Authentic contexts in mathematics textbooks in secondary pre-vocational education (VMBO)

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Introduction

In Dutch mathematics education at secondary schools, textbooks play a very important role. The Dutch mathematics textbooks are full of colour; the content areas are integrated (no separate books for geometry, algebra etc.). The didactical approach is 'problem driven' and contexts play an important part. Each new subject starts with a more or less realistic problem and unfolds through series of problems. There is not much theoretical background in the books. Attention is paid to (self) assessment; often tests and quizzes are provided as well as summaries.

Over the years, the textbooks have been made to be almost teacher proof. Sequences of tasks are carefully designed by teams of teachers, writing for commercial publishers, to have students' effectively learn the mathematics that is intended. There are not much 'obstacles' for students in the textbooks, so they can easily work on their own without much guidance from their teachers. To support teachers textbooks come with assessments, answer guides and other additional materials. Teachers fully depend for their teaching on the textbooks.

This may to some seem ideal, but the situation also has some severe drawbacks. Textbooks are taken for granted, they are not tailored to fit the needs for specific (small) groups of students, they are rather aimed at the average student, the possibility to make connections between different subjects is not facilitated by the textbooks, textbooks are never really up-to-date (it is hard to relate to students' actual reality).

In 2003 a small study was carried out to investigate whether Dutch mathematics textbooks facilitate authentic learning. In the next paragraphs, the study and its results are summarized. At the end of this paper, an update of the situation in the Netherlands with respect to teaching materials is given and new research questions are posed.

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The situation

Students in the practical-oriented learning trajectories in the last two years of Dutch pre-vocational secondary education[1] often experience problems with the general subjects. In school the students live in some respect in two different worlds: on the one hand they follow a practical pre-vocational program addressing professionally-oriented subjects which are mostly taught in the workplaces at school. The most important objective for this program is that students orientate themselves on 'competencies' needed for their following vocational study and professional practice (Onstenk, 2002). On the other hand, there are the general subjects, which are typical 'school subjects'. The students sit in a classroom and work mostly on school tasks. The link between vocational and general subjects is weak in several aspects: different teachers; different locations in school, no connections between the contents of the general and vocational subjects and different didactical practices. On top of that: the specific professional languages of the various domains differ, which makes making tuning the subjects to each other even more problematic.

Nowadays, authentic learning is seen as a core element of a powerful learning environment, especially in (pre)vocational education. Using authentic situations and tasks that reflect professional practice, gives meaning to learning for students; they become more motivated; develop more understanding and are better prepared for being able to apply what they have learned (Bransford et al 2000, p. 134). There is also a movement in Dutch pre-vocational education towards more authentic teaching and learning, especially in the practical vocational subjects (see among others van der Sanden, 2003, p. 27 etc., and Kemme, 2001). Various departments use in their teaching what we will refer to here with the general term 'business simulations'. In such a simulation, a business situation is imitated in the workplace at school: for example a restaurant for the 'consumables' department. Students have roles and tasks in this simulation environment with the responsibilities that go with them. This way of teaching supports authentic learning. The use of business simulations, only happens for the practical vocational subjects, without connections being made to the general subjects. In addition, the other way round, there is no reference to these business simulations or parts of them in the general subjects.

1 Pre-vocational secondary education in the Netherlands is the lowest level in compulsory education for students aged 12-16. It lasts four years and it is intended as a foundation course as regards both the general and the pre-vocational component. In the last two years pupils choose one of four different types of learning pathways and a specific vocational department. Part of the program is a practical orientation on a professional sector and the other part consists of general subjects. It provides a basis for further vocational training.

Mathematics is one of the general subjects in pre-vocational education. It is a subject that seems eminently suitable to be taught authentically (Forman & Steen, 2000). Especially if you consider mathematics to be a subject one must actively 'do' (Romberg, 1994) and if connections are made to the students' reality. Dutch mathematics textbook series, both for primary and for secondary education, claim that they are based on the theory of Realistic Mathematics Education (RME). In many aspects, the influence of RME is clearly visible: the extensive use of contexts, the sequencing of instructional activities and the use of certain models. The textbooks have certainly helped to implement some of the ideas of RME into mathematics teaching in Dutch classrooms[2].

There seem to be, however, characteristics of the Dutch textbooks that prevent the adoption of the most important principle of RME: give students the opportunity to actively construct their own mathematics and use it in an authentic way. Specifically this problem was noted for pre-vocational education. For these students to get involved in mathematics the contexts should be familiar and the use of mathematics should be authentic to them.

For mathematics to be authentic for the students a connection should be made between the practical vocational subjects – that form part of the student's reality – and mathematics in the pre-vocational education.

The question may be asked, whether mathematics education in pre-vocational education is reflecting the recent insights in the importance of authentic learning. Do the students get the mathematics they really need, and therefore deserve? Is there authentic learning in mathematics education in pre-vocational education?

