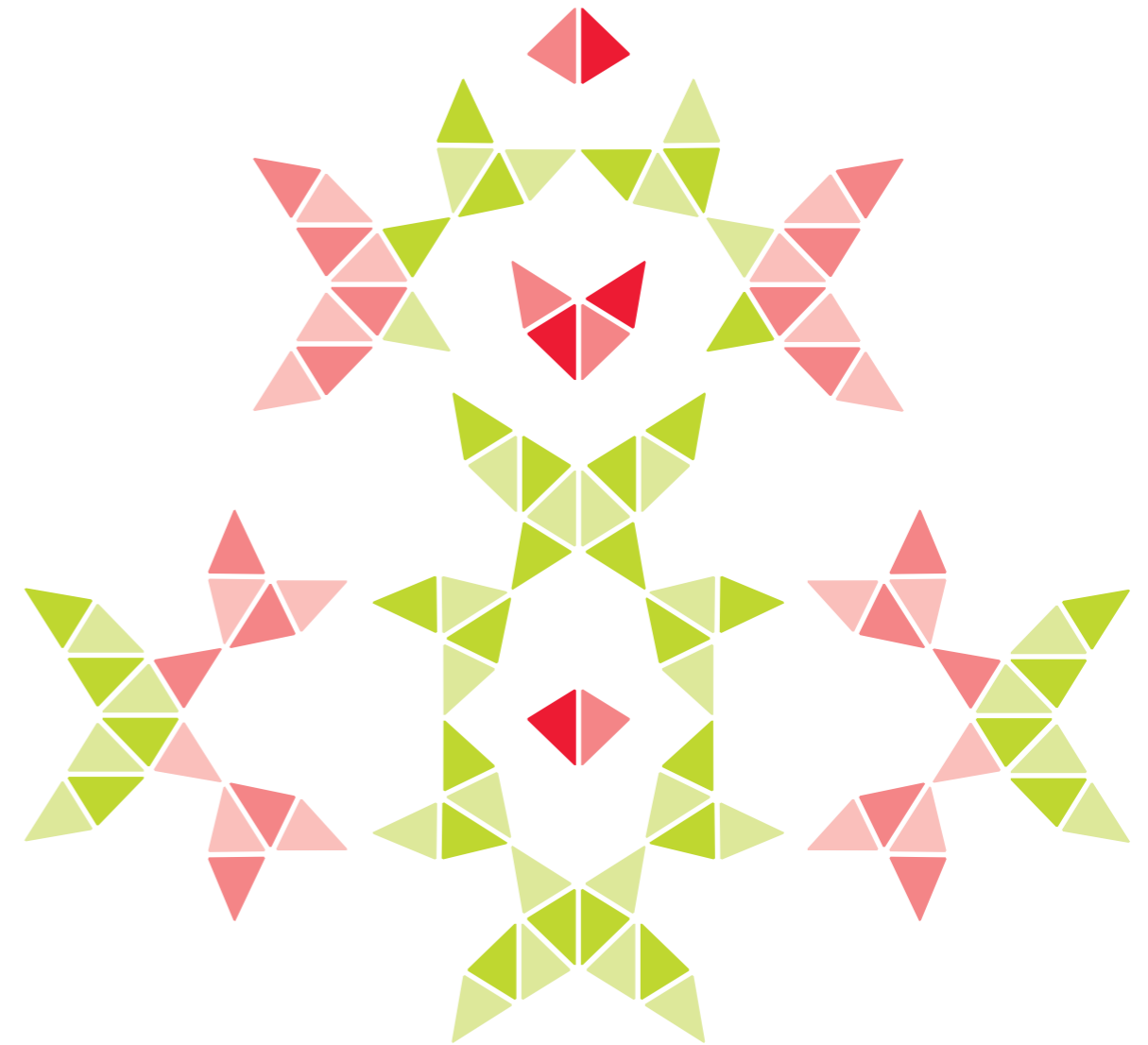
The teacher introduced the scenario and asked students to take the role of engineers and conduct their own piloting and investigation, in order to make a good parachute for airdropping humanitarian aid. Various materials were available in the classroom (plastic, cloth, different types of rope, different masses, scissors, metres...) and students made their own plans and experimental designs. From the very beginning, a high level of motivation and ownership was observed, since students got actively engaged in the task. For 2 hours they worked in groups of 2-4 people investigating gravity and air force resistance while trying out several loads, materials, sizes and shapes. They were asked to describe their experiments in a report, trying to use science and maths to explain their outcomes and back up their final parachute. Finally, in the next lesson they presented their work and best parachute to the rest of the class.

The analysis and discussion of the experience with the teacher students made them aware of the importance of systematic work and the control of variables in order to draw appropriate conclusions. The teacher also took the chance to have a science discussion on the effects of gravity and air force resistance and how it could be used to slow down falling objects. He also drew attention on how the activity had supported students in the development of inquiry skills.

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Link to the mascil website:

www.fi.uu.nl/toepassing/28519



Counting people



Children start counting from a very early age. Counting is a key mathematical competency in Early Childhood Education and the first grades of Primary School. However, counting goes beyond school and it is an essential activity in many professional fields such as biology, ecology, geology, medicine, journalism, sociology. In contrast with the ‘world of school’, where counting is normally an unproblematic and straightforward activity, in the ‘world of work’ counting can become a really challenging task. In this activity, students will have to devise their own plan for counting the number of people in a big public area.

