Teaching Quantum Mechanics

at the secondary school level

Kim Krijtenburg - Lewerissa

TEACHING QUANTUM MECHANICS AT THE SECONDARY SCHOOL LEVEL

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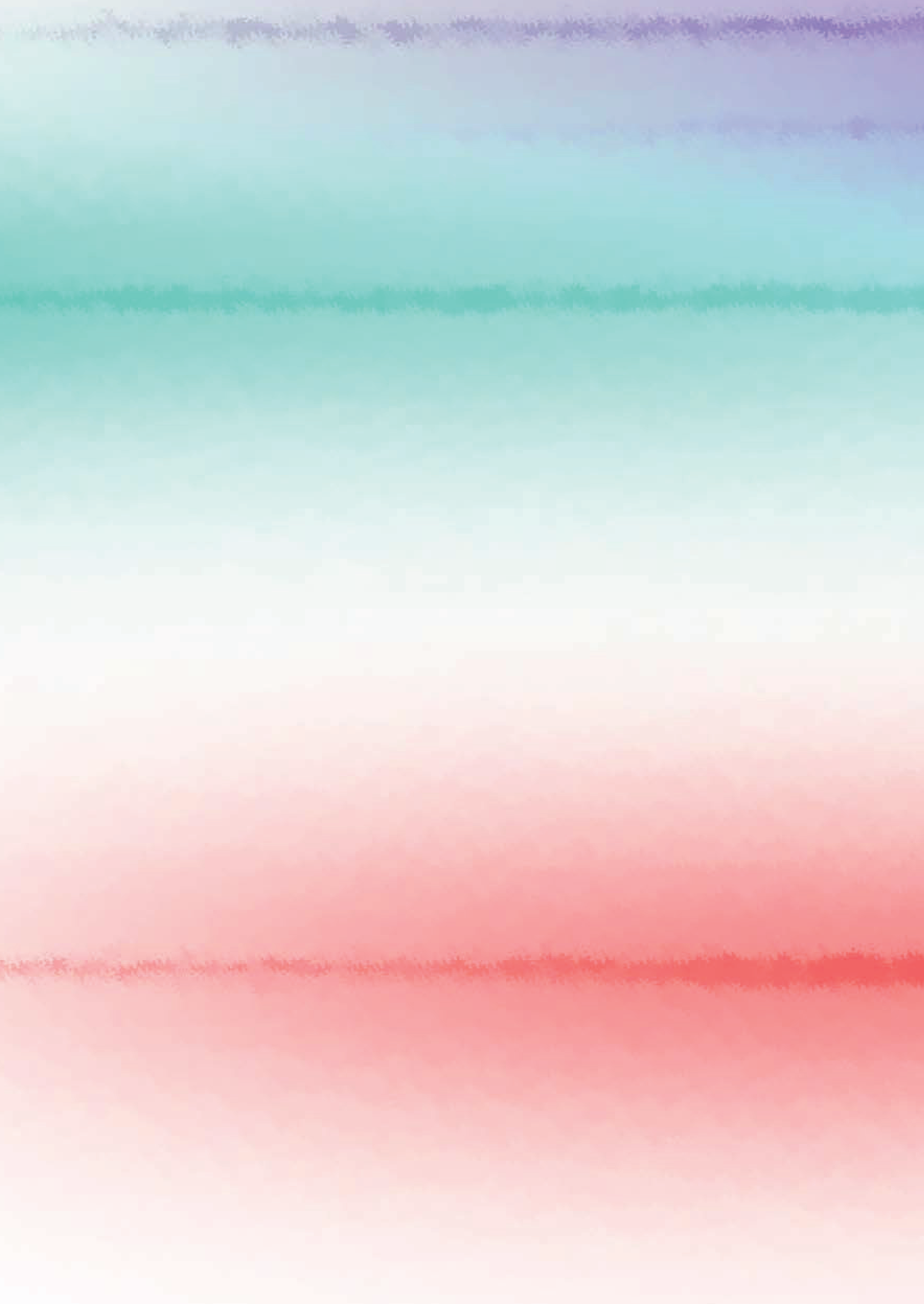

1

Introduction

1.1 INTRODUCTION

'The development of quantum mechanics early in the twentieth century, obliged physicists to change radically the concepts they used to describe the world.'

Alain Aspect[1]

At the end of the 19th century, physicists were convinced they had understood the nature of matter. Matter was seen as composed of particles composed of atoms, and atoms were considered to be point particles. For describing their behavior, we had classical mechanics. According to classical mechanics, a particle's motion could be exactly described and predicted by the laws that Newton had formulated in 1685. Based on these laws physicists had built an elaborate mathematical system consisting of conservation laws involving energy, momentum and angular momentum. Classical mechanics was an excellent theory for describing macroscopic phenomena. In the 19th century, the laws of electrodynamics were added to this system and the feeling was that physics was more or less complete.

However, physics was apparently unable to explain several phenomena that puzzled physicists at the end of the 19th century: *black body radiation*, the *photoelectric effect*, and *atomic spectra*. These phenomena required analyzing the level of microscopic particles. The atoms apparently did not behave according to the well-known laws.

BLACK BODY RADIATION – ENERGY QUANTIZATION

Black body radiation is the electromagnetic radiation emitted by an ideal black body as a function of its temperature. The daily life phenomenon that corresponds with this is that a piece of metal will glow (i.e. emit light) when it is heated. Classical mechanics predicted that a black body would emit more radiation at shorter wavelengths, which is for higher frequencies of the light wave (see dotted line in Figure 1). According to the Rayleigh-Jeans law, the emitted energy was proportional to frequency squared. This implied that the total emitted energy of black bodies would be infinite and that matter would radiate all of its energy in a short time, which

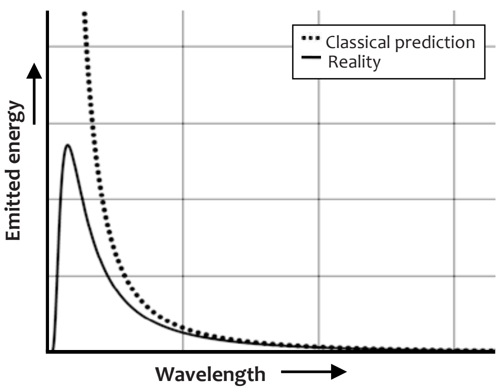

FIGURE 1 The energy emitted by a black body depending on the wavelength, based on classical mechanics and on measurements.

was clearly not true. Measurements also showed that the classical approach did not correspond to reality; for decreasing wavelength the energy first increased, then reached a maximum, and finally decreased (see continuous line in Figure 1). Classical mechanics and electromagnetism could not explain this phenomena, and physicists were searching for solutions. A creative solution was proposed by Max Planck, who stated a theory based on the idea that atoms emit and absorb finite portions of energy (energy quantization)[2]. For Planck, this was a purely formal assumption, he made no assumption about the nature of light itself. Planck's theory was very successful. Using his formalism the spectrum of the black body could be explained.

The photoelectric effect - the particle behavior of light

The idea of energy quantization was taken further by Einstein, who recognized energy quantization as an important concept when trying to explain the photoelectric effect[2].

When light hits a metal, electrons can be emitted. According to classical theory, this was caused by the transfer of energy from light to an electron. In classical mechanics light was considered to be a wave. This would imply that there would be two ways of increasing the number of emitted electrons; by increasing the frequency of the light wave, and by increasing the intensity. Both would lead to more energy transferred to the electrons in the metal. However, measurements showed that this was not the case, that below a certain threshold frequency no electrons were emitted, not even for high intensities or long exposure times. In order to explain this, Einstein proposed that radiation energy is not continuously distributed within a light ray, but that it consisted of finite portions of energy: *photons*. He suggested that energy of a single photon depends on the frequency of the light, in the same way as done by Planck. When an electron absorbed one photon, the amount of energy of one photon needed to be large enough to expel the electron from the metal. Below a certain frequency, the energy would be too low to emit an electron, which would explain the threshold frequency found in measurement. In 1916 Millikan[3] provided experimental proof of Einstein's theory, which implied that light can be approached as energy quanta, and therefore exhibits behavior that is associated with particles instead of waves.

Atomic spectra – the wave behaviour of particles

Based on the relation between energy and wavelength, as presented by Planck and Einstein, in 1913 Bohr proposed his atomic model[4]. In this atomic model Bohr stated that there were specific permitted orbits for electrons, which could explain the spectrum of hydrogen. Electrons would jump from one orbit to another, causing light to be emitted, with a wavelength that corresponded to the change in energy. Although Bohr's atomic model corresponded with the experimentally observed spectral lines of hydrogen, it gave no physical explanation for the existence of the specific permitted orbits. De Broglie related these specific permitted orbits and the stable motions of electrons in the atom to wave behaviour[5], and was the first to assign wave properties to particles in order to explain these orbits[6]. De Broglie's ideas inspired Schrödinger to derive a wave equation for the hydrogen atom[7], which led to de development of the Schrödinger equation and the wave function[8]. This wave function corresponded with observed spectral lines of hydrogen, and explained the observed energy quantization. However, the interpretation of this wave function was unclear. According to Schrödinger, this wave equation described the behaviour of tiny wave packets, but soon after, Born proposed a statistical interpretation[9].

The statistical interpretation

Born stated that the wave equation was not related to physical properties of particles, but that its amplitude was a measure for the probability of a particle being found in a certain place. In technical terms: the normalized, squared wave function describes a probability density[10], which predicts the possibility of finding the particle at a certain location. This interpretation led to completely new concepts, such as:

(1) tunneling: a particle can pass a barrier that was assumed to be impenetrable;
(2) superposition: if two states are a solution to the Schrödinger equation, the sum of these states is a solution too;
(3) the uncertainty principle: the position and momentum of an object cannot be both measured simultaneously with exact precision, and
(4) entanglement: combinations of particles can be created, that cannot be described independently, even when separated by a large distance.

The latter raised objections by Einstein, Podolski and Rosen[11]. In 1935, they showed with a mathematical thought experiment that the result of a measurement of one particle of an entangled two-particle quantum system has direct effect on the second particle, even when they are at great distance. This would violate the classical ideas of causality (i.e. an effect cannot occur from a cause that is not in the past) and locality (i.e. an object can only be influenced by its immediate surroundings) and led Einstein, Podolski and Rosen to conclude that QM is an incomplete theory that should be supplemented with additional (hidden) variables. In 1964 Bell[12] conducted a similar thought experiment, in which he formulated physical consequences of QM without, and QM with local hidden variables. Bell showed that both theories predict different experimental results. In the years after, several research groups have shown that experimental outcomes are in accord with QM without local hidden variables[13-16] and that refute either locality or realism (i.e. the assumption that entities have well-defined properties, independent of measurement). Current research is now using entanglement for quantum cryptography and quantum encryption. Superposition, entanglement and teleportation are important topics of research in order to create better understanding of quantum effects, and to manipulate quantum systems for new materials and applications. Still, there is no consensus on the interpretation of QM and the wave equation, not even among scientists[17]. QM is a theory that corresponds with reality and that has predicted things beyond expectation, but it remains under discussion wat QM implies for the way we understand what physical reality is.

1.2 Teaching quantum mechanics

'Lately, a lot is going on in physics, and I think there is also a widespread feeling among teachers, that they stand for an evolution.'

Adriaan Fokker (1926)[18]

Quantum mechanics has changed our world view, and, for more than a century, lays at the base of many important developments in physics research and the development of new technologies. One would expect that a topic of this importance would have been an important, and well-evaluated part of the Dutch secondary school curriculum for decades. However, reality is far from this. The development of the physics curriculum was a tedious process, especially regarding the introduction of modern physics. Around 1900, physics and mathematics were closely related, scientist researched a combination of mathematics and physics. In Dutch secondary education, mathematics was the main topic, physics played a marginal role[18]. In the beginning of the 20th century the difference between research in mathematics and physics increased. Mathematical theories became more and more abstract, whereas physics thrived on experimental and practical research. Because of this, in the 1920's a discussion started on the renewal of physics education and the relation between physics and mathematics. A committee was formed to improve the teaching of physics in secondary education: the Fokker committee. Fokker proposed a physics curriculum that used experiments, and that addressed up-to-date scientific insights. Of course, at that moment QM was still in full development, and Fokker also raised the question what parts of modern physics should be in the secondary school curriculum. In 1937 a few of the ideas of the Fokker committee were implemented in Dutch secondary schools, but the implementation of modern physics evolved slowly. Later on, there have been initiatives for an introduction of quantum mechanics at the pre-university level. Not until 1976, QM was introduced as an optional part of the curriculum, but this topic was discontinued in 1982. Then, from 1996 to 2005, 40 schools took part in a modern physics project, which included the wave-particle duality, and the most straightforward form of the wave function: the wave function for a particle trapped between two infinite barriers (the 1D infinite potential well)[19]. The wave-particle duality was introduced historically, and was used to show the non-deterministic character of QM. The 1D infinite potential well was used to illustrate quantization, estimate the order of magnitude of atoms, and determine the absorption spectrum of a coloring agent. Based on the experience with this project and a pilot, in 2016 QM became part of the Dutch secondary school curriculum. The QM topics in the current Dutch secondary school curriculum are: (1) the wave character of light, (2) wave-particle duality, (3) the photoelectric effect, (4) Heisenberg's uncertainty principle, (5) the 1D infinite potential well, (6) the hydrogen atom, and (7) tunneling.

1.3 Difficulties in teaching quantum mechanics

'When you ask what are electrons and protons I ought to answer that this question is not a profitable one to ask and does not really have a meaning. The important thing about electrons and protons is not what they are but how they behave, how they move.'

Paul Dirac[20]

Quantum mechanics is based on a complex mathematical formalism, is different from what students have learned before, and its implications for the way we see physical reality is still under debate. What does this mean for teaching QM at the secondary school level?

Non-deterministic thinking

Since the Dutch secondary school curriculum does not include the mathematical tools for a formal, mathematical approach to quantum mechanics, QM needs to be taught at a more conceptual level. However, when looking at the history of QM, one can see that a conceptual approach to QM raises a problem; the implications resulting from the QM formalism are counterintuitive. Where students previously have learned that physics can precisely predict the outcome of an experiment, in QM they learn that physics can only predict a probability distribution. Where they have learned that objects cannot pass impenetrable barriers, they now have to accept there is tunneling. These counterintuitive fundamental ideas from QM are based on complex mathematical formalism. Physicists have done mathematical derivations, thought experiments, and real experiments in order to verify of falsify the implications of QM theory. Making secondary school students understand QM concepts, experiments and thought experiments without introducing complex mathematical formalism is a challenge.

Students' learning

When students enter a classroom, they do so with existing ideas about the world around them. These ideas are based on their own experiences, and on what they have learned previously. When students learn new concepts and theories, they need to implement these into an existing mental model or framework. During learning, students can interpret concepts incorrectly, and develop misunderstandings.

Research has shown that there are some misunderstandings that can easily be overcome, while other difficulties are more persistent[21]. Chi[22] describes that robust misunderstandings appear when there is a need for an *ontological shift*. According to Chi there are three ontological categories; entities, processes, and mental states. When students have placed a concept in an incorrect category, an ontological shift is needed. In QM, students need to understand that particle behaviour can be described with wave properties, whereas particles are entities and waves are processes. This even goes further that correcting a miscategorization, because students sometimes need to reason from particle properties, and other times from wave properties. Hence, students need to switch between different perspectives[23] and have a flexible ontology[24, 25]. In order to do so, students need to become more aware of the limitations of physics models, and capable of choosing an appropriate model for a specific situation.

Additionally, research has shown that new concepts are not always implemented constructively, and that the students' existing mental model can be based on incorrect ideas. During the process of learning, students can relate new knowledge to unrelated prior learning, or interpret concepts intuitively, in a non-scientific way[26]. When students incorporate new concepts into an incorrect or incompatible prior knowledge without making the complete framework consistent, this leads to fragmentation. But, in the search for coherence and consistency, students often create new and incorrect frameworks that have some internal consistency. Such models are called synthetic models[27, 28]. It is important to take into account the existence and development of these synthetic models.

Challenges for teaching quantum mechanics

For a student used to classical reasoning, quantum mechanics is counterintuitive, and often seen as strange and incomprehensible. This does not necessarily imply that QM is too complex to understand, or impossible to teach. However, there are challenges that need attention. Students need to become familiar with a new, non-deterministic way of thinking. Also students need to gain deeper insights into scientific models and their limitations, in order to be capable of switching between different models and representations. And finally, there has to be emphasis on the essential prior knowledge needed to understand QM. In order to bridge the gap between students' existing prior knowledge and QM, it is important to know what prior knowledge supports QM understanding.

1.4 The need for teaching quantum mechanics

'Look at the leap between horse-and-cart, and Schiphol Airport. Take that leap and increase it by a factor of ten, when talking about quantum technology. Then realize that we are talking about something that will be very useful to society, and can become incredibly disruptive to society.'

Vincent Icke[29]

From 1900 the world has changed. The 'birth' of quantum mechanics caused drastic changes in the way physics describes the world at the atomic and subatomic scale. The changed understanding of (sub)atomic particles and chemical interactions caused the rise of the fields of laser and semiconductor physics: the first quantum revolution. Semiconductors are used to produce the basic components of modern electronic devices, such as smartphones and computers.

Currently, the second quantum revolution is taking place[30]; a revolution that applies the laws of quantum mechanics in order to engineer on a subatomic scale, and develop new technologies, such as quantum electronic devices, quantum cryptography, and quantum information technology. Research groups are working on ways to control photons, atoms, ions, and electrons in order to create state-of-the-art materials, quantum information processors, and quantum cryptographic keys. All of this can lead to radical new technologies, such as quantum simulations, quantum sensors, and quantum computers. Recently researchers reported of a quantum processor that outperformed the most powerful conventional computers[31], which is a giant step towards the development of a quantum computer.

Quantum technology has had a very great impact on society, and its influence is only increasing. This makes that teaching QM is not only important for knowledge of the development of physics, scientific models, and modern physics, but also for decision-making and the understanding of the impact of QM on society. This gives rise to the question of what secondary school students should learn. The answer to this question is not only dependent on secondary school students' prior knowledge and (mathematical) skills. It is also related to why we teach QM at secondary schools. When we want our students to learn high-end technologies and ground-breaking research related to QM physics, they should learn about entanglement, quantum states and non-locality. When we want our students to learn about the development of scientific theories, they should be confronted with the problems that researchers encountered which led to changing ideas, and learn about the wave behaviour of particles, the photoelectric effect and atomic spectra. Well-considered choices need to be made, based on the complexity of the subtopic, the mathematical formalism needed, the goals for teaching QM, and the way students learn.

1.5 Aim of this thesis

In order to design a well-balanced curriculum, Duit et al.[32] stated that it is important to clarify and analyze the subject matter, investigate student and teacher perspectives, and design and evaluate learning materials and learning sequences. Despite the experience with QM at the secondary school level and the evaluation of the pilot and the modern physics project, there is still need for more research into the three aspects specified by Duit, especially for the analysis of the subject matter, and the investigation of student and teacher perspectives. Therefore, in this dissertation we present our research in which we investigate the following research questions:

(1) What is the current state of research on students' understanding, teaching strategies, and assessment methods for the main concepts of QM aimed at secondary education?

(2) In the view of experts, what are the essential topics that secondary school students need to learn in order to develop an appropriate image of quantum mechanics in terms of research, developments and applications? And what are the experts' arguments for choosing their topics?

(3) What misunderstandings do Dutch students have after learning QM? And what are the underlying difficulties and causes that lead to these misunderstandings?
(4) Is it possible to increase students' understanding of QM by addressing these underlying difficulties?

1.6 OUTLINE OF THIS THESIS

To address the research questions, four studies were conducted. In the following chapters we present the results of these studies.

First, in Chapter 2 we present the results of a literature review, in which we have investigated what difficulties students have shown in previous research. We also listed the tools that are designed to investigate these difficulties and the multimedia applications that are currently available for teaching QM. Finally we give an overview of the teaching strategies that have been used and investigated for their effect. Because the Dutch secondary school level is not entirely identical to secondary schools in other countries, we have extended our investigation to secondary and lower undergraduate education.

In Chapter 3 we present the results of a Delphi study into expert opinions regarding the teaching of QM at secondary schools. In this study we asked experts in the field of QM and related research fields what they think is important to teach at the secondary school level, and why.

Chapter 4 shows the results of an investigation into students' difficulties regarding the potential well and tunneling. For this investigation we administered a test on conceptual understanding of QM and conducted interviews. The test results and interview transcripts were analyzed and resulted in an overview of difficulties that Dutch secondary school students have after being taught QM.

In chapter 5 the influence of students' understanding of prior knowledge on their understanding of QM is presented. For this study, instructional materials were created in order to increase students' understanding of potential energy. The effect of this increased understanding was then investigated in a quasi-experimental intervention.

Finally, in chapter 6 we draw conclusions from all four conducted studies. The results of the review, the Delphi study, the study into students' difficulties, and the effect of prior knowledge will be used to outline the challenges there are for teaching QM at the secondary school level.

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2

INSIGHTS INTO TEACHING QUANTUM MECHANICS IN SECONDARY AND LOWER UNDERGRADUATE EDUCATION

This study presents a review of the current state of research on teaching quantum mechanics in secondary and lower undergraduate education. A conceptual approach to quantum mechanics is being implemented in more and more introductory physics courses around the world. Because of the differences between the conceptual nature of quantum mechanics and classical physics, research on misconceptions, testing, and teaching strategies for introductory quantum mechanics is needed. For this review, 75 articles were selected and analyzed for the misconceptions, research tools, teaching strategies and multimedia applications investigated. Outcomes were categorized according to their contribution to the various subtopics of quantum mechanics. Analysis shows that students have difficulty relating quantum physics to physical reality. It also shows that the teaching of complex quantum behavior, such as time dependence, superposition and the measurement problem, has barely been investigated for the secondary and lower undergraduate level. At the secondary school level, this review shows a need to investigate student difficulties concerning wave functions and potential wells. Investigation of research tools shows the necessity for the development of assessment tools for secondary and lower undergraduate education, which cover all major topics and are suitable for statistical analysis. Furthermore, this review shows the existence of very diverse ideas concerning teaching strategies for quantum mechanics and a lack of research into which strategies promote understanding. This review underlines the need for more empirical research into student difficulties, teaching strategies, activities and research tools intended for a conceptual approach for quantum mechanics.

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2.1 INTRODUCTION

Quantum mechanics has gained a strong position in physics research and its applications. Developments in medical imaging, nanoscience, laser physics and semiconductors are all based on quantum phenomena. Moreover, quantum mechanics is the foundation of completely new and promising technologies: quantum computers, quantum encryption and quantum entanglement. Quantum mechanics has been an important part of university physics and engineering education for a long time, but the often abstract and mathematical teaching practices used have been in dispute for several years[1]. Currently, more emphasis is placed upon visualization and conceptual understanding [2, 3]. This conceptual approach to quantum mechanics has made it possible to introduce quantum mechanics at an earlier stage, and therefore it has become part of the secondary school curriculum in many countries. Quantum mechanics has been part of the upper secondary school curriculum in England [4], Germany [5], Italy [6] and the USA [7] for several years. More recently, quantum mechanics has been incorporated in the Dutch [8] and the French [9] secondary school curricula, and in Norway new teaching modules have been designed and tested in the ReleQuant project [10].

Because quantum mechanics led to fundamental changes in the way the physical world is understood and how physical reality is perceived [11], quantum mechanics education is faced with several challenges. For instance, the introduction of probability, uncertainty and superposition, which are essential for understanding quantum mechanics, is highly non-trivial. These concepts are counterintuitive and conflict with the classical world view that is familiar to most students. A radical change in thinking is needed [12] and ways to instigate conceptual change [13, 14] should be investigated.

Several initiatives have been taken to improve students' understanding of quantum mechanics and resolve problems encountered in teaching quantum mechanics, including a review of misconceptions of upper level undergraduate students [15]. This review by Singh and Marshman gives a good overview of students' difficulties on an abstract and mathematical level. Introductory quantum mechanics courses mainly focus on the introduction of the main concepts and students' conceptual understanding hereof. Therefore, we reviewed articles covering educational research on quantum mechanics for the secondary and lower undergraduate level, aiming to answer the following question:

What is the current state of research on students' understanding, teaching strategies, and assessment methods for the main concepts of quantum mechanics, aimed at secondary and lower undergraduate education?

More specifically, we researched the following questions:

(1) What learning difficulties do secondary and lower undergraduate level students encounter while being taught quantum mechanics?

(2) What instruments have been designed and evaluated to probe students' understanding on a conceptual level?
(3) What teaching strategies aimed at the secondary and lower undergraduate level have been tested, implemented and evaluated for their influence on students' understanding?

The overview presented in this article therefore comprises (1) students' misconceptions and difficulties, (2) research-based tools to analyze student understanding, (3) assessed instructional strategies, activities and multimedia applications that improve student understanding.

2.2 METHOD

For this review study three databases were searched: Scopus, Web of Science and ERIC. The following query was used to find appropriate articles, published in journals: '(quantum OR "de Broglie" OR "photoelectric effect") AND (student OR instruction) AND (concept OR understanding OR reasoning OR difficulties)'. This search resulted in 471 articles from ERIC, Web of Science and Scopus, published between 1997 and the present.

Subsequently the results were filtered using the following criteria: (1) The article addresses the understanding of quantum concepts for secondary or undergraduate students in an educational setting, (2) the article includes an implementation and evaluation of its impact on understanding, (3) the article does not expect students to be familiar with mathematical formalism (e.g. Dirac notation, Hamiltonians or complex integrals), and (4) the article mainly emphasizes physical aspects.

A total of 74 articles matched these criteria. These articles were analyzed for detected student difficulties, used research-based tools which measure student understanding, and assessed instructional strategies, multimedia applications and activities. The following sections present these difficulties, tools, and teaching approaches, all categorized and analyzed for content, research methods and value for teaching quantum mechanics in secondary and lower undergraduate education. Where needed, additional literature has been used to clarify or evaluate the findings in the selected literature.

2.3 LEARNING DIFFICULTIES

For the development of effective teaching strategies, it is important to know what difficulties students have with quantum mechanics. Therefore this section gives an overview of findings for the first sub-question: "What learning difficulties do secondary and lower undergraduate level students encounter while being taught quantum mechanics?" To answer this question, the selected articles were all scanned for misconceptions concerning the topics shown in table 1. These topics

Table 1 Quantum topics used for the analysis of the selected articles

Wave/particle duality	Wave function	Atoms	Complex quantum behavior
Dual behavior of photons & electrons	Wave functions & potentials	Quantization & energy levels	Time dependent Schrödinger equation
Double slit experiment	Probability	Atomic models	Quantum states
Uncertainty principle	Tunneling	Pauli principle & spin	Superposition
Photoelectric effect			Measurement

were based on (1) the learning goals formulated by McKagan, Perkins and Wieman [16], which were based on interviews with faculty members who had recently taught modern physics; and (2) learning goals determined in a Delphi study among Dutch experts in quantum mechanics [17], a method which uses consecutive questionnaires to explore consensus among experts [18]. The topics in Table 1 encapsulate the main topics found in introductory quantum mechanics curricula around the world [4-10]. This section gives an overview of misconceptions and learning difficulties found in the reviewed articles, organized by the topics in Table 1. See Appendix A for more information concerning the research methods for articles discussed in this section.

2.3.1 Wave-particle duality

The fact that tiny entities show both particle and wave behavior is called wave-particle duality. This phenomenon is in conflict with prior, classical reasoning. Several selected articles addressed the understanding of wave-particle duality [1, 4, 5, 16, 19-34]. Ireson and Ayene, Kriek and Damtie researched existing student-views of undergraduate students using cluster analysis [20, 24, 25]. Three clusters emerged: (1) Classical description, in which students describe quantum objects exclusively as particles or waves; (2) mixed description, in which students see that wave and particle behavior coexist, but still describe single quantum objects in classical terms; and (3) quasi-quantum description, in which students understand that quantum objects can behave as both particles and waves, but still have difficulty describing events in a non-deterministic way. Similar categories of understanding were found by Greca, Freire, Mannila, Koponen and Niskanen [22, 26]. These clusters all depend on the extent to which students hold on to classical thinking and constitute a spectrum from misplaced classical thinking to correct quantum thinking. Table 2 gives an overview of misconceptions and learning difficulties encountered in the reviewed research, divided into these three clusters. In the following sections, the listed misconceptions are discussed in more detail.

PHOTONS AND ELECTRONS

In many cases electrons display particle properties, but that is not the entire picture. Electrons also exhibit wave properties, such as diffraction and interference. Conversely, light shows wave and particle behavior. Light diffracts, refracts and shows interference, but additionally its energy is quantized, i.e. transferred in "packages". The reviewed literature showed that students have a range of different visualizations of photons and electrons, and many have difficulty juxtaposing wave and particle behavior. Research showed that many secondary and undergraduate students erroneously see electrons exclusively as particles and photons as bright spherical balls with a definite location or trajectory [4, 5, 22-25, 29].

The wave-like behavior of electrons is hard to define, for electrons appear as bright spots on fluorescent screens in most of the textbook experiments. The wave-like behavior of electrons only appears in the distribution of these bright spots. Quantum mechanics does not describe an electron's path, only the probability of finding it at a certain location. Müller and Wiesner[5] observed that students sometimes falsely considered this wave behavior to be a cloud of smeared charge. McKagan, Perkins, Wieman [16] and Olsen [29] reported that several secondary and undergraduate students considered the wave behavior of electrons to be a pilot wave, which forces the electron into a sinusoidal path.

Photons are also sometimes considered to move along sinusoidal paths[30], but Olsen observed that students showed less difficulty assigning both wave and particle behavior to light than to electrons [29]. Sen [31] observed that most students had a more scientific way of describing photons than electrons and ascribed this to the fact that photons are introduced later in the curriculum, which he believes to result in fewer misconceptions of photons at the start of undergraduate education.

DOUBLE SLIT EXPERIMENT

The double slit experiment is used to illustrate the wave-like behavior of photons, electrons, buckyballs and other small objects. These objects pass through a double slit, fall onto a detection screen and cause an interference pattern. For electrons, this interference pattern appears only in the distribution of the bright spots. Understanding of the double slit experiment depends in part on the students' understanding of the wave and particle behavior of quantum objects. If students see photons as classical particles with definite trajectories, this influences their comprehension of this experiment. This can be seen by the fact that some secondary school students considered photons to deflect at the slit edges and move in straight lines towards the screen[21]. Another common problem depends on incomplete understanding of the de Broglie wavelength. Students do not always understand the influence of velocity and mass on wavelength and the influence of wavelength on the interference pattern [21, 34].

UNCERTAINTY PRINCIPLE

The uncertainty principle states that there are certain properties that cannot simultaneously be well-defined. An example thereof is the relation between position and momentum, for which the uncertainty principle is described as $\Delta x \Delta p \geq \frac{h}{4\pi}$. This equation shows that when one of the properties is determined with high precision, the outcome of a measurement of the other property becomes less certain. The uncertainty principle for position and momentum can intuitively be related to the wave behavior of small entities. For example, a strongly localized wave package is a superposition of many waves with varying wavelength and momentum. Ayene, Kriek and Damtie [20] observed four categories of depictions of the Heisenberg uncertainty principle: (1) Uncertainty is erroneously described as a measurement error due to external effects, (2) uncertainty is wrongly described as a measurement error due to error of the instrument, (3) Uncertainty is falsely thought to be caused by measurement disturbance, and (4) uncertainty is correctly seen as an intrinsic property of quantum systems. Only a small number of students had views that fell within the fourth, correct, category. Müller, Wiesner[5] and Singh[32] also observed that secondary and undergraduate students attributed uncertainty to external effects. They reported that some students stated that uncertainty is caused by the high velocity of quantum particles.