This question has been researched by a team of researchers from The Freudenthal Institute, by order of the Dutch Society of Mathematics Teachers (NVvW)[3]. The question has been narrowed, since it would be impossible to answer its full scope within the framework of the short study that has been carried out. It was decided to do a thorough analysis of the three most commonly used mathematics textbook series, and investigate to what degree they are suitable for and support authentic learning. This means that an analysis of the learning processes of students

2 In the US a mathematics textbook series based on RME principles has been developed by the Freudenthal Institute in collaboration with WCER (University of Wisconsin, Madison). It is called 'Mathematics in Context' (Romberg, De Lange, 2004), and is a standards based mathematics curriculum, developed for American middle schools.

3 This study has been financed from the budget made available by the Ministry of Education (OC&W) to LPC for Shortterm Educational Research (Kortlopend Onderwijsonderzoek), which is performed based on questions from the field of education. Project number 02.1.3.1

and the way in which teachers and students use the textbooks were not part of this study. Some recommendations will be made, however, on how to use the findings from this study to improve the teaching practice, for instance by changing the teaching materials.

Summarizing, the research question is:

> *Do the existing mathematics textbooks that are commonly used in pre-vocational education, support authentic learning and what more, if anything, is needed to realize this?*

Elaborating the problem definition

To make authentic learning possible, suitable authentic learning activities are needed. Based on Dutch literature on authentic teaching and learning (Roelofs & Houtveen, 1999) the following relevant characteristics for authentic tasks can be formulated. Authentic tasks should:

- Relate to students' personal environment, (students should be able to identify with them);
- Be relevant/meaningful for learning in situations outside school;
- provide opportunities for constructing knowledge;
- as closely as possible reflect the reality of professional vocational practice.

The current generation of Dutch mathematics textbooks contains much of so-called problems-in-context that can make mathematics more meaningful and authentic for students, for example because they can relate them to their own experiences and because the tasks contain a 'real problem'. Furthermore, these problems-in-context provide the possibility for 'coloring' the mathematics to reflect the professional vocational program of the department the students have chosen. In addition realistic mathematics education (RME), the Dutch approach to mathematics education (Gravemeijer, 1990), intends to have the students learn actively and through discovery, while constructing knowledge (Bransford et al., 2000, p. 62). This is realized among other things by posing open 'complex' problems in which students are free to choose their own approach. Creating context-rich teaching is a development that can also be seen in related general subjects such as physics and chemistry (Goedhart et al, 2001).

The characteristics of authentic learning activities that were mentioned earlier can be translated to and applied on the mathematics textbooks. This leads to the following research sub questions:

- Question 1: Are the contexts used in the textbooks recognizable for the students, can they make sense of them?
- Question 2. To what degree are the context-rich problems relevant/ meaningful for situations outside the school?
- Question 3: Is a complex problem stated that allows the student to construct knowledge by selecting his/her own approach?
- Question 4: To what degree do the tasks match those in professional vocational practice?

Finally, the teachers association NVvW also asked: 'what, if anything, extra is required for using the existing mathematics textbooks to realize authentic learning?' This leads to the final sub question of the study.

- Question 5: To what degree do the existing mathematics textbooks have the potential to be used to realize authentic learning?

Operationalisation

In order to be able to answer these questions by analyzing the textbook-problems the questions had to be operationalized in scoreable categories.

Recognisability

Question 1: Are the contexts used in the textbooks recognizable for the students, can they make sense of them?

'Recognizable for the students' concerns the choice and use of contexts. We define the context of the textbook problem as that part of the description of the situation to which the set task applies. To clarify this definition of 'context' we will use a sample problem with explanation from 'The PISA 2003 Assessment Framework' (OECD-PISA, 2003, p. 32).

> ***Savings account***
> *€ 1.000,- can be placed in a savings account with a local bank. There is a choice of two options: you will either get an annual interest rate of 4% or you will immediately be paid € 10,- and the annual interest rate will be 3%. Which option yields the most money after 1 year? And after 2 years?*

The situation of 'banking', is probably recognizable for all Dutch students. The context is concrete money (euros) that can be put in a savings account in a bank and that 'grows' by annual interest.

In the study we want to avoid too limited a view of connecting contexts to students' own reality, the main point is recognisability of the context: can the students relate to it. For example, an earthquake as a context will be something that most of the students can recognize or even relate to, even if they have never experienced one themselves.

Thus, the first characteristic by which the textbook problems will be analyzed is: 'Is the task embedded in a context that the students will recognize?'. This characteristic is referred to as '*recognisability*'.

Relation to everyday life and obviousness of the problem

Question 2. To what degree are the contextual problems relevant/meaningful for situations outside school?

We can interpret relevant/meaningful as a characteristic of the context itself, but also as a characteristic of the task as a whole. For operationalisation we will therefore split this question into two sub questions, both of which will lead to a scoreable characteristic.

First, we will consider 'relevant/meaningful in situations outside school' as a characteristic of the context:

Question 2a: Is the context relevant/meaningful in situations outside school?

This relates to whether the context, as it is stated in the problem, can occur in situations outside school, i.e. in the students' everyday life, in society etcetera. To that end, we ask questions such as: 'Do or will students ever encounter something like this? Does it happen to him/her? Can he/she make sense of it outside of school?'.

This leads to the second characteristic that can be scored for each problem, which we have called 'relation to *everyday life* (outside school)'. The earthquake context will score slightly lower on this aspect than it will on the characteristic of recognisability.