Inquiry Based Learning and the World of Work

This is an open task that tries to involve the students in many aspects of inquiry based learning, such as developing questions, researching for finding an answer and decision making. The task is a problem of estimating how many people there are in a crowd. When it is difficult to count each every individual person in a crowd, many interesting (mathematical) ideas may arise from solving such a problem. The role that the students adopt here is that of a journalist, who wants to present information on the number of people in a huge crowd without only relying on the data provided by the organizers or authorities. Students may work in small groups and negotiate their estimations because the correct answer to this problem can only be estimated. Critical aspects to this inquiry are the area measurement and the estimation of the space that one person or a small group of people occupy when they are standing. The first aspect – the area – may be investigated through an online service based on Google maps, while the latter is connected with the concept of density, bringing the need to observe the area of the crowd and consider what the density could be. Here students can also model this themselves and take measurements. The role of the teacher is that of scaffolding the inquiry by the students and giving help for example

about the instruments the students need for investigating the problem. A debate among the different answers that the students may bring is critical for the development of the activity.

Classroom practice from Greece

The task took place in a grade 9 class (14 years old) in a public secondary school in the center of Athens. The mathematics lesson lasted two teaching hours (90 minutes). Twenty-one students participated in the activity working in groups of three to four (6 groups in total). In the beginning the teacher provided the students with two photos of an antiracist demonstration in the center of Athens. The photos were taken at different times showing people standing in three different streets (where the density of the people varies). Students are asked to adopt the role of a journalist and to provide information about the number of demonstrators. The resources they could use were the internet, calculators, rulers and a measuring tape. In the first teaching period the students worked in groups on the problem while the teacher was moving around the groups. In the second teaching period the problem of calculating the density was negotiated in the classroom interaction, while towards the end the groups presented their findings.



- Three successive steps were followed to solve the problem:
- a. using google earth to measure the dimensions of the three streets for calculating their area
 - b. estimating the number of people that can stand in specific areas and
 - c. calculating the total number of demonstrators.

Concerning the measurement of the dimensions, three different strategies emerged. Four groups used the scale of google earth (a given length on the computer screen and its real value) and the

ruler to measure the dimensions of the streets on the computer screen to find the real ones while one group used the measuring tool of google earth. An unexpected strategy of one group was the construction of the scale by comparing a known length (the length of a car) with the sides of the streets.

Concerning the number of people in a given area, the problem posed by one of the groups was: “how many persons can fit in one square meter?” This problem started to be shared among different groups and finally it was discussed in the



classroom. The students simulated the problem by forming a square meter on the floor using a measuring tape, standing inside to find out the number of persons that fit and measuring the density. Then this problem was used as a key idea to calculate the density in the three streets of the photo.



In the classroom discussion, it appeared that the students wanted to confirm their solution. They were surprised that each group had a different estimate that was found acceptable by the teachers. They insisted in asking for the correct answer as they could not cope with this uncertainty. A possible interpretation for this incident is that most tasks posed in the classroom are of a closed type and that students act according to the existing classroom norms.

An important feature of this task appeared to be its openness that engages students in decision making through their own appropriate data. The students wondered which data are needed, how these could be selected and worked out. The task could be extended to other contexts and situations from the microcosm (e.g. bacteria) where the students had to select a representative sample and use methods from probability and statistics.

Authors: Areti Efstathiou, Vasilis Karagiannis and Chara Papakanderaki

Link to mascil website:
www.fi.uu.nl/toepassing/28343

Holiday kennel for dogs



The design of a boarding kennel for dogs is a complex task involving many variables. Designers have to think about the structure, the size of dog houses, the number of dogs in a dog house, free space, etc. The minimal space required for dogs in a dog house depends on the size of the dog. The task presents a table with the height of a dog as a variable. Other sources take the length of a dog into account (a link is provided). How do you measure height or length of a dog? In this task students work as boarding kennel constructors on the design of a dog kennel. The plan of the kennel and the distribution of dogs in dog houses have to be determined. Students have to understand the constraints and work with them for their design.

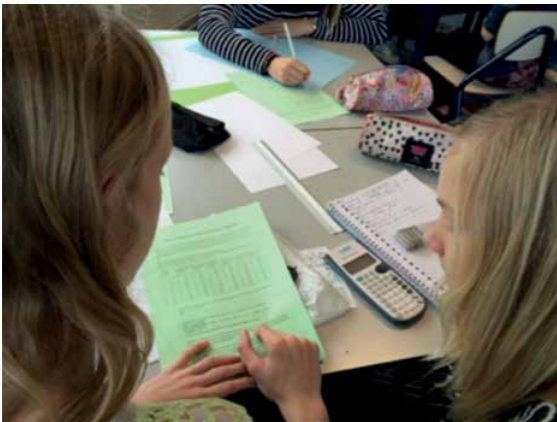
Inquiry Based Learning and the World of Work

This is an open and unstructured task suitable for inquiry based learning. The task focuses on spatial design of a dog boarding kennel within certain constraints. The students adopt the role of architects with a specific focus on animal housing. The design is commissioned by a couple who wants to start exploiting a holiday kennel for dogs in their large garden. Students explore the constraints of the task and make their own decisions on lay-out and measurements using mathematics. They have to think about which variables to take into account and they

have to decide for instance, based on existing regulations, what the measurements of individual dog-spaces will be and how these relate to the type of dogs they want to accommodate. The students collaborate in small teams and discuss their ways of working. There is not one correct answer: different teams may come up with different designs based on different decisions they made during the design process. The teacher provides structure in the lesson. It is helpful for the process to have students report and reflect on their approaches in a whole class discussion after a brief orientation on the problem in their small groups (about 15 minutes). The students deliver a scale drawing of the dog kennel, with an explanatory report – for the client – in which they present and justify their decisions and highlight the characteristics of the resulting product. They present this product in class as well.