PHOTOELECTRIC EFFECT

The photoelectric effect is the phenomenon by which materials can emit electrons when irradiated by light of sufficiently high frequency. This effect is used to show the particle-like behavior of light. This particle-like behavior emerges from the observation that the energy of the emitted electron depends solely on the frequency of the incident light, whereas the intensity of the light determines only the number of emitted electrons. For this subject Asikainen and Hirvonen [19] observed that some students confused the photoelectric effect with ionization. Their research also showed that certain students had difficulty with fully understanding how light and electrons interact, and how various aspects (work function, kinetic energy, cutoff frequency and material properties) together constitute the photoelectric effect. McKagan *et al.* [27] observed that some undergraduate students could not distinguish between intensity and frequency of light, were unable to explain why photons are related to the photoelectric effect, falsely believed that an increase of light intensity will increase the energy transferred to a single electron, or incorrectly believed that a voltage is needed for the photoelectric effect. This last incorrect believe was also observed with secondary school students by Sokolowski [33]. Özcan [30] observed that undergraduate students' different models of light influenced their understanding of the photoelectric effect. Students who used the wave model falsely described the energy transfer in terms of vibrations, which were caused by wave fronts striking the metal. These students believed an increase in light intensity would lead to an increase in the number of wave fronts. Oh [28] observed that some undergraduate students wrongly thought that light reacts chemically with an electron, and others falsely believed that the intensity of light could influence if electrons were ejected or not.

TABLE 2 Misconceptions about wave-particle duality organized into three categories ranging from classical to quantum thinking.

	Classical description	Mixed description	Quasi-quantum description
Photons & electrons	Electrons/photons are depicted as classical particles [1, 4, 5, 16, 20, 22-25]	Electrons/photons follow a definite sinusoidal path [16, 29, 30]	Electrons are smeared clouds of charge [5, 24, 25]
	Electrons/photons have definite trajectories [1, 4, 5, 16, 20, 22-25]	Electrons are either a particle or a wave depending on other factors [21, 29]	Electrons/photons are waves and particles simultaneously [20, 30]
	Light always behaves like a wave [24, 25]	Equations of properties of light also apply to electrons [21]	
Double slit experiment	Light has no momentum[1]	There is no relation between momentum and de Broglie wavelength [21, 34]	There is no relation between momentum and interference pattern [21, 34]
	Photons/electrons deflect at a slit and subsequently move in a straight line [21]	No interference pattern appears with single photons/electrons [24-26]	
Uncertainty principle	Uncertainty is due to external effects, measurement errors or measurement disturbance [5, 20, 32]		
Photoelectric effect	Energy is transmitted by wave fronts, more wave fronts cause more energy [30]	Light collides with electrons [19, 28]	
	The intensity of light influences the energy transferred to a single electron [27, 28]		

2.3.2 WAVE FUNCTIONS

In this section the observed misconceptions concerning wave functions, potential wells, tunneling and probability found in the selected articles [35-44] are presented. Articles matching our search criteria, that addressed the understanding of wave functions, described difficulties of undergraduate students only.

WAVE FUNCTIONS AND POTENTIAL WELLS

Wave functions represent the state of particles. The wave function ψ is not a physical wave, but a mathematical construct, which, for a bound electron, is specified by four quantum numbers, *n*, *l*, *m* and *s*. ψ contains all information of a system and predicts how particles will behave given a specific potential. $|\psi|^2$ can be interpreted as the probability density. Similar to wave-particle duality, students often describe the wave function as a sinusoidal particle path [41]. Table 3 presents reported misconceptions, divided into the two categories observed by Singh, Belloni and Christian [42, 43]: (1) misunderstanding due to overgeneralizations of prior concepts, and (2) difficulty distinguishing between closely related concepts [40-43], which results in a mix-up of energy, wave functions, and probability. The first category corresponds with the work by Brooks and Etkina[36], who concluded classical metaphors cause misconceptions and promote misplaced classical thinking. This over-literal interpretation of classical metaphors was also observed by McKagan, Perkins and Wieman [38]. These authors noticed that many students were likely to have difficulties in understanding the meaning of potential well graphs, and saw potential wells as external objects. McKagan *et al.* also observed that students mixed up wave functions and energy levels. Domert, Linder and Ingerman [40] ascribed this to the use of diagrams combining energy levels and wave functions as illustrated in Figure 1. However, McKagan *et al.* showed that eliminating these diagrams does not automatically prevent misconceptions.

TUNNELING AND PROBABILITY

Wave functions are not limited to classically permitted regions, they can extend past classical boundaries. This effect causes particles to have a probability of existing at positions that are classically impossible. An important result thereof is the phenomenon called tunneling; a small particle can end up on the other side of a classically impenetrable barrier. In this phenomenon no energy is lost and no work is

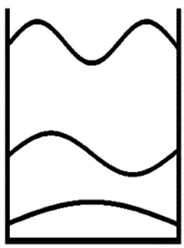

FIGURE 1 A typical diagram as found in many textbooks, which simultaneously shows wave functions and energy levels.

done. In understanding of tunneling, the false belief that energy is lost during the process is prominent [37, 38, 44]. McKagan *et al.* [38] reported that students falsely attributed this energy loss to (1) work done on or by the particle inside the barrier; or to (2) the decrease of wave function amplitude. The same research also showed other misconceptions caused by a mix-up of physical quantities. Several students confused the wave function and energy. For example, some students erroneously believed that a decrease in amplitude causes an increase in energy, or the energy was partly reflected by the barrier. McKagan *et al.* also observed difficulty in understanding plane waves, which led to a mix-up of ensemble and single particle description. Domert, Linder and Ingerman [40] observed that some students believed that only the tops of the waves, which supposedly were higher than the barrier, could pass the barrier. They also stated that misunderstanding of probability is an obstacle to the appropriate understanding of scattering and tunneling. They reported that many students had difficulty to distinguish between energy and probability, which they attributed in part to diagrams which mix wave functions and energy levels (see figure 1). Bao, Redish and Wittmann [35, 39] observed that students can have difficulty with the predictability and stochastic nature of probability. Students falsely believed that the preceding distribution of outcomes influenced the subsequent outcome of single events, and tended to use classical arguments in their reasoning. This tendency was attributed to the lack of experience students have with probabilistic interpretations in physical systems.

TABLE 3 Misconceptions about wave functions and potentials, categorized into two categories

	Overgeneralization of prior concepts	**Mix-up of related concepts**
Wave functions & potentials	Wave functions describe a trajectory [35, 41]	Change in amplitude causes change in energy [38]
	Potential wells are objects [36, 37]	The amplitude or equilibrium of the wave function is mixed up with energy [38]
	Height in potential graphs means position [35]	There is difficulty to distinguish between energy and probability [40]
Tunneling & probability	The amplitude of wave functions is a measure of energy [36, 38, 41]	Only the tops of the waves, which overtop the barrier, will pass [38, 40]
	Probability is described with classical arguments (e.g. velocity) [35, 40]	Part of the energy is reflected at a barrier during tunneling [38, 40]
	Energy or effort is needed to tunnel through a barrier [37, 38, 44]	A single particle is described as an ensemble of particles [38, 39]

2.3.3 Atoms

The following section describes students learning difficulties related to the understanding of atomic structure, quantization and spin, as found in the reviewed articles [12, 24, 25, 31, 45-56].

Atomic structure and models

The quantum atomic model describes the probability of observing the electron at a certain position, but it does not describe a temporal trajectory of an electron inside the atom. Research shows that secondary and undergraduate students hold on to various atom models [12, 24, 25, 31, 45-55] and can develop hybrid models consisting of combinations of different models [45]. Papageorgiou, Markos and Zarkadis [56] reported that the use of these models is influenced by the context of the task. The context of the question or previous questions influenced students' descriptions, which was also observed by McKagan, Perkins and Wieman [48]. Based on a questionnaire administered to 140 undergraduate students, Ke, Monk and Duschl [46] divided the different atomic models into three different stages: (1) An early, planetary, quantum model, in which the electron orbits in a circle of constant radius, (2) a transitional model, in which the electron moves along a sinusoidal path, and (3) a probabilistic model, in which the position of the electron is uncertain. These stages are similar to the categories Ireson [24] observed. Additionally Dangur, Avargil, Peskin and Dori [54] divided the probabilistic model into a visual conceptual model based on probability distributions, and a mathematical model, in which students understand that the state of a particle can be described by a specific mathematical model. Although researchers used different classifications, one difficulty emerged in the majority of articles: Secondary and lower undergraduate students have difficulty letting go of Bohr's planetary atomic model [12, 25, 45-51, 53, 55]. Kalkanis, Hadzidaki and Stavrou [12] ascribed this to many students believing that scientific content they learned previously is scientifically correct. This is in agreement with Stefani and Tsaparlis [50], who observed that models are sometimes seen as replicas of reality. Ke *et al.*[46] and Wang and Barrow [53] reported that more experienced students understood the difference between various models and could switch between them. McKagan *et al.*[48] claimed the solution is in comparing and contrasting different models, but also reported that students had difficulty understanding the reasons for the development of new atom models, which Taber [47] also reported in his research related to energy levels.

Energy levels, quantization and spin.

To explain atomic spectra, current atomic models include energy levels. These energy levels cannot be arbitrary, but they have certain, specified values. These quantized energy levels can only be explained by considering them as bound wave functions and taking into account boundary conditions. Taber [47] observed that several secondary students did not understand the necessity of introducing quantization, because they did not see the planetary model as insufficient. Some students also had difficulty in forming an adequate concept of orbitals and confused orbitals with planetary orbits or concentric shells. Didiş, Eryılmaz and Erkoç [55] reported that some undergraduate students did not understand that energy quantization is a natural phenomenon that occurs only when boundary conditions apply.

The distribution of electrons over the available energy levels in a system depends partly on electron spin. Spin is an intrinsic property of small particles and is a form of quantum angular momentum. But, in contrast to its classical counterpart, it is not a factual rotation. With regard to spin, Zhu and Singh [57], Taber [47] and Özcan [52] observed that many students falsely believed that quantum spin is an objects rotation around its axis or around the core. Özcan indicated that there seemed to be a relation between the understanding of atomic models and spin. Those students who believed that quantum spin is an actual movement often used the classical atomic model. For students who described spin correctly, the use of the quantum atomic model was more dominant.

2.3.4 Complex quantum behavior

The concepts discussed in the previous sections all are reductions from the fundamental principles of quantum mechanics. A wave function is a solution of the Schrödinger equation and represents a certain quantum state, which can be described by a set of quantum numbers. Little research has been done into misconceptions regarding these more complex subjects, such as quantum states, superposition and time evolution, for the secondary school level. Michelini, Ragazzon, Santi and Stefanel [58] developed and evaluated materials on quantum states and superposition, and concluded that secondary students' difficulties in accepting non-determinism often cause a fall back to classical reasoning, and are an obstacle to understanding quantum states. Passante, Emigh and Shaffer [59] also researched understanding of quantum states and observed that undergraduate students find it hard to distinguish between pure superposition and mixed states. They also researched student understanding of time dependence, mainly focusing on upper division undergraduate level students [60]. One observation that could be useful for secondary and lower undergraduate education was that many students believed that for a time-dependent wave function, the probability of finding a particle in a region must also be time-dependent. Regarding time dependence, Zhu and Singh [43, 61] observed some students who falsely believed that after measurement the wave function will remain the same or, after collapsing, will eventually go back to its initial state.

2.4 Research tools

This section answers the second sub-question: "What instruments have been designed and evaluated to probe student understanding on a conceptual level?" and presents an analysis of the questionnaires and instruments intended for secondary and lower undergraduate education that were observed in the 75 reviewed articles. The research tools are analyzed on how they are designed and evaluated, and on the topics which they cover. Table 4 presents a summary of this analysis.

2.4.1 MULTIPLE-CHOICE CONCEPT TESTS

Several concept tests have been designed and used to uncover students' difficulties, but a substantial part was only aimed at the upper undergraduate level and emphasized mathematical formalism[43, 62-64]; other tests were not sufficiently evaluated[65]. The selected literature included three evaluated multiple choice questionnaires[2, 16, 66] suitable for secondary and lower undergraduate level students, which will be described in this section.

QUANTUM MECHANICS VISUALIZATION INVENTORY

Cataloglu and Robinett[2] designed the Quantum Mechanics Visualization Inventory (QMVI), based on existing materials and commonly used text books. Alterations to the preliminary inventory were made based on student feedback, comments from faculty colleagues and an item analysis. The QMVI consists of 25 questions and focuses on the interpretation of various diagrams. Although many of the questions require mathematical reasoning, approximately one-third of the questions address conceptual understanding of the influence of the potential energy on probability and the wave function. These questions can provide useful information on the student difficulties discussed in section IIIB. The test was validated for content by content experts and Ph.D. candidates and analyzed for reliability and item difficulty in two pilot studies. The test was found to be reliable, but slightly difficult ($\alpha = 0.83$, mean item difficulty = 0.45). Afterwards the QMVI was administered to students ranging from sophomore level to graduate level. Analysis showed there was a large correlation between the students' confidence in, and correctness of their answers. Analysis also showed differences in understanding for the three different levels of instruction, which matched expectations. No articles were published on the evaluation of the QMVI at the secondary school level.

QUANTUM MECHANICS CONCEPTUAL SURVEY

The Quantum Mechanics Conceptual Survey (QMCS) was designed to elicit student difficulties on topics covered in most courses on quantum mechanics[16]. For the preliminary version, textbooks were reviewed, students were observed and faculty interviews were held to determine the topics. This preliminary version addressed wave functions, probability, wave-particle duality, the Schrödinger equation, quantization of states, the uncertainty principle, superposition, operators and observables, tunneling, and measurement. Over a period of three years this 25-item survey was altered, surveys were analyzed and interviews were held with students. Finally, 12 questions proved to be useful for detecting student difficulties. The final questionnaire addresses the conceptual understanding of a broad range of topics discussed in section III, i.e. wave-particle duality, wave functions, potential wells, atom structure and quantization. Because of the small number of questions however, the QMCS is not appropriate for proper statistical analysis and researchers suggested that more questions should be developed. The QMCS was tested at different levels, and the researchers concluded that the QMCS is a useful posttest for the upper undergraduate level. Preliminary results indicated it could also be suitable to investigate learning gains of lower undergraduate level students, but this needs to be verified in future research.

QUANTUM PHYSICS CONCEPTUAL SURVEY

Wuttiprom, Sharma, Johnston, Chitaree and Soankwan[66] developed the Quantum Physics Conceptual Survey (QPCS) to test student understanding of basic concepts of quantum mechanics. The researchers studied syllabi and consulted experts in order to determine topics and create survey questions. The QPCS addresses conceptual understanding of the photoelectric effect, wave-particle duality, the de Broglie wavelength, double slit interference, and the uncertainty principle, of which student difficulties were discussed in section IIIA. The questions were trialed with different groups of students and each version of the survey was critiqued by a group of discipline or teaching experts to establish validity. Subsequently, the final survey, consisting of 25 items, was administered to 312 lower undergraduate students at the University of Sydney. The results were statistically analyzed for item difficulty, discrimination of single items, discrimination of the entire test and the consistency among the questions. Analysis showed that two items were likely to be too difficult and three items too easy (item difficulty index > 0.9 or < 0.3), five items also turned out to be poor discriminators (item point biserial coefficient < 0.2). Still, the KR-21 reliability index and Ferguson's delta were found to be satisfactory (KR21 = 0.97, δ = 0.97). The researchers concluded that even though several items needed improvement, these results indicated that the QPCS is a reliable survey.

2.4.2 OTHER TOOLS

Besides multiple choice concept tests, there are other strategies to investigate students' difficulties. The reviewed literature included four other, evaluated, research tools, which emphasize students' reasoning, mental models, and underlying causes of misunderstanding[24, 25, 31, 47, 51].

MULTIVARIATE ANALYSIS

Ireson[24, 25] designed a 40-item Likert-scale questionnaire, of which 29 items tested conceptual understanding of wave-particle duality, atom structure and quantization. This questionnaire was administered to 338 lower undergraduate students. The analysis was based on the assumption that understanding can be represented by clustering the conceptions of a group of students. First, the responses were subjected to cluster analysis, which clusters individuals and gives insight into understanding at the group level. This resulted in three clusters, which were labelled quantum thinking, intermediate thinking and mechanistic thinking. Second, Ireson used multidimensional scaling, which was used to map the response in multiple dimensions. This resulted in a two-dimensional model, of which the dimensions represented students' dual and non-deterministic thinking. This two-dimensional model confirmed the existence of three clusters; Ireson concluded that this method can be used to gain insight in students thinking and clusters or dimensions in their understanding.

CONCEPT MAP STRATEGY

Sen[31] used a concept map strategy to evaluate the learning process, diagnose learning difficulties and map the progression of students' cognitive structure. Training in creating concept maps was provided to 88 undergraduate students, from

Table 4 Overview of research tools appropriate for probing conceptual understanding of secondary and lower undergraduate level students.

Researchers	Year	Research Tool	Level	Country	Content	Design & evaluation
Cataloglu, E & Robinett, R.W. [2]	2002	QMVI	Undergraduate students	US	Wave functions, potential wells, quantization	Content based on existing materials and commonly used text books. Modified after student and faculty feedback and item analysis. Results suggested QMVI scores may be a reasonable measure of student understanding
Ireson, G.[24, 25]	1999	Multivariate analysis	Undergraduate students	UK	Wave-particle duality, atomic structure, quantization	Items based on previous research on students conceptions [67, 68]. Multivariate analysis resulted in a holistic picture. Findings were consistent with other research, using different methodology.
McKagan *et al.*[16]	2010	QMCS	Undergraduate students	US	Wave-particle duality, wave functions, potential wells, atomic structure, quantization, measurement	Content based on literature, faculty interviews, textbook reviews and student observations. Modified after interviews, surveys and discussions. QMCS is too small to adequately probe student understanding. Useful as pre and posttest for undergraduate students, but not for graduate students.

Sen, A.I.[31]	2002	Concept map strategy	Undergraduate students	Turkey	Wave-particle duality, atomic structure	Strategy based on Ausubel's theory on cognitive and meaningful learning[69, 70]. Reliability and validity were analyzed using Crohnbach's α and factor analysis. Results were consistent with another, questionnaire-based, study.
Taber, K.[47]	2005	Typology of learning impediments	Upper secondary students	UK	Atomic structure	Typology based on consideration of the influence of prior knowledge[71]. Proposed modification: include substantive learning impediments categorized as analogical, epistemological, linguistic, pedagogical, or ontological.
Tsaparlis, G. & Papaphotis, G.[51]	2009	Questionnaire	Upper secondary students	Greece	Atomic structure	Content based on questions in an earlier study[72], which were judged for content validity by chemistry teachers.
Wuttiprom, S. *et al.*[66]	2009	QPCS	Undergraduate students	Australia	Wave-particle duality	Content based on expert opinions and students difficulties Modified after trials with students and experts. Reliability was analyzed with item analysis, KR21 reliability test, and Ferguson's delta.

three different educational levels. At the end of the semester, the students each individually constructed a paper and pencil concept map. The concept map had to contain three main concepts (the atom, electron and photon) and students were instructed to pay attention to the hierarchical order and links among concepts. Sen scored the concept maps for the number of valid concepts, relationships, branching, hierarchies, and cross links. The scoring of the concept maps was tested for reliability, Cronbach's α was 0.67. Additionally, the scoring scheme was analyzed for construct validity by factor analysis. This analysis showed that the five scoring categories were correlated to separate single factors. The researcher also observed that the concept maps resembled results from a questionnaire-based study on the same subject. Results showed significant differences in the number of concepts and branches for the three different educational levels. Sen concluded that the results suggest that concept mapping can be used to investigate cognitive structures and the development thereof. However, the interpretation of the scores needs to be evaluated empirically[73].

TYPOLOGY OF LEARNING IMPEDIMENTS

Taber[47] constructed and evaluated a typology of learning impediments, which he used to analyze underlying causes for students' difficulties. The typology was based on the Ausubelian idea that, for meaningful learning, students need to relate new concepts to prior knowledge. Four types of learning impediments were defined: (1) Students lack prerequisite knowledge; (2) students fail to make required connections; (3) students interpret the material inappropriately, because of their intuitive ideas; and (4) students interpret the material inappropriately, because of their cognitive structures. Taber used this typology to analyze data from an interview-based study on the understanding of chemical bonding of pre-university students. The researcher identified all four types of learning impediments and concluded that the typology is a useful heuristic tool, which can be used to interpret data on student learning. Still, Taber also recommended a refinement, that takes into account misconceptions based on analogies or epistemological assumptions.

QUESTIONNAIRE ON ATOMIC STRUCTURE

Tsaparlis and Papaphotis[51] designed a questionnaire for a study into the deep understanding and critical thinking of first-year undergraduates with regard to the quantum atom model. The questionnaire was based on a preliminary questionnaire, that had been validated for content by chemistry teachers in a previous study[72]. It consisted of 14 open-ended questions; 9 of them were designed to test conceptual understanding, and the other questions were aimed at algorithmic knowledge. The questionnaire was administered to 125 students, as part of a qualitative study. The researchers only drew conclusions about student understanding, the questionnaire itself was not evaluated.

2.5 TEACHING STRATEGIES

This section addresses the sub-question: "What teaching strategies aimed at the secondary and lower undergraduate level have been tested, implemented and evaluated for their influence on student understanding?" and presents approaches promoting the understanding of quantum mechanical concepts that have been investigated in the selected literature. The following section presents the teaching strategies found in the selected articles, divided in instructional and multimedia-based strategies. There are several other activities described in literature, e.g. the hands-on activities from Visual Quantum Mechanics[74], the Dutch approach using the particle in a box[8], and the approach starting with qubits[75], but this review only discusses strategies which were implemented and evaluated in an educational setting.

2.5.1 INSTRUCTIONAL STRATEGIES

There are still many questions concerning the teaching of introductory quantum mechanics. The introduction using wave-particle duality, for example, is still under discussion. Several alternative ways to introduce quantum mechanics have been used[58, 76, 77], but these alternatives have not been properly evaluated and compared to the use of wave-particle duality. However, several articles did describe investigations into the influence of teaching methods on student understanding. This section describes implemented and evaluated instructional strategies that were found within the selected literature[12, 22, 36, 48, 49, 54, 76, 78-89], organized into four groups.

FOCUS ON INTERPRETATION

Because of quantum mechanics' indeterminacy, many interpretations are possible. Today's quantum experts do not support one single interpretation, although the Copenhagen interpretation is often considered to be the standard interpretation[90]. Baily and Finkelstein[78, 79] researched the influence of addressing interpretations of quantum mechanics on student interpretations. Results showed that undergraduate students tended to prefer a local and deterministic interpretation if there was no emphasis on ontology. Baily and Finkelstein also presented results of the implementation of a new curriculum[76], which addressed the topic of "physical interpretation" explicitly. This curriculum included in-class discussions and experimental evidence, and aimed for understanding of different perspectives, their advantages, and limitations. Results of the use of this curriculum showed a clear change in student interpretation and the researchers concluded this confirms the importance of emphasis on interpretation. Greca and Freire[22] also researched the influence of teaching on undergraduate students' interpretations. For this purpose an interpretation was chosen that suited their didactic strategy, which emphasized a phenomenological-conceptual approach. The researchers used a realistic interpretation of the Copenhagen interpretation, in which the probability density function does not predict the probability of finding a particle, but the probability of the particle being present at a certain position. Comparison with a control group showed that in the experimental groups more students developed reasonable understanding. These examples showed the importance of an emphasis on interpretation in the design of new curricula.

FOCUS ON MODELS
Research showed that students tend to hold on to Bohr's planetary description of the atom[45, 46, 51, 53], because it corresponds to students' classical worldview. Several approaches were evaluated to address this problem. Kalkanis *et al.*[12] presented an approach that emphasized the differences between classical and quantum mechanics. An instructional module focusing on the hydrogen atom was developed, which contrasted the classical and quantum models, and used the Heisenberg uncertainty relation as the basic principle. The module was taught to 98 pre-service teachers and evaluated with pre- and posttests and semi-structured interviews. Results showed that a vast majority described the hydrogen atom correctly and could appropriately apply Heisenberg's uncertainty principle. The students had also become more aware of the process of learning and showed a change in worldview.

Strategies based on the historical development of the atomic model were evaluated by Unver and Arabacioglu[88] and McKagan *et al.*[48]. Unver and Arabacioglu developed a teaching module focusing on observations and experiments that led to alterations of the atomic model. The module was implemented in a course for pre-service teachers (N=73). Pre- and posttest comparisons showed a significant change in understanding. McKagan *et al.* designed an undergraduate course focusing on model building and reasoning for each model. Results showed that emphasis on the analysis of the predictions of each model, and the explanation of reasoning behind the development of the model resulted in an increase in the use of the Schrödinger model.

Classical analogies are also used to promote understanding of the quantum atom model. Budde, Niedderer, Scott and Leach[80] developed the Bremen teaching approach for upper secondary schools, which is based on similarities between the quantum atom model and liquids. Nine students were taught that atoms consist of Electronium, a liquid substance, to promote the idea that an atom has a continuous nature, in which electrons are not moving. Budde *et al.* observed that some students described Electronium as having a particle nature, but students still developed the conception that electrons are not moving. The researchers concluded that its focus on plausible aspects lead to high acceptance of the Electronium model.

FOCUS ON MATHEMATICAL OR CONCEPTUAL UNDERSTANDING
Lower undergraduate and secondary students do not have extensive mathematical skills, which are an important part of quantum physics. This raises the question to what extent mathematical skills are needed for good understanding of quantum concepts. Studies have been done into the relation between mathematical and conceptual understanding of quantum concepts. Koopman, Brouwer, Heck and Buma[84] observed that undergraduate students in a Quantum Chemistry course lacked mathematical skills, and they designed a remedial program. This program consisted of a diagnostic test, a pre-lecture, and online mathematics assignments. Students' results were monitored and commented upon. Students could consult a tutor and, if needed, additional explanation was scheduled. Koopman *et al.* observed a positive correlation between students' scores on the math assignments and the

final exams (*N*=29). From a comparison with student's grades for Calculus, the researchers concluded that mathematical skills are necessary, but not sufficient for conceptual understanding. Papaphotis and Tsaparlis[49, 86] researched the relation between algorithmic and conceptual understanding in high school chemistry. The study was conducted on 125 science students at the start of their first year at university. Students completed a questionnaire that addressed procedural knowledge and conceptual understanding. No correlation was found between their levels of procedural and conceptual performance. To investigate the effect of a non-mathematical approach on student understanding of the atomic structure, Dangur, Avargil, Peskin and Dori[54, 82] developed a teaching module focusing on real-life applications and visualization. This module was used for 122 secondary students and 65 undergraduate students. Results showed a significant improvement of understanding for both secondary and undergraduate students. Comparison with mathematically oriented undergraduates showed that the undergraduate test-group scored significantly higher on textual and visual understanding. This research suggests a conceptual, non-mathematical, approach for teaching quantum mechanics can lead to adequate understanding.

USE OF ACTIVITIES

Active learning has become increasingly important in research into student engagement and understanding[91]. As a consequence, several reviewed articles described investigations into the influence of student activities on conceptual understanding. One example of active learning is the use of peer interaction. Shi[87] researched the influence of peer interaction on student understanding of duality and atomic models. Peer interaction was used once or twice a week during an undergraduate course on quantum mechanics. Students in the experimental group scored significantly higher than the control group on the posttest. Deslauriers and Wieman[81] investigated the effect of two different teaching methods on students' learning. One group (*N*=57) was taught traditionally, while the other (*N*=67) experienced interactive engagement methods (quizzes, simulations, clicker questions). The QMCS was used to test understanding, and comparison of the results for the two groups showed that the use of interactive engagement methods resulted in significantly higher scores. Yildiz and Büyükkasap[89] researched the influence of writing on understanding of the photoelectric effect. Pre-service teachers (*N*=36) had to write a letter to senior high school students in which they explained the photoelectric effect. Results showed that these students scored significantly better on the posttest and exams than the control group. Gunel[83] explored differences in learning gains for two different writing tasks on Bohr's atomic model and the photoelectric effect (*N*=132). The study indicated that secondary students who created a PowerPoint presentation had significantly higher learning gains than those who completed a summary report. Muller, Sharma, Eklund and Reimann[85] explored how well undergraduate students (*N*=40) could learn from watching a video of a student-tutor dialogue on quantum tunneling. Results were compared to students who watched a traditional explanation. The students who watched the dialogue performed significantly better on the posttest. These results all suggest that active learning can contribute to better understanding of quantum concepts.

2.5.2 Multimedia

Numerous multimedia applications have been designed for teaching quantum mechanics, but not all have been thoroughly evaluated. An overview of useful multimedia for quantum mechanics education was provided by Mason *et al.*[92]. The following subsection discusses evaluated multimedia found in the reviewed articles[5, 27, 32, 33, 38, 57, 58, 77, 93-100]. First PhET, QuILT and QuVis are treated, which are databases covering a large number of topics. Then other, separate, simulations and teaching sequences using simulations will be discussed.

PhET

McKagan *et al.*[98] described 18 simulations on fundamental principles, historical experiments or applications of quantum mechanics developed in the PhET (Physics Education Technology) project. Most of them were developed for use in an undergraduate level course. These simulations were developed based on previous research, student interviews and classroom testing. The interviews and classroom testing mainly focused on finding problems in the simulations, but some results of interviews and exams showed that several simulations ("Davisson-Germer: Electron Diffraction" and "Photoelectric Effect") resulted in better understanding. The researchers also noted that student interviews on the simulation "Quantum Tunneling and Wave Packets" suggested that guided activities could improve students' learning path when using the simulations. However, more research could still be done into the learning gains seen with the use of these simulations. The simulations on the photoelectric effect and tunneling were described more extensively. The simulation "Photoelectric Effect" was used for curriculum improvement[27]. This curriculum, based on active engagement techniques, resulted in better understanding of the photoelectric effect. However, students had difficulty linking this experiment to the particle behavior of light. The simulation "Quantum Tunneling and Wave Packets" was also part of an improved curriculum[38] that led to greater insight into students' difficulties on tunneling.

QuILT's

Singh[32] described the development of QuILT's, Quantum Interactive Learning Tutorials covering a broad range of subtopics. These tutorials, which were developed for undergraduate courses, consist of a combination of tasks, homework, Java applets and pre- and posttests. QuILT's were designed based on knowledge of student difficulties, and evaluated using pretests, posttests, and student interviews. The multimedia applications used in the QuILT's were adapted from different sources (e.g. PhET[98] and Physlets[101]) Results of the pre-experimental evaluation of QuILT's on time-development, the uncertainty principle and the Mach-Zehnder interferometer, showed a substantial change in performance. Zhu and Singh also evaluated a QuILT regarding the Stern-Gerlach experiment[57] and quantum measurement[100]. Both resulted in distinct improvement of understanding. Comparison of the results for students who went through the tutorial on quantum measurement with those for a control group showed that the QuILT resulted in better scores on the post-test.

QuVis

Kohnle *et al.*[96, 97] reported on the development of QuVis, which is a collection of interactive animations and visualizations for undergraduate students. Student interviews and observation sessions were used to optimize the interface design. Subsequently, the researchers investigated the influence of two simulations (the potential step and the finite well) on student understanding in a quasi-experimental setting. Two groups of students completed a diagnostic test: an experimental group, which worked with the animations, and a control group. Statistical analysis of the test results showed a significant relation between having worked with the simulations an performance on questions covering the corresponding subjects. In more recent work, Kohnle, Baily, Campbell, Korolkova and Paetkau[95] presented simulations regarding two-level quantum systems. They evaluated the learning gains resulting from use of a simulation on superposition states and mixed states. Results showed a substantial change in understanding.