The next example illustrates another aspect of the difference between the characteristics 'recognisability' and 'everyday life'. A problem on 'renting bicycles' has a context that is recognizable for students, and will therefore score high on the first characteristic, '*recognizability*'. The score on '*everyday life*' will be lower, it can after all not be expected that every student has (had to) deal with this in practice. That score will be even lower if the renting costs in the textbook problem are presented with a formula or a graph, rather than as amounts per hour, since that is not done in reality.

Secondly, relevance or meaningfulness also relates to the task as a whole. It then relates to whether, within the chosen context, a problem is posed for the student that is relevant and meaningful for situations outside the school. This leads to:

Question 2b: Is a problem posed that is relevant/meaningful for situations outside school?

When a recognizable and meaningful context is used in a good way, a specific question or a specific problem emerges from it almost naturally. For instance, an obvious and relevant problem in the renting-a-bicycle-context is 'what does it cost if I want to rent a bike for half a day?' Using mathematics to solve this problem seems natural. Thus we could say the use of mathematics in this case would be obvious and therefore authentic.

To answer question 2b, we check whether the problems in the textbook pose a meaningful problem that emerges naturally from the context and that needs the use of mathematics to be solved. This third characteristic in the classification system is referred to as '*obviousness*'.

Problems that will not score high on this characteristic, are for example ones that are formulated as '*draw the graph*' or '*calculate the price for t = 4*'. The first of these questions does not follow naturally from a context, the second one has not been formulated as a meaningful problem in terms that fit the context. In context terms it would be posed as 'What is the cost for a four hour rent?'.

Please note that we do not claim that such problems are not useful. This study concentrates on characteristics that determine the authenticity of the learning activities, rather than on other characteristics.

Problem solving

Question 3: Is a complex problem stated that allows the student to construct knowledge by selecting his/her own approach?

This third research question explores the way problems in textbooks are posed, especially regarding the degree to which they leave students room to choose their own approach. When students can choose their own approach to solving a problem, and can or have to justify that choice, there is an opportunity for construction of knowledge.

For answering this question, we score the textbook problems according to the degree in which there is an opportunity for students to choose their own problem solving approach. This is the fourth characteristic in the instrument, and it is referred to as '*problem solving*'.

To continue with the earlier quoted example on renting bicycles: if, after an authentic presentation of the bike rental context, the question '*What does it cost if I want to rent a bike for half a day?*' is asked, there is an obviously relevant problem, with an opportunity for the students to choose their own approach. If, however, in this textbook problem there is first a sub question in which the task is to make a table with hours and costs, then a sub question on how many hours half a day is, and finally the sub question '*Determine, using the table, what renting a bike for half a day would cost?*', then the score on this characteristic would be significantly lower.

Relation to professional vocational practice

Question 4: To what degree do the tasks (the textbook problems) relate to professional vocational practice?

This research question about the correspondence between mathematics problems and professional vocational practice has been interpreted as the degree in which the problems relate to tasks and activities in the practical vocational program at school. After all, students in pre-vocational education, do not fully engage in professional practice yet, but only orientate themselves to it at school. This happens during the practical vocational program the students take in the department of their choice. The four departments students can choose are: care and healthcare, business, engineering and technology, agri-culture.

Every textbook problem is also scored on this fifth characteristic, the degree in which it has a '*relation to professional practice*'. For example, a problem on calculating the amount of paint needed to paint a room relates to vocational subjects in the technology and engineering department; the same holds for reading a construction drawing. The more authentic this drawing looks, and the more 'real' the activities the student has to do, are, the higher the score on this characteristic will be.

Potential

Question 5: To what degree do the existing mathematics textbooks have the potential to be used to realize authentic learning?

For this characteristic, the expertise of the researchers in the area of curriculum development and as teachers is of crucial importance. This characteristic relates to whether a paragraph, with its contexts and the specified mathematics, can be adapted to raise the scores on the other characteristics for authentic learning activities. To that end, the characteristic '*potential*' has been scored at paragraph level; the researchers also had to indicate what formed the basis for each high 'potential' score they gave. This could be done by adding a comment for example specifying possible adaptations to change a paragraph to better support authentic learning.

Results of the analysis

For a discussion of the results in this paper, we investigate the scores on paragraph level. We are treating the paragraph as a unit of analysis, because problems from one paragraph cover the same mathematical topic, and a paragraph often forms a unit for teaching time as well. Analysis on the level of individual problems would be too detailed for an overview.

The dataset contains the data on the following numbers of paragraphs[4]:

Table 1 : *Number of paragraphs in the researched textbook series*[5]

Textbook series name1	*Algebra*	*Number and Measurement*	*Geometry*	*Statistics and Data handling*	*Number of paragraphs*
Getal en Ruimte (G&R)	29	27	37	19	112
Moderne Wiskunde (MW)	55	29	59	26	169
Netwerk	50	23	45	20	138
Total (n = 419)					419

We discuss the results based on the extreme values, i.e. the scores 0 and 4. There are two reasons for this. In the first place, it turned out during the scoring and consultation within the research team, that (as is to be expected) the scores on the extreme values, 0 and 4 had the best inter-subjective reliability. Secondly, it turned out to be the case, during the detailed analysis of the scoring results, that these were represented well by the extreme scores. Including the intermediate scores in the analysis provides a more nuanced picture, but it does not essentially change the overall results.