Classroom practice from the Netherlands

This activity was tried out in a grade 9 class (13 year olds) in a secondary school in the Netherlands in a mathematics lesson of 70 minutes. The class consisted of 24 students, who are used to working on open ended tasks in small groups. For this activity the students worked in groups of four.



The teacher introduces the task by showing a video of a dog kennel. She explained the way in which the product should be presented: a scale design of a dog kennel on a piece of coloured paper. The teacher then told the students to read

the task in silence without further instructions. Since the students are used to working on this type of task, after a brief check by the teacher to see if everything was clear about the task and the constraints, they could continue to work on their design for the rest of the lesson.

Most of the groups immediately divided the task into sub-tasks, they investigated the constraints presented in the problem and planned the layout, while making decisions on for example the measurements, the number of dogs etcetera. They made a scale model applying several mathematical skills.

A lot of creative ideas were used in the design like adding a swimming pool or parking spaces. During the process the teacher decided to invite



students to design a flyer and become explicit about the arguments underlying their decisions, like: why did you choose this specific lay out? What makes your design visually attractive? Is it efficient? Can it be used flexibly to accommodate different numbers and different types of dogs? It turned out that, contrary to expectations, most teams in their design used one size for all the smaller spaces for dogs, while In the assignment data was presented about area requirements for different types of and numbers of dogs. It was expected that the students would use these data. It could be useful to explicitly set a requirement for including at least three different sized kennels in the design. This may further enhance the level of mathematics in the task. An extension of this task can be to have students include a budget and specify the costs for the clients.

Authors: Lysanne Smit, Monica Wijers, Michiel Doorman

Link to mascil website:
www.fi.uu.nl/toepassing/28343



Designing your non-standard bookshelf

In this activity students have to design their own non-rectangular bookshelf for a wall with given sizes. The design must be feasible with their available tools/possibilities and must contain a detailed plan for realizing the bookshelf. In a less open formulation the bookshelf has to possess several properties like having regular polygonal cells, using a modular construction, and being symmetrical.

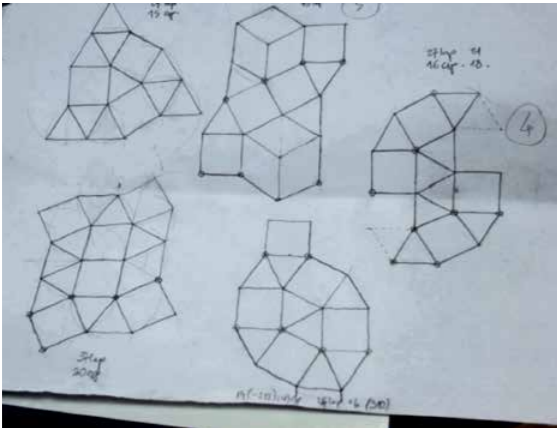


Inquiry Based Learning and the World of Work

The activity is open and can be easily adapted in order to emphasize different mathematical content areas (e.g. plane tiling with several regular polygonal shapes, basic geometry of regular polygons, Diophantine equations) depending on the level of the students. The task focuses on the one hand on the creative design of the bookshelf and on the other hand on the execution, the assembly of the pieces having in mind a series of practical and functional criteria like stability, capacity, complexity of the assembling procedure and how to secure the bookshelf to the wall. Students adopt the role of designers of modular furniture. They have to think of the affordances and the constraints of the available materials and to develop a strategy for designing a new and spectacular bookshelf (within the available time). Students are expected to experience that a good plan and division of work is essential for creating and finishing a design. The World of Work dimension can be extended to include artisanal skills (like carpentry) if a real product, in this case a bookshelf on the wall of the classroom, is wanted. Moreover this activity provides a great opportunity to focus on how practical demands (like modular construction, minimum number for different type of pieces) influence the design, beyond the geometry of the design.

Classroom practice from the Romania

This activity was tried out in a grade 9 and a grade 10 class (15 year olds) in a secondary Art School in Miercurea Ciuc, Romania as collaboration of the mathematics teacher, the mascil team and a professional architect, who is teaching technological subjects at this school. The mathematics teacher had 2 introductory lessons (each of them 50 minutes) about the angles in a regular polygon, and about tessellations with regular polygons. For this activity the teacher used Polydron frameworks in order to have students investigate and experiment with different tessellations of the plane with regular polygons. After this the students had the opportunity to choose between two projects to work on. One of these was the design and the manufacturing of a non-standard bookshelf; the other one was a functional design for the tiling of a new classroom's floor with regular polygons. For the bookshelf project the architect started with a 50 minutes lesson about designing the bookshelf, choosing materials, designing the nodes to assure modularity and minimize the types of different cuttings. At this stage the students observed that if they choose to have polygonal cells whose angles are multiples of 30° , and all sides of every polygon have the same length, then they can build the shelf with only two different types of pieces: a disk shaped metallic spacer with 12 equally



distributed notches on it, and a rectangular plywood piece with four tiny cuts on it. The plywood and the metallic spacers were bought in the local store, and with the help of a local manufacturer the notches were made on the spacers and the plywood was cut into the right pieces.

After this they had 2x50 minutes to assemble different designs they sketched before on paper. They assembled each model they designed on paper to test the stability of the construction (without being secured to the wall) and they choose one model to be constructed and secured on the wall of a classroom to store their drawings. During this construction phase students modified some designs because the constructed bookshelves were not stable enough.