Simulations on atomic structure

Several simulations were designed to improve understanding of the atomic structure. Chen, Hsiao and She[93] investigated the different effect of static and dynamic representations on understanding of atomic orbitals. The researchers compared two groups of secondary students. One group completed a learning activity using static 3D representations, while the second group worked with a dynamic 3D representation. Analysis of a pre- and posttest showed that both representations increased conceptual understanding. However, the researchers concluded that students who worked with the dynamic representations had more sophisticated mental models of the atom. Ochterski[99] used research-quality software (GaussView) and designed and evaluated two activities (N=95, N=71) to introduce orbitals and molecular shape to high school students. Pre- and posttests for both activities showed an increase in understanding; Ochterski concluded that research-quality software can be effective, even if students have little background in chemistry.

Teaching sequences using simulations

Other simulations were evaluated within the context of the design of a course. Malgieri, Onorato and De Ambrosis[77] described a teaching sequence using the Feynman sum over paths method. This sequence used simulations in GeoGebra, which included the photoelectric effect and the double-slit experiment. The eight hour course was tested on pre-service teachers (N=12) and evaluated with a pre- and posttest. Results showed a good level of understanding of the role of measurement and the single photon interpretation of the double-slit experiment. However, the understanding of the uncertainty principle was still not adequate. Müller and Wiesner[5] designed and implemented a secondary school course using virtual experiments with the Mach-Zehnder interferometer and the double slit. Interviews and a questionnaire showed that students (N=523) who took part in the course developed better quantum understanding than the control group. Michelini *et al.*[58] proposed a secondary school teaching sequence using PEC strategies (Prevision-Experiment-Comparison). This sequence included simulations on light interaction with Polaroids and Malus law. Analysis of student worksheets (*N*=300) and a group discussion (*N*=17) showed that the approach stimulated learning for at least 75% of the students. The researchers concluded that software simulations can help students in building a phenomenological framework, but are not sufficient.

QUANTUM COMPUTER GAMES

A different way of using multimedia is the use of quantum computer games. Gordon and Gordon[94] developed the computer game "Schrödinger cats and hounds" to teach quantum mechanical concepts in a fun way. Game-aided lectures were given to 95 undergraduate students. Analysis of a pre- and posttest showed an increase in understanding.

2.6 CONCLUSIONS

In this review we presented an overview of existing knowledge on student difficulties, research tools for investigation of conceptual understanding and teaching strategies. The conclusions of this literature review will be presented in this section.

2.6.1 STUDENT DIFFICULTIES

Analysis of the selected articles shows that secondary and undergraduate students have many difficulties when they learn quantum mechanics. Much research has been done into misunderstanding of wave-particle duality, wave functions, and atoms. However, not much research has been done into student difficulties with complex quantum behavior, and no research was found concerning secondary students' understanding of the wave function. Research into the understanding of wave-particle duality showed that undergraduate students' understanding can be clustered according to the extent of classical thinking[20, 22, 24-26]. Researchers also observed misplaced classical thinking in understanding of the wave function; several students displayed an over-literal interpretation of classical metaphors[36, 38], or used classical reasoning in describing the process of tunneling[38, 44]. Research into students' understanding of the quantum atomic model also indicated that both secondary and undergraduate students hold on to previously learned, semi-classical, models[12, 25, 45-51, 53, 55]. From these results we can conclude that many difficulties that students experience are related to the inability to connect quantum behavior to the physical reality as they see it, which results in a mix-up of classical and quantum concepts. Although this has been researched mainly for the undergraduate level, the existing research shows similarities in secondary and undergraduate students' understanding of duality and atomic models. This suggests that the mix-up of classical and quantum concepts is also an important issue at the secondary level. Researchers have proposed several ideas concerning solutions for the mix-up of classical and quantum concepts; e.g. analogies should be well-defined[36], diagrams should be unambiguous[38, 40], and students should have more knowledge of the use of models in physics[12, 48, 88]. However, the impact of these proposed solutions remains to be investigated.

2.6.2 Research tools

The research tools discussed in section IV all include conceptual questions, that could be useful probing the understanding of secondary and lower undergraduate level students. The topics addressed in these tools are: wave-particle duality, wave functions, quantization, atomic structure and measurement. Table 5 gives an overview of the topics covered by each research tool. As can be seen, none of the instruments covers the complete spectrum of quantum mechanics. Furthermore, only the research tools from Ireson, Taber and Tsaparlis, regarding duality and atomic structure, are used in secondary school settings. The QMVI addresses conceptual understanding only in part, and therefore some questions can be appropriate for the secondary and lower undergraduate level. The QMCS, which covers most of the topics, aims to probe conceptual understanding, but has not been thoroughly evaluated for secondary and lower undergraduate education. Moreover, the QMCS includes too few questions for statistical analysis. These results imply that the development and evaluation of more questions is needed, not only to cover all major topics from quantum mechanics, but also to make statistical analysis possible.

2.6.3 Teaching strategies

Various methods and approaches have been designed and used to promote understanding in introductory courses on quantum mechanics, both at the secondary and undergraduate level. Still, only a small selection of these methods have been evaluated for their impact on students' understanding. These evaluations show that:

(1) emphasis on interpretations influences undergraduate student perspectives, and should be taken into account in the development of curricula and teaching sequences;
(2) emphasis on the development of and the differences between various atomic models, can result in better understanding of undergraduate students;
(3) a non-mathematical, conceptual approach can lead to adequate understanding for secondary and undergraduate students;
(4) active learning contributes to the understanding of quantum mechanical concepts.

However, there is a need for more empirical research into the teaching of quantum mechanics and teaching strategies should be researched for both secondary and undergraduate education.

Table 5 Topics covered by the research tools.

		QMVI	QMCS	QPCS	Sen*	Ireson	Taber	Tsa-parlis
		Lower undergraduate education (•)				Secondary education (■)		
Wave-particle duality	Photons & electrons		•	•	•	• / ■	■	
	Double slit experiment		•	•		• / ■		
	Uncertainty principle		•	•	•			■
	Photoelectric effect			•	•			
Wave functions	Wave functions & potential wells	•	•					
	Tunneling	•	•					
	Probability	•	•					■
Atoms	Atomic structure		•		•	• / ■	■	■
	Energy levels, quantization & spin	•	•		•	• / ■	■	■
Complex QM behavior	Quantum states							
	Superposition							
	Time evolution & measurement	•	•					

* Dependent on individual student responses

TABLE 6 Overview of quantum mechanical topics covered by the multimedia applications.

		PhET	QuILT†	QuVis	Malgieri	Gordon	Chen	Ochterski	Müller	Michelini
		Lower undergraduate education (•)					Secondary education (■)			
Wave-particle duality	Photons & electrons	•	•	•	•	•			■	■
	Double slit experiment	•	•		•				■	
	Uncertainty principle	•	•	•	•				■	■
	Photoelectric effect	•		•					■	
Wave functions	Wave functions & potential wells	•	•	•					■	
	Tunneling	•	•	•						
	Probability	•	•	•	•				■	■
Atoms	Atomic structure	•	•	•			■	■	■	
	Energy levels, quantization & spin	•	•	•			■	■	■	
Complex quantum behavior	Quantum states	•	•	•	•	•			■	■
	Superposition	•	•	•	•	•			■	■
	Time evolution & measurement	•	•	•						

†Tutorials using simulations of other sources

Furthermore, many multimedia applications have been designed for teaching quantum mechanics. Table VI shows that for undergraduate education all quantum topics are covered by the multimedia applications found in the reviewed articles. For secondary education there are fewer applications and most topics are covered. Most of the applications were evaluated for practical use; only some of the simulations were also evaluated for their influence on student understanding. Singh and Zhu[32, 57, 100] have made a start with the design and evaluation of tutorials using multimedia, but more research into how these applications can be used to promote understanding is needed.

2.6.4 IMPLICATIONS FOR RESEARCHERS

This review shows the current state of research into learning difficulties and teaching strategies for quantum physics at the secondary and lower university level. Analysis of 75 articles showed there are many groups researching student understanding, teaching strategies or assessment methods, mostly aiming at undergraduate education.

LOWER UNDERGRADUATE LEVEL

For lower undergraduate students, several learning difficulties were observed in the selected articles, but little research has been done into the conceptual understanding of complex quantum behavior. Although these topics are also difficult for upper-graduate students, it would be good to investigate to what extent these topics can be taught conceptually. More research should also be done into the underlying difficulties and causes of observed student difficulties. Several assessment methods have been designed for the undergraduate level, but there is still need for tests that cover more topics and are suitable for statistical analysis. More empirical research is needed for the further development of lower undergraduate level courses on quantum mechanics, in which teaching strategies are evaluated and compared using proper assessment tools. This research should also include investigations into ways to promote students' understanding using multimedia applications and experiments.

SECONDARY SCHOOL LEVEL

With regard to quantum mechanics at the secondary school level, more empirical research into teaching strategies is also needed. But, although many learning difficulties that were found in research at the undergraduate level were confirmed for secondary school students, several topics have not yet been thoroughly investigated and more research into learning difficulties is needed. For the secondary school level, there is a need for more research into the understanding of wave functions and potential wells, topics that are part of several secondary school curricula. Research into the teaching of quantum states at a conceptual level is also needed, because this is part of some secondary school curricula.

To thoroughly investigate teaching strategies, multimedia applications and experiments suitable for secondary school students, research tools are needed. The existing concept tests primarily focus on the undergraduate level, and therefore, it remains to be investigated whether these assessment tools are also applicable at the secondary school level.

2.6.5 Implications for teachers

Analysis of the current research shows that students have many difficulties while learning quantum mechanics. Although most of the research has been conducted at the undergraduate level, overlapping research shows similar difficulties at both levels addressed in the studies reviewed. Therefore, both lower undergraduate and secondary school teachers can benefit from the research discussed in this review. This review shows that there has been little empirical research into ways to promote understanding, but teachers should be aware that students tend to hold on to classical thinking, which leads to the misinterpretation of unfamiliar quantum concepts, and the mix-up of classical and quantum physics. It can be helpful to emphasize differences and similarities between quantum concepts and students' preconceptions, which has proved to be useful in the teaching of the quantum atomic model at the undergraduate level. Teachers should also be aware that it is important to specify the limitations of metaphors, because they can lead to over-literal interpretations.

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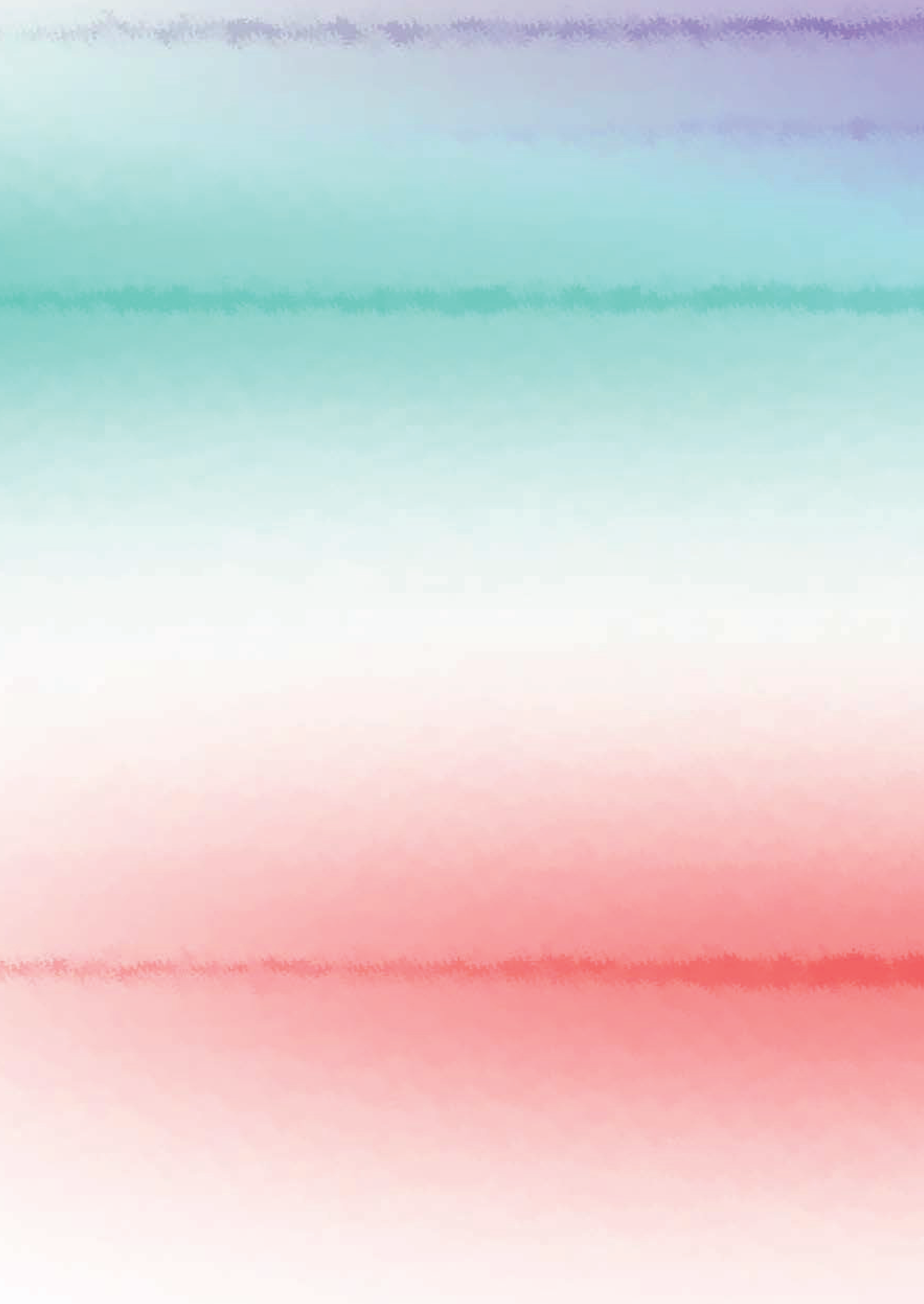

3

Key topics for teaching quantum mechanics at secondary schools: A Delphi study into expert opinions

This chapter describes a Delphi study aiming to investigate which quantum mechanics topics experts consider to be important to teach at the secondary level, and what arguments these experts give. A series of three questionnaires was administered to experts in the fields of quantum physics, mathematics, chemistry and biophysics (n = 17, 12, 11 for the first, second, and third questionnaires, respectively; the number of participants changed due to attrition). Several experts from this group (n = 9) were also interviewed. Results show that there is consensus on the topics considered to be important, i.e. duality, wave functions and atoms. Experts mainly based their topic ranking on relations between concepts, and on what quantum mechanics topics they consider to be fundamental. The topics that were considered less important were often described as too difficult or too complex.

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3.1 INTRODUCTION

Quantum mechanics is an important theory underpinning many areas of physics research, and plays a vital role in current technologies, such as medical imaging, nanoscience, laser physics and semiconductor technology. Quantum mechanics is also the foundation for several emergent technologies including quantum computers, quantum encryption and quantum teleportation. Quantum mechanics has been an important part of university physics education for a long time. Traditionally, it has primarily been taught in a rather formal and mathematical way[1]. Because of its theoretical and practical importance, quantum mechanics has found its way into the secondary school curriculum. Because the mathematical skills of secondary school students fall short of what is needed for a more formal, mathematical approach, this introduction of quantum mechanics in secondary schools often aims for qualitative understanding. Such a qualitative approach has become more and more important in physics education[2], and the currently available visualization techniques and multimedia have made it possible to introduce complex and abstract topics, such as quantum mechanics, in a more qualitative way[3, 4]. Quantum mechanics has been part of the upper secondary school curriculum in England[5], Germany[6], Italy[7] and the USA[8] for several years. More recently, quantum mechanics has been incorporated in the Dutch[9], Norwegian[10] and French[11] secondary school curricula.

Because quantum mechanics entails fundamental changes in the way the physical world is understood and conflicts with students' classical thinking[12], there is need for a research-based instructional strategy that aims for conceptual understanding, comprising the key topics of quantum mechanics[13]. However, there is no generally accepted opinion on what to teach in introductory quantum mechanics courses, and a wide variety of topics has been explored for use in a more conceptual approach to quantum mechanics. Examples of introductory topics that have been used at the secondary and undergraduate level are: wave-particle duality [6, 11, 14], entangled photons[10], the infinite potential well[9], quantum states[15], spin[16], and path integrals[17]. While the primary reason for using these topics in most cases was to find a way to introduce quantum mechanics conceptually and visually, the researchers also presented various other arguments for the use of these approaches, ranging from their importance for the understanding of quantum mechanics to their relevance for our daily life.

The current study was conducted in the context of the introduction of quantum mechanics in Dutch secondary schools, which is the result of a curriculum reform [18] aiming to promote scientific literacy. More specifically, this reformed curriculum aims to promote scientific skills and thinking, and to give a good perspective on the relevance of science and technology in society and the interaction between scientific research and technological developments. This is in line with the current emphasis on scientific literacy and STS (science-technology-society) in secondary education[19-22]. Although many researchers investigating introductory topics for quantum mechanics often presume the chosen topics to be relevant, little systematic research has been done into the topics' relevance for development of a good perspective regarding the importance of quantum mechanics for science, technology and society.

According to Duit *et al.*[23], investigation of the relevance of a topic is important in science curriculum design. They proposed the Model of Educational Reconstruction, which consists of three components: (1) clarification and analysis of science content, (2) research on teaching and learning, and (3) design and evaluation. The first step of this model includes the analysis of key topics, related applications, and their scientific and social implications. This knowledge, together with knowledge of students' preconceptions and difficulties, can provide a basis for the design of a curriculum [24]. Based on the Model of Educational Reconstruction, Laherto[25] investigated the educational relevance of nanoscience in secondary education, and Sakhnini and Blonder (2015) used a Delphi study among teachers and experts in nanotechnology to explore key topics in nanoscience for secondary schools[26].

Following this lead, it becomes clear that research is needed on which subtopics of quantum mechanics are relevant for promoting scientific literacy. This article describes our investigation using the Delphi method to determine which subtopics of quantum mechanics (which will be called 'topics' throughout this article) experts consider relevant for teaching in secondary education, and an analysis of the experts' arguments. In contrast to the study by Sakhnini and Blonder (2015), we only consulted experts in quantum physics and related research fields, because teachers do not necessarily understand quantum mechanical topics[27, 28], and experts have more experience with scientific research and technological developments related to quantum mechanics.

3.2 Background

In this section, an overview is given of the existing research into what topics are important when teaching introductory quantum mechanics. The phrase 'scientific literacy' is also clarified, and a framework of goals for scientific literacy is presented. This framework gives an overview of all goals that can be addressed in curricula aiming for scientific literacy.

3.2.1 Research into key topics of quantum mechanics

In previous research, there have been attempts to determine which topics form the basis for quantum mechanics and should be taught in introductory courses. At the undergraduate level, McKagan, Perkins, and Wieman[29] asked eight faculty members which three quantum mechanics topics were most important, in order to determine which concepts should be addressed in their concept test. These interviews resulted in a list of nine topics, but there was high variability in the faculty members' choices; the researchers noted that this list does not reflect a general opinion. Additionally, Wuttiprom, Sharma, Johnston, Chitaree and Soankwan[30] analyzed university syllabi and consulted experts from a single university to identify important topics for their concept test. This yielded two main topics for their concept test: quantization and uncertainty. Both investigations were aiming at determining the important topics of quantum mechanics at the undergraduate level, but although the topics obtained were useful for developing concept tests, these topics did not reflect a general opinion. Furthermore, no emphasis was put on the educational relevance of these topics for promoting scientific literacy, which is an important reason for introducing quantum mechanics at the secondary level.

3.2.2 SCIENTIFIC LITERACY

As we intend later to analyze reasons given for including aspects of QM in the school curriculum against the aim of promoting scientific literacy, it is necessary to consider in a little more depth what the term 'scientific literacy' might mean. Scientific literacy is a very popular term in contemporary science education. It refers to 'the public understanding of science' and has been used in very different contexts and perspectives, varying from awareness of the impact of science on society to understanding of the scientific method. Holbrook and Rannikmae[31] stated that there are two points of view on scientific literacy; the first view regards scientific literacy as the fundamental ideas in science that everyone should know, while the second view considers scientific literacy to be the science-related knowledge and skills needed to function in society. For PISA 2006, a model was developed that included both points of view[32]. In this model, scientific literacy is based on scientific knowledge, scientific competencies and attitude toward science. Scientific knowledge is defined as both knowledge *of* science and knowledge *about* science, scientific competencies are defined as the ability to identify scientific issues, explain phenomena scientifically and use scientific evidence, and attitude toward science is defined as a person's interest in and support for scientific inquiry. Table 1 gives an overview of the categories used in PISA 2006, which was used as the basis for the PISA assessment in 2006, 2009 and 2012.

Table 1 The categorization used by in PISA 2006[32].

Goals for scientific literacy		
Competencies	**Knowledge**	**Attitude**
Identifying scientific issues	Scientific concepts	Interest in science
Explaining phenomena scientifically	The nature of science	Support for scientific inquiry
Using scientific evidence		Responsibility towards resources and environments

For a broader overview of existing goals for scientific literacy, these three categories can be complemented with the different aspects of scientific literacy described by DeBoer[21]. In his review he showed that, historically, there have been nine separate goals that are related to scientific literacy:

(1) Teaching and learning about science as a cultural force in the modern world;
(2) Preparation for the world of work;
(3) Teaching and learning about science that has direct application to everyday living;
(4) Teaching students to be informed citizens;
(5) Learning about science as a particular way of examining the natural world;
(6) Understanding reports and discussions of science that appear in the popular media;
(7) Learning about science for its aesthetic appeal;
(8) Preparing citizens who are sympathetic to science;
(9) Understanding the nature and importance of technology and the relationship between technology and science.

These goals, together with the goals developed for PISA 2006, give a good overview of the different aspects of scientific literacy, and can be used to analyze argumentation, development processes and curricula. Table 2 shows a framework based on the descriptions of aspects of scientific literacy by DeBoer and PISA. To create this framework, first the nine goals given by DeBoer were placed within the three main categories of PISA 2006. Then the descriptions in DeBoer and PISA 2006 were compared for overlaps. For the categories "knowledge" and "attitude", the goals mentioned by DeBoer were extensions refining the descriptions from PISA therefore five goals were placed beside the goals of PISA.

3.3 Purpose of this study

For a systematic investigation into which quantum topics are considered important for secondary education for scientific literacy, a Delphi study was conducted among a number of Dutch experts in quantum physics and related research fields. The selection procedure and the expertise of the selected experts will be specified in the next section of this article. This research method is intended to find consensus among experts concerning the topics that are important within the Dutch context, in which the curriculum renewal aims to create a better understanding of the importance of science for research and technology. Therefore the questions under investigation are:

(1) In the view of experts, what are the essential topics that secondary school students need to learn in order to develop an appropriate image of quantum mechanics in terms of research, developments and applications?
(2) What are the experts' arguments for choosing their topics and to what extent do these arguments correspond to the different categories and sub-goals for scientific literacy?

Table 2 Overview of aspects of scientific literacy guiding the topic choice for a curriculum, based on OECD[32] and DeBoer[21].

	Aspects of scientific literacy	Description
Competencies	Identifying scientific issues	Enabling students to recognize scientific issues and key features of scientific investigation.
	Explaining phenomena scientifically	Enabling students to apply scientific knowledge in a given situation, to interpret scientific phenomena and identify appropriate descriptions.
	Being able to make informed decisions	Enabling student to identify, interpret and be critical about scientific evidence-related evidence in media and conversations, reflect on the societal implications and make informed decisions.
Knowledge	Understanding of scientific concepts	Promoting knowledge and understanding of topic content and relations between topics which are considered fundamental for students to know.
	Understanding the nature of science	Promoting the understanding of scientific inquiry, data analysis, scientific explanations and models, and limitations of scientific knowledge.
	Knowledge of science as a cultural force	Promoting knowledge of the historical development of scientific ideas, current understandings in science and their effect on science and society.
	Knowledge for future careers	Promoting knowledge about and needed for future careers or further studies in science.
	Understanding the relationship between science and technology	Promoting understanding of the nature of technology and the interdependence of science and technology (e.g., technological applications based on scientific inquiry).
Attitude	Interest in science	Promoting students' engagement in science-related social issues, their willingness to acquire scientific knowledge and skills, and their consideration of science-related careers.
	Support for scientific inquiry	Promoting students' appreciation of and support for scientific inquiry.
	Responsibility towards resources and environments	Promoting students' sense of personal responsibility for maintaining a sustainable environment, and willingness to take action.
	Seeing the influence of science in everyday life	Prompting students to see the applications of science in their daily lives, and have a more informed and intelligent experience with the natural world.
	Appreciating the beauty of science	Promoting students' appreciation of and fascination for the natural world.

This article will give an overview of the research conducted and its results. First, the Delphi approach and the research method used in this study are explained, then an overview is given of the results and conclusions.

3.4 Method

The Delphi method is a systematic approach to researching expert opinions on a specific topic[33, 34] and is often used to exchange knowledge between experts, determine expert opinions, determine the assumptions leading to those opinions, find consensus, and create rankings of different alternatives.

This method uses multiple consecutive questionnaires in which experts can give their opinion together with their arguments. In this succession of questionnaires, the experts' previously stated opinions and arguments are summarized and shared. Before completing the current iteration of the questionnaire, the experts can read the different arguments and reconsider their previous response. This method is useful when opinions or predictions are being investigated, and when it is difficult to bring the experts together in person. It has the advantage that experts participate anonymously, which prevents group behaviour and places emphasis on their reasoning. The Delphi technique can be used for curriculum design [35-37]; in this specific study it was used to explore expert opinions on the key topics of quantum mechanics that are suitable for developing the scientific literacy of secondary school students. Figure 1 shows the procedure used in this research, which is based on the approach described by Okoli and Pawlowski[34].

3.4.1 Expert selection

First, we identified relevant research fields and institutions, related to research and technologies in which quantum mechanics plays a crucial role. Forty-eight experts from various Dutch universities and institutions were then invited to participate in this Delphi study. The responding experts were researchers in the field of quantum physics, quantum mathematics, quantum chemistry and biophysics, from eight different universities. There was some attrition; the number of respondents in every round, categorized for the various research fields, is listed in Table 3.

3.4.2 First round

In the first round the responding experts completed an online survey. The experts were asked which quantum mechanics topics they considered necessary to address in order to give secondary school students an appropriate image of current research and technological developments. To ensure a connection with current technologies and everyday life, which is important for the Dutch curriculum renewal, we chose to explicitly ask for applications. Therefore, the experts were asked to give at least 5 concepts and 5 applications, together with a description of the chosen topics (concepts and applications), and an explanation of their topic choice. The responses were analyzed and the coding was checked for interrater reliability ($\kappa = 0.81$) with the

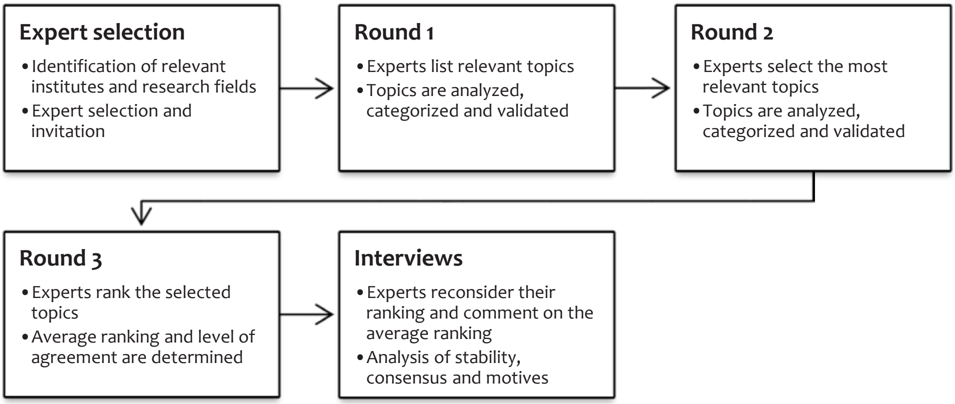

FIGURE 1 The procedure followed in this Delphi study

TABLE 3 Overview of the experts' research fields.

Research field	Expertise	Number of experts round 1	Number of experts round 2	Number of experts round 3	Number of expert interviews
Physics	High energy physics	2	1	1	1
	Quantum physics	3	3	2	2
	Solid state physics	2	2	2	2
	Particle physics	2	0	1	1
Mathematics	Quantum mathematics	2	2	1	1
Chemistry	Solid state chemistry	1	1	1	1
	Polymer chemistry	1	0	0	0
Biophysics	Nano photonics	2	2	2	1
	Biophysics	2	1	1	0
Total		17	12	11	9

help of the second author. Then the codes were categorized in cooperation with the third author, who is an expert in quantum physics and nanophysics. The experts' descriptions of the topics (concepts and applications) in each category and the experts' argumentations were summarized and the third author verified that the content was correct and corresponded with the experts' responses.

3.4.3 Second round

The second round also involved an online survey. In this survey the experts of round one were asked to read the summary of the descriptions of the topics in each category and the summary of the experts' arguments. Then the respondents were asked for each topic if they considered it considered appropriate for secondary schools. The responses were analyzed and categorized. Topics that were chosen by at least two-thirds of the experts were used for the following round, together with a list of the experts' arguments.

3.4.4 Third round

In the third round the experts were asked to rank the selected topics, from indispensable to dispensable. For this, the experts had to place each topic in one of the following categories: (1) indispensable, (2) desirable, (3) optional, or (4) dispensable. The number of topics that could be placed within each category was limited. Within each category the topics were also ranked. The experts' categorization was analyzed, the rankings were used to create an average ranking, and consensus was analyzed using Kendall's w.

3.4.5 Interviews

After the third round, interviews were conducted with several experts of the previous round to investigate the stability and validity of the experts' rankings, and to explore the reasoning and arguments on which the experts based their rankings. Transcripts of the interviews were analyzed for stability, consensus and the underlying arguments. For stability and consensus the experts were asked if they would alter something in their individual ranking and if they agreed with the final ranking. For the analysis of the arguments, the arguments were compared to the goals of scientific literacy in Table 2.

3.5 Results

3.5.1 First round

In round one, the experts stated which five quantum mechanics topics and applications they considered necessary for scientific literacy. Their responses were analyzed, which resulted in a list of 89 topics, accompanied by explanations and arguments. The 11 topics listed in Table 4 were proposed by more than 50% of the experts. Because of the large number of topics, the 89 topics were categorized. In cooperation with the third author, an expert in quantum physics, the topics with related content were grouped. Seven groups were formed: wave-particle duality, wave functions, atoms, subatomic particles, materials, nonlocality and history. These categories are shown in Table 5, together with a reduced summary of the experts' descriptions. Table 5 also shows the different aspects of scientific literacy which were used in the experts' arguments.

3.5.2 SECOND ROUND

In the second round, the experts selected topics from the list of 89 topics and explained their choices, after reading the corresponding explanations and summaries. Analysis of their responses showed that experts often labelled the topics as concepts, examples and applications. This led to a change in categorization, in the analysis and following rounds the topics were divided into three groups; concepts, examples and applications. The experts' arguments also showed some topics coincided; these topic were merged into one topic, which resulted in a list of 84 topics. Table 6 shows these topics, together with the number of experts who selected the listed concepts, examples and applications. From this table can be seen that the applications were considered less important for secondary education than the concepts and examples. The 37 topics chosen by at least eight experts were used in round three.