Recognizability and relation to daily life

We discuss the results for the characteristics '*recognizability*' and '*relation to everyday life*' in relation to each other. It would seem reasonable to assume that 'relation to everyday life' can be seen as a refinement of 'recognizability'. A context that students do not recognize, is unlikely to occur (and be recognized) in the students' everyday life outside school, or to put it the other way around: a contextual problem that the student encounters in everyday life, is certain to be recognizable.

4 The paragraphs of which the problems were scored, are the paragraphs that contain the core subject matter. So paragraphs with diagnostic tests or extra tasks were not part of the study

5 The three main textbooks series for mathematics are: Getal en Ruimte (Numbers and Space); Moderne Wiskunde (Modern Mathematics) and Netwerk. See appendix A for references to the analyzed volumes.

These two characteristics have been scored at the problem level, meaning that each individual problem got a score on each of these characteristics on a three point Likert scale 0, 1, 2. These scores were then summarized at the paragraph level, on a five-point scale (with the values 0, 1, 2, 3 and 4). During the scoring there was the opportunity to include a comment. This option was hardly used by the researchers.

As explained before, for this summary of results we only take into account those paragraphs from the total set of 419 that were scored as '4' (the maximum) on each of the two characteristics

Table 2: *Recognizability (number of paragraphs scoring 4).*

	algebra	*number*	*geometry*	*statistics*	*tot.*	*percentage*
g & r	3	10	3	6	22	20%
mw	12	6	8	10	36	21%
netwerk	1	6	3	0	10	7%
(n=419)	16 (12%)	22 (28%)	14 (10%)	16 (25%)	90	16%

In table 2 we can see that 16% of the paragraphs of all three mathematics textbooks series together contain contexts and problems that are highly recognizable to students. The scores for Getal en Ruimte and Moderne Wiskunde are comparable on this characteristic, the low score for Netwerk is notable. When we go through the books for Getal en Ruimte, the large number of photographs is striking. The inclusion of 'real' photo's strengthens recognizability. Another remarkable point is that both Getal en Ruimte and Moderne Wiskunde regularly use the same context in a series of problems rather than in just one problem as is the case in Netwerk. If this is a recognizable context, the likelihood of the whole paragraph scoring high on this characteristic increases. Bare problems, that is to say, problems without context, score 0 on this characteristic. The two domains Number and Statistics clearly get higher scores on recognizability than the domains Algebra and Geometry.

We can deduce from table 3 that the problems in 9% of the paragraphs of all three textbook series together, show a strong connection to students' everyday lives. Here too, there are differences between the textbook series.

Table 3: *Relation to everyday life (number of paragraphs with score 4)*

	algebra	*number*	*geometry*	*statistics*	*tot.*	*percentage*
g en r	5	6	0	1	12	11%
mw	3	7	3	5	18	11%
netwerk	2	1	1	2	6	4%
Total (n=419)	10 (7%)	14 (18%)	4 (3%)	8 (13%)	36	9%

The percentages are lower than those in table 1. Relation to everyday life can be seen, as mentioned earlier, as a possible refinement of 'recognizability'. The relatively low number of geometry paragraphs that score a 4 for the relation with everyday life is remarkable. We had expected a higher score, since much of 3D-geometry relates to ways of seeing and to orientation and navigation, which are also activities in everyday life. On the other hand regarding 2D geometry, there are very few 'ideally shaped' objects in daily life, such as exact circles, pentagons, etc. This 'pure' geometry may very well get more attention in these grade levels in pre-vocational education than the 3D geometry of orientation and navigation. This may explain the low score for geometry.

We also studied the number of paragraphs that scored 0 for these two characteristics, See tables 4 and 5.

Table 4. *Recognizability, score= 0*

	Number of paragraphs	percentage
g & r	8	7 %
mw	22	13 %
netwerk	4	3 %
Total	39	8 %

Table 5. *Everyday life, score = 0*

	Number of paragraphs	percentage
g & r	22	20 %
mw	76	45 %
netwerk	13	9 %
Total	124	26 %

It can be noticed here again that the characteristic 'relation to everyday life' can be seen as a refinement of 'recognizability'. The textbook series Netwerk has remarkably few paragraphs that got score '0'. Since this series also had few paragraphs that scored a '4', we can conclude that the majority of paragraphs in this textbooks series presents problems in contexts that only to a limited degree are recognizable and relate to the students' everyday life.

Obviousness of problems and problem solving

The characteristics '*obviousness*' and '*problem solving*' are discussed together. Both characteristics provide information on the type of task that the student has to perform. As mentioned before, obviousness of a problem is connected to the problem or task as a whole, and is about whether a problem is posed that follows obviously from the context or not.

The 'problem solving' score indicates how the posed problem can be solved by the students: can the students choose their own approach (high score)? Or do they merely have to perform an action – that often has not been given any meaning (low score)?

In the problem in figure 1 about Inez making pie soup for six people, based on a recipe she has for pie soup for four persons this is the case. The obvious problem: *How much of each ingredient will Inez need?* is posed. No further information apart from the authentic recipe is presented. The way to approach the problem has been left open, so students can choose their own. This problem scores high on both 'obviousness' and 'problem solving'.