In the different lessons student were working in different ways: there were some plenary discussions (about the materials, the modularity and the universality of the nodes), individual work (for the design of bookshelves), work in small groups (experimenting with tessellations) and working in a larger group (assembling the different bookshelves). Moreover for the different stages they worked with different teachers, and during some activities there were several teachers in their classroom. During the final activity four teachers participated as observers. All stages of the project were photographed and videotaped and the video can be accessed at the link below. The video was first presented in the school for a larger community. Later it was uploaded to a video sharing portal and was also used in mascil PD courses. Moreover the architect participating in the project presented it

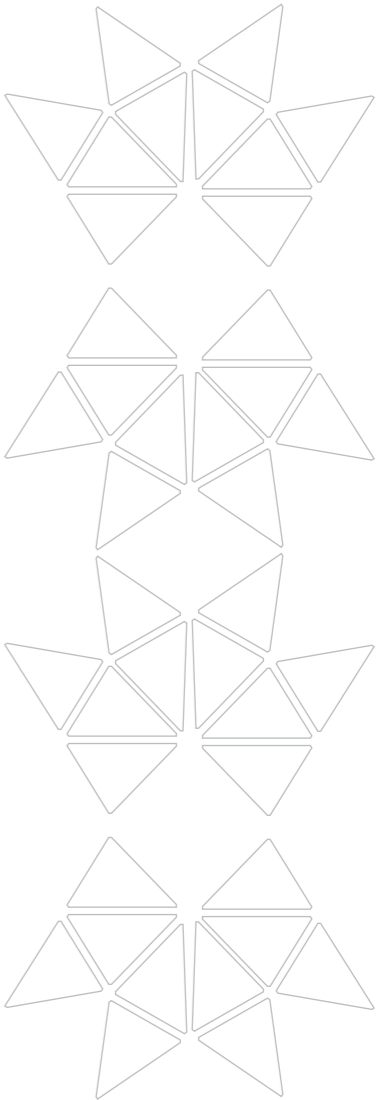


to fellow architects and the general public on two occasions: at the Transylvania 2015 Architecture Biennial (October 3-19, 2015, Cluj-Napoca), and at an exposition organized in Bucharest on April 6, 2016 by the Balassi Institute and Zeppelin Magazine.

Authors: Gabriella Zsombori, Szilárd András

Link to video:
www.youtube.com/watch?v=Xv1nNLaqM6I

Link to mascil website:
www.fi.uu.nl/toepassing/28204



Chocolate chip mining



This task is designed to model some of the issues faced in extracting useful minerals from their ores. The original task was linked in particular to copper mining, with the chocolate chips representing the copper and the cookie the ore. Aspects of materials science feature to greater or lesser extents in most science curricular, from an early age through to the end of compulsory schooling. Hence this task can be adapted and positioned in most contexts, though it has been generally used at secondary school level. The task is explicitly linked to the environmental impacts of mining and problems of waste management.

Inquiry Based Learning and the World of Work

This basic task has a clear starting point and aim: the students are positioned as mining engineers who receive a rock sample (chocolate chip cookies) and are charged with finding efficient ways to separate the valuable mineral (chocolate) from the rock and dispose of the waste material that remains. At the end of the task the students have to present a report to the directors of the mining company. In inquiry terms, the emphasis is on being systematic in experimenting and trialing possible solutions, evaluating the outcomes of the investigation and communicating the results. Hence the activity can be seen as a guided inquiry. The teacher will need to make decisions concerning the apparatus that is available to support the extraction and provide encouragement and support for the way data is recorded and presented (for example in producing tables or calculating yields). In carrying out the investigation the students work together in small teams. This aspect of the process helps to model the way a group might tackle such a problem in a mining company, with students taking on particular roles in the investigation and producing the report. There is much potential to take this activity further in customizing it to a particular group and curriculum need. The example that follows shows how this was done in one iteration.

You are cookie miners!





You work at Maryland Quarry and you want to mine as much chocolate as you possibly can.

Work out a method which produces the greatest yield of chocolate but with the lowest environmental cost and price.



Classroom practice from England

The task was used with two parallel Year 8 classes (12-13 years old) in a state secondary school in the East Midlands. The four teachers (two experienced and two undertaking initial training) involved had been encouraged to take the basic cookie mining task and adapt it for use in the context of their school curriculum needs and requirements. Hence, while the basic extraction work took place in one 80 minute lesson, the whole activity acted as the basis for teaching a section of materials science over

<p>You are an environmental consultant!</p> <p>For your work you assess air, land and water contamination, environment impact, waste management and make sure the environment is looked after as a priority you are an eco-warrior who protects the environment above else!</p>		<p>You are a mine owner!</p> <p>You are the owner of a large mining site. You manage lots of employees and mining areas. The more you mine, the more money you make!</p>	
<p>You are in charge of setting up new mine sites!</p> <p>You are one of the mine employees. You are responsible for finding areas where new mines are built. These areas must have plenty to mine but you must also make sure that you are not destroying too many natural areas.</p>		<p>You are in charge of selling materials to electronics companies!</p> <p>You sell sell sell! Once the materials have been mined and processed, you sell them onto electronics companies in the local area to some profits.</p>	

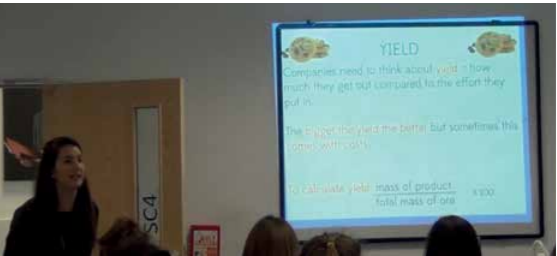
a three week period (approximately 280 minutes of class time plus homework). The lessons were led by a single teacher, in both of the Year 8 classes this was a teacher undertaking their teaching practice, with a supervising teacher

and/or mascil team member present. The four teachers took the chocolate cookie mining materials and used these as the basis for developing a section of their materials science work related to the extraction of metals.