TABLE 4 The most frequently proposed quantum mechanics topics in round one (top 11 out of 89 items, N = 17).

Topic	Number of experts
Spectral lines	16
Tunneling	12
Photoelectric effect	11
Probability	11
Wave-particle duality	11
Double slit experiment	10
Energy levels and quantization	10
Hydrogen atom	10
Heisenberg's uncertainty principle	9
Lasers	9
Wave function	9

Table 5 The categories resulting from analyzing the experts' responses in round 1.

Categories	Experts' descriptions	Experts' arguments
Wave-particle duality	A particle shows both wave and particle behaviour. Single photon detection, the double slit experiment, the photoelectric effect, and the delayed choice experiment can illustrate this dual behaviour.	Understanding of scientific concepts, understanding the relation between science and technology, seeing the influence of science in everyday life.
Wave functions	A particle can be described by a wave function Ψ. The wave function can be a superposition of all possible wave functions. Ψ^2 is a measure of the particle's probability distribution.	Understanding of scientific concepts, understanding the nature of science, understanding the relation between science and technology, identifying and explaining scientific issues, seeing the influence of science in everyday life, appreciating the beauty of science.
Atoms	Electron's energy levels are quantized, which determines spectra and colors of atoms. Quantization can be explained with Bohr's atomic model, and with the quantum atomic model. The electron configuration also depends on Pauli's exclusion principle.	Understanding of scientific concepts, knowing science as a cultural force, understanding the relation between science and technology, seeing the influence of science in everyday life.
Subatomic particles	Subatomic particles have properties, which can be described by quantum numbers. An important property is spin, which is important in magnetism and electron configuration, and can be illustrated by the Stern-Gerlach experiment.	Understanding of scientific concepts, understanding the relation between science and technology.
Materials	Molecules and metals have energy bands and band gaps, which determine material properties such as strength, structure, color and resistance.	Understanding of scientific concepts, knowing science as a cultural force, understanding the relation between science and technology, seeing the influence of science in everyday life.
Nonlocality	QM violates local realism, which can be illustrated by the fact that entangled particles, when separated, cannot be described independently. This phenomenon shows the counterintuitive character of QM and is important in information technologies.	Understanding of scientific concepts, understanding the relation between science and technology.
History	QM plays an important role in the history of science and was one of the most important scientific revolutions of the 20th century. It shows the nature of science.	Knowing science as a cultural force, understanding the relation between science and technology, seeing the influence of science in everyday life.

Table 6 Overview of the topics selected by the experts in round two (N = 12), together with the number of experts who wanted the topics to be taught at secondary schools.

Concepts	**No. of experts**	**Examples**	**No. of experts**	**Applications**	**No. of experts**
'de Broglie' wavelength	12	Double slit experiment	12	Solar cells	9
Particle behaviour of light	12	Atomic structure	12	Quantum information	9
Probability	12	Periodic table	12	STM	8
Energy levels and quantization	12	Spectral lines	12	Lasers	8
Wave-particle duality	11	Photoelectric effect	11	LEDs	8
Wave function	11	Hydrogen atom	10	Quantum computers	8
Heisenberg's uncertainty principle	11	Bohr's atomic model	10	Single photon detection	7
Tunneling	11	Color	10	Spectral analysis of stars	7
Pauli's exclusion principle	11	Magnetism	10	Transistors	7
Spin	11	Orbitals	9	Quantum cryptography	7
Momentum	10	Material properties	9	Atomic clock	6
Fermions and bosons	10	1D infinite well	8	Fluorescence	6
Superposition	8	Radioactive decay	8	Neon lamps	6
Time evolution	8	Schrödinger's cat	8	MRI	6
Quantum numbers	8	Bonds	8	IC's and chips	6
QM at a macroscopic scale	7	Semi-conduction	8	Quantum teleportation	6
Entanglement	7	Conduction	7	GPS	5
History of QM	7	Heat radiation	7	Microwaves	5

Table 6 continued

Concepts	**No. of experts**	**Examples**	**No. of experts**	**Applications**	**No. of experts**
Complementarity	6	Polarization	6	CCD	5
Zero point energy	6	Energy bands	6	Giant magneto resistance	5
Subatomic particles	6	Super-conduction	6	CT scan	4
Standard model	6	Chemical reactions	5	SEM	3
Bohr versus Einstein	6	Stern-Gerlach experiment	5	Random generators	3
Foundations of QM	6	Delayed choice Experiment	4	Single molecule microscopy	3
Schrödinger equation	5	Crystal structures	4	Flash memory	3
Stationary states	5			Bennet-Brassard protocol	3
Measurement	5			PET scan	2
EPR paradox	5				
Development of atomic models	5				
Free vs. localized particle	4				
Locality and causality	3				
Bell's inequalities	3				

Table 7 Mean expert ranking in the third round (N = 11) on the importance of the selected quantum topics for the secondary school curriculum. Rank 1 is considered most important.

Rank	Concepts	Mean rank	Examples	Mean rank	Applica-tions	Mean rank
1	Wave/particle duality	2.10	Double slit experiment	2.10	Solar cells	2.70
2	Particle behaviour of light	3.50	Spectral lines	4.20	STM	3.10
3	Wave function	4.20	Photoelectric effect	4.30	LEDs	3.60
4	De Broglie wavelength	4.60	Atomic structure	4.60	Lasers	3.70
5	Probability	4.80	1D infinite potential well	6.00	Quantum informatio n	3.90
6	Energy levels and quantization	6.80	Hydrogen atom	6.30	Quantum computers	4.00
7	Heisenberg's uncertainty principle	7.40	Periodic table	6.60		
8	Superposition	9.10	Bohr's atomic model	8.20		
9	Spin	9.40	Radioactive decay	9.40		
10	Tunneling	9.70	Bonds	11.40		
11	Pauli principle	10.60	Orbitals	11.60		
12	Momentum	11.20	Magnetism	11.70		
13	Quantum numbers	11.50	Schrödinger's cat	12.20		
14	Fermions and bosons	11.70	Color	12.20		
15	Time evolution	13.40	Material properties	12.40		
16			Semi-conductors	12.80		

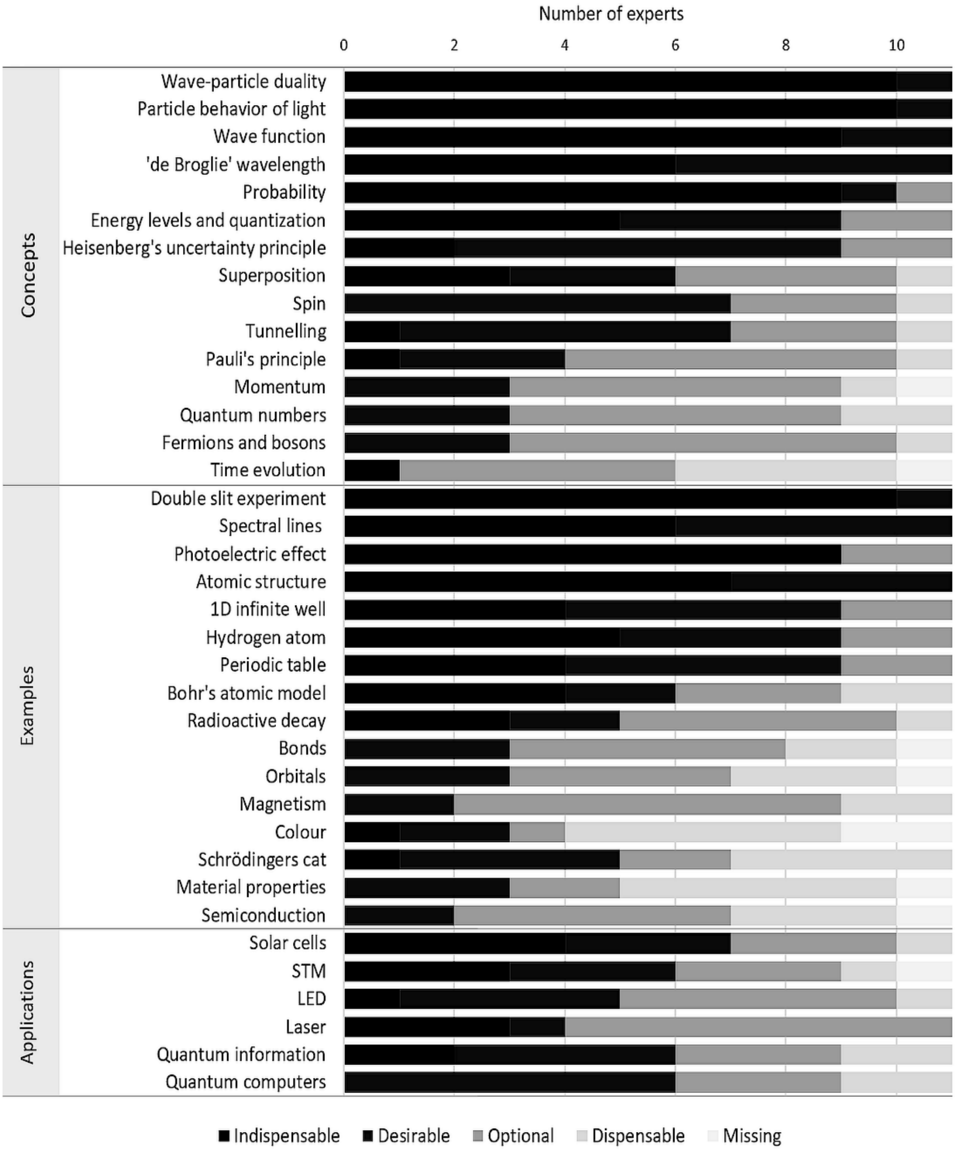

Figure 2 The number of experts in round three (N = 11) who considered the listed topics indispensable, desirable, optional or dispensable.

3.5.3 Third round

In the third round of the Delphi study, the experts placed the 37 remaining topics into categories going from indispensable to dispensable and ranked them, after reading the experts' arguments used in round two. Kendall's *w* was used to determine the average ranking, which is shown in Table 7, and the level of agreement on this ranking. The experts showed moderate to strong agreement (Kraska-Miller, 2013; Schmidt, 1997) on the exact ranking of the concepts ($w = 0.61$) and examples ($w = 0.58$), but there was no significant agreement on which applications should be treated in secondary schools to establish scientific literacy. The placement of the 37 topics within the four categories was also analyzed. As can be seen in Figure 2, the first seven concepts and examples in Table 7 are considered indispensable or desirable by at least nine experts. Furthermore, none of the other experts considered these concepts and examples dispensable, which leads to the conclusion that there is a strong agreement on the importance of these 14 topics.

3.5.4 Interviews

After the third round, semi-structured interviews were conducted with 9 experts from the previous round. The main objectives of the interviews were to investigate the stability of the experts' categorizations in the third round, the experts' level of agreement with the final rankings, and their underlying arguments.

To investigate the stability of the rankings, the experts were shown their own personal final rankings and were asked if there were topics they would change in rank or category. Seven experts proposed changes, but only two of these changes involved a change of category. These changes caused a slight change in the values shown in Figure 2 for superposition (a shift of from optional to indispensable) and the Pauli principle (a shift from dispensable to desirable). The changes also caused a slight change in the average ranking shown in Table 7 (spin and tunneling are switched, and orbitals and bonds are switched). Still, these are minor changes and the ranking can be considered stable, especially the ranking of the topics which are considered indispensable.

The consensus was investigated by discussing the average ranking. The majority of the experts perceived the average ranking to be similar to their own ranking (6 out of 9 experts), especially the upper part of the ranking of concepts and examples:

> *'The first parts are almost exactly the same'*

> *'They are a little… they are rather similar'*

Two of the experts who did not mention that the average ranking was similar to their own ranking stated that they considered the average ranking sensible. The differences in ranking that were perceived as striking were mainly in the lower part:

> *'I think it is surprising that superposition is at position 8'*

'The only thing that surprises me is the fact that Schrödinger's cat has a low position.'

Only two topics from the upper part of the ranking were mentioned by an expert as showing a difference:

'… I ranked the photoelectric effect, I ranked it lower'

'I would not know what essential topics should be explained with the uncertainty principle … I don't think that it is essential'

The fact that the majority of the experts perceived the average ranking similar to their own ranking, especially the upper part of the ranking, demonstrates that there is a high level of agreement, especially for the topics that are considered essential and desirable. The level of agreement was also determined for the rankings based on the interviews (w_{concepts}=0.61, w_{examples}=0.58), and showed a moderate to strong agreement.

The arguments used by the experts were analyzed using the goals for scientific literacy from Table 2 as codes. These codes were assigned to fragments in the transcripts, a fragment being a line of reasoning mainly addressing one single issue (e.g. a subtopic or category of quantum mechanics, a goal for scientific literacy or statement the expert wants to make). Table 8 gives an overview of the arguments used by the experts, together with the topics that were discussed. Since most experts did not distinguish between 'identifying scientific issues' and 'explaining phenomena scientifically', these two categories were merged into one category. The results showed that the experts based their rankings mainly on the understanding of scientific concepts, and that over 75% percent of the fragments are related to knowledge.

When looking to the arguments about understanding of scientific concepts in more detail, there were several underlying categories. Besides content reasoning based on what concepts the experts consider to be the fundamental concepts of quantum mechanics and the relation between these different concepts, experts also based their arguments on the conceptual complexity of the topic, and the extent to which a topic demystifies quantum mechanics. The complexity of the topic was addressed especially often (21 out of 67 fragments):

'I would like to introduce quantum information, but I think it is too abstract.'

'I consider superposition to be a central element … but I do understand

that it is too difficult to explain.'

'Quantum computers … they are fascinating, but there is a lot of mathematics involved'

However, five of these experts also stated that students should have basic knowledge of complex topics in order to be able to interpret new developments presented in the media and distinguish fact from fiction in discussions. Some experts stated that you have to avoid the applications that cannot be explained to secondary school students, others stated you can refer to these applications, but shouldn't try to explain them. This conflict between importance and difficulty may explain the lack of consensus for the applications, most of which are both complex and prominent in the media.

Eight of the experts used the argument that the chosen topics show students that quantum mechanics forms the basis for our everyday life:

'So everything, really everything is quantum'

'… they think it is fascinating, that something that fundamental, that it [radioactive decay] is a deep quantum mechanical phenomenon.'

The experts stated that students should be aware that quantum mechanics is the foundation of everything we perceive, and that many technologies we use in our daily lives are based on quantum mechanics. During the interviews, the experts showed they were fascinated by the way quantum mechanics determines the natural world themselves and two experts explicitly stated that it is fascinating for students too. Other experts were not explicit, but used phrases that show they aim for more than being informed about quantum physics in our everyday life:

'As long as the message of quantum mechanics sinks in … that it is not a classical world, but a quantum world'

'But when you see that it [everyday life] is not at all self-evident, that a strange theory is needed to understand it…'

Table 8 The arguments regarding scientific literacy used by the experts (N = 9) during the interviews.

	Goals for scientific literacy	No. of experts	No. of fragments	Topics mentioned
Competencies	Identifying and explaining scientific issues	5	7	Heisenberg's uncertainty principle, energy levels and quantization, Schrödinger's cat, quantum information, quantum computers.
	Being able to make informed decisions	-	-	-
Knowledge	Understanding of scientific concepts	9	67	All
	Understanding the nature of science	3	4	Double slit, wave function.
	Knowing science as a cultural force	4	4	Material properties.
	Being aware of career opportunities	1	1	Quantum information, quantum computers.
	The relationship between science and technology	2	2	Wave-particle duality, probability, semiconductors.
Attitude	Interest in science	-	-	-
	Support for scientific inquiry	1	1	Quantum information, STM.
	Responsibility towards resources and environments	-	-	-
	Seeing the influence of science in everyday life	8	11	Wave-particle duality, 'de Broglie' wavelength, Heisenberg's uncertainty principle, quantization and energy levels, tunneling, atoms, 1D infinite potential well, radioactive decay, spin, fermions/bosons, material properties, lasers.
	Appreciating the beauty of science	2	3	Wave-particle duality, spin, tunneling, quantum information, quantum computers.

So, even though the goal 'appreciating the beauty of science' was not often mentioned specifically, this goal seems closely related to 'seeing the influence of science in everyday life'.

Other goals were mentioned less often, and the goals mentioned mainly focused on understanding and explaining of quantum mechanical concepts, but the understanding of physical models, the importance of quantum mechanics for technological developments and its impact on society were also mentioned. Even though the goals mentioned by the experts were mainly content based, Table 8 shows there are many topics of quantum mechanics considered appropriate for promoting scientific literacy; in particular, quantum information and wave-particle duality were mentioned often.

3.6 Conclusions

In this article we presented an analysis of key quantum mechanics topics, which is the first step in developing a curriculum on quantum mechanics for the secondary level, based on an investigation of relevant topics, and students' preconceptions and difficulties. For this analysis we investigated: (1) which topics experts considered essential for obtaining an appropriate image of quantum mechanics in terms of research, developments and applications, and (2) what arguments experts used for choosing these key topics. In this section, we give an overview of the main conclusions that can be drawn based on the Delphi study and the interviews, together with recommendations for further research and curriculum development.

3.6.1 Key topics

In contrast to the results of McKagan *et al.*[29], which showed no consensus on key topics, this study shows there is a moderate to strong agreement on what quantum mechanics topics are considered to be important. The Delphi study showed that the majority of the experts considered the following topics essential:

(1) **Duality**: The wave-particle duality, the particle behaviour of light, the 'de Broglie' wavelength, Heisenberg's uncertainty principle, the double slit experiment and the photoelectric effect.
(2) **Wave functions**: The wave function, probability and the 1D potential well.
(3) **Atoms**: Energy levels, quantization, atomic structure, spectral lines, the hydrogen atom and the periodic table.

These topics were considered important by a majority of the experts in rounds two and three, and the interviews also showed that the experts considered the upper part of the average ranking similar to their personal ranking.

3.6.2 Arguments

The arguments used for the ranking were mainly based on knowledge, especially on 'the understanding of scientific concepts', for example, the relation between the different concepts and their position within quantum mechanics. This is in in accordance with the fact that the consulted experts were all academic scientists and researchers, who are more likely to embrace *wish-they-knew* and *need-to-know* science[38, 39]. The lack of addressing the other goals for scientific literacy may be partly due to the predominantly unstructured nature of the interviews, in which the different goals were not specifically mentioned. Moreover, the enquiry emphasized specifically research and technological developments, which is appropriate for the Dutch curriculum, but may have interfered with our focus on scientific literacy.

An important argument for finding a topic appropriate for secondary education was its complexity. Most topics that were described as too complex or abstract were considered less essential. Although the experts mainly reasoned about content knowledge, the goal of 'seeing the influence of science in everyday life' was also mentioned by the majority of the experts. Additionally, the interviews showed that there are various aspects of quantum mechanics that can be used to address the different goals for promoting scientific literacy.

3.6.3 Implications

The ranking of quantum mechanics topics found in this study is based on the opinions and expertise of academic scientists and researchers. These experts can be considered content experts, who have a good view of quantum mechanics and its position within the fields of research and development. Still, these experts are all part of a specific sub-group of academic scientists and researchers, which may have biased the outcomes; the results of this study are likely to be a sub-set of views on what students '*need-to-know*' and what we '*wish-they-knew*'. However, the knowledge of the general public, industry, policy-makers, and even secondary school teachers about quantum mechanics is rather limited, which makes it difficult to take their opinion into consideration without first teaching them the basics of quantum mechanics.

Since quantum mechanics is a rather new field for secondary school curriculum policy-makers and researchers in the Netherlands, this ranking provides a good starting point for the development of a research-based curriculum. Still, the ranking resulting from this study is rather unspecific, because the listed topics all consist of various subtopics and can be taught in many different ways. Also, the results of this study do not give insights into the experts' exact interpretation of the understanding of the chosen topics. For the development of a quantum mechanics curriculum, not only insights into what students should learn, but also knowledge of the feasibility of teaching these topics at secondary school level is needed. Therefore, there is a need for practice based research into students' understanding of quantum mechanics, in which the feasibility of teaching the various subtopics of quantum mechanics to secondary school students is investigated. The knowledge of learning difficulties, underlying problems, and needs for prior knowledge obtained from this research into feasibility and students' learning difficulties can form the basis for the design of instructional materials.

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4

Secondary school students' misunderstandings of potential wells and tunneling

In order to investigate students' misunderstandings of potential wells and tunneling, a conceptual knowledge test was administered to Dutch secondary school students after they were taught about quantum mechanics. A frequency analysis of responses to the multiple choice questions (n=98) and coding of the responses to the open ended questions and explanations (n=13) showed that Dutch secondary school students experienced difficulties similar to those reported for undergraduate students. The students' underlying difficulties were analyzed using a typology of learning impediments. Results of this analysis showed that students have difficulty connecting knowledge of potential wells and tunneling to their prior knowledge. Students mainly have creative and epistemological learning impediments, which caused eight incorrect synthetic models.

BASED ON: K. KRIJTENBURG-LEWERISSA, H.J. POL, A. BRINKMAN AND W.R. VAN JOOLINGEN, SECONDARY SCHOOL STUDENTS' MISUNDERSTANDINGS OF POTENTIAL WELLS AND TUNNELING. (SUBMITTED)

4.1 INTRODUCTION

Since quantum mechanics (QM) plays a fundamental role in physics research and its applications, it has become part of the secondary school curriculum in many countries. Teaching QM at the secondary school level is challenging, because secondary school students have not learned to use the mathematical tools needed for a formal, mathematical approach to QM. Therefore, in secondary schools, QM needs to be taught at a conceptual level. Teaching QM at the secondary school level is also difficult because QM is fundamentally different from the classical physics that secondary school students have encountered [1]; daily life experiences are usually not associated with QM [2], which makes it counter-intuitive. Students have the tendency to describe quantum phenomena deterministically [3], which conflicts with QM principles. Research has also shown that students tend to incorrectly generalize their prior knowledge of classical concepts [4]. For a good implementation of QM at secondary schools, knowledge of students' difficulties when learning QM is needed

Research [5] has shown that there are QM topics that are taught in most international curricula, for instance, wave-particle duality and discrete energy levels. Most research into QM education at the secondary school level has focused on these topics [6]. However, less research has been conducted regarding topics that are taught less frequently. This can be seen by the scarcity of research conducted on students understanding of the philosophical aspects of QM [2, 7], or mathematical representations, for instance the 1D infinite potential well, which is taught in the Netherlands [8]. A review [6] of the current knowledge of students' misunderstandings of QM showed that more research is needed into secondary school students' understanding of QM and their underlying difficulties, especially for students' understanding of the wave function, potential wells and tunneling.

In this paper, we present our research into secondary students' (mis)understandings of the 1D infinite potential well and tunneling, which recently have become part of the Dutch secondary school physics curriculum [9]. To investigate Dutch students' understanding, a conceptual knowledge test was administered and the results were analyzed. To explore the underlying problems related to the observed misunderstandings, interviews were conducted.

4.2 BACKGROUND

In presenting our investigation of students' difficulties, first we will give an overview of existing research on students' difficulties regarding potential wells and tunneling. Additionally, we will discuss relevant conceptual change theories and the typology of learning impediments created by De Jong and Taber [10], which will be used to analyze the observed learning difficulties.

Table 1 Undergraduate students' incorrect ideas regarding wave functions, potentials, tunneling and probability. Reprinted from K. Krijtenburg-Lewerissa *et al.*[6].

	Overgeneralization of prior concepts	**Mix-up of related concepts**
Wave functions & potentials	Wave functions describe a trajectory	Change in amplitude causes change in energy
	Potential wells are objects	The amplitude or equilibrium of the wave function is mixed up with energy
	Height in potential graphs means position	There is difficulty to distinguish between energy and probability
Tunneling & probability	The amplitude of wave functions is a measure of energy	Only the tops of the waves, which overtop the barrier, will pass
	Probability is described with classical arguments (e.g. velocity)	Part of the energy is reflected at a barrier during tunneling
	Energy or effort is needed to tunnel through a barrier	A single particle is described as an ensemble of particles

4.2.1 Students' difficulties with potential wells and tunneling

There has been research into students' understanding of potential wells [11] and tunneling [12-14], but mainly at the undergraduate level. This research has shown that students have difficulty understanding potential wells and tunneling, and often use classical reasoning. Research into other topics of QM has shown that students often describe the wave function as a classical particle moving over a sinusoidal trajectory [15-17]. This classical reasoning also causes students to describe potential wells as external objects, and to describe tunneling in terms of interaction of a particle with the barrier [12]. Singh, Belloni and Christian observed that this misplaced classical thinking can be caused by a mix-up of related concepts, and overgeneralization of previously learned concepts [4, 18]. In our review [6] we assigned the different incorrect ideas found in literature to the two categories observed by Singh *et al.* (see Table 1). That review of the current knowledge of students' misunderstandings showed that undergraduate students experience difficulty with learning QM, because they are not able to connect quantum behavior to the physical reality as they see it.

4.2.2 Conceptual learning of QM

QM is fundamentally different from classical mechanics, because at the quantum level, objects behave like waves in certain circumstances, and like particles in other circumstances. In a theory of conceptual change for learning science concepts, Chi [19] specified three ontological categories: 1) entities, which, for example, have weight and occupy space, 2) processes, which occur over time, and 3) mental states, such as emotions or intentions. In classical mechanics a particle would belong to the 'entities' category, and waves would belong the 'processes' category. However, in QM a quantum entity is both an entity and a process. This requires an ontological shift, in which the quantum entity belongs to a new ontological category, having both wave and particle properties [20]. To use this new ontology, students need the capability to move back and forth between wave and particle representations. Therefore, conceptual learning of QM is best approached not as the acquisition of static knowledge, but as a process of exploring, developing and evaluating alternative explanatory models [21]. This should lead to a change in students' conceptual profile [22]. Students need to become aware of the limitations of different models and need to be capable of deciding which model or description is appropriate in a specific situation [23, 24]. In order to understand the limitations of models and representations, students need to have the newly learned concepts correctly embedded in their existing knowledge structures. When these newly learned concepts are incorrectly integrated within students' prior knowledge, this leads to inconsistent and incorrect models, which are called synthetic models [25, 26]. For example, Vosniadou, Vamvakoussi and Skopeliti [26] describe synthetic models of the shape of the earth, in which children have combined their idea of a flat earth with the spherical model of the earth. Many children create mixed, synthetic models, and e.g. come up with a model of a flattened or hollow sphere. Since QM by its very nature is inconsistent with students' prior knowledge, these synthetic models are likely to be formed. When looking at Table 1, the observed mix-ups of related concepts and overgeneralizations of prior concepts are incorrect integrations of QM into students' prior knowledge, and hence are synthetic models. The misconception 'Wave functions describe a trajectory' for example, is a synthetic model in which students combine their prior ideas of particles and waves. Students learn that a QM entity shows both particle and wave behavior and therefore create a model in which a particle moves like a wave.

Students' difficulties and synthetic models can be classified using the typology of De Jong and Taber[10], which is shown in Figure 1. This typology is based on the ideas of Ausubel[27]; deep learning will only happen when students can relate the new knowledge to their existing knowledge. In the first version of this typology[28], Taber explains that these different learning impediments are based on a consideration of what can go wrong when students try to include new knowledge into their existing framework. His main line of reasoning is similar to that of Vosniadou[25]. However, he does not only describe that there are incorrect models, but he also gives categories of underlying principles that impede deep learning. The typology is based on a consideration that students cannot make sense of a new concept when they lack prior knowledge, or do not recognize how new concepts relate to prior knowledge. They can also form alternative frameworks, which, for example, can be caused by their everyday experience, misinformation in society, teachers' misunderstanding or teachers inability to present the new concept adequately. In a later study, Taber also

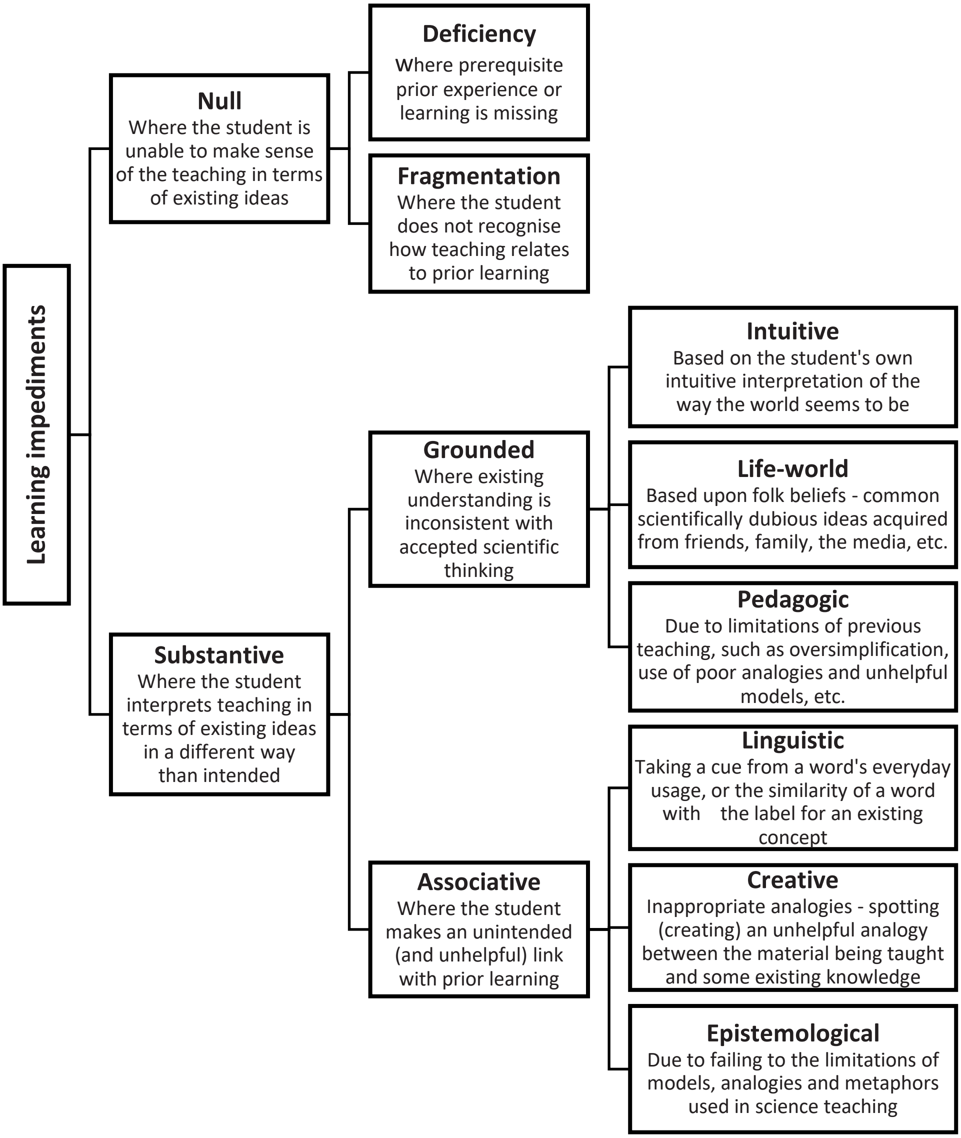


FIGURE 1 The typology of learning impediments as presented by De Jong and Taber[10].

observed that linguistic cues and students' epistemological assumptions play a role [29]. The typology presented Figure 1 was created during a study into difficulties that students encountered while learning particle theory. Several of the misconceptions of Table 1 can be related to a learning impediment. The misconception 'height in a potential graph means position' for example, can be related to an creative analogy to potential diagrams that students have encountered while learning classical physics. The misconception 'potential wells are objects', can be related to the phrase 'well' and therefore may be caused by a linguistic impediment. Since at first glance De Jong and Taber's typology can be related to QM misconceptions, this typology is a good starting point for analyzing students' understanding of QM. Additionally, this typology also helps to investigate the underlying principles that impede deep learning.