10 Inez wil voor zes personen erwtensoep maken. In haar kookboek vindt ze in een recept de benodigdheden voor vier personen.
Hoeveel gram spliterwten, schouderkarbonades, knolselderij, prei en winterpeen heeft zij nodig?

Erwtensoep
voor 4 personen
300 g spliterwten
400 g schouderkarbonades
3 kruidenbouillontabletten
200 g knolselderij + het blad
2 aardappels
2 uien
200 g prei
300 g winterpeen
2 laurierblaadjes

Fig. 1. *Problem from Netwerk, volume 4 vmbo kader*

The score for the problem in figure 2 is lower than the score for the problem in figure 1. The question in figure 2 is:

> *A bag of 250 g of potato crisps costs €1.29. Make a table and calculate how much 100 g cost?*

The problem solving approach is partly prescribed by 'make a table'. We note that there is no meaningful or obvious problem posed. Actually no problem is posed at all, only a task is set: 'make... and calculate...'.

12 Een zakje van 250 gram chips kost € 1,29.
Maak een tabel en bereken hoeveel 100 gram kost.

Fig. 2. *Problem from Netwerk, volume 4 vmbo kader*

There are two reasons to explore the two characteristics in combination: in the first place we expect the scores to be connected. After all, if no problem has been posed, there will generally speaking be no opportunity to choose an approach. In the second place, the scores for both characteristics determine to what degree a problem will be meaningful to students.

It is the case for both characteristics that only very few textbook problems scored '4's. Therefore, we will not split up the scores into mathematical domains, but it will be enough to study the totals over all domains (shown in tables 6 and 7).

Table 6: *Obviousness, score = 4*

	Number of paragraphs	percentage
g & r	7	6%
mw	15	9%
netwerk	5	3%
Total (n=419)	26	6%

Table 7: *Problem solving, score = 4*

	Number of paragraphs	percentage
g & r	1	1%
mw	4	2%
netwerk	0	0%
Total (n=419)	5	1%

All three textbooks series score low for both characteristics; there are some differences, but these are very small. It was found during the analysis at problem level that a large majority of problems in all textbooks contains a series of sub questions , see figure 3 for an example with five sub question.

18 Sita verkoopt zelfgemaakte kaarsen voor €5,– per stuk. Haar winst bereken je met de formule $5k - 30 = w$. Hierbij is k het aantal verkochte kaarsen en w de winst in euro's.

a Hoeveel euro heeft haar materiaal gekost?

b Bereken w als Sita 11 kaarsen verkoopt.

c Neem de tabel hiernaast over en vul hem in.

d Zet pijltjes onder de tabel. Vul onder de pijltjes in welk getal er steeds bijkomt of afgaat.

e Hoeveel kaarsen moet Sita verkopen om winst te maken?

k	0	1	2	3	4	5	6
w in euro's	..	..	..	..	..	..	..

Fig. 3. *Problem from Moderne Wiskunde, volume 3a vmbo basis/kader*

This means that in textbook problems there is a large number of 'step-up' questions, which are not obviously evoked by the context, and have not been posed as meaningful problems. These are often questions that require a specific action and are intended to put the student on the right track and guide them towards a specific approach. Examples of such sub questions are: 'fill in the table', 'draw the graph', 'draw the lines of vision', 'measure the angle' etc. The actual obvious and relevant problem is sometimes only posed as the final sub question. What can be seen in a number of cases is that, because of the division into sub questions, the obvious problem is not presented at all; the core problem has been split up into the sub questions and dissolved. This explains the often low score for both 'obviousness' and 'problem solving'.

A final aspect that resulted in low scores on obviousness is the formulation of the obvious problem not in terms of the context, but in terms of mathematics. For instance, a graph may be presented to determine for which amount of something the cost is lowest (which can be an obvious problem coming from the context), while the problem has been formulated in such a general mathematical way that the relation to the context is lost, as in: '*for which value of p does the graph have a minimum*'.

The problem in figure 3 is about Sita selling homemade candles. Part of the text shows a mixture of mathematical and contextual language, it says:

> *For calculating her profit, you can use the formula* $5k - 30 = w$. *In which k refers to the number of candles sold and w is the profit in euros.*

The formula is presented, rather than a description in terms of the situation. Sub question b also uses a mix of contextual and mathematical language:

> *Calculate w if Sita sells 11 candles.*

The combination of this use of mixed language (which makes problems less obviously related to the context) and the splitting into sub questions, lowers the score on 'obviousness'.

The scores on the characteristic 'problem solving' are even lower than those on 'obviousness'. This has probably to do with the mutual dependence on these characteristic as was mentioned before.

Another finding is that the textbooks contain more 'sums' than problems. The earlier mentioned phenomenon of sub questions results in real relevant and obvious problems either not being posed at all, or not until the final sub question. By then, the problem solving method or approach

has already been fully laid down in the previous sub questions, which does not leave much more for solving the problem than performing a final pre-structured step. Here too it is the case that often, as the result of the way the problem is phrased, it does not live up to its promise. Rather than posing the problem as a problem, an order to do something is given, for instance 'calculate', 'draw' or 'find on the graph'. These are 'orders' rather than questions, and they are definitely not problems which leave room for students own individual problem solving approaches.