They chose this area as one that was often seen as rather separate from the lives of the pupils and unrelated to their own lives. Hence, in producing an overall plan for cookie mining a key to their approach was to produce character cards designed to give students roles to inhabit over the whole three week period. This meant that they needed to explore the kinds of work carried out by their characters. Further resources were developed to provide background to the industrial issues, with supporting worksheets, videos and case studies drawn from web-based resources. These resources were designed to both support the investigation and its relevance to the world of work and also ensure that key concepts from the science curriculum were being covered across the sequence. An emphasis was given to issues of yield, profitability, global factors affecting demand and the environment.

At the start the teachers used a variety of prompts, including video, to raise some of the issues and link to the students' prior experiences. They were then given the roles that they needed to focus on throughout the sequence. This meant that there was already some background in place when the mining task was introduced. It also meant that the much more could be drawn out from the activity than could be realized from the basic task itself. The task was further extended by the requirement for the students to do presentations in the final lesson, in line with the character roles they had been given, in addition to producing

a written report. In trialing various methods of extraction, the basic resources (knives, spatulas, sieves, balances, measuring cylinders etc) were provided and other requests met (within reason and safety issues allowing). During the course of the lesson sequence, the students were encouraged to research further, both during the lessons and as part of homework. Many students drew on the Internet in so doing, in some cases going far beyond basic expectations.



During the time when the students were extracting the chocolate the teachers visited the groups and gave guidance when required. At particular points in the process they stopped to explain things to the whole group, for instance in going through yield calculations. This emphasis on yield, together with issues of profit, was developed further by giving costs for the chocolate ore at the start and selling prices, which were graded according to the purity of the chocolate produced. This purity decision was taken by the teacher leading the group.

You are intrepid chocolate miners! You are aiming to mine as much chocolate as possible from the Maryland Quarry at the lowest environmental and fiscal cost. Using a range of tools, your team should come up with a method that you believe will give you the greatest %yield.

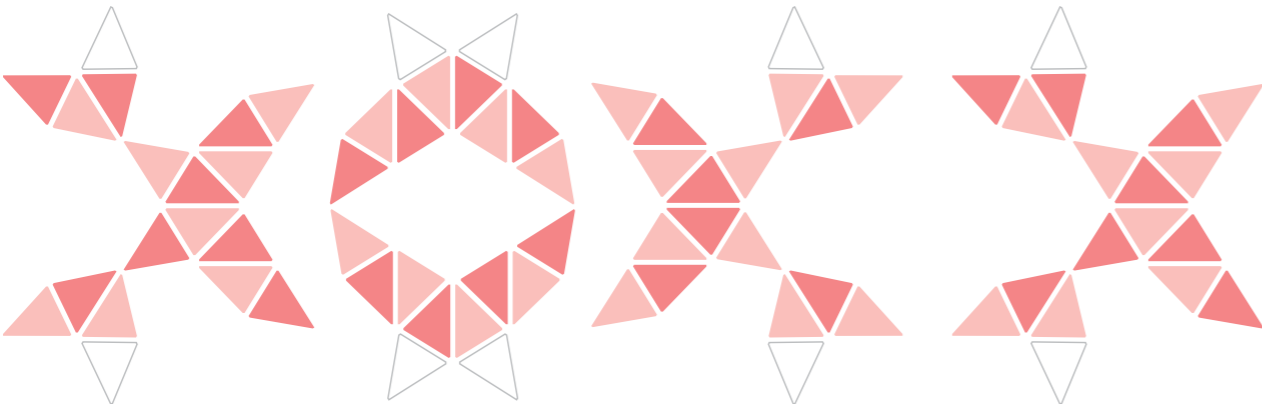
Chocolate ore (per cookie) has a cost of £50,000.

The selling price of mined chocolate has a varying price depending on purity.

Purity Score –	1	£20,000 per gram
	2	£13,000 per gram
	3	£3,000 per gram

Your independent Chocolate Inspectors Mr O’Farrell & Miss Wilkinson will assign your chocolate a purity score.

Use the box below to make notes and calculate the best method to obtain the maximum yield and profit.



A time element was introduced following the initial trialing of extraction methods. The teachers also chose to manipulate the ore cost and selling prices to illustrate the issues of market fluctuations and their impact on profitability.

The teachers also ensured that assessment for learning approaches was built in as an integral part of the process. Thus, alongside attention to the expectations of the English National Curriculum, in terms of assessments of levels achieved in relation to the concepts covered, was assessment of process, contributions and presentation skills. This included the use of peer assessment. Elements of collaboration within groups and competition between them were evident at times during process!

Author: Pete Sorensen, drawing on the work of Lucy Hughes, Robin Wood, Belinda Wilkinson and Jake O’Farrell.