4.3 Method

In order to investigate students' understanding of the 1D infinite potential well and tunneling, a test was created. A review of existing tests on QM showed that there were only two multiple choice (MC) tests that partly addressed tunneling at a level that was appropriate for the Dutch secondary school level; the QMCI [30] and the QMCS [15]. No suitable test questions were found regarding the 1D infinite potential well. One of the questions on tunneling could be used at once, four other questions were slightly adapted. The questions were translated and verified by a content expert and two experts in physics pedagogy. The other four MC question were created by the authors, based on misconceptions found in their review [6]. To investigate not only the presence of known misconceptions, but also the underlying difficulties, the authors also created open-ended (OE) questions. These OE questions were mainly explanations; for each MC question, students were also asked to explain their choice. The OE questions were created to investigate if students were able to explain what the wave function and 1D potential well represented.

The test was given to 98 students during a physics class, at five different secondary schools after they were taught QM. The students were 17-18 years old and in their last year of pre-university education, The groups were chosen by convenience sampling. The test included the QM topics that are part of the secondary school physics curriculum in the Netherlands: (1) the wave character of light, (2) wave-particle duality, (3) the photoelectric effect, (4) Heisenberg's uncertainty principle, (5) the. 1D infinite potential well (i.e. the particle-in-a-box model), (6) the hydrogen atom, and (7) tunneling. In this article we discuss only the results of the 12 questions regarding the potential well and tunneling. Of these 12 questions, seven questions addressed the understanding of potential wells and wave functions, and five questions addressed the understanding of tunneling. The questions addressed the topics shown in Table 2. The translated questions used in the test can be found in Appendix B. The answers of the MC questions were used for a frequency analysis, while the explanations of the MC questions and responses to the OE questions were used for qualitative analysis.

For the qualitative analysis, 16 students were selected for interviews. This selection was based on an analysis of the MC questions. For the four schools participating in the interviews, we selected at least one student who had a low score, one student who had an intermediate score and one student who had a high score. Due to absences, 13 of the selected students took part in the interviews. The interviews were conducted within six weeks after taking the test. During the interviews the students were asked to explain their answers to questions 2, 3, 5, 8, 9 and 12 on the test. They were asked to: (Q2) explain what wave functions are and interpret a graph of a wave function, (Q5) explain the incorrect representation of the 1D infinite potential well, (Q3) compare two wave functions, (Q8) describe what happens with the energy of a particle during tunneling, and (Q9 and Q12) describe the influence of changing the width and height of the barrier on tunneling. For the 13 interviewed students, the interviews, explanations of MC questions during the test, and answers to OE questions were analyzed using open coding. Then, related codes were grouped together into categories of observed misunderstandings. To investigate the underlying problems, these categories of misunderstandings were analyzed using the typology of learning impediments of De Jong and Taber[10], leading to a more detailed framework for students' difficulties while learning QM. Finally, this

TABLE 2 The content of the conceptual knowledge test.

	Question	Question type	Source	Topic
Potential wells and wave functions	Q1	OE	-	Explain the particle-in-a-box model
	Q2	OE	-	Interpret a wave function
	Q3	MC	-	Interpret a wave functions in terms of energy
	Q4	MC	-	State what is a measure of the energy level in the particle-in-a-box model
	Q5	OE	-	Explain the incorrect representation of the particle-in-a-box model
	Q6	MC	-	Interpret a wave functions in terms of probability distribution
	Q7	MC	QMCI Q9	State what property of a wave function is a measure of the energy level
Tunneling	Q8	MC	QMCS Q7	State how tunneling influences the energy level
	Q9	MC	QMCI Q2	State how barrier height influences tunneling
	Q10	MC	QMCI Q6	State how the energy level influences tunneling probability
	Q11	MC	-	Interpret how the potential barrier influences the energy
	Q12	MC	QMCI Q3	State how barrier width influences tunneling

framework was used to conduct a frequency analysis on the responses of the complete group of 98 students.

4.4 QUANTITATIVE RESULTS

The online conceptual knowledge test (Appendix B) was given to secondary school students after they had been taught QM (n = 98). The reliability of the MC part of the test, addressing potential wells and tunneling was determined with Cronbach's alpha: $\alpha = 0.747$. Table 3 shows the percentage of students that answered the MC questions correctly. Since question 9 and 12 addressed the understanding of both probability and energy, these questions are presented as having two parts.

The results for the MC questions on potential wells and wave functions were analyzed to investigate the presence of known difficulties. Analysis of these questions showed that most students knew how the wave function relates to the position of the particle (Q6) and understood that wave functions with different frequencies must have different energy levels (Q3). However, when students were asked what defines the energy level (Q4, Q7), 24% of the students believed that the amplitude of the wave function influences the energy level. The questions addressing students' understanding of tunneling showed that approximately 50% of the students believed that energy decreases after tunneling, a difficulty that has been previously reported for undergraduate and graduate students [11, 12, 31] . However, secondary school students seem to have more difficulty understanding tunneling. Question 8 was answered correctly by only 39% of the students, which is significantly less that the results for this question in the QMCS [15], where 75% of the graduate students answered this question correctly. What also stood out was the difference between students' ideas concerning the influence of the width and height of the barrier on the energy level; 20% of the students believed that the height of the barrier influences the energy level, whereas 45% of the students believed that the width of the barrier influences the energy level. This is in line with the results of McKagan and Wieman [32], who found that 19% of the graduate students believed that the width influenced the energy of the particle, whereas 11% believed that the height of the barrier influences the energy.

TABLE 3 Results MC questions on potential wells and tunneling (n = 98)

Topic	Question	Subtopic	Students with a correct answer (%)	Remarks
1D infinite potential well & wave functions	Q3	Frequency is related to the energy level	80	10% chose the option that wave functions with different frequencies have the same energy level
	Q4	Number of nodes of the standing wave is related to the energy level	56	24% chose amplitude, and 12% the area under the curve as a measure of the energy level
	Q6	$\|\Psi\|^2$ is the probability distribution	93	
	Q7	Frequency is related to the energy level	63	24% chose amplitude as a measure of the energy level
Tunneling	Q8	Total energy remains equal after tunneling	39	51% chose the option that the energy after tunneling is decreased
	Q9	Increasing the barrier height decreases the probability of tunneling	64	
		Increasing the barrier height does not influence the particle's energy level	80	
	Q10	Probability of tunneling does not solely depend on the particle's and barrier's energy levels	76	12% chose the option that the probability of tunneling is 0, and 11% that it is 0.5
	Q11	Energy remains equal after tunneling (visual)	38	48% chose the option that the energy after tunneling is decreased
	Q12	Increasing the barrier width decreases the probability of tunneling	74	
		Increasing the barrier width does not influence the particle's energy level	55	

The results of the MC questions give an overview of incorrect knowledge, but they give no information about the underlying ideas that cause students' difficulties. Therefore, the incorrect beliefs found in this study were examined more thoroughly in the qualitative analysis of the explanations of the MC questions, the responses to the OE questions, and the interview transcripts.

4.5 QUALITATIVE RESULTS

In order to investigate if there are more difficulties, and to find underlying problems, the explanations, OE questions and interviews were analyzed of the 13 selected students. First, open coding was used to analyze these students' responses. This led to 299 codes, which described students' correct and incorrect lines of reasoning. After merging overlapping codes, 160 codes remained, of which 77 described incorrect ideas. These 77 codes were analyzed and grouped into the 12 codes shown in Table 4. The areas of difficulty found for the Dutch secondary school students are similar to the undergraduate students' incorrect ideas presented in Table 1. Most of these incorrect ideas can be considered to be synthetic models in which students have created incorrect links to prior knowledge. Table 5 shows which students used these incorrect ideas in their reasoning during the test and in the interviews. In this table, it can be seen that high scoring students can still use incorrect explanations. In the interviews the students were asked to explain question 2, 3, 5, 8, 9 and 12 of the test, to probe more deeply the underlying problems behind the incorrect views and synthetic models. In the following sections, a more detailed description of students' incorrect views will be given, together with an analysis of the types of learning impediments that play a role.

TABLE 4 *The incorrect ideas on potential wells and tunneling observed in the explanations of the MC questions, the responses to the OE questions, and in the interviews (n = 13)*

Topic	Subtopic
1D infinite potential well & wave functions	The model gives information about the particle's height
	The well is a physical object
	The particle has classical wave properties
	The well is linked to resistance
	The equilibrium is a measure of the energy level
	The amplitude is a measure of the energy level
	Incorrect use of amplitude and wavelength in energy equations
Tunneling	The particle loses energy during tunneling
	After tunneling the particles energy is increased
	The particles energy needs to exceed a threshold for tunneling
	Either the width or the height of the barrier solely influences tunneling
	Incorrect reasoning with frequency or amplitude of the wave function

4.5.1 The 1D potential well & wave functions

To gain understanding of students' difficulties, the quotes from MC explanations, OE questions, and the interviews belonging to an incorrect idea were compared, grouped and categorized within the framework of learning impediments (Figure 1). In this section we describe the students' responses regarding the 1D potential well and wave functions. We also illustrate these descriptions with quotes, and explain to which learning impediments these responses correspond.

THE MODEL GIVES INFORMATION ABOUT THE PARTICLE'S HEIGHT

When confronted with the incorrect wave/energy representation of Q5, six students stated that the straight horizontal lines represent the height of the particle. When looking deeper into these students' reasoning, several of them seem to have difficulty connecting the learned concepts with their previously learned models and representations. As a result, four students used a previously learned semi-classical model to explain the 1D potential well:

> *S6: '… these are electrons in different shells … closer to the nucleus the energy is lower.'*
>
> *S11: 'I would say it is higher, but I do not know if I can explain it with physics. I'd better think in terms of chemistry… I would look at those [refers to shells which were mentioned earlier]…'*

This use of an inappropriate model can be seen as an **epistemological learning impediment.**

The other two students tried to explain the 1D potential well by mixing it with prior knowledge of classical waves:

> *S12: 'As I learned from physics… isn't that the average that the particle moves around? So it is the average distance from the nucleus.'*
>
> *S13: 'When the particle gets higher, it goes faster and creates more standing waves.'*

This mix-up of different representations can be seen as **a creative learning impediment**.

THE WELL IS A PHYSICAL OBJECT

When the students were asked what the vertical lines in the 1D potential well represented, two students tried to explain the vertical lines, using classical, deterministic thinking. One of these students could only explain the figure within the analogy of a physical well:

> *S12: 'The edges are the sides of the well?'*

TABLE 5 The distribution of the incorrect ideas over the 13 selected students. S1-S4 scored high, S5-S9 intermediate, and S10-S13 low on the MC questions.

		High scoring				Intermediate scoring					Low scoring				
		S1	**S2**	**S3**	**S4**	**S5**	**S6**	**S7**	**S8**	**S9**	**S10**	**S11**	**S12**	**S13**	**Totals**
1D potential wells & wave functions	The model gives information about the particles height			•	•		•					•	•	•	6
	The well is a physical object					•							•		2
	The particle has classical wave properties		•				•		•	•	•	•	•	•	8
	The well is linked to resistance												•		1
	The equilibrium is a measure for the energy level	•		•			•	•	•		•				6
	The amplitude is a measure of the energy level	•	•	•						•	•	•	•	•	8
	Incorrect use of amplitude and wavelength in energy equations							•						•	2
Tunneling	The particle loses energy during tunneling		•		•	•	•		•	•	•	•	•	•	10
	After tunneling the particles energy is increased							•	•			•			3
	The particles energy needs to exceed a threshold for tunneling	•		•		•	•	•	•			•			7
	Either the width or the height of the barrier solely influences tunneling		•	•			•			•	•		•		6
	Incorrect reasoning with frequency or amplitude of the wave function									•	•				2
	Totals	3	4	5	2	3	6	4	5	5	6	6	7	5	61

This can be seen as **a linguistic learning impediment**, which is caused by the use of the word “well”.

The other student uses her knowledge that the potential is a simplified atomic model, describing the vertical lines as boundaries of an atom.

> *S5: ‘… those are the boundaries, the radius of the atom.’*

But this deterministic reasoning also led to incorrect ideas:

> *S5: ‘The bottom represents the nucleus of the atom.’*

These examples show that this student linked the potential well to the atomic model, but took this too far. This can be seen as an **epistemological learning impediment**.

Only three students related the vertical line to the potential energy of the system. Two students did not know what the figure represented and could not relate the energy diagram to their prior knowledge. Still, seven of the students were aware that the infinite potential determined the possible positions of the electron.

THE PARTICLE HAS CLASSICAL WAVE PROPERTIES

When explaining the 1D infinite potential well, students were also reasoning about the wave character of the electron within the well. Most of the students related the wave function to prior knowledge of classical waves, but because the electron also behaves like a particle, this knowledge of waves was combined with a deterministic description of the electron’s path. This led to a mixed model-up in which the electron vibrates or moves along a sinusoidal path:

> *S8: ‘that is the equilibrium which it moves around.’*
>
> *S13: ‘It has to do with a vibration or how fast it moves.’*
>
> *S11: ‘It moves along these lines.’*
>
> *S12: ‘The particle moves like a wave and has a tone.’*

This mix-up of different models can be seen as a **creative learning impediment**. However, these creative, mixed and incorrect models seem to be caused by the inability of students to incorporate the correct, non-deterministic representation in their thinking:

> *S12: ‘I find that a difficult question… because I think this line represents a probability, our teacher stated that yesterday. And the particle is there, not straight, but a little bit as a wave… but, maybe it moves like that. Its position however, is completely random.’*

This student had been taught that the wave function gives information about the probability distribution of the electron. Since this student does not know how the probability distribution relates to his prior knowledge, he keeps thinking in terms of movement and position, which can be seen as a **fragmentation learning impediment**.

Other students showed that their ideas were incomplete, when they were asked to elucidate their descriptions:

> *S9: 'I think this one moves more.'*
>
> *I: 'What does that mean?'*
>
> *S9: 'I don't know… position?'*
>
> *I: 'You say it moves more, or more often.'*
>
> *S9: 'More, so it has more energy.'*
>
> *I: 'And…'*
>
> *S9: 'So it has a… I don't know.'*
>
> *I: 'I don't understand what you mean by "movement"'*
>
> *S13: 'That this is the highest velocity it can have, but that… that is not true.'*

These students could not relate the wave behavior of electrons to their prior, classical, knowledge, which is a **fragmentation learning impediment**.

THE WELL IS LINKED TO RESISTANCE

When the students were asked to explain the potential well, one student linked it to resistance:

> *S12: '…the particle cannot escape. This way the concept of "resistance" can be clarified.'*

In other parts of the interview, the student talked about collisions. Therefore, it is likely that the word 'resistance' refers to forces or interactions working on the particle. This mix-up of resistance, forces and potential energy can be seen as a **creative learning impediment**.

THE EQUILIBRIUM IS A MEASURE OF THE ENERGY LEVEL

When students were asked what is a measure of the energy level in the 1D potential well, four students stated that the equilibrium of the wave function represents the energy level, and not the wavelength:

> *S3: 'I assume that the x-axis [student points to equilibrium] lies at y=0, so it has a low energy level. '*

> *S10: 'The higher the equilibrium, the higher the particle's potential energy'*

This is a mix-up of two representations (the equilibrium and the energy level), which can be seen as a **creative learning impediment**.

However, the second statement is correct for representations similar to Figure 2. Two other students specifically referred a representation as shown in Figure 2. During the interviews these students were confused, because in Q3 of the test, the wave functions with different energy levels both intercepted *y* = 0.

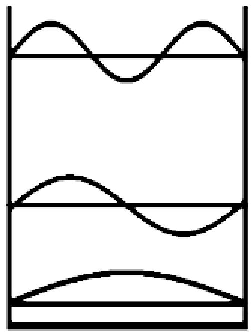


FIGURE 2 The incorrect 1D potential well representation, which simultaneously shows wave functions and energy levels.

> *S1: 'This [student points to the equilibrium] should be drawn higher.'*

Since both students specifically used the incorrect 1D potential well representation in their reasoning, this can be seen as a **pedagogic learning impediment**, resulting from the use of this representation in text books. This shows that the pedagogic learning impediment can lead to a creative learning impediment.

THE AMPLITUDE IS A MEASURE OF THE ENERGY LEVEL

When students were asked to explain the figures belonging to Q2 and Q3, students linked the displacement or amplitude to the energy level:

> *S10: 'The vertical axis is the energy level. The amplitude of both wave functions is equal, so the energy level is equal.'*

> *S11: 'The electron is moving and gets a higher energy, because the second part of the sine is higher than the first.'*

> *S12: 'The bigger the displacement, the higher the particle's energy level'.*

Two students mixed up the wave representation with a deterministic atomic model, and linked the sinusoidal path to the movement between two different energy states or shells. While pointing at point A in Figure 3, one of these students said:

> *S2: '… the excited state is located at this position…'*

When we asked why the student linked this to an excited state, the student stated that she believed so, because the figure had to do with the particle's position. These results show that these incorrect ideas are **creative learning impediments**. The idea that the y-axis represents both position and energy can be caused by the incorrect potential well representation of Figure 2 and can be seen as a **pedagogic learning impediment** as well.

INCORRECT USE OF AMPLITUDE AND WAVELENGTH IN ENERGY EQUATIONS

Two students showed they did not understand the relation between energy equations they had learned and the potential well. Both students used an incorrect quantity as a parameter in an energy equation. This can be considered as a **fragmentation learning impediment**, since these students did not see how the energy equations relate to the potential well model.

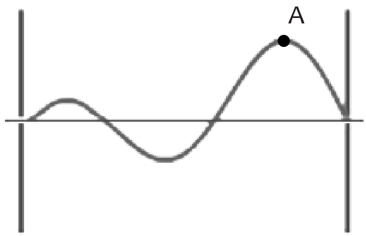


Figure 3 A standing wave within the 1D potential well

4.5.2 Tunneling

To gain understanding of students' difficulties regarding tunneling, the quotes from the test and the interviews were compared, grouped and categorized within the framework of learning impediments (Figure 1). In this section we describe the students' responses regarding tunneling. We also illustrate these descriptions with quotes, and explain to which learning impediments these responses correspond.

THE PARTICLE LOSES ENERGY DURING TUNNELING

When the students were asked to compare the energy before and after tunneling, 10 students stated that the energy after tunneling is lower, because the particle loses energy. Three students referred to some sort of interaction:

> *S12: 'The particle loses energy, because of collisions.'*
>
> *S5: '… it is harder for the particle to get through the barrier.'*

Six students described the path of the particle during tunneling:

> *S11: 'The particle has to cover a longer distance'*
>
> *S8: 'When the barrier gets wider, the particle has to bridge a longer distance.'*

These students used the classical description of a particle, to describe a non-deterministic phenomenon. Since these students used a correct model in the wrong context, these approaches can be regarded as **epistemological learning impediments**.

One student mixed up the potential well and the barrier:

> *S13: 'Because it keeps moving back and forward [points to the barrier] … but it will tunnel through…'*

This can be considered a **creative learning impediment,** in which two models are being mixed up and used to create a new, incorrect model.

AFTER TUNNELING THE PARTICLE'S ENERGY IS INCREASED

Three students stated that the particle's energy after tunneling is higher. One student showed a **creative learning impediment** and thought that the particle must have a higher final energy level, to be able to stay on that side of the barrier:

> *S8: 'The particle's energy must be larger after tunneling, otherwise it would fall back.'*

Two of these students also believe that energy is lost during tunneling, one of them explained that he thinks that the energy initially becomes larger and then decreases:

> *S7: 'It [the particle] needs a lot of energy for tunneling and afterwards the energy decreases, but stays higher than the energy at the beginning…'*

These students described this process in a classical way, which can be seen as an **epistemological learning impediment.**

THE PARTICLE'S ENERGY NEEDS TO EXCEED A THRESHOLD FOR TUNNELING

Three students who answered Q9 correctly, explained their answer by saying that the particle needs more energy when the barrier is higher.

> *S5: '… the particle needs to have more energy to get across.'*

These students knew that the height of the barrier relates to the tunneling probability, but still reasoned deterministically. This deterministic reasoning can be seen as an **epistemological learning impediment**.

When students were asked to choose the tunneling probability when the particle's energy level is half the barrier's energy level, two students stated that the particle's energy needs to be higher than the barriers energy level:

> *S6: 'The particle's energy is only half of the barriers energy, so it can never go through the barrier, because the barrier is too big.'*

The other two students thought that there is a specific amount of energy needed to go through the barrier. When we asked what will happen if the particle's energy level is higher, but still lower than that of the barrier, one of them said:

> *S11: 'I just think it needs a specific energy to go through. If it has an energy higher than that, maybe it will go through a little bit easier.'*

These incorrect ideas are related to students' knowledge of energy and barriers in classical systems, so this can be seen as an **epistemological learning impediment**.

EITHER THE WIDTH OR THE HEIGHT OF THE BARRIER SOLELY INFLUENCES TUNNELING

Five students believed that only the width of the barrier influences tunneling. When asked why, these students reasoned deterministically, which can be seen as an **epistemological learning impediment**:

> *S10: 'The particle doesn't go over the barrier, but through it. When the barrier becomes higher, the distance that the particle has to bridge doesn't get longer.'*
>
> *S6: 'When the barrier becomes wider, the particle has more time to lose energy.'*
>
> *S12: 'The height doesn't increase the resistance, only the width does.'*

Two students stated that only the height of the barrier influences tunneling. These students reasoned that only the difference between the energy level of the particle and the barrier influences tunneling;

> *S2: 'The energy difference stays the same … so that doesn't make a difference.'*

These students only reasoned with energy and lacked knowledge of the influence of the barrier width on the wave functions, which is helpful in understanding tunneling. Since the influence of the barrier on the wave function is not part of the curriculum, this can be seen as a **pedagogic learning impediment**.

INCORRECT REASONING WITH FREQUENCY OR AMPLITUDE OF THE WAVE FUNCTION

While explaining tunneling, several students used the difference between the wave function on both sides of the barrier. Some students had difficulties interpreting the wave function. One student linked the energy level to the amplitude of the wave function, which led to the believe that the energy is lower after tunneling. In accordance with section A, this can be seen as a **creative learning impediment**. Another student falsely stated that the frequency of the wave function is higher after tunneling and therefore concluded that the energy after tunneling is higher. This is an error in recollection of the shape of the wave function, not in understanding.

Table 8 The arguments regarding scientific literacy used by the experts (N = 9) during the interviews.

	Goals for scientific literacy	No. of experts	No. of fragments	Topics mentioned
Competencies	Identifying and explaining scientific issues	5	7	Heisenberg's uncertainty principle, energy levels and quantization, Schrödinger's cat, quantum information, quantum computers.
	Being able to make informed decisions	-	-	-
Knowledge	Understanding of scientific concepts	9	67	All
	Understanding the nature of science	3	4	Double slit, wave function.
	Knowing science as a cultural force	4	4	Material properties.
	Being aware of career opportunities	1	1	Quantum information, quantum computers.
	The relationship between science and technology	2	2	Wave-particle duality, probability, semiconductors.
Attitude	Interest in science	-	-	-
	Support for scientific inquiry	1	1	Quantum information, STM.
	Responsibility towards resources and environments	-	-	-
	Seeing the influence of science in everyday life	8	11	Wave-particle duality, 'de Broglie' wavelength, Heisenberg's uncertainty principle, quantization and energy levels, tunneling, atoms, 1D infinite potential well, radioactive decay, spin, fermions/bosons, material properties, lasers.
	Appreciating the beauty of science	2	3	Wave-particle duality, spin, tunneling, quantum information, quantum computers.

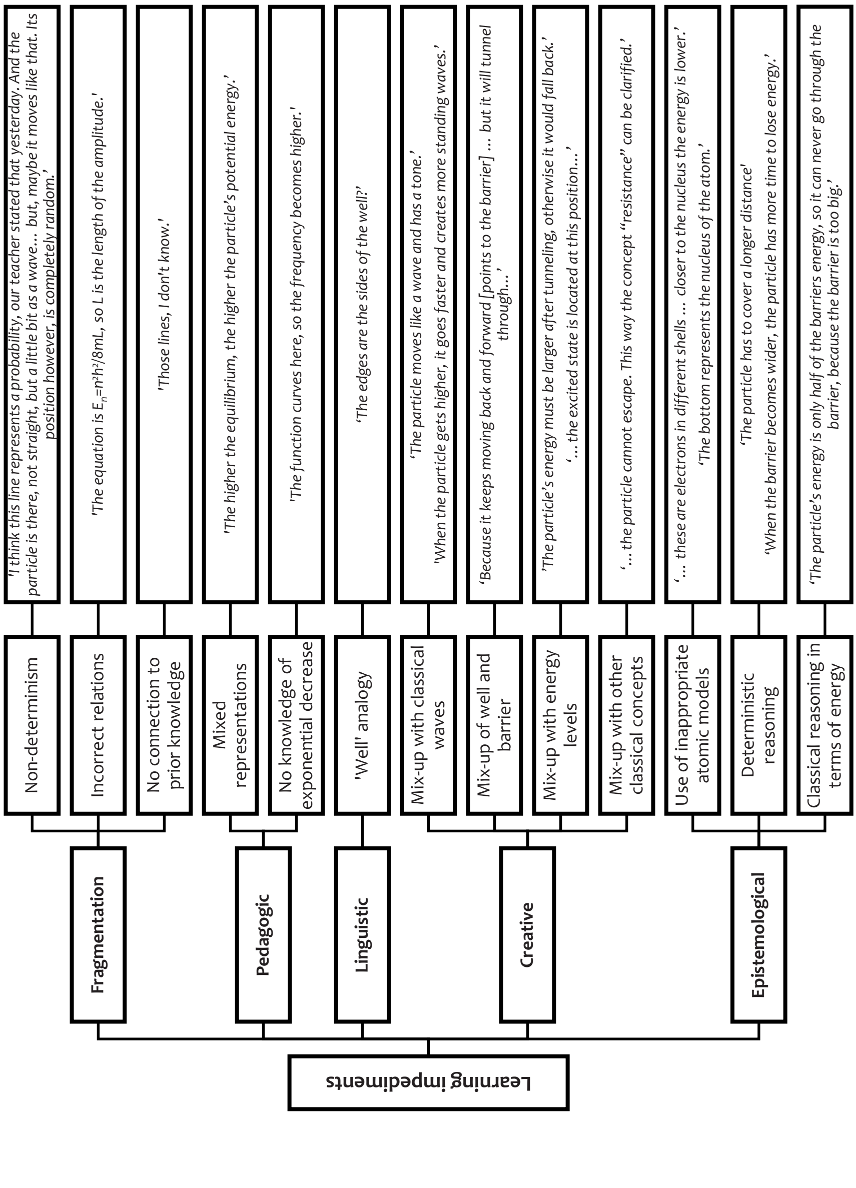


Figure 4 An overview of the observed learning impediments

4.5.3 Overview of observed learning impediments

In the previous paragraphs we identified several observed learning impediments. Table 6 shows the learning impediments that were found for each misunderstanding. We did not observe **deficiency**, **intuitive** and **life-world learning impediments,** which is probably caused by unfamiliarity with, and the abstract nature of, potential wells and tunneling. There were three types of **fragmentation learning impediments**: students could not relate non-deterministic concepts (probability and the wave function) to their deterministic worldview, did not know how the energy equations related to the different representations, and could not relate energy diagrams correctly to their prior knowledge. Two types of **pedagogic learning impediments** were found: students believed that the y-axis of the 1D infinite potential well represented both position and energy, and did not know what happens with the wave function during tunneling. Because flaws in teaching can lead to other learning impediments, sometimes pedagogic learning impediments co-existed with other learning impediments. One **linguistic learning impediment** was found: one student interpreted the 1D infinite potential well literally. The observed **creative learning impediments** can be divided into four types. Students mixed up: the quantum particle’s wave behavior with properties of classical waves, the 1D infinite potential well and the barrier, the amplitude of the wave function with energy states, and the potential well with other classical concepts. Three types of **epistemological learning impediments** were found: students used inappropriate atomic models, reasoned classically in terms of energy, and reasoned deterministically. A schematic overview of the observed learning impediments can be found in Figure 4, illustrated with students’ quotes.

4.5.4 Analysis open ended questions and explanations (n = 98)

Finally, the responses to the explanations of the MC questions and the responses to the OE questions and explanations were analyzed for the complete sample, using the learning impediments in Figure 4. The coding scheme, used for this analysis, is shown in Appendix C. The coding was checked for inter-rater reliability, Cohen’s kappa was 0,67. Table 7 shows the occurrence of the different types of learning impediments within the complete group of students. 14% of the explanations and open ended questions were not answered, or answered by saying it was a guess.

As can be seen in Table 7, few students showed fragmentation learning impediments. This is partly due to the fact that students who could not make sense of a topic or question often did not explain their reasoning. Still, the three categories found in the subset, were also present in the complete group of students. Furthermore, no other students were found within the compete group that had a linguistic learning impediment.

Pedagogic learning impediments were found more often. This type of learning impediment was difficult to discern, since both associated difficulties are implicit and

TABLE 7 The learning impediments found in the explanations, OE questions, and interviews

Learning impediment	Code	Specific difficulty	Students with this difficulty (%)
Fragmentation	F1	Non-determinism	7
	F2	Incorrect relations	6
	F3	No connection to prior knowledge	8
Pedagogic	P1	Mixed representation	19
	P2	No knowledge of exponential decrease	7
Linguistic	L1	'Well' analogy	1
Creative	C1	Mix-up with classical waves	24
	C2	Mix-up of well and barrier	7
	C3	Mix-up with energy level	35
	C4	Mix-up with other classical concepts	4
Epistemological	E1	Use of inappropriate atomic models	6
	E2	Deterministic reasoning in terms of movement	47
	E3	Classical reasoning in terms of energy	32

result in other learning impediments. Difficulty caused by the mixed representation of the potential well and the wave function (Figure 2) was assigned when students specifically linked the y-axis of the wave function to energy. Still, many students mixed up the amplitude and the energy level without specifically doing this, but this does not necessarily rule out a pedagogic learning impediment. This was also the case for knowledge of the exponential decrease of the wave function within the barrier, since many students had difficulty explaining tunneling and often just stated trivialities. In the cases where this impediment was observed, students tried to explain why the height and width did or did not influence tunneling probability.

Many students showed creative or epistemological learning impediments. The creative learning impediments were often found when students were interpreting the wave function; students mixed up the amplitude with energy, or described electrons as particles that vibrate or move like a wave. Mix-ups with other (semi) classical concepts were mix-ups with nuclear fusion and cell walls. The epistemological learning impediments were mainly found when students were reasoning about tunneling. Many students reasoned with distance, or stated that it would take more time or effort for a particle to go through a wider barrier. Also, many students reasoned that a certain amount of energy is needed, some because they reasoned that energy is lost, others because they reasoned that the particle's energy level needs to be higher than the barrier's energy level.