Link to professional (vocational) practice?

For the 'relation to professional practice' we study how often the maximum score of '4' on this characteristic was given to a paragraph. The description of the characteristic clearly states that relation to professional practice includes the relation to the vocational programs in school.

Since the number of paragraphs scoring '4' on this characteristic is very low, we will once again confine ourselves to the totals for each textbook series and not split up the scores for the mathematical domains:

Table 8. *Relation to professional practice, score = 4*

	number of paragraphs	*percentage*
g & r	13	8 %
mw	16	9 %
netwerk	0	0 %
Total (n=419)	29	7 %

Table 8 shows that a strong relation to professional practice is hardly found in the textbook problems. This can be explained by the fact that the same mathematics textbooks are being used by students in all vocational departments; they prepare for the same final examination. Often mathematics classes are formed with students from several departments. The mathematics examination program and the central written examinations are the same for all students in the same learning trajectory irrelevant of the department they have chosen. Thus these examinations cannot reflect specific professional practices other than in a very general sense. The low score on this characteristic is to be understood against this background. In addition, if we look at all scores and not just the '4's we see that the relation to professional practice is absent or very weak for about 70% of the paragraphs.

Table 9. *Overview of scores for relation with professional practice*

Score	*0*	*1*	*2*	*3*	*4*
Number of paragraphs (n=419)	186	103	78	32	20
Percentage	44 %	24 %	18 %	7 %	5 %

Potential to realize authentic learning

This characteristic is crucial to this study, since it offers a starting point for improving the mathematics textbooks towards more authentic teaching and learning.

Here too we look only at the number of paragraphs that score a '4' on this characteristic: 'potential'.

Table 10. *Potential (number of paragraphs with score '4').*

	algebra	*number*	*geometry*	*statistics*	*tot.*	*percentage*
g & r	3	15	9	6	33	29%
mw	5	11	17	5	38	22%
netwerk	0	8	9	3	20	14%
Total (n=419)	8 (6%)	34 (43%)	35 (25%)	14 (22%)	91	22%

It is a hopeful sign that 22% of the paragraphs show a very high potential for realizing authentic teaching and learning (after adaptation). We do see differences between the textbook series here as well. The order is comparable to that for the other questions.

What can also be seen is that the domain of Number clearly has the highest potential (score 4 for 43% of the paragraphs). This was no surprise for the researchers, since there is much of 'useful' and everyday mathematics in this domain. Many recognizable, meaningful and relevant contextual problems for many vocational subjects and for daily life outside school relate to this domain (which also includes measurement and estimation). The scores on the domains Geometry and Statistics are fairly similar, around 25%. Earlier we found a low score for relation to everyday life in Geometry. The relatively high score for potential allows us to conclude that there are possibilities for improvement. We think the potential for Statistics could even be higher if the content of this domain were to be changed. Now the domain consists mainly of simple descriptive statistics and of data processing, which is limited to dealing

with network-graphs. These network graphs do not occur often either in daily life or in professional situations. A different content for Data processing, more linked to representing and interpreting information in everyday life (media) and vocational and professional practice, would seem to be eminently possible. The low score for algebra can be explained from the necessarily often somewhat abstract character of the problems in this domain.

Conclusions and recommendations

We classify the conclusions along the lines of a number of possible improvements to the mathematics textbooks. This means we will summarize our research in the form of recommendations to publishers and authors of textbook series for mathematics.

A problem as an authentic learning task

Most problems in the various textbooks are set up with a series of sub problems. Often this structure guides the student towards the solution of the final and usually most important sub problem. An example of such a problem can be seen in figure 4.

1 Je verbruikt in een jaar heel wat elektriciteit. De ene maand verbruik je meer dan in de andere maand.
In de grafiek hiernaast kun je zien hoeveel elektriciteit de familie Galema gedurende een jaar verbruikt.

a Welke betekenis hebben de letters bij de horizontale as?

b Waarom is er op de verticale as een zaagtand gebruikt?

c In welke twee maanden verbruikt de familie 220 kWh elektriciteit?

d In welke maand is het stroomverbruik het hoogst? Waardoor komt dat volgens jou?

e In welke maanden is het verbruik het kleinst?

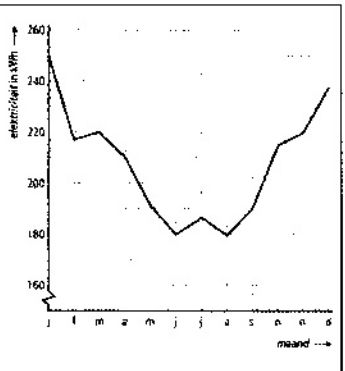

Fig. 4 *From Moderne wiskunde, volume 3a vmbo basis/kader*

This is the first problem in a new chapter. The context is stated in the first section of the problem:

> *You use a lot of electricity in a year. In one month you will use more than in an other month.*

The context electricity use (at home) is recognizable and conceivable for all students, so the problem scores highest with a '2' (on a scale of 0-1-2) for the characteristic 'recognizability'. Although 'electricity use' is an everyday phenomenon, the question remains whether and how students would encounter this in their own reality. This might be through the meter readings, or maybe through the bill of the electricity company. In this problem, however, attention centers around a graph.