Link to Mascil website:
www.fi.uu.nl/toepassing/22016



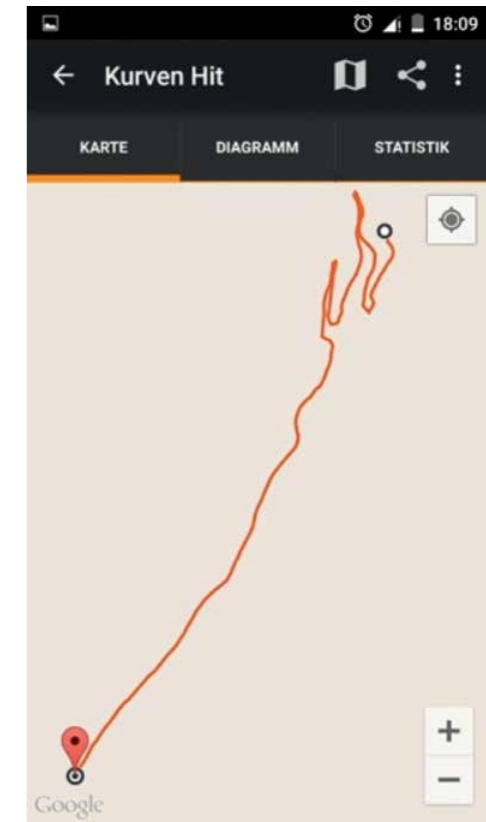
GPS mathematics puzzle: Holiday in Morocco



Being able to deal with smartphone data is an important skill in the modern world of work. In this task, this can be experienced by solving a GPS puzzle. The GPS puzzle has been created by someone who had spent his holiday in Morocco and tracked himself nine times during this holiday using GPS on his smartphone. The outcome consisted of seven different pieces of information for each of these tracks. The information is printed on individual cards, which have to be put together again. The first piece of information consists of texts, describing the situation, the second one of pictures from the situation, the third of the tracks, but without showing a map of the area. The fourth through sixth piece of information show the height and/or velocity diagram. Finally the seventh card shows the statistics. For all nine different situations the seven different pieces of information have to be gathered together. The task could be done at a lower level, by having fewer different pieces of information, i.e. using a sub-set of cards.

Inquiry Based Learning and the World of Work

This task trains skills needed to solve open tasks like interpreting and evaluating diagrams and communicating and presenting solutions. As students use a smartphone often, the context is meaningful to students. Interpreting data from a computer or a smartphone is a skill that is needed in many different fields of the world of work. There is not one specific way to get to the solution, and also not only one solution strategy. Finding the solution in a team is much easier than alone. Therefore the task clearly supports collaboration and communication.



Classroom practice from Germany

This activity was tried out in grade 5 to 10 classes (10 to 16 year olds) in a secondary school in Germany in mathematics lessons of 90 minutes. The classes consisted of about 30 students, who were used to working independently. For this activity the students worked in groups of three or four.

The teacher told the students in the beginning of the lesson, that he had spent his last holidays in Morocco and tracked himself with the GPS in nine different situations there. Then he described the pieces of information he had about those situations (see above) and gave the students the handouts. The students built groups, chose their level and then read their task silently. They were very engaged in finding their solutions and presented them on a poster. Afterwards they compared their solutions to those from their classmates' in a gallery walk.

While the students in the lower grades could only choose between level one and two, the older students were allowed to choose between all levels. The students up to grade nine had to cut out the pieces, whereas the tenth graders were allowed to give numbers in order to match the pieces. Interestingly also, the tenth graders preferred to cut out the pieces instead of giving numbers. It was not easy for the students to find the correct solutions, but all of them enjoyed the activity very much – and most of them had to look at diagrams really carefully for the first time.



Homework

Create your own GPS mathematical puzzle for three everyday situations. You can see an example of a student's solution to the homework on the right.

Authors: Anika Weihberger, Patrick Bronner

Link to mascil website:
www.fi.uu.nl/toepassing/28425



Brine



In this activity the students get a brine (with selected impurities) and have to develop an experimental setup to separate the different substances from each other. The experiment should result in a clean substance (table

salt) which might be used for cooking. The experimental setup is compared to production processes in industry and evaluated.

Inquiry Based Learning and the World of Work

This is an open and unstructured task suitable for inquiry based learning. The task focuses on separating substances, an important issue in the chemical industry. The students adopt the role of chemical engineers. They work in groups in a controlled environment on a clear task, and the activities are related/similar to what happens in a laboratory/plant. As engineers at a salt production company, they are instructed to develop a process that will purify a sample of impure salt brine. The final product is meant to be table salt that might be used in the kitchen.

Students are introduced to the problem by being shown part of a video introducing JOZO table salt production in the Netherlands. The video is stopped when the brine arrives at the salt production company, and students are faced with the problem of continuing the process from there. As further motivation, they can be told that the whole video will be shown at the end of the lesson. Students then explore the constraints of the task and plan their investigation. They have to decide, for instance, which selection of equipment they will use to conduct their experiment. The students collaborate in small teams of 3-4 and discuss their ways of working. There is not one single correct answer: different teams may come up with different processes

based on different decisions they made during planning. The teacher provides structure in the lesson, and ensures that all groups know how to proceed. It might be helpful for the process to have students report and reflect on their experimental set-ups in a whole class discussion before they conduct their experiments. After conducting their experiments, student groups present their work and findings, and discuss and evaluate their processes in plenary.



The choice of substances should be closely related to the students' level of competence. For elementary students, physical processes such as filtering or water evaporating would suffice to develop an experimental set-up and discuss results. For more advanced students, calcium sulfate as an impurity can be included in addition to sodium chloride, sand and stones, in order to discuss how can be chemically removed from the brine. Here, the different processes of salt production come into play and can be compared with and evaluated against those the students conducted.

Depending on the students' experimental skills, the providing of equipment to conduct experiments can be more or less scaffolded. Ideally, students should be encouraged to select equipment of their choice, but low-achieving groups may need a list of materials and equipment they are allowed to use (but not necessarily have to). Depending on students' skills, the list can also be complemented with superfluous equipment so that students have to choose the most appropriate set-up.