4.5.5 Overview of observed synthetic models

The observed learning impediments show that students have difficulty integrating QM in their prior knowledge. The observed fragmentation learning impediments and the second pedagogic learning impediments (P2) were based on missing knowledge. The other pedagogic, linguistic, creative, and epistemological learning impediments were expressed as incorrect models in which students had added the new concepts incorrectly to their existing framework. When looking at Table 7 and Figure 4, P1 and C3 are related learning impediments, which correspond to a similar incorrect model. Hence, there are 8 main synthetic models that were observed: 1) the potential well is a physical well, 2) a mix-up with classical waves, 3) a mix-up of the potential well and the barrier, 4) a mix-up with the energy level, 5) a mix-up with other classical concepts, 6) the use of inappropriate atomic models, 7) deterministic reasoning in terms of movement, and 8) classical reasoning in terms of energy. Table 8 gives an overview of these synthetic models, together with a visual representations that is based on students' wording.

4.6 CONCLUSIONS

In this study we investigated secondary school students' understanding of potential wells and tunneling. In this section, we give an overview of the main results and draw conclusions based on these results. Additionally, we will describe the implications for researchers and teachers.

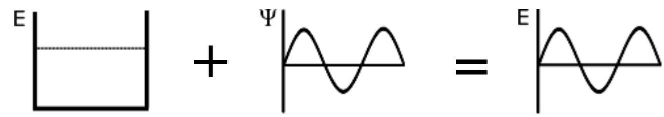
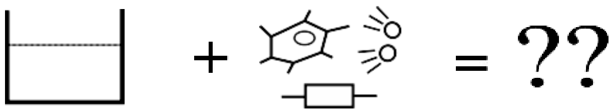
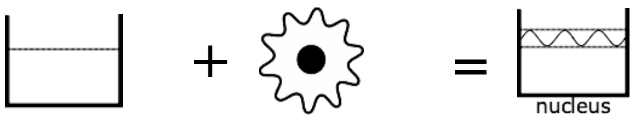
4.6.1 Students' understanding of potential wells and tunneling

Analysis of the conceptual knowledge test showed that Dutch secondary school students experienced difficulties that were also reported for students at the undergraduate level; students mixed up classical and quantum models, and overgeneralized prior classical knowledge.

In the MC questions the students showed two main difficulties: 1) 24% of the students believed that the amplitude or equilibrium of the wave function is related to the energy level, 2) approximately 50% of the students believed that energy was lost during tunneling. We also observed that students believed that there was a difference in the influence height of the barrier in comparison to the influence of the width of the barrier.

In a qualitative analysis of students' explanations and answers to the open ended questions we found several underlying difficulties. Regarding the 1D infinite potential well and wave functions, the major difficulties were related to wave functions. While reasoning about the 1D infinite potential well, most of the students knew that this model represented a limited space in which a particle is contained. However, several students explained the model incorrectly, referring to semi-

Table 8 An overview and a visual representation of the eight synthetic models found in this study

Synthetic model	Visual representation and quote
1 the potential well is a physical well (L1)	 *‘The edges are the sides of the well?’(S12)*
2 a mix-up with classical waves (C1)	 *‘The particle moves like a wave …’ (S12)*
3 a mix-up of well and barrier (C2)	 *‘Because it keeps moving back and forward [points to the barrier] …' (S13)*
4 a mix-up with energy level (C3/P1)	 *The electron is moving and gets a higher energy, because the second part of the sine is higher than the first.’ (S11)*
5 a mix-up with other classical concepts (C4)	 *‘This way the concept “resistance” can be clarified.’ (S12)* *“… waves with a smaller wavelength are further from the cell wall’ (S37)*
6 the use of in-appropriate atomic models (E1)	 *‘The nodes and antinodes show the shape of the shell in which the particle is located’ (S59)*
7 deterministic reasoning in terms of movement (E2)	 *‘The particle doesn’t go over the barrier, but through it.’ (S10)*
8 classical reasoning in terms of energy (E3)	 *‘… the particle needs to have more energy to get across.’ (S5)*

classical atomic models, and only a few students described the 1D infinite potential well model in terms of potential energy. Additionally, the interviews showed that several students did not know what the vertical lines in this model represented. These results show that students still have difficulty relating the 1D infinite potential well model to their prior knowledge of atomic models and potential energy. The latter can be explained by the fact that there is little emphasis on potential energy other than gravitational energy in the Dutch secondary school curriculum. While interpreting wave functions, students often showed creative learning impediments. About one-fourth of the students created a mixed-up model of the wave and particle behavior in which the particle vibrated or moved across a sinusoidal pathway. Over one-third of the students mixed up the amplitude or equilibrium of the wave function with the energy level. These mix-ups show that students have difficulty integrating the wave-particle duality within their existing knowledge structures.

With respect to tunneling, students mainly showed epistemological learning impediments. Students often reasoned deterministically and stated that a particle needed more time, effort or distance to tunnel through a wider barrier. Many students had difficulty reasoning with energy and stated that the particles' energy needed to be higher than the barrier's energy, or at least needed a minimum energy level. Students had difficulty integrating the concept of probability density in their deterministic thinking. Additionally, students lacked knowledge of the behavior of the wave function in potential barrier, which made it difficult to reason about what happens with the energy and probability.

In retrospect, different learning impediments were found for potential wells and wave functions, and tunneling. The main learning impediment for potential wells and wave functions was creative, while for tunneling it was epistemological. This can be explained by the way QM is implemented in the Dutch secondary school curriculum. For the 1D infinite potential well, there is emphasis on both the energy representation and the wave function, and when students try to make sense of it, they try to integrate these two representations into one model. For tunneling, however, only the energy representation is used, causing students to connect this new topic only to their prior knowledge of energy or energy diagrams, the latter of which students have mainly seen while learning classical mechanics.

From the results of this study we can conclude that secondary school students have difficulty incorporating new knowledge of the 1D infinite potential well and tunneling into their thinking. Mix-ups (creative learning impediments) arise mainly when students have to work with both energy and wave function representations, while the use of inappropriate classical reasoning (epistemological learning impediments) often occurs when students describe potential energy diagrams.

4.6.2 Implications

The research presented here showed that Dutch secondary school students have several difficulties in understanding the 1D infinite potential well and tunneling after being taught QM. The main problems were related to incorrect connections with prior knowledge. Some of the synthetic models found in this study are related to a lack of knowledge of the wave function, other synthetic models are related to students' inability to interpret potential energy diagrams. According to Vosniadou, Vamvakoussi and Skopeliti [26], synthetic models can be avoided when instruction shows students how to connect their prior knowledge to the new concept. For students' understanding of QM, this could imply that teachers should support understanding by introducing the wave function for tunneling, and connect it to prior knowledge the wave function, which students have already encountered in the context of the 1D infinite potential well. It could also imply that teachers should also make an effort to promote students' understanding of classical potential energy diagrams in order to improve their prior knowledge, and connect this to QM energy diagrams.

However, since QM is inconsistent with the classical models that students have learned, showing how to integrate QM with students' prior knowledge may not be so straightforward. At present, it is not clear what influence greater prior knowledge of underlying concepts, such as waves and potential energy, has on students' understanding of QM. Additionally, it is not clear to what extent students need to integrate QM within their prior knowledge in order to be able to decide which model is needed. Therefore, there is a need for more research into the influence of prior knowledge on students' understanding of QM, and into the prerequisites students need to be able to make an appropriate choice between representations.

4.7 Acknowledgements

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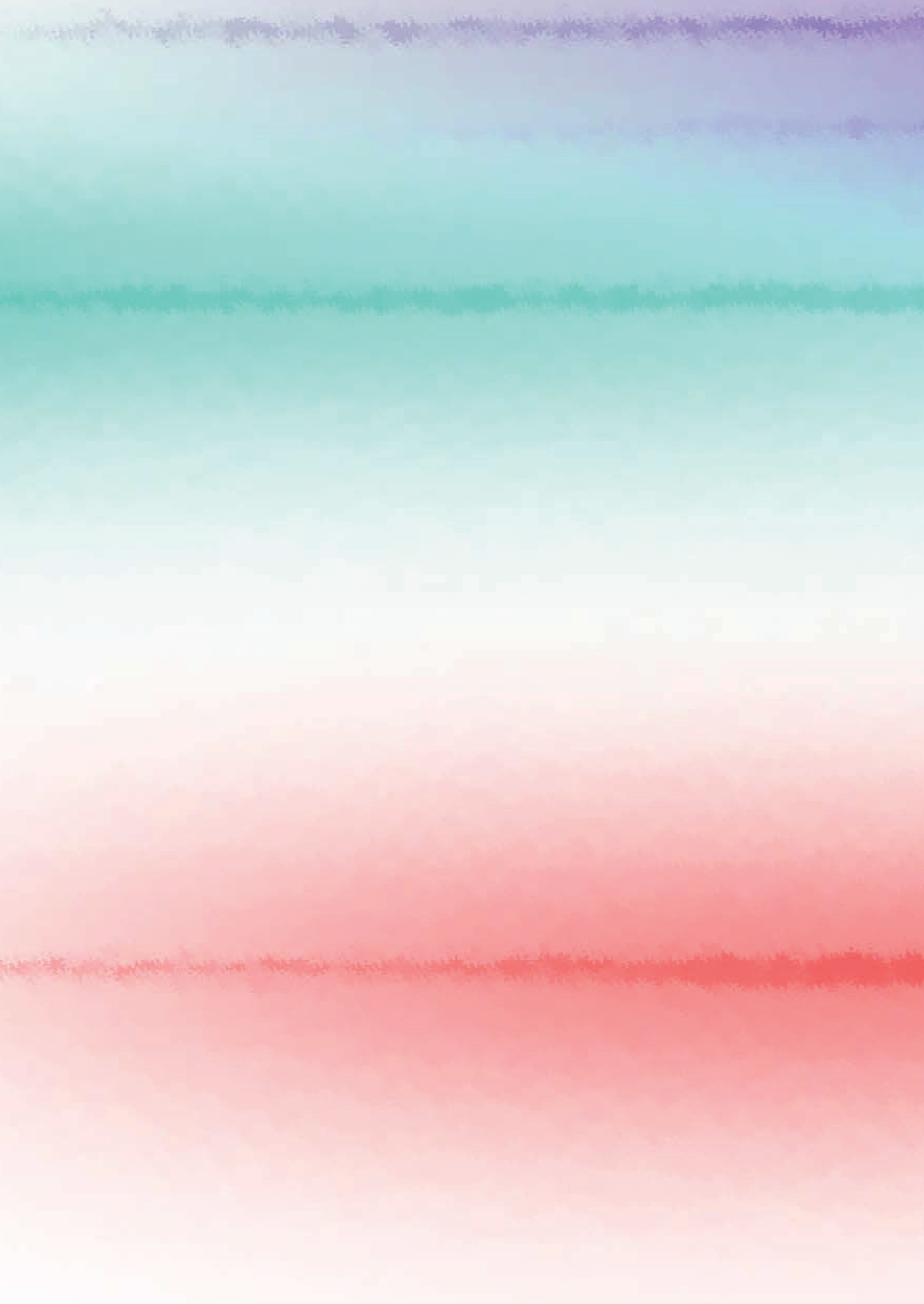

5

PRIOR KNOWLEDGE OF POTENTIAL ENERGY AND THE UNDERSTANDING OF QUANTUM MECHANICS

In this study we investigated the relations between the understanding of energy diagrams and the understanding of the potential well and tunneling. For this a quasi-experimental intervention was used, in which the experimental group received additional lessons on classical energy diagrams before being taught quantum mechanics. Two tests were developed in order to determine students' understanding of potential energy and quantum mechanics. The potential energy test was used after the lessons on potential energy, and before quantum mechanics instruction. The potential energy test addressed students' understanding of energy in relation to force, position and velocity. The quantum mechanics test was used as a pre- and post-test, and focused on the understanding of the influence of tunneling on energy and probability, and on the understanding of the relation of potential energy to energy and probability. The results of the tests showed that the experimental group not only had better understanding of potential energy diagrams, but also of quantum mechanics even before they were being taught quantum mechanics. Analysis of the tests also showed that there was a significant correlation between the understanding of potential energy diagrams and the understanding of quantum mechanics.

BASED ON: K. KRIJTENBURG-LEWERISSA, H.J. POL, A. BRINKMAN AND W.R. VAN JOOLINGEN, PRIOR KNOWLEDGE OF POTENTIAL ENERGY AND THE UNDERSTANDING OF QUANTUM MECHANICS. (SUBMITTED)

5.1 Introduction

In recent years, quantum mechanics (QM) increasingly has become part of secondary school curricula[1]. Since QM is rather abstract and counterintuitive, this has resulted in an increased interest into the investigation of methods for introducing QM at a more conceptual level[2]. Recent research into the introduction of QM at the secondary level has focused mainly on better understanding of students' difficulties regarding the counterintuitive wave-particle duality[3-9], and some research has focused on two-level quantum states[10, 11]. Another way of introducing QM, which has been investigated less frequently, is to introduce the infinite 1D potential well and tunneling[12]. The potential well and tunneling have been investigated for the undergraduate level[13-15]. However, even though experts consider this topic important[16], there has been little research into secondary school students' understanding of the potential well and tunneling[17]. In the Netherlands, the potential well and tunneling have recently been introduced at the secondary level. The introduction of the wave behaviour of quantum entities by using the potential well seems rather abstract and difficult for students to understand. But, in contradiction to the wave-particle duality, the potential well offers ways of approaching QM that are already familiar to secondary school students in the classical context. Students already are familiar with other forms of potential energy (PE), such as gravitational and elastic energy, which can be more easily connected to real-life experiences than QM. Therefore, this approach could be used to create better understanding of QM in terms of energy, by reducing the gap between students' prior understanding and QM. In previous research[18], we have observed that several difficulties in learning QM are related to students' inability to interpret PE diagrams. Therefore we have investigated if students' understanding of QM is influenced by their prior knowledge on PE diagrams.

5.2 Background

Teaching QM at the secondary school level is a challenge, because it is counterintuitive and conflicts with students' classical thinking[19]. When learning classical mechanics, students have learned about particles and waves, which are intrinsically different concepts. Particles have properties such as position, mass and size, whereas waves have properties such as wavelength and amplitude. In QM an electron can have both particle and wave properties, which is inconsistent with students' prior learning. From the perspective of learning theory, this raises difficulties. According to Chi[20], there are three ontological categories; entities, processes and mental states. Robust misconceptions occur when new concepts are miscategorised and students need to 'move' a concept from one ontological category to another. Since particles belong to the ontological category 'entities' and waves to the category 'processes', there is a need for a new ontological category for learning QM. Students need to embrace a new, flexible, ontology[21], in which the quantum entity can have particle or wave properties, depending on the context. The need for this new ontological category, and the overlap with students existing ontological categories makes learning QM a complex process.

In conceptual change theory, the most common conceptual change strategy is to create a cognitive conflict[22], which shows that students' prior thinking is incorrect. Therefore, many research focuses on showing the conflict of the double slit experiment with students' expectation based on prior, classical, knowledge in order to show students the need of a new theory. But according to Posner *et al.*[23], in order to create conceptual change, there is also need for a new theory, which is understandable, logical and useful. So, even when students see that classical mechanics is not capable of explaining quantum phenomena, they still need to accept that quantum mechanics does explain it. For the wave-particle duality, this remains a challenge, because students have to learn that there is a new ontological category. An issue that makes it even harder, is the difficulty of interpreting the wave-particle duality. In order to make QM understandable, logical and useful, students need to see that some classical concepts still apply in QM. Vosniadou and Skopeliti[24] propose to design curricula aiming to reduce the gap between students' prior knowledge and the new knowledge. Upper-level secondary school students are familiar with potential energy in the context of gravitational and elastic energy, and they are able to relate this to real-life experiences. Therefore, introducing a model system, such as the "infinite potential well" and connecting it to compatible prior knowledge on energy diagrams could be a way to reduce the gap between initial knowledge and QM. At the undergraduate level, there has been some research into students' understanding of potential energy and atomic-molecular interactions. Becker and Cooper[25] observed several intuitive and incorrect interpretations of PE. They concluded that it is important to promote prior knowledge of PE and help students to make connections between PE and atomic-molecular interactions.

In previous research into students' understanding of QM[18] we have found that students have several difficulties in understanding the PE diagrams of the 1D infinite potential well and tunnelling. The main problems were related to incorrect relations with prior knowledge, which resulted in inconsistent thinking. Some of the students' difficulties were related to lack of knowledge of the wave function, other difficulties were related to students' inability to interpret energy diagrams. In order to make it easier to relate QM to prior knowledge of energy diagrams, instructional materials were developed to promote students' prior knowledge of energy diagrams in a classical context. A quasi-experimental intervention was designed to investigate whether there is an actual relation between understanding energy diagrams and understanding QM. This research aimed to answer the following questions:

(1) Can we improve students' understanding of PE?

(2) Does an increase in understanding of PE lead to a better understanding of QM?

(3) Is there a relation between the understanding of PE and QM?

Table 1 Overview of the module on PE and energy diagrams

Chapter	Themes
1. Introduction	Work and energy
	Energy conservation
2. Earths' gravitation	Gravitational force and energy on earth
	Interpreting PE diagrams: the height of a ball
	Advanced exercises: roller coasters
3. Elastic energy	Elastic force and energy
	Interpreting PE diagrams: a mass-spring system
	Advanced exercises: bungee run and bungee trampoline
4. Universal gravitation	Gravitational force and energy
	Interpreting PE diagrams: a satellite launch
	Advanced exercises: space travel
5. Force and PE	Comparison of a F,x- and E,x-diagram
6. Electric energy	Force and energy of point charges
	Force and energy in homogeneous electric fields
	Advanced exercises: alpha decay

5.3 METHOD

An intervention was conducted at Dutch secondary schools, in the final year of pre-university education. Teachers of ten different secondary schools were willing to participate in our study. 13 classes (with in total 234 students) were used as experimental groups, 11 classes (*n*=157) as control groups. In order to create difference in understanding between the experimental and control groups, instructional materials on PE were created. Tests were used to compare students' understanding of PE and QM.

5.3.1 CREATION OF INSTRUCTIONAL MATERIALS

We created a module regarding potential energy and energy diagrams as an addition for teaching quantum mechanics. The module was created in order to; 1) refresh students' knowledge on gravitational energy, elastic energy and electric energy, 2) explain that these are all types of PE, and 3) learn students to interpret energy diagrams in terms of velocity, position, and force. The materials were pre-tested with a small group of secondary school students. Evaluation with a preliminary pre- and post-test gave a first indication that students had more knowledge of PE after they worked with the materials. Based on student and teacher feedback, the materials were adjusted. A schematic overview of the final module can be found in Table 1.

5.3.2 DESCRIPTION OF THE TESTS

To determine students' understanding of PE diagrams, potential wells and tunneling we created two tests; 1) a test regarding students' understanding of energy and 2) a test regarding students' understanding of the potential well and tunneling. The energy test focused on the ability to relate energy and energy diagrams to the position (**PE↔x**) and velocity (**PE↔v**) of, and forces (**PE↔F**) working on an object.

Table 2 Overview of the energy test

Topic	nº	Addressed understanding of PE	Description of the question
Swing	PE1	PE ↔ x	Compare PE for two different heights
	PE2	PE ↔ x	Determine the amplitude based on PE and E
	PE3	PE ↔ F	Determine at which position $\sum \vec{F} = 0$
Falling stone with a spring	PE4	PE ↔ x	Determine the lowest position in a system
	PE5	PE ↔ v	Determine the maximum KE
	PE6	PE ↔ F	Determine at which position $\sum \vec{F} = 0$
Bungee jump	PE7	PE ↔ v	Choose the correct KE diagram
	PE8	PE ↔ x	Draw the gravitational energy
	PE9	PE ↔ x/v	Describe the movement based on PE
Lennard-Jones potential	PE10	PE ↔ F	Determine where there is an attractive force
	PE11	PE ↔ F	Determine where there is a repulsive force
	PE12	PE ↔ F	Compare F at different positions
	PE13	PE ↔ v	Compare KE at different positions

Table 3 Overview of the QM test

Topic	nº	Addressed understanding of PE	Description of the question
1D infinite potential well	QM1	PE ↔ KE	Determine KE in- and outside the well
	QM2	PE ↔ P	Determine P in- and outside the well
	QM3	PE ↔ KE	Determine KE at different regions in a ‘step well’
Tunneling	QM4	PE ↔ E	Compare E before and after tunneling
	QM5	PE ↔ KE	Compare KE before and after tunneling
	QM6	PE ↔ P/E	State how barrier height influences P and E
	QM7	PE ↔ P/E	State how barrier width influences P and E
	QM8	PE ↔ KE	State which answers will change if PE on the right side of the barrier is higher than on the left

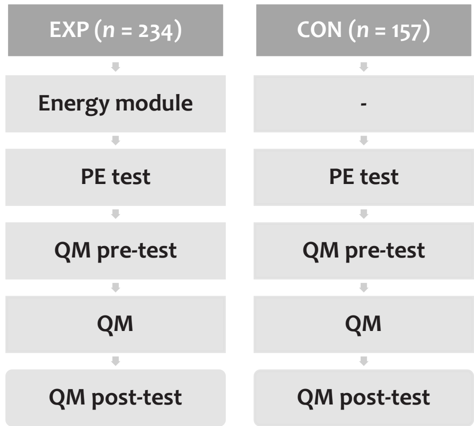

Figure 1 Experimental Procedure

This test existed of 13 questions in four different contexts. The questions of the energy test are described in Table 2, the complete test can be found in Appendix D. The QM test on potential wells and tunneling consisted of seven questions; 3 questions regarding the potential well and 5 questions on tunneling. The QM test focused on the ability to relate the PE diagrams to probability, kinetic energy and total energy. The questions of the QM test are described in Table 3, the complete test can be found in Appendix E.

5.3.3 Procedure

The module and tests were used at ten different secondary schools, in the final year of pre-university education. The group sizes varied between 14 and 28 students. The quasi-experimental intervention consisted of the implementation of the energy module and the use of the energy and QM test (see Figure 1). The experimental groups (n=234, 13 classes) worked with the module and then took the tests, the control groups (n=157, 11 classes) immediately started with the tests. After the students had taken the energy- and pre-test, teachers would go back to their normal program of teaching QM. The books and methods used for teaching QM varied for the different teachers and schools. Afterwards a post-test was given to determine students' final understanding of potential wells and tunneling.

5.3.4 Data analysis

In order to investigate if the developed tests could be used to determine understanding of the three different aspects of the understanding of PE and the understanding of QM, we did an explorative factor analysis using principle component analysis[26] (PCA). PCA is a method for dimension reduction, which can be used to reduce a large set of correlated variables into a smaller set of unrelated principal components. These principal components are linear combinations of the original variables. To explore the differences in understanding of PE and QM between the experimental and control group, we performed an independent sample t-test of the different tests, and calculated the effect size[27]; Cohen's *d*. The p-value of the t-test will give information on the existence of a significant difference, the d-value will give information on the size of this difference between the experimental and control group. The relation between the understanding of PE diagrams and the understanding of potential wells and tunneling, was investigated by calculating the Pearson correlation coefficient between the results of the tests, and conducting a path analysis[28]. A path analysis is a visual representation of the different variables, in which the regression coefficients of the different relations between these variables are shown.

5.4 Results

5.4.1 Test evaluation with principal component analysis

A PCA was used to analyse the tests. During the analysis of the PE test, four questions were found to be outliers and were omitted. The analysis showed that the remaining PE test consisted of three components (see Table 4). These components were in line with the content of the questions:

Component E1 – Understanding of the relation between PE and force (***PE↔F***);
Component E2 – Understanding of the relation between PE and position (***PE↔x***);
Component E3 – Understanding the relation between PE and movement or velocity (***PE↔v***).

For the PCA of the QM test, we used the results of the post-test. The PCA resulted in two components (see Table 5), which were reasonably consistent with the content of the questions. What stood out is that one of the questions on tunneling was strongly related to the questions on the potential well. This can be explained by the fact that this question was not focusing on tunneling itself, but on the influence of the shape of the PE diagram on kinetic energy and probability, which was also the focus of the questions regarding the potential well. Hence, the two components are:

Component Q1 – Understanding of the influence of tunneling on energy and probability (**TU↔E/P**);
Component Q2 – Understanding the relation of PE to energy and probability (***PE*↔E/P**).

The components found in this analysis were used in the further analysis of students' understanding.

Table 4 Pattern matrix of the PE test

	Component		
	E1	E2	E3
EN1		,66	
EN2		,66	
EN4		,50	
EN5			,70
EN7			,63
EN8		,36	
EN9			,44
EN10	,87		
EN11	,88		

Table 5 Pattern matrix of the QM test

	Component	
	Q1	Q2
PO1		,74
PO2		,60
PO3		,67
PO4	,67	
PO5	,57	
PO6	,68	
PO7	,77	
PO8		,42

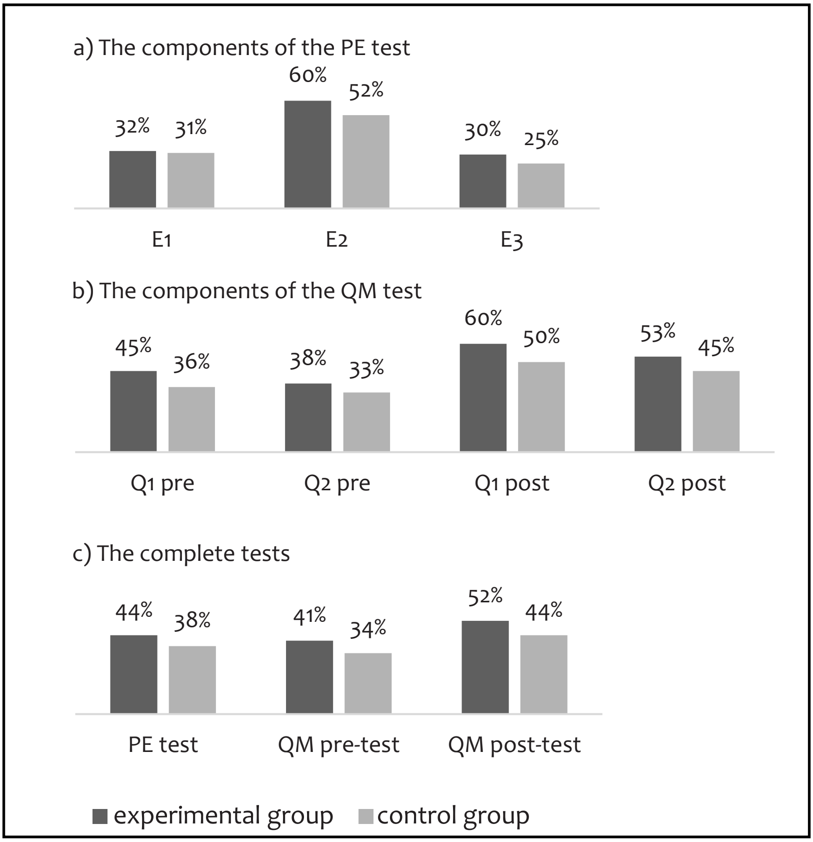

Figure 2 Scores of the experimental and control group

Table 6 Results of the independent sample t-test for the scores of the experimental and control group

			Group		T-test		Effect size
			EXP	CON	t-value	p	Cohen's d
Components of PE test	**E1**	**M** **SD**	,32 ,40	,31 ,39	,26	,797	0,03
	E2	**M** **SD**	,60 ,22	,52 ,21	3,74	,000	0,39
	E3	**M** **SD**	,30 ,25	,25 ,23	2,12	,035	0,22
Components of QM pre-test	**Q1**	**M** **SD**	,38 ,29	,33 ,28	1,55	,122	0,16
	Q2	**M** **SD**	,45 ,25	,36 ,28	3,37	,001	0,34
Components of QM post-test	**Q1**	**M** **SD**	,51 ,34	,44 ,33	2,12	,035	0,22
	Q2	**M** **SD**	,53 ,29	,45 ,31	2,46	,015	0,25
Complete tests	**PE test**	**M** **SD**	,44 ,17	,38 ,15	3,48	,001	0.36
	QM pre-test	**M** **SD**	,41 ,20	,35 ,22	3,19	,002	0.33
	QM post-test	**M** **SD**	,52 ,26	,44 ,256	2,80	,005	0,29

5.4.2 Differences between the experimental an control group

To determine the differences in understanding of the experimental and the control group, we analysed the test scores for the questions categorised into the different components found in the previous paragraph. The students' scores for the different tests are shown in figure 2. As can be seen, the experimental group outperformed the control group, both on the separate components as on the complete tests. An independent-samples t-test was conducted to compare the scores of the experimental and control group. The results of the t-test and the effect sizes are shown in table 6. The t-test showed that there was a significant difference in understanding between the experimental and control group for component E2 (d=0,39, p=0,000) and E3 (d=0,22, p=0,035) of the PE test, Q2 (d=0,34, p=0,001) of the pre-test and Q1 (d=0,22, p=0,035) and Q2 (d=0,25, p=0,015) of the post-test. Also can be seen that there is a significant difference in understanding for all the complete tests. However, the effect sizes are relatively small.

5.4.3 Relation between the understanding of energy diagrams and the understanding of potential wells and tunneling

In order to analyse if there is a relation between the understanding of potential energy diagrams and the understanding of potential wells and tunneling, we calculated the Pearson correlation coefficient between the results of the tests. The results (Table 7) show that there is a significant, but relatively small, correlation between the scores of the PE test and the scores of the QM test, especially for component E2: the understanding of the relation between PE and position.

To examine the relation between the understanding of the different aspects of energy diagram more thoroughly, a path analysis was conducted. In Figure 3 the grey arrows show the regression coefficients of the three components of the PE test for the results of the QM pre-test. This represents the influence that the different components have on the prior knowledge of QM. The black arrows show the regression coefficients of the three components of the PE test and the pre-test for the results of the QM post-test. This represents the influence of the prior understanding of energy diagrams and QM on the final understanding of QM. This figure shows that students' score for interpreting energy diagrams in terms of position has de largest direct and indirect influence on the final understanding of QM.

Table 7 The correlation between the understanding of energy and QM

			Components of QM pre-test		Components of QM post-test		Complete QM test	
			Q1	Q2	Q1	Q2	Pre	Post
Components of PE test	E1	r		,13			,12	
		Sig.		,008			,023	
	E2	r		,29	,13	,27	,20	,24
		Sig.		,000	,013	,000	,000	,000
	E3	r			,12			,11
		Sig.			,019			,034
Complete PE test		r		,27	,15	,20	,20	,21
		Sig.		,000	,002	,000	,000	,000

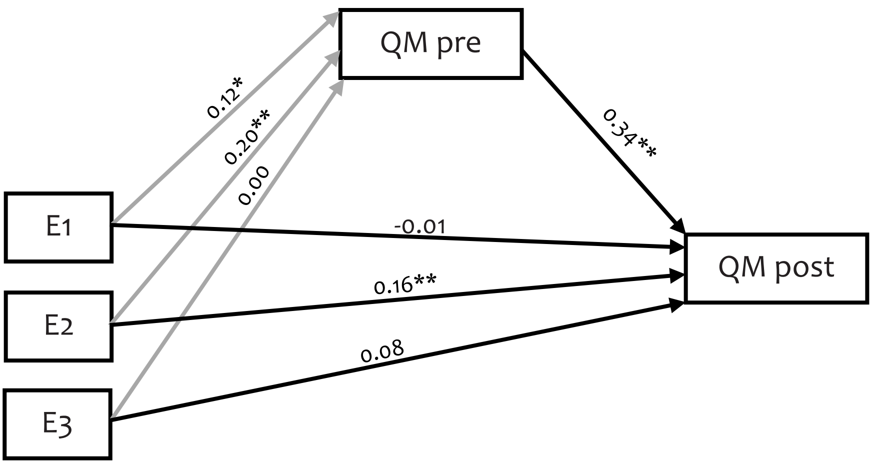

Figure 3 A path analysis of the understanding of energy diagrams, prior and final understanding of QM.