> *In the graph you can see how much electricity the Galema family uses in a year.*

In reality, students will not be confronted with this kind of graph. Thus the problem scores a '1' (doubtful) for the characteristic 'relation to everyday life'.

The first three sub questions of this problem are referring to the graph:

- *what is the meaning of the letters along the horizontal axis?*
- *why is a zig-zag used on the vertical axis?*
- *in which two months the family uses 220 kWh of electricity?*

No 'obvious problems' arising naturally from the context are posed. These sub questions are probably intended to put students on the right track and focus them on the interpretation of the graph. The final two questions,

- *in which months is the electricity use the highest? What do you think causes this?*
- *in which months the use is lowest?*

are well suited to the context, they are in a sense directly evoked by it. In none of the questions however a real (complex) problem is posed, so there is no clear opportunity for an individual problem solving approach by the student. The only opportunity is in finding and formulating reasons for high electricity use in winter (which seems rather trivial). This results in a score of 1 (doubtful/partial) for the characteristics 'obviousness' and 'problem solving'. In this case, it would be easy to convert the problem into a more authentic learning task, that gives more room to the student, therefore this problem has high 'potential'. A possible simple adjustment could be:

> *Look carefully at the graph. It shows how much electricity the Galema family used in 2002. Describe what their electricity use looked like in that year. In addition give an explanation.*

Another option would be to entirely leave out the graph and ask the students to draw the graph of the electricity use during the year for the Galema family and describe why they made the graph as they did.

Often a problem can already gain in authenticity by only using the final sub question, since that is often the problem that is naturally evoked by the context. Leaving out the introductory sub questions would also give students more of an opportunity to choose their own approach.

Beware of quasi contexts

Many problems in the Dutch mathematics textbooks arise from or are embedded in a 'realistic' context. This is definitely true for the different trajectories in pre-vocational education. A number of the contexts even seems to clearly be derived from the vocational subjects. This does not mean, however, that the 'authenticity' criteria for an authentic learning task are automatically fulfilled.

For instance, a textbook problem may have a real context; within that context a graph may serve as a useful tool to clarify the situation. Despite this, there could still be a problem: the questions may wander outside the context. For example, there may be a question about the maximum in the graph, which is a mathematical formulated question not obviously relating to the context. Within the context this question would better be formulated as for example: 'for what amount are the costs highest?' or 'what are the maximum costs?'. Therefore, it is a question of language, but it is the precise wording that will reveal a problem's shortcomings. We then may not be dealing with an authentic problem, but with the mathematical task of reading a graph. The context in that case is only a superficial wrapping, it is a quasi context. An example of such a quasi context can be seen in figure 5.

13 Gerard en Marianne gaan naar de disco.
Voordat ze weggaan, haalt Marianne twee pizza's uit de diepvriezer en zet ze in de magnetron.
In de tabellen zie je hoe de temperatuur van de pizza's stijgt.

Pizza 1:

tijd (min.)	0	1	2	3	4
temp. (°C)	−18	−11	−8	−3	0

Pizza 2:

tijd (min.)	0	1	2	3	4
temp. (°C)	18	−12	−6	0	6

Bij welke tabel is er een lineair verband?

Fig. 5 *Problem from Netwerk, volume 4 vmbo, basis*

The introductory text in this problem is:

> *Gerard and Marianne go dancing. Before they leave, Marianne takes two pizzas from the freezer and places these in the microwave. The tables show how the temperature of the pizzas is rising.*

The question that follows is:

> *Which table represents a linear relationship?*

As you can see the context plays no part whatsoever, in this case it is even doubtful whether the tables really reflect what happens to the two pizzas in reality. All that is asked for in the problem is recognizing a linear relationship based on the characteristics of a table. The problem could equally well, and maybe even better, have been posed with just the two tables, with x and y (or other meaningless variables) as row-titles, without ever mentioning a context.

Number of different contexts in a single paragraph

Paragraphs in Dutch mathematics textbooks usually fill one or two lessons of 50 minutes, and cover one specific mathematical (sub)topic. The problems in a paragraph are in a well thought out didactical order, interspersed with short sections of theory. To achieve enough depth and abstraction, often the choice is made to offer a wide variety of contexts in the problems. An understandable approach, which, however, comes at the cost of authenticity. The lifespan of the contexts is so short that they seem to function as different wrappers for the same mathematical 'trick'. This offers very little opportunity for the authentic relation between mathematics and reality to occur.

Make a link with a practical subject

It would be good to teach mathematics in pre-vocational education at least partly as arising from the vocational subjects[6]. There is no better way to convince students of the value of mathematics than when they have learned to use it within the context of their future profession. This argues in favor of a better link between mathematics and the practical vocational program. The teachers of the vocational subjects[7] also are in

6 This is increasingly the case in both middle and higher vocational education. Generally, there is a form of problem oriented teaching where the solution to a practical problem has a central position. A result of this is that the position of mathematics as an autonomous discipline is under severe pressure in these types of education.

7 This comment refers to a part of this study that is not reported on here, in which interviews with teachers were held on this topic.

favor of this and see the possibilities. They are often in need of a better connection between subjects as well.