JOZO video:
www.youtube.com/watch?v=VM7HeserH0U

Classroom practice from Norway

The target group for this activity is lower secondary school, or age group 11 -15 years, and the discipline is elementary chemistry. Suggested duration is 90 min. The activity was tried out with great success in a grade 9 class (14 year olds) in Norway, and later used by the teacher as an example task during a professional development session of colleagues at the same school. The professional development session consisted of 13 teacher colleagues, teaching 5th to 10th grade. For this activity, the participants worked in groups of three or four.



The teacher dressed up in lab coat and glasses, adopted the role as chemical engineer and leader of the Akzo Nobel Institute. He introduced the task to the participant teachers by showing them the start of the JOZO video. The participants were given roles as factory employees (engineers), and each group was given a sample of brine. The brine consisted of concentrated sodium chloride solution, sand, gravel and dirt from the schoolyard, but this was not revealed to the 'engineers'. Their task was to recover as much

and as good salt as possible from the solution. At the end of the lesson, the 'engineers' were supposed to hand in their product to their leader whom would be responsible for quality assurance of their table salt. Their samples would then be evaluated according to the following criteria: taste, smell, dryness and purity. Also, the effectiveness of group work was considered important. The evaluation criteria were written on the blackboard, with the scores to be filled in at the end of the lesson.

The participants showed great enthusiasm and eagerness. The different groups decided quickly on equipment, and all groups started to sift their brine. Many creative ideas were used in the sifting process. Different strainers were chosen, some used filter paper and others not. Then they heated the filtrate using a gas burner. To control the working progress, the teacher asked each group to present their plan prior to their experimental work and a status quo report in the middle of the lesson. At the end of the lesson, the teacher compared the products from each group. He tasted and smelled them, judged their dryness and purity, and considered group effectiveness, assigned points to each criterion – and finally scored a winner.



It turned out that all participant teachers were really engaged. Most of them did not have any previous experience of this kind of role play in a science lesson, and the role play seemed to be a motivating factor. The requirement that their work should result in a product deliverable was highly engaging. As the teacher/leader evaluated their samples, they all observed with anticipation who the winner group was going to be. In the reflections after the lesson, the teachers had several suggestions on how to adapt the activity to lower grades. Security concerns could e.g. be handled by evaporating without heating.

Alternatively, the teacher could handle the gas burners after students had stated their hypothesis. Another idea was to let students propose methods to distinguish fresh water from brine. In fact, they claimed that the activity was adaptable to grades as low as 1 to 4, to illustrate how salt is produced. The teachers were convinced that this activity would be something students would remember, feel motivated by, and something they would talk about, also to their parents.

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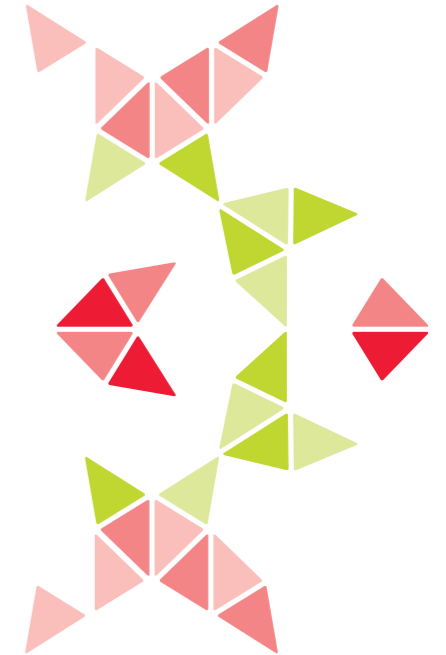
Link to mascil website:
www.fi.uu.nl/oepassing/28121

How do smartphone apps work?



Part of a software developer's job is being able to understand how the software of the competitors functions. This aspect of a software developer's job is recreated in this task. In it, students find out how four different smartphone apps function:

- “Smart Ruler” is an app where a ruler appears on the display of the smartphone, so that you can measure with the smartphone as if it was a regular ruler.
- “Smart measure” is an app that measures the distance to an object. After entering the height of the smartphone and the height of an object and focussing the camera on the top or the bottom of an object, the app will tell you the distance to the object.
- “Smart distance” also measures the distance to an object. You enter the height or width of the object and mark the object's height or width on the screen and then focus the camera on the object .
- “Smart speed gun” measures the velocity of an object. For that the object has to move in a straight line and you enter the smallest distance to the object. Then you follow the object with the smartphone during its movement.



For all four apps the students have to take into account, that the app can read some information from the smartphone, like the number of pixels, the size of the screen and the focus distance. By that, the smartphone “knows” some image properties. The students have to know that the problems can be simplified by assuming that the camera is a pinhole camera, which in reality it is not. With the intercept theorems, the apps then calculate the distance to or the height of an object.

Inquiry Based Learning and the World of Work

This is an open and unstructured task suitable for inquiry based learning. If the students are not able to solve the open problem, clues that come along with the task can be added to slightly close the task. Finding out how the smartphone apps function is quite demanding for the students, since they will have to plan their own investigations, apply mathematics, interpret and evaluate their results, communicate and present them. Depending on what information they assume is gathered on the smartphone, they can develop solutions involving trigonometry or the intercept theorems. Therefore the task evokes multiple solution strategies.