5.5 Conclusions

5.5.1 The relation between the understanding of potential energy and quantum mechanics

We have investigated the relationship between the understanding of energy diagrams and the understanding of the potential well and tunnelling. Analysis of an PE test and a QM test showed that there was a significant difference in understanding between the control and experimental group. The experimental group scored significantly better on the PE test and on the QM pre- and post-test. Remarkable was the fact that the experimental group had better understanding of QM even before students were taught QM. These results clearly show that QM understanding is supported by a good understanding of the classical concept of potential energy.

When looking at the different components of the understanding of energy and the understanding of QM, there was also a significant difference in test scores. The experimental group scored significantly better on their understanding on energy diagrams in terms of position (E2) and velocity (E3). However, no significant difference was found for the understanding of energy diagrams in terms of forces acting on an object (E1). Analysis of the two components of the QM test showed that the experimental group scored significantly better on the understanding of the relation of PE to energy and probability (Q2) in the pre- and post-test. For the understanding of tunnelling (Q1), there was only a significant difference for the scores on the QM post-test.

The analysis of the Pearson correlation between the different components of the energy and QM test showed that there was a significant correlation between the scores on the energy and the QM pre- and post-test. The most prominent correlation was found between the understanding of energy diagrams in terms of position (E2) and the understanding of the relation of PE to energy and probability (Q2). The path analysis confirmed that the understanding of energy diagrams in terms of position had the greatest influence on the understanding of QM before and after QM instruction.

In this investigation we have seen that students who received additional lessons on potential energy scored significantly better at the QM test. We also have seen that there is a significant correlation in students' understanding of PE and QM. Therefore, we can conclude that an increase in understanding of PE diagrams does lead to better understanding of QM. Knowledge of PE has a distinct and significant influence on the understanding of QM. The results therefore suggest that understanding PE is an important part of understanding the potential well and tunnelling, and can be used to reduce the gap between students' prior knowledge and QM.

5.5.2 Limitations and implications of this study

The intervention used in this study consisted of providing instructional materials on PE, without teacher training or instructional materials relating PE specifically to QM. Additionally, the books and methods used for teaching QM varied for the different teachers and schools. This may have influenced the outcomes of this study and diminished the effect sizes and correlations. However, this leads to the expectation that effects might be even higher when performing the intervention under more controlled conditions.

This leads to an opportunity for researchers in the field of QM education. This study shows that there is a relation between understanding PE and QM, but the materials used in this study are not yet refined and optimized. In order to improve QM teaching at the secondary school level, there is a need for design-based research. Materials, stimulating knowledge of PE, need to be designed, implemented, analysed and improved. There is also a need for research in which is investigated how QM can be adequately connected to students' prior knowledge on PE. Additionally, the role of the teacher should be taken into consideration. Teachers could play a major role in connecting QM to students' prior knowledge. Teachers should be aware of this, not only in the context of QM, but for teaching physics in general.

This research also has implications for curriculum development in physics education. It shows the importance of prior knowledge for learning QM and for physics in general. Additionally, this research showed the importance of students' understanding of energy, which is a central concept in physics. This raises the question of the importance of the central concepts of physics (e.g. energy, force, and momentum) for the understanding of other topics. More emphasis on these central concepts within the physics curriculum, as binding principles between all physics domains, could increase cohesion , and may lead to students that are more aware of the nature of physics and have deeper understanding. Therefore, curriculum developers need to consider: 1) what prior knowledge is needed for the different topics within the curriculum, and 2) how the different topics in the curriculum are related to the central concepts of physics. A curriculum in which the topics build on previous topics and in which connection between related topics are made, will lead to better physics understanding.

5.6 Acknowledgements

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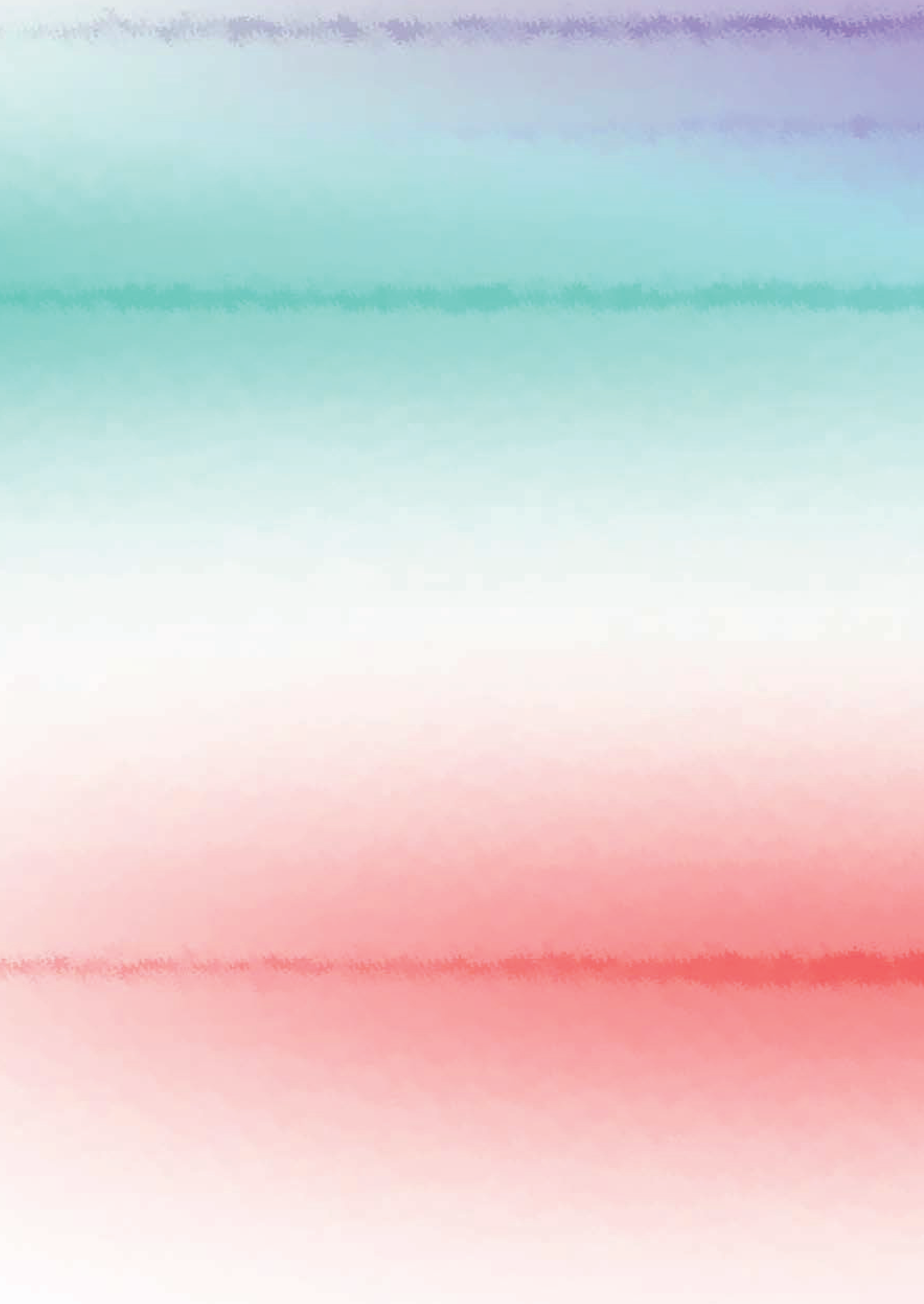

6

Conclusion & Discussion

6.1 Introduction

Quantum mechanics (QM) has had an major impact on scientific thinking, and still has an enormous impact on society, present-day research, and the development of high-end technology. For this reason, QM has become part of secondary school curricula in many countries. In order to design a well-balanced QM curriculum at the secondary school level, there is a need to investigate what QM topics we should teach, what difficulties secondary school students encounter while learning QM topics, and how we can help students to incorporate QM concepts into their existing knowledge structures. Therefore, in the four studies presented in this dissertation, we have investigated:

(1) the current state of research on students' understanding, teaching strategies, and assessment methods for QM at the secondary school level;
(2) what topics Dutch experts (in the field of QM and related research fields) consider to be important to teach at secondary schools;
(3) the misunderstandings Dutch students have after learning QM, and;
(4) the relation between prior knowledge of potential energy and QM understanding.

In this chapter we will first summarize these studies and reflect on the main findings of each separate study. After this, we will reflect on the overall results of the four studies. Finally, we will consider what the results of our studies imply for educational research and curriculum design.

6.2 Reflections on the results of each study

6.2.1 Insights into teaching quantum mechanics in secondary and lower undergraduate education

Our literature review showed that there has been done much research into students' understanding of the wave-particle duality and atoms at the secondary and undergraduate level. Less research has been done into students' difficulties in understanding of the wave function and complex quantum behavior. Moreover, the research for students' understanding of the wave function only focused on the undergraduate level. Research shows that the main problem in learning QM is that students tend to hold on to their classical way of thinking. For the wave-particle duality this led to a mix-up of wave and particle behavior, for wave functions this led to an overliteral interpretation of analogies, and for the atom this led to students holding on to semi-classical atomic models. Overall, students have difficulty to incorporate the QM knowledge into their existing classical framework.

The review also showed that there are several research tools and concept tests, but mainly at the undergraduate level, and mainly addressing only parts of QM. Only one test, the QMCS, covers wave-particle duality, wave functions, atoms and complex QM behavior. Still, this test has not been thoroughly evaluated for secondary education, and includes too few questions for statistical analysis.

The results of the first study also showed that various methods have been used to address difficulties in learning QM. Only some of these methods were evaluated for their impact on students' understanding. These evaluations showed students do not necessarily need a mathematical approach for understanding QM. Additionally, three approaches have been shown to improve students' understanding: (1) emphasis on interpretations; (2) emphasis on the development of and differences between different atomic models, and (3) active learning. Also, many multimedia applications have been designed. These applications were mainly evaluated for practical use, and more research into their influence on students' understanding is needed.

Much has been written and published regarding teaching introductory quantum mechanics. But, because QM has just recently entered many secondary school curricula, not much empirical research has been conducted. For this reason, this review included many non-empirical studies. Our literature review therefore gives a good overview of common difficulties, but there is still much to learn about students' underlying difficulties and the impact of specific teaching strategies. What is clear, is that students have difficulty with the non-classical and non-deterministic way of thinking. This review showed that this difficulty plays a role in all of the subtopics of QM. There are several suppositions of how to effectively address these difficulties, but there is a need for more empirical research into the effect of different teaching strategies. A reason for the lack of empirical research can be the absence of adequate research tools for the secondary and undergraduate level. In order to advance research into the teaching and understanding of QM, the design of an appropriate and well-evaluated concept test is of the utmost importance.

This study gives a clear overview of the current knowledge of teaching QM at the secondary school level, and the challenges there are for research. The research into teaching QM at the secondary school level is an upcoming field and there is still much to be done.

6.2.2 Key topics for quantum mechanics at the secondary schools

One aspect that needed more research was which quantum topics are important to teach at the secondary school level. Therefore, in the second study, experts in the fields of QM and related research fields were asked what topics they considered to be important for teaching QM at secondary schools. For this purpose, the Delphi technique was used. In three rounds the experts were asked to propose, select and rank QM topics. The Delphi study showed that there was moderate to strong consensus on the inclusion of the following concepts: (1) wave-particle duality, (2) particle behaviour of light, (3) wave functions, (4) de Broglie wavelength, (5) probability, (6) energy levels and quantization, and (7) Heisenberg's uncertainty principle. The following examples were considered important by the majority of the experts: (1) the double slit experiment, (2) spectral lines, (3) the photoelectric effect, (4) atomic structure, (5) the 1D infinite potential well, (6) the hydrogen atom, and (7) the periodic table. There was no consensus about which applications should be part of the curriculum. Interviews showed that the experts' opinions were based mainly on the idea that students should have a certain understanding of important scientific concepts. Topics that were considered too complex or abstract were viewed as less essential.

When looking at the Dutch physics curriculum and the international core curriculum[1], one can see that these corresponds substantially with the topics that experts considered to be important. However, experts mainly reasoned from the 'knowledge' perspective. For curriculum design other aspects, such as students' understanding of the importance of QM for the technological development of society, should be considered too. However, there was little consensus on the topics and applications related to up-to-date technological developments, such as quantum information, semiconductors, quantum computers, and fermions and bosons. This was mainly due to the fact that the experts considered these topics to be both important and complex. The question is to what extent experts are able to assess if the topic is too difficult. Since content experts are not necessarily didactical experts, the feasibility of teaching the proposed topics at the secondary school level should be investigated, also topics that were considered to be too complex. Especially if one of our goals is to give secondary school students more insight into quantum technology and its effects on society, addressing these complex topics conceptually is still worth investigating.

6.2.3 Secondary school students' misunderstandings of potential wells and tunneling

In the third study, we researched Dutch students' misunderstandings after they learned quantum mechanics. For this a conceptual understanding test was administered, which was based on the topics of the Dutch physics curriculum. Quantitative analysis of the test showed that Dutch secondary school students experience the same difficulties that were reported for undergraduate students in Chapter 2. Students mixed up classical and quantum models, and overgeneralized prior classical knowledge. A qualitative analysis of the open ended questions, explanations and interviews showed that Dutch students have difficulty connecting knowledge of the 1D infinite potential well and tunneling to their prior knowledge. For the 1D infinite potential well, students often integrated the wave representation and the energy representation into one model. This resulted in creative, incorrect models. For example, students mixed up the amplitude or equilibrium with the energy level, or described a particle that vibrated or moved across a sinusoidal pathway. For tunneling, students often reasoned deterministically. For instance, students described a particle moving through or over a barrier, or reasoned in terms of effort or distance. The main problems found in this study were related to incorrect connections with prior knowledge.

There has already been research into undergraduate students' understanding of the 1D infinite potential well and tunneling. This study confirmed the assumption that students at the secondary school level experience difficulties that are similar to those of undergraduate students. Additionally, the perspective of learning impediments gave more insight into the underlying problems. When students learned both energy and wave function representations, this led primarily to creative mix-ups. When students only learned to work with the energy representation, this led to inappropriate classical reasoning. Both difficulties are the result of unintended links between new concepts and prior learning. Therefore, the results of this study imply that it is important to help students to incorporate the new QM concepts into their existing framework. Whether this can be done adequately by changing students' prior knowledge, or showing them how QM relates to their prior understanding of physics, needs to be investigated.

6.2.4 Prior knowledge of potential energy and the understanding of quantum mechanics

In Chapter 5 we investigated if an increase of understanding of prior knowledge on potential energy leads to a better understanding of quantum mechanics, using a quasi-experimental intervention. Results of this intervention showed that students of the experimental group had a significant better understanding of QM before QM instruction. This results shows that QM understanding is supported by a good understanding of potential energy (PE). Statistical analysis of a PE-test and a QM pre- and post-test showed that there is a significant correlation between the understanding of PE and QM. The observed correlation can mainly be attributed to the correlation between students' prior knowledge of 'the relation between PE and position' and students' understanding of QM.

The intervention presented in this study did not consist of teacher training, the instructional materials were not evaluated thoroughly, and the connection between QM instruction and the instructional materials were not yet optimized. The fact that we have found a correlation between prior knowledge of PE and the understanding of QM in this uncontrolled setting, therefore implies that: 1) the observed correlations may be stronger if the materials and the intervention are optimized, and 2) there can also be significant correlation between the other components of PE and the understanding of QM. In order to investigate this more thoroughly, there should be emphasis on improvement of the instructional materials based on design-based research. Additionally, the results of this study also raise the question of whether there are other skills or central concepts of physics that are important for the understanding of QM. An important question that needs research therefore is: What basis do students need to be able to implement QM in their existing knowledge structures?

6.3 Overall reflections

When looking back at the four studies, there are several points to consider. First of all, what did these studies add to existing research? The literature review showed that there was no research into secondary school students' understanding of the wave function and tunneling and that there was little research into underlying problems. Based on study three, we now can say that secondary school students experience the same difficulties that undergraduate students experience. Additionally, we have a more detailed overview of what students' difficulties are, which can be used to prevent or address these difficulties. In our review we also learned that there were three teaching strategies that proved to be useful. Based on study four, we can add a fourth teaching strategy, namely addressing students' prior understanding of potential energy. In previous research we also observed that there were many ways that QM was introduced, and that there were different opinions into what should be taught. The second study gives an answer from the perspective of content experts. This does not give a direct answer to the question what should be taught, it is a starting point for the further development of a QM curriculum.

6.3.1 Generalizability of the presented studies

Another aspect is the impact of the results from these studies. To what extent can the results of the studies that were presented in this thesis be generalized for the international research field?

In chapter 3 the Delphi technique was used in order to find QM topics that were important to teach at secondary schools. The expert selection was based on the experts' relation to relevant institutes and research groups, and publically-available information on the panelists' accomplishments and expertise. However, the experts were all Dutch and shaped by the Dutch education system. This makes their assessment of the difficulty of QM topics not appropriate for all contexts. Still, the panelists are experts in the international field of scientific research and developments, which makes their assessment of key topics based on their importance in QM generally applicable. Regarding the question formulation, the question that was given to the experts did not focus on the nature of science in general, but on the relevance from the perspective of up-to-date research and technological development, which is in accordance with the Dutch curriculum, but not necessarily with other curricula. Finally, we used interviews to verify consensus and explore experts' argumentations, which confirmed the results from the Delphi study. In summary, since the topic choice mainly was based on what experts considered to be the fundamental concepts of QM, these results are generally useful as a starting point for creating a QM curriculum at the secondary school level. The argumentation regarding the nature of science and complexity of the topic gives insight into substantive argumentations, but partly depends on the context.

In chapter 4, results were based on a quantitative and qualitative analysis of a conceptual understanding test and interviews. Several test questions were used from existing tests, and other questions were created by the authors. The test questions were selected and created based on students' misunderstandings found in the review study. To ensure the quality of the questions, the questions and the translations were checked by a content expert and experts in physics pedagogy. However, the test was not thoroughly evaluated by using item analysis or comparing it to other test results. This sufficed, because the main goal of this study was to get an overall view of students' difficulties and the underlying problems. Because of the explorative and qualitative character of this study, the types of difficulties found in this study are still partly generalizable. The difficulties found in this study are mainly related to incorrect connections between new and prior knowledge, and therefore are likely to be found in other contexts. However, difficulties related to oversimplification, folk beliefs, language and intuition, are related to students' background, nationality and religion, and are therefore context dependent. Thus, the epistemological and creative learning impediments found in chapter 4 are likely to be found in other context, whereas the linguistic and pedagogic learning impediments are more dependent on the specific context.

In the study on the influence of prior knowledge, described in chapter 5, instructional materials were designed, and two tests were used. In section 6.2.4 we already stated the limitations of the intervention, but how does that influence the generalizability of the study? The results of the fourth study show that there was a significant correlation between students' understanding of potential energy and QM. There was a significant difference in understanding of the experimental and control group, with a small to medium effect size. The generalizability of this effect size depends mainly on the sample population. The students partaking in the intervention were selected using convenience sampling, but the Dutch educational system reduces variance in compulsory attendance, curriculum standard, academic standards, and funding[2]. Therefore, the differences between Dutch students are small, and results are generalizable for Dutch secondary school students. However, PISA 2015[3] showed that there are significant differences between countries in students' performance and equity in science education. So, although we established a significant correlation between understanding PE and QM, we cannot generalize the effect size of this intervention. This depends on students' prior knowledge and background, which varies between countries.

6.3.2 Future research

The research presented in this thesis gives rise to some interesting follow-up questions. In the first place, this research raises the question of what the results of the Delphi study imply for the future focus areas of QM research. In the study presented in Chapter 3, the content experts considered topics like quantum information and superposition too difficult. However, these experts were not necessarily experts in physics teaching or aware of relevant educational research, and the experts were possibly biased by the education they have received themselves. It may be worthwhile to investigate what students' difficulties are for these more complex topics, and how to address these topics more conceptually. A motivation for doing so are the rapid developments in quantum computing, quantum informatics and quantum encryption that are on the verge of becoming applicable in practical situations. Students who are currently in primary or secondary education must be prepared to interpret such new developments in the media and to distinguish fact from fiction. A way of approaching complex quantum behavior conceptually, is using the spin-first approach. This approach has already been used in undergraduate education[4-6], and comparison with a traditional position-first approach showed that this approach led to increased performance. The approach has also been used in secondary education[7], and results suggested that this approach is within the capacity of upper-level secondary school students. Still, for the secondary school level there is a need for the development of instructional methods, and a comparison with the position-first approach.

Second, the study presented in Chapter 5 gives rise to questions on how we teach QM within the context of physics as a whole. The influence of prior knowledge has been established; in this study it proved to be very helpful to strengthen students' knowledge of (potential) energy in order to understand QM concepts. Energy is a central concept in physics, which raises the question of what influence other central concepts, such as momentum, forces and fields, have on students' understanding of QM. Currently the model-based nature of physical theory and the role that central concepts play within these models is underemphasized in the way physics is usually taught. A consequence might be that QM is seen as something that is completely at odds with classical physics. Emphasizing the model-based nature of physics and the way physical concepts are taken from the classical world view to the QM view may help students to see the continuity of the change in theory, deepen their understanding of physics as a science, and see why classical theory and QM can co-exist. Based on our results we can hypothesize that such an approach to teaching QM within the physics curriculum might be fruitful.

Even more so, understanding of central concepts in physics could play an important role in understanding physics in general. Some studies into the emphasis on energy as a central concept in science education are available[8, 9], but results showed that understanding energy is difficult and abstract[10]. More study is necessary to come to grips with such an approach. In order to adequately implement this approach into QM teaching, design-based research should be used to design and evaluate instructional materials that improve students' understanding of the model-based nature of physics and of central concepts like energy and momentum within both classical and quantum mechanics.

Third, an important aspect for research into the teaching of quantum mechanics is how to address its philosophical aspects, in particular its interpretations. Research suggests that students who have understood the (mathematical) basics of QM not necessarily accept it as a genuine description of reality[11]. This may be caused by the fact that students tend to hold on to a realist view[12] (i.e. the assumption that entities have well-defined properties, independent of measurement). This does not correspond with the Copenhagen interpretation, and other interpretations of QM. In order to conceptually understand QM, students need to be aware of both its accuracy and its philosophical implications, especially when we want students to distinguish fact from fiction. There has been an increase in the research of the role of addressing interpretations in teaching QM[1, 13, 14], and more research into this topic is important.

Finally, in our review study we observed that there is not much empirical research into students' difficulties and appropriate teaching strategies. Therefore we want to emphasize that there is a need for more empirical research in which researchers systematically analyze students' difficulties and teaching strategies. This does not only apply to future research that we have recommended in this thesis, but also for research aiming to strengthen and validate the existing knowledge on teaching QM, as presented in Chapter 2. In order to create comparable and generalizable results, it is also important that there are appropriate tests on conceptual understanding of QM. Hence, there is a need for the design and validation of a test for the understanding of QM at a conceptual level, covering wave-particle duality, wave functions, atomic models and complex quantum behavior.

6.3.3 Implications for the physics curriculum

The results and considerations presented in this dissertation also have influence on curriculum design. The results presented in Chapter 3 showed that the topics that were considered important by content experts substantially corresponded with the Dutch physics curriculum at secondary schools. This research also showed that the content experts in this study mainly based their opinion on the knowledge of scientific concepts they wanted students to have. Both results are not surprising, since the Dutch physics curriculum is strongly defined by opinions of academics, who are more likely to embrace *wish-they-knew* and *need-to-know* science[15]. Since the physics curriculum at the secondary school level aims to do more than just increasing knowledge of scientific concepts, it would be recommendable to increase the role of other groups of interest.

Still, the key topics presented in Chapter 3 are only part of the answer of what we should teach. When we want to promote scientific literacy, we also have to consider which topics promote students' knowledge of the nature of science, their interest in science and support for scientific inquiry, and their ability to understand scientific issues and make informed decisions. The first aspect can be addressed using the historical approach[16], and recent studies also emphasize on the importance of addressing philosophical aspects in order to promote knowledge of the nature of science[12, 17]. The current Dutch physics curriculum and the international core curriculum at the secondary school level both consist of topics that are appropriate for a historical or philosophical approach. However, these topics are mainly related to developments that took place in the 20th century. In order increase students' attitude towards QM research, and their competencies for explaining and decision-making, it is important to relate QM to current technological developments. For this, students should learn about spin, quantum states, and entanglement. Therefore it is recommendable to include these topics in the design of a physics curriculum.

In Chapter 4 we have seen that students have difficulty relating the new QM concepts to their prior knowledge. Additionally, Chapter 5 showed the importance of understanding potential energy for understanding QM, that here is an interplay between PE and QM that can be used to strengthen the coherence of the curriculum and deepen students' understanding. In order to improve students' understanding of QM, it is important to give students the possibility to relate QM to concepts that they have learned previously. Therefore, it is important to rethink the topics that are taught before introducing QM. Central topics in classical physics that also play a role in QM need to be addressed thoroughly, in order to create deep understanding of relevant prior knowledge before teaching QM. This is not only the case for energy, but also for other topics such as momentum, forces, fields, interference, superposition, and probability distribution. Chapter 4 also showed that students mix up and overgeneralize scientific models they have learned previously. In order to prevent this, there should be emphasis on the model-based nature of physics. When we make students aware of the limitations of different models, teach them how to choose appropriate models, and show where QM relates to previously learned concepts and models, this will lead to less mix-ups and overgeneralizations. Moreover, increased emphasis on model thinking and central concepts will deepen students' understanding of physics in general. Therefore, curriculum developers need to carefully consider the central concepts and scientific skills needed to understand QM, and need to give these concepts and skills a prominent place within the secondary school physics curriculum.

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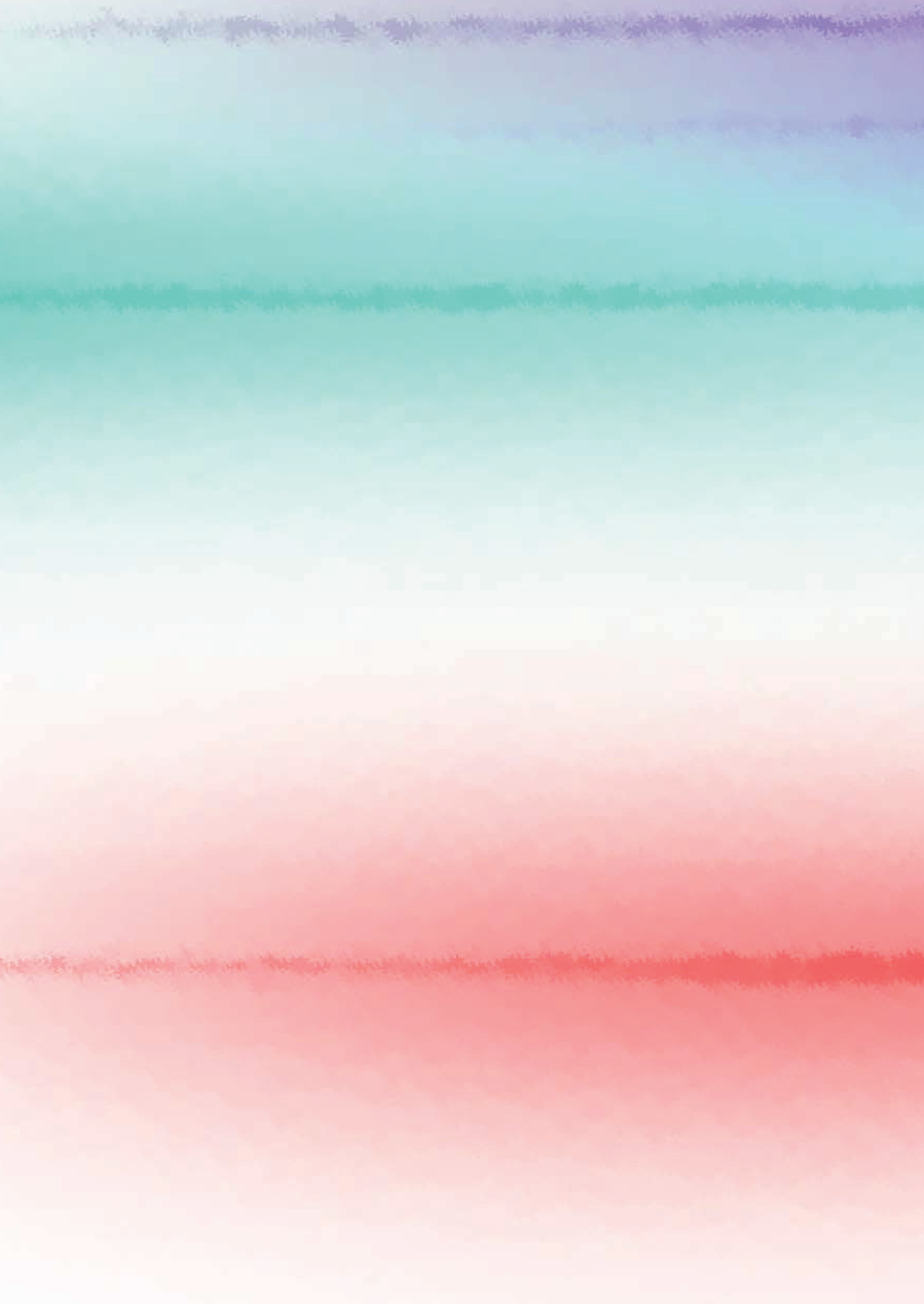


APPENDICES

Appendix A: Details of the selected articles on student difficulties described in Chapter 2

Part	Researchers	Topic	Level	Country	Methodology & analysis
A	Asikainen, M. A. and Hirvonen, P. E. (2009). [19]	Photoelectric effect	Undergraduate students & physics teachers	Finland	A case study, using pre- and post-test and semi-structured interviews, was carried out with pre-service (N=8) and in-service (N=17) teachers. Test responses were categorized, interviews were used for validation.
A	Ayene, M., Kriek, J. and Damtie, B. (2011). [20]	Wave-particle duality, uncertainty principle	Undergraduate students	Ethiopia	Semi-structured interviews were conducted with undergraduate students (N=25). Responses were categorized.
A	Dutt, A. (2011)[21]	Wave-particle duality, double slit experiment, photoelectric effect, quantization	Upper secondary students	Australia	Test and worksheet data from grade 12 students were analyzed and interviews were held with 6 volunteering students.
A	Greca, I. M. and Freire, O. (2003). [22]	Wave-particle duality, uncertainty principle, probability distribution, superposition	Undergraduate students	Brazil	Concept tests and conceptual problems were used (N=89), field notes were collected during classes. Responses were categorized using hierarchical clustering and multidimensional scaling.
A	Hubber, P. (2006). [23]	Light	Upper secondary students	Australia	Three semi-structured interviews conducted and two questionnaires were administered (N=6). Responses were categorized.