Integration or connection

Those involved are aware of the advantages of introducing a better connection between mathematics and the vocational subjects. It is, however, important to take a close look at the optimal way to realize this connection. There are several objections to fully integrating mathematics with a vocational subject. These objections are both formal and related to content.

The examination program for mathematics is independent of the department students are in, that is to say: the mathematical content is the same for students in all vocational sectors. That in itself would not obstruct 'sectoral coloring' of math, were it not that the central final examination also observes this neutrality towards the vocational sectors. Thus a too strong connection with a specific vocational program, might be in the way of a proper preparation for the final central examination in mathematics.

The mathematical content of some of the domains in the examination program are hard to connect with a practical vocational subject. This would mostly be true for the algebra domain. For instance, interviews with teachers of the practical vocational program in the technology and engineering department revealed that students do not make use of formulas in their vocational courses. Great importance however is attached to the use of tables.

Although the role of mathematics within the vocational program is not always immediately visible for the students, some students indicate that there is some use in having separate lessons for mathematics.

"You learn different things in mathematics." "You just need it to get on in life."

Aside from the aspect of usefulness, often referred to as numeracy or mathematical literacy, mathematics also has an intellectual value. This is true at all levels of schooling. The understanding that having two graphs representing two different rates for renting bicycles gives a quick and handy overview of the situation is such an authentic intellectual achievement of mathematics. Maybe this aspect of mathematics is what students refer to when they say they do not want to abolish mathematics lessons just like that.

Applicability of the classification system for other domains

The classification system used in this study to score problems on aspects of authenticity has been specifically designed for research into the problems in mathematics textbook. The question arises whether this instrument

might be used for other domains, such as science (Goedhart et al, 2001) as well.

We did not explicitly take this question into account when defining the instrument, so an experiment would be in order to shed some light on this issue. However, we do think that the characteristics 'recognizability', 'relation to daily life' and 'relation to profession' are without doubt clearly scoreable dimensions for other knowledge domains where the learning tasks are presented as 'problems in contexts'. The characteristics 'obviousness' and 'problem solving' would seem to need a closer exploration.

New developments and future research issues

Since this research was carried out in 2003, textbooks for mathematics have changed. Publishers designed mixed media editions. They integrated the use of IT in their textbook series, often by adding cd-roms with extra IT materials such as software activities, mini-games and electronic tests.

In addition, Dutch (pre)vocational education is changing more and more towards competence-based learning. Authentic tasks for the vocational programs are more common than they were at the time of this study. Furthermore, also general secondary education shows a small move towards competence-based and performance-based forms of learning. For general subjects like mathematics this means that curriculum developers, software designers and teachers are designing new teaching materials (both paper and digital) as well as new ways of teaching or coaching the students in their learning process. In this type of education students have more responsibility and are more in control of their own learning. They can often arrange or select their own tasks and have more control over sources and other materials they use (like books, and software and tools). Meaningful, complex, rich problems in which mathematics can be used in an authentic way need to be at the heart of the learning process. The responsibility on what is being learned is no longer just a matter of textbook and teacher.

This situation asks for rethinking textbooks research. First, textbook seems to be a too limited concept: teaching materials is better suited to cover the scope of the types of materials used for teaching mathematics. Furthermore, teaching materials in itself become less and less important factors in shaping teaching and learning. The users (teachers and students) and the way they use the materials need to be the focus of research.

The Freudenthal Institute therefore in its recently formulated research program focuses on two broad themes:

- the role of technological tools in teaching and learning;
- the role of the teacher.

In both themes the research on good teaching materials (the role of the context, the search for meaningful learning, etc.) will be continued.

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Appendix

The mathematics teaching materials that were investigated

The mathematics teaching materials that were investigated are:

Getal en Ruimte, EPN, Houten, 2001, first edition, 3rd impression.
The following volumes have been analysed: 3B1, 3B2, 4B1, 3K1, 3K2, 4K1.
For these volumes for all chapters the problems from the paragraphs containing the core content have been analyzed. The summary, mixed problems and repetition have been left out of the analysis. The practical problems and integrated mathematical activities have been analyzed separately at the paragraph level. Volumes 4B2 and 4K2 consist of direct preparation and training for the central final examination. They contain no new materials and, as such, are not relevant for the research.
Moderne Wiskunde, Wolters Noordhoff, Groningen, 2000, seventh edition.
The following volumes have been analysed: 3 vmbo basis, 3a vmbo basiskader, 3b vmbo basiskader, 4 basis, 4 kader.
For these volumes for all chapter the problems in the introductory paragraph and the four paragraphs of core content have been analyzed. The summary, the diagnostic test, the extra practice, the mixed problems and the plus paragraphs have been left out of the analysis. The integrated mathematical activities and the orientation on learning and work have been analyzed separately at the paragraph level.
Netwerk, Wolters Noordhoff, Groningen, 2000, first edition.
The following volumes have been analyzed: 3 vmbo basis, 4 vmbo basis, 3 vmbo basis/kader, 4 vmbo kader.
For these volumes for all chapters the problems of the four core chapters with core content have been analyzed. Test yourself, summary and repetition and examination training have been left out of the analysis. The practical assignments have been analyzed separately at the paragraph level.