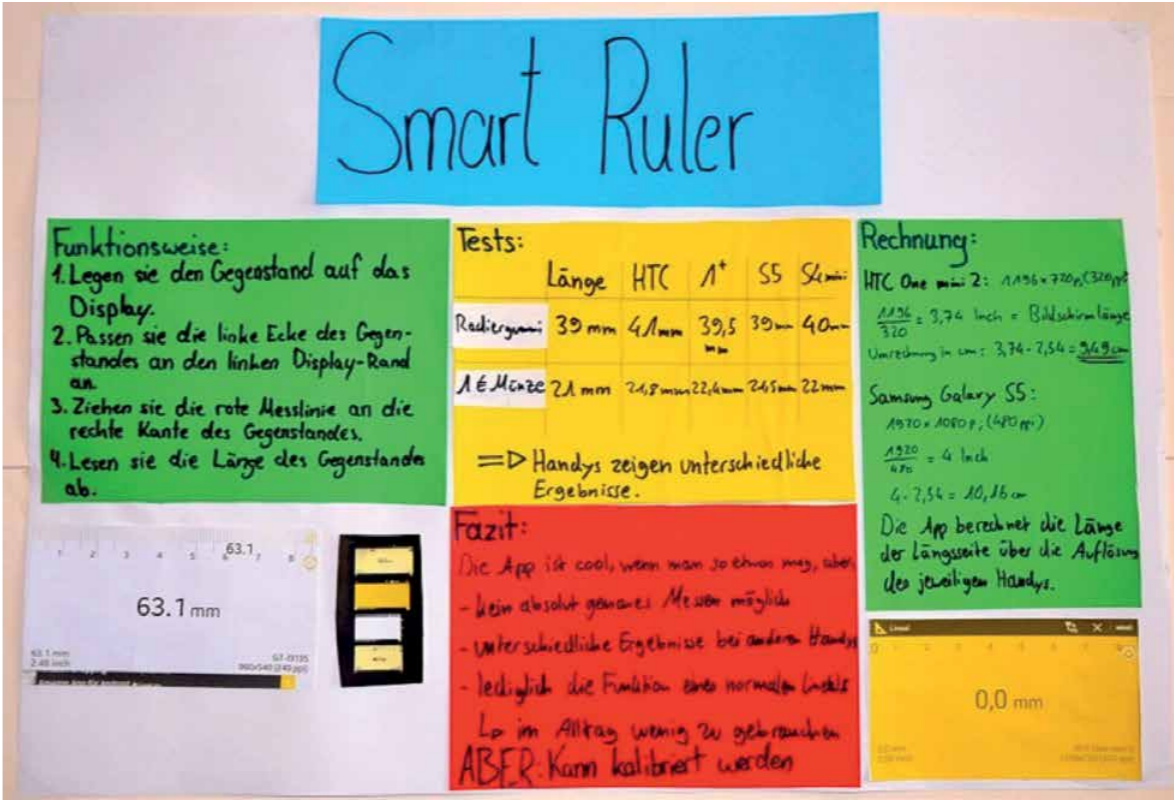
In this task the students take the role of a software developer. Nowadays in a lot of professions software development is involved. Understanding the inner workings of computers is very important for the students' future world of work. It is also interesting to understand how components of apps that students use in their everyday life function. The task does not specifically ask for a product, but it involves a big step for producing a product, i.e. for creating an app.



Classroom practice from Germany

This activity was tried out in grade 9 classes and a grade 10 class (14 to 16 year olds) in a secondary school in Germany. The mathematics

lessons lasted 90 minutes, and in an extra lesson of 45 minutes the results were presented. The classes consisted of about 30 students, who were used to working on open ended tasks in small groups. For this activity the students worked in groups of three or four. In the picture on the right you can see how a student is



measuring the height of another student with a smartphone app. The teacher asked the students to download all four smartphone apps “smart ruler”, “smart measure”, “smart distance” and “speed gun” prior to the lesson. The apps only work on Android smartphones, but this did not turn out to be a problem, as there was a student with an Android smartphone in every group.

All groups usually had two smartphones they could work with. In the beginning of the lesson the teacher told the students that they are working for Apple and are supposed to find out how the Android apps work, so that they can develop iOS versions of the apps. In order to work more efficiently, each student group only had to find out how one of the apps works. While it is very easy to find out how the “smart ruler” works, it is quite difficult to find out how the “speed gun” works. The teacher explained the different levels of difficulty and then let the class read the task in silence. After that the students chose their task individually and then formed groups of three or four.

The “smart ruler” was chosen by students, who are not so good in mathematics. They were usually very quick in finding out that the app gets the information on the dimensions of the smartphone and the number of pixels from the smartphone itself and is therefore able to calculate with the rule of three how many pixels fit in one centimetre. This was very motivating

for the students. But they had some problems formulating their answers in a mathematically correct way. It took a while for the other groups to find out how the smartphone apps work. Some of them also used the clue cards. Once they understood their app, they wrote down their answers rather quickly.

All groups were supposed to put their answers on a poster. Some groups were not able to finish their poster within 90 minutes, but had to finish it as homework. In the next lesson the posters were presented in a gallery walk. The students were asked to put remarks on the other posters about what they liked and disliked. Afterwards they could improve their own poster and it was then graded by the teacher. The students found out different ways to program the apps: Either with trigonometry or with the intercept theorems. Even though they had been quite familiar with the fact that there might not be an exact solution to a problem, they were a bit surprised that different mathematical tools can lead to the same solution.

It was a very motivating experience for the students to use their smartphones in class. But it was not only new for the students, but also for the teacher: There was one group that didn’t start the task right away, but discussed something else. It was an interesting experience for the teacher to ask the students to work with their smartphones instead of asking them to put the smartphones away as usually.

In a series of lessons about intercept theorems and trigonometry, this was a perfect application from the world of work.

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