Part	Researchers	Topic	Level	Country	Methodology & analysis
A/C	Ireson, G. (1999).[24]	Wave-particle duality, atoms	Undergraduate students	UK	A questionnaire was given to the students (N=338). Responses were analyzed with cluster analysis and multidimensional scaling.
A/C	Ireson, G. (2000).[25]	Wave-particle duality, atoms	Undergraduate students	UK	A questionnaire was given to the students (N=338). Responses were analyzed using cluster analysis and multidimensional scaling.
A	Johnston, I. D., Crawford, K. and Fletcher, P. R. (1998).[1]	Wave-particle duality	Undergraduate students	Australia	Students (N=33) were given wo short-response quizzes. Responses were categorized and analyzed for correctness.
A	Manilla, K., Koponen, I.T. and Niskanen, J. A. (2001).[26]	Wave-particle duality	Undergraduate students	Finland	Intermediate level students (N=29) answered 8 open-ended questions. Modified concept maps were created for each response, compared to a “master map” based on experts’ conceptions, and categorized.
A	Masshadi, A. and Woolnough, B. (1999).[4]	Wave-particle duality	Upper secondary students	UK	Students (N=83) were given a semi-structured questionnaire. Responses were categorized.
A	McKagan, S. B. *et al.* (2009).[27]	Photoelectric effect	Undergraduate student	USA	After a reformed course, students’ responses to two exam questions were analyzed (N=465, N=188).

Part	Researchers	Topic	Level	Country	Methodology & analysis
A	McKagan, S. B., Perkins, K. K. and Wieman, C. E. (2010).[16]	Wave-particle duality, double slit experiment	Undergraduate students	USA	Interviews were conducted (*N*=46) during the design and evaluation of the QMCS.
A	Müller, R. and Wiesner, H. (2002).[5]	Wave-particle duality, atoms, uncertainty principle, non-determinism	Secondary and undergraduate students	Germany	A questionnaire was administered to secondary students (*N*=523) and interviews were conducted with secondary students (*N*=27) and undergraduates (*N*=37). Responses were categorized.
A	Oh, J. Y. (2011).[28]	Photoelectric effect	Undergraduate students	South Korea	Three groups of students (*N*=31, *N*=49, *N*=49) were given a pre and a posttest, which were validated by interviews. Responses were categorized.
A	Olsen, R. V. (2002).[29]	Wave-particle duality	Upper secondary students	Norway	Students from 20 different schools (*N*=236) were given a test. Multiple choice questions were analyzed quantitatively, open-ended questions were categorized.
A	Özcan, Ö. (2015).[30]	Photoelectric effect, blackbody radiation, Compton effect	Undergraduate students	Turkey	Pre-service physics teachers (*N*=110) were given a questionnaire. Responses were categorized, and analyzed for correctness.

Part	Researchers	Topic	Level	Country	Methodology & analysis
A/C	Sen, A. I. (2002).[31]	Wave-particle duality, atoms	Undergraduate students	Turkey	Students (*N*=88) created a concept map. These maps were analyzed for number of concepts, relationships, branches, hierarchies and cross-links.
A	Singh, C. (2008).[32]	Uncertainty principle, time development, Mach-Zehnder interferometer	Undergraduate students	USA	A pre- and posttest were given to students (*N*=12) who did the QuILT. Examples of students' responses were provided.
A	Sokolowski, A. (2013).[33]	Photoelectric effect	Upper secondary school	USA	A group of students (*N*=15) answered one conceptual question during an assignment. Examples of responses were provided.
A	Vokos, S. *et al.* (2000).[34]	Double slit experiment	Undergraduate students	USA	Written problems were given to students (*N*=450) in various physics undergraduate courses and interviews were conducted (*N*=14). Students' reasoning was analyzed and categorized.
B	Bao, L. and Redish, E. F. (2002)[35]	Probability	Undergraduate students	USA	Interviews were conducted with physics students (*N*=16). The observations were summarized.
B	Brookes, D. T. and Etkina, E. (2007)[36]	Potential wells	Undergraduate students		Students (*N*=4) were observed while working on homework problems. Examples of students' reasoning were shown and analyzed.

Part	Researchers	Topic	Level	Country	Methodology & analysis
B	Domert, D., Linder, C. and Ingerman, Ǻ. (2005).[40]	Probability, tunneling	Undergraduate students	Sweden	Students (*N*=12) were interviewed while working with a computer simulation. Observations were categorized and examples were given.
B	McKagan, S. B., Perkins, K. K. and Wieman, C. E. (2008).[38]	Tunneling	Undergraduate students	USA	Data was collected for eight courses, consisting of observations, responses to essay questions, interviews and a concept test (QMCS). Observations were categorized and illustrated, test results were reported.
B	Özcan, Ö. (2011).[41]	Wave functions, operators	Undergraduate students	Turkey	Semi-structured interviews were held with preservice physics teachers (*N*=34). Observations were categorized.
B	Özcan, Ö., Didiş, N. and Tasar, M. F. (2009)[37]	Potential wells	Undergraduate & graduate students	Turkey	A concept test was given to undergraduate (*N*=95) and graduate (*N*=15) students. Semi-structured interviews were held with 10 students. Student responses were presented.
B/D	Singh, C. (2008)[43]	Wave functions, probability, measurement	Undergraduate & graduate student	USA	Surveys were administered to graduate students (*N*=202), interviews were held with graduate and undergraduate students (*N*=15). Results were categorized and examples were given.

Part	Researchers	Topic	Level	Country	Methodology & analysis
B	Singh, C., Belloni, M. and Christina, W. (2006).[42]	Wave functions, probability, measurement	Undergraduate & graduate students	USA	Surveys were administered to graduate (N=200) and undergraduate (N=89) students. Examples of difficulties were presented.
B	Wittmann, M. C., Morgan, J. T. and Feeley, R. (2006).[39]	Probability	Undergraduate students	USA	Students (N=42) were given a pre- and posttest. A series of questions were given also during the semester. Students' responses were presented.
B	Wittmann, M. C., Morgan, J. T. and Bao, L. (2005).[44]	Tunneling	Undergraduate students	USA	Written examination questions, ungraded quizzes, surveys and interviews were analyzed by content analysis, interpretation of diagrams and descriptions of students' actions.
C	Dangur, V. *et al.* (2014).[54]	Atomic structure	Upper secondary & undergraduate students	Israel	Pre- and posttest were used to probe secondary (N=122) and undergraduate (N=65) student understanding. A rubric was designed to analyze the 3-item test.
C	Didiş, N., Eryılmaz, A. and Erkoç, Ş. (2014).[55]	Light, energy, angular momentum	Undergraduate students	Turkey	Interviews were conducted, a test was administered and exams were analyzed (N=31). The interviews were coded and mental models were constructed.

Part	Researchers	Topic	Level	Country	Methodology & analysis
C	Kalkanis, G., Hadzidaki, P. and Stavrou, D. (2003)[12]	Atomic structure, models	Undergraduate students	Greece	A concept test was given to the test group (*N*=98) and a control group (*N*=102). Semi-structured interviews were conducted with a sample of the test group. Difficulties found during the interviews were summarized.
C	Ke, J. L., Monk, M. and Duschl, R. (2005).[46]	Atomic structure	Upper secondary – PhD. students	Taiwan	A questionnaire was given to students from high school to PhD. level (*N*=140). Responses were categorized. 28 students were interviewed using concept cards in order to refine the categorization.
C	McKagan, S. B., Perkins, K. K. and Wieman, C. E. (2008).[48]	Atomic structure, models	Undergraduate students	USA	One exam question was analyzed for four courses (*N*=591). Responses were categorized.
C	Özcan, Ö. (2013).[52]	Spin	Undergraduate students	Turkey	Interviews were conducted with introductory (*N*=24) and advanced (*N*=25) students. The results were categorized.

Part	Researchers	Topic	Level	Country	Methodology & analysis
C	Papageorgiou, G., Markos, A. and Zarkadis, N. (2016).[56]	Atomic structure	Upper secondary students	Greece	Students (N=421) were given two cognitive tests measuring field dependence and reasoning abilities. A third test was used to assess students' representations of the atom. These representations were categorized and the influence of student characteristics thereon was investigated.
C	Papaphotis, G. and Tsaparlis, G. (2008).[49]	Atomic structure, uncertainty principle	Undergraduate students	Greece	A questionnaire was given to first-year students (N=125). Student difficulties were summarized and illustrated with examples.
C	Petri, J. and Niedderer, H. (1998).[45]	Atom structure	Upper secondary students	Germany	Observations, questionnaires, interviews and written materials were analyzed to describe the learning pathway of one student within a course. The data were analyzed for change in conceptions and meta-cognitive beliefs.
C	Stefani, C. and Tsaparlis, G. (2009).[50]	Atomic structure	Undergraduate students	Greece	Interviews were held with 2nd year students (N=19). The responses were categorized.
C	Taber, K. S. (2005).[47]	Atomic structure	Upper secondary students	UK	Semi-structured interviews were conducted with students (N=15). A typology of learning impediments was used to categorize the response.

Part	Researchers	Topic	Level	Country	Methodology & analysis
C	Tsaparlis G. and Papaphotis, G. (2009). [51]	Atomic structure, uncertainty principle	Undergraduate students	Greece	A questionnaire was given to first-year students (N=125). Semi-structured interviews were conducted with a sub-sample (N=23). Students' discussions were summarized and illustrated with examples.
C	Wang, C. Y. and Barrow, L. H. (2013).[53]	Atomic structure, chemical bonding	Undergraduate students	USA	Three diagnostic tests were used to analyze student understanding (N=159). Interviews, using a think-aloud protocol and interview-about-events, were conducted with a sub-sample (N=48). Representations of conceptual frameworks were created and analyzed by axial coding.
C	Zhu, G. and Singh, C. (2011).[57]	Spin, Stern-Gerlach experiment	Undergraduate & graduate students	USA	Surveys were administered (n>200) and semi-structured interviews were conducted with a subset of students. Results were used to design a tutorial.
D	Emigh, P. J., Passante, G. and Shaffer, P. S. (2015)[60]	Time dependence	Undergraduate	USA	Four tasks were used to assess student understanding (N_1=416, N_2=439, N_3=285, N_4=215). The tasks were examined to identify difficulties, and these difficulties were categorized.
D	Michelini, M. *et al.* (2004).[58]	Quantum states, nonlocality	Upper secondary students	Italy	Students (N=17) took part in group discussions of worksheets. Examples of student reasoning and a summary of the discussion were presented.

Part	Researchers	Topic	Level	Country	Methodology & analysis
D	Passante, G., Emigh, P. J. and Shaffer, P. S. (2015).[59]	Superposition	Undergraduate & graduate students	USA	A multiple choice question was used to explore the understanding of sophomores, juniors and graduate students. Juniors (N=32) were asked to consider four statements. Results for the multiple choice question and an overview of student reasoning regarding these statements were provided.
D	Zhu, G. and Singh, C. (2012).[61]	Measurement	Undergraduate & graduate students	USA	Concept tests, quizzes and tests were analyzed over several years. Interviews and informal discussions were conducted with a subset of students to investigate students' reasoning. An overview of the responses and students' reasoning is presented.

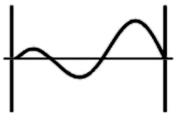
Appendix B: Conceptual knowledge test on potential wells, wave functions and tunneling

Question 1

During your physics lessons you have encountered the particle-in-a-box model. Explain the meaning and the use of this model.

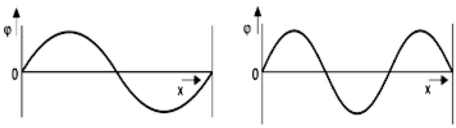
Question 2

In the figure below you see a possible wave function of an electron within an atom. Explain what you can about the position, velocity and energy of this electron, based on the figure below.



Question 3

In the figure below you see two wave functions belonging to a particle-in-a-box. The scale of both figures is equal.



What can you say about the energy level of the particles corresponding with these wave functions? Explain your choice.

A. The energy is equal
B. The energy of the particle corresponding to the left diagram is bigger
C. The energy of the particle corresponding to the right diagram is bigger
D. You cannot tell anything based on these figures

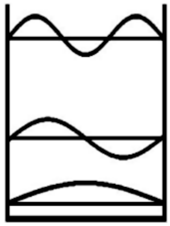
Question 4

In the particle-in-a-box model, what is a measure of the energy level? Explain your choice.

A. The amplitude
B. The area under the curve
C. The height of the equilibrium
D. The number of nodes and antinodes

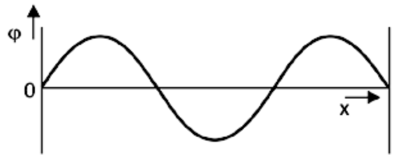
Question 5

In the figure below you see a representation of standing waves belonging to the infinite potential well. Explain the use of this model and explain how this model is related to the real world.



Question 6

In the figure below you see the wave function belonging to a particle in a box.



What can you say about the position of this particle? Explain your choice

A. The particle is located at one of the positions where the wave function has a maximum displacement
B. The particle is located at one of the positions where the wave function is zero
C. The highest probability of finding the particle is at one of the positions where the wave function has a maximum displacement
D. The highest probability of finding the particle is at one of the positions where the wave function is zero
E. You cannot tell anything about the particle's position based on this figure

Question 7 (Based on QMCI Q9)

A number of students discuss the relation between the wave function and the energy level. Which one of the statements below is correct? Explain your choice.

A particle with a higher energy level…

A. … has a wave function with a bigger amplitude
B. … has a wave function with a higher frequency
C. … has a wave function with a bigger amplitude AND a higher frequency
D. … has a wave function with a bigger wave length
E. … can have the same wave function as a particle with a lower energy level

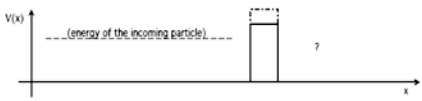
Question 8 (QMCS Q7)

The total energy of an electron after it tunnels through a potential energy barrier is…

A. ...greater than its energy before tunneling
B. … equal to its energy before tunneling
C. … less than its energy before tunneling

Question 9 (Based on QMCI Q2)

A particle with a certain energy level may tunnel through a barrier. Then the barrier becomes higher (see the figure below)

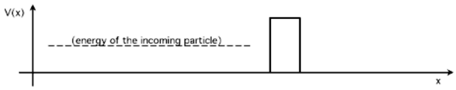


How does this change influence the tunneling process? Explain your choice.

A. The energy of the transmitted particle decreases
B. The probability of tunneling decreases
C. Answer A and B are both true
D. None of the above answers is true

Question 10 (Based on QMCI Q6)

A particle encounters a barrier, as shown in the figure below. The particle’s energy is 50% of the barrier’s energy.

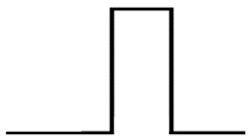


What is the probability (P) that this particle will tunnel through the barrier? Explain your choice.

A. P = 100%
B. P = 50%
C. P = 0%
D. 0% < P < 100%

Question 11

An electron tunnels through the barrier shown below:

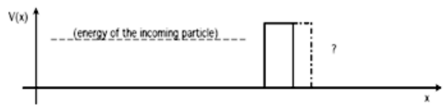


What can you say about the energy before and after tunneling? Explain your choice.

A. $E_{before} > E_{after}$
B. $E_{before} = E_{after}$
C. $E_{before} < E_{after}$

Question 12 (Based on QMCI Q3)

A particle with a certain energy level may tunnel through a barrier. Then the barrier becomes wider (see the figure below)



How does this change influence the tunneling process? Explain your choice.

A. The energy of the transmitted particle decreases
B. The probability of tunneling decreases
C. Answers A and B are both true
D. None of the above answers is true

Appendix C: The coding scheme based on students' learning impediments

Fragmentation	**F1 - Non-determinism**	Students cannot relate non-deterministic concepts to their deterministic worldview, and cannot describe the non-deterministic concept of describe these concepts in deterministic wordings. e.g. students state that: – The probability distribution is related to an exactly determined position – The wave function is related to an exactly determined position – They do not know how to describe the probability distribution
	F2 – Equations & relations	Students do not know how to use energy equations and use the wrong quantities or relations between quantities. e.g. students: – use the wrong quantity in an equation – reason with an incorrect relation/proportionality
	F3 - Energy diagrams	Students do not know that the potential well and the barrier represent energy and are not capable of describing the well/barrier.
Pedagogic	**P1 - Mixed representations**	Students believe the **y-axis** of the infinite potential well represented both position and energy: – Equal amplitude or A^2 (~probability) implies equal energy – Conclusions on energy based on the position on the y-axis (in a diagram representing the wave function)
	P2 - Wave functions during tunneling	Students do not know what happens with the wave function during tunneling. e.g. students state that: – Only differences between the particle's and barrier's energy level influence tunneling – Only the height of the barrier influences tunneling
Linguistic	**L1 - 'well'-analogy**	Students interpret the infinite potential well literally. e.g. students refer to: – a wall (wand) – an edge (rand)

Creative	**C1 - Mix-up with classical waves**	Students mix up the quantum particle's wave behavior with properties of classical waves. e.g. students state that: – The particle moves like a wave – The particle has properties of classical waves – The particle vibrates
	C2 - Mix-up of well and barrier	Students mix up the infinite potential well and the barrier. e.g. students state that: – The wave function during tunneling is a standing wave – The particle moves back and forth within the barrier – The particle escapes from the (infinite) well during tunneling
	C3 - Mix-up with energy level	Students mix up the amplitude/equilibrium of the wave function with energy, or the energy before/after tunneling with an energy level. e.g. students state that: – The energy level is the **equilibrium** of the wave function – The amplitude of the wave function is an energy level or excited state – The wave function decreases exponentially, therefore the energy decreases too.
	C4 - Mix-up with other classical concepts	Students mix up the potential well with other classical (e.g. resistance) or semi-classical (e.g. ionization) concepts.
Epistemological	**E1 - Use of inappropriate atomic models**	Students use inappropriate atomic models. e.g. students state that: – The energy level in the potential well is a shell – The particle moves in an orbit – The bottom of well is the nucleus
	E2 - Deterministic reasoning in terms of movement	Students explain tunneling deterministically in terms of position, movement and time. e.g. students state that: – The particle collides with a barrier – The particle bridges a distance – The particle arrives at a certain height – Tunneling takes a certain time
	E3 - Classical reasoning in terms of energy	Students explain tunneling deterministically in terms of energy. e.g. students state that: – The particle needs to have more energy than the barrier – The barrier absorbs energy

APPENDIX D: THE ENERGY TEST

QUESTION 1 SWING

Liz is swinging. Figure 1 shows her highest position (A) and lowest position (B). In this task we do not consider friction forces.

PE1) What do you know about the potential energy E_p in position A and B?

A. $E_p(A) > E_p(B)$
B. $E_p(A) < E_p(B)$
C. $E_p(A) = E_p(B)$
D. Based on this information, nothing can be said about that.

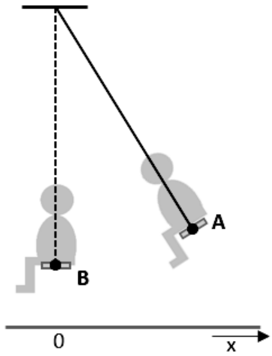


FIGURE 1: LIZ ON A SWING

In figure 2 you see the gravitational energy E_z and the total energy E_t of Liz plotted against the location x.

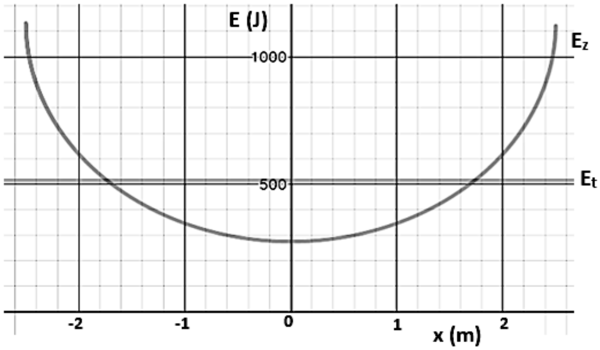


FIGURE 2 THE *E,x*-DIAGRAM OF THE SWINGING MOTION

PE2) What are the possible positions for Liz?

A. Between x = 0 and 1,7 m
B. Between x = -1,7 and 1,7 m
C. Between x = -2,5 and 2,5 m
D. Based on this information, nothing can be said about that

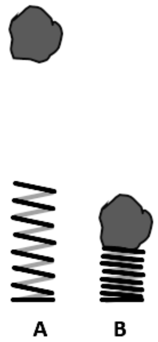
PE3) At what position is the resulting force on Liz equal to 0?

A. At x = 0 m
B. At x = -1,7 and 1,7 m
C. At x = -2,5 and 2,5 m
D. Based on this information, nothing can be said about that.

>> Next page >>

QUESTION 2 A FALLING ROCK

Jasper drops a 3,0 kg stone from a height of 1,0 m onto a spring (Figure 3, situation A). At the moment that Jasper releases the stone, it has a speed of 0 m/s. At a certain moment in time, the stone reaches its lowest point (situation B). Figure 4 shows the gravitational and spring energy of the stone and the spring. In this task we ignore friction forces.



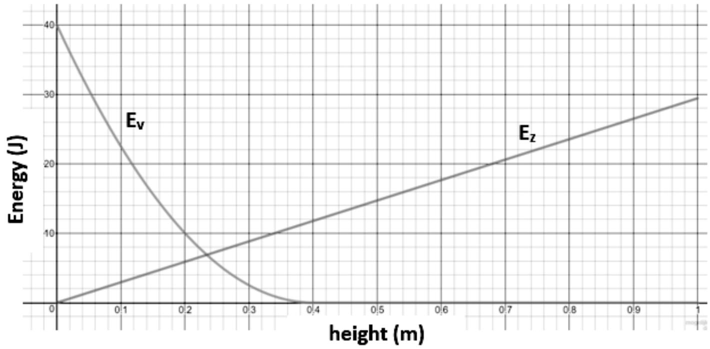


FIGURE 3 A STONE FALLS ONTO A SPRING

FIGURE 4 THE GRAVITATIONAL ENERGY (E_Z) AND THE SPRING ENERGY (E_V) OF THE SYSTEM

PE4) At what height is the stone when it reaches its lowest point? At:

A. h = 0,00 m
B. h = 0,07 m
C. h = 0,23 m
D. h = 0,34 m
E. h = 0,39 m
F. Based on this information, nothing can be said about that.

PE5) At what height does the stone move at its greatest speed? At:

G. h = 0,00 m
H. h = 0,07 m
I. h = 0,23 m
J. h = 0,34 m
K. h = 0,39 m
L. Based on this information, nothing can be said about that.

PE6) At what height is the resulting force on the stone 0 N?

A. At the maximum height of $E_v + E_z$ in Figure 4
B. At the height at which the slope of $E_v + E_z$ in Figure 4 equals 0
C. At the height at which E_v and E_z in Figure 4 are equal
D. None of the above

>> Next page >>

QUESTION 3 BUNGEE JUMP

Marijke is out bungee jumping. Figure 5 shows the gravitational and spring energy of Marijke during the bungee jump. In this task we leave friction forces out of consideration.

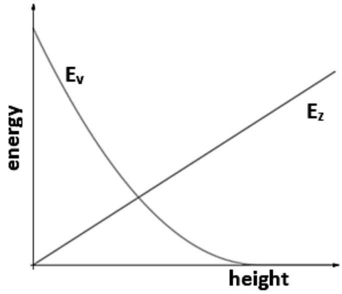
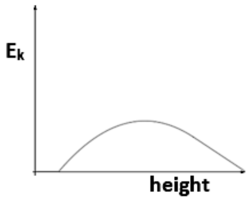


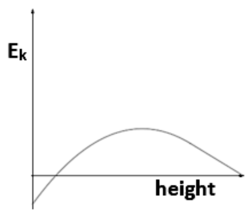
FIGURE 5 THE GRAVITATIONAL (E_Z) AND SPRING ENERGY (E_V) OF MARIJKE DURING THE BUNGEE JUMP

PE7) Which of the diagrams below, correctly represents Marijke's kinetic energy on the bungee cord?

A.

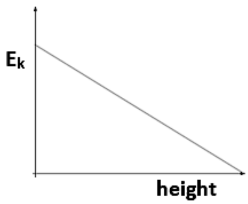


B.

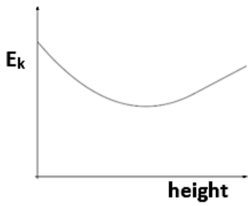


C.

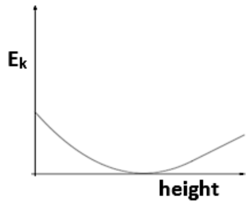
D.



E.



F.



Pascal, Marijke's father, also wants to bungee jump. He is twice as heavy as his daughter.

PE8) Draw the gravitational energy of Pascal for a bungee jump starting at the same height.

PE9) Explain, using spring and gravitational energy, why it is very unwise for pascal to bungee jump under the same conditions as Marijke.

>> Next page >>

QUESTION 4 THE LENNARD-JONES POTENTIAL

Figure 6 shows the Lennard-Jones potential for two Argon atoms. The Lennard-Jones potential is the potential energy E_p of two atoms plotted against the mutual distance r.

This potential energy is caused by two forces; the repelling electrical force due to the negatively charged electron cloud, and the attracting Van der Waals force.

Figure 6 shows 4 different areas (A to D) and 3 distances (r_1, r_2, r_3).

PE10) On your answer sheet, tick the area(s) where the potential energy is primarily caused by the repulsive electrical force

PE11) On your answer sheet, tick the area(s) in which the potential energy is mainly caused by the Van der Waals force.

Two atoms are in energy state E_{at}, as shown with the dotted line in figure 6.

PE12) What do you know about the size of the resulting force between the atoms when they are at a distance r_1, r_2 or r_3 from each other?

A. $F(r_1) < F(r_2) < F(r_3)$

B. $F(r_3) < F(r_2) < F(r_1)$

C. $F(r_2) < F(r_3) < F(r_1)$

D. $F(r_1) < F(r_3) < F(r_2)$

PE13) What do you know about the size of the kinetic energy of the atoms when they are at a distance r_1, r_2 or r_3 from each other?

A. $E_k(r_1) < E_k(r_2) < E_k(r_3)$

B. $E_k(r_3) < E_k(r_2) < E_k(r_1)$

C. $E_k(r_2) < E_k(r_3) < E_k(r_1)$

D. $E_k(r_1) < E_k(r_3) < E_k(r_2)$

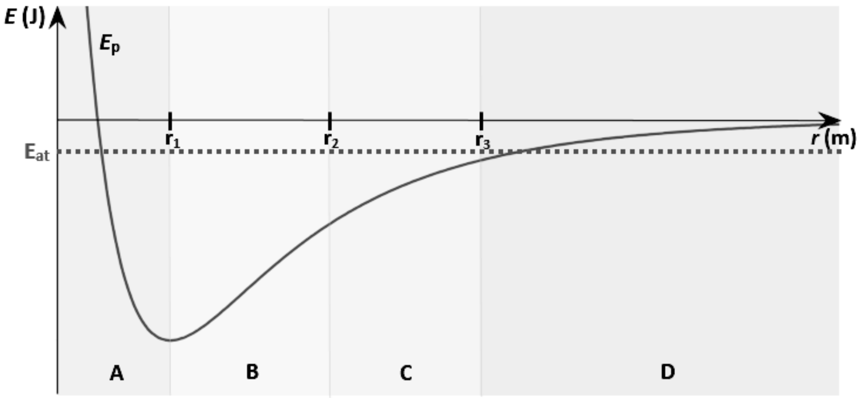


FIGURE 6 THE LENNARD-JONES POTENTIAL

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APPENDIX E: THE QUANTUM TEST

QUESTION 1

Figure 1 shows a representation of the potential energy of an electron in a limited space: the particle-in-a-box model. This representation is also called the infinite potential well.

Between $x = 0$ and $x = L$ the potential energy is 0, beyond that E_p is infinite.

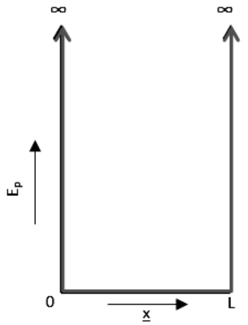

FIGURE 1 THE POTENTIAL ENERGY OF AN ELECTRON IN A LIMITED SPACE

QM1) What can you say about the kinetic energy E_k of this electron?

	Between 0 and L	Beyond 0 and L
A.	$E_k = 0$	$E_k = 0$
B.	$E_k = 0$	$E_k \neq 0$
C.	$E_k \neq 0$	$E_k = 0$
D.	$E_k \neq 0$	$E_k \neq 0$

QM2) What can you say about the probability P of finding the electron at a certain position?

	Between 0 and L	Beyond 0 and L
A.	$P = 0$	$P = 0$
B.	$P = 0$	$P \neq 0$
C.	$P \neq 0$	$P = 0$
D.	$P \neq 0$	$P \neq 0$

QUESTION 2

Figure 2 shows another diagram of the potential energy of an electron in a certain limited space. The electron is in the ground state.

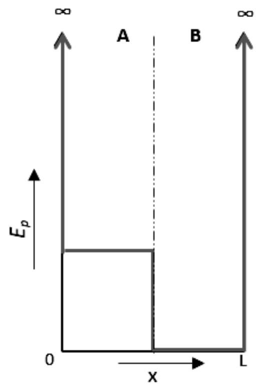

FIGURE 2 THE POTENTIAL ENERGY OF AN ELECTRON IN A DIFFERENT LIMITED SPACE

QM3) What do you know about the kinetic energy of the electron in area A (E_{kA}) and area B (E_{kB})?

A. $E_{kA} > E_{kB}$
B. $E_{kA} < E_{kB}$
C. $E_{kA} = E_{kB}$
D. Based on Figure 2 nothing can be said about that.

>> Next page >>

Question 3

At a certain moment t_1 a small particle is located at the left side of a potential barrier. Figure 3 shows the particles total energy E_t and the potential energy E_p. After some time, at time t_2, the particle turns out to be on the right side of the barrier, this is called tunneling.

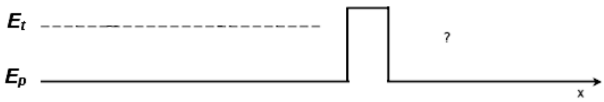

FIGURE 3 A POTENTIAL BARRIER

QM4) What can be said about the total energy before (E_{t1}) and after (E_{t2}) tunneling?

A. $E_{t1} > E_{t2}$
B. $E_{t1} < E_{t2}$
C. $E_{t1} = E_{t2}$
D. Based on this information, nothing can be said about that.

QM5) What can be said about the kinetic energy before (E_{k1}) and after (E_{k2}) tunneling?

A. $E_{k1} > E_{k2}$
B. $E_{k1} < E_{k2}$
C. $E_{k1} = E_{k2}$
D. Based on this information, nothing can be said about that.

We look at the same situation, but now potential barrier becomes higher, as shown in Figure 4 below.

QM6) How does this change influence the tunneling process?

A. The energy of the particle decreases after tunneling
B. The probability of tunneling decreases.
C. Answer A and B are both true.
D. None of the above answers is true.

We look at the same situation, but now the potential barrier becomes wider, as shown in Figure 5 below.

QM7) How does this change influence the tunneling process?

A. The energy of the particle decreases after tunneling
B. The probability of tunneling decreases
C. Answer A and B are both true
D. None of the above anwers is true

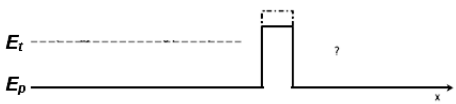

FIGURE 4 A HIGHER POTENTIAL BARRIER

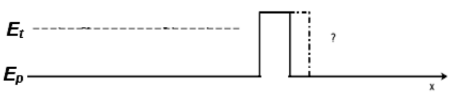

FIGURE 5 A WIDER POTENTIAL BARRIER

>> Next page >>

QUESTION 4

QM8) Assume the situation is slightly different, as shown in Figure 6. Which of your answers of Question 3 you would answer differently for this situation?

FIGURE 6 AN ASYMTETRIC POTENTIAL BARRIER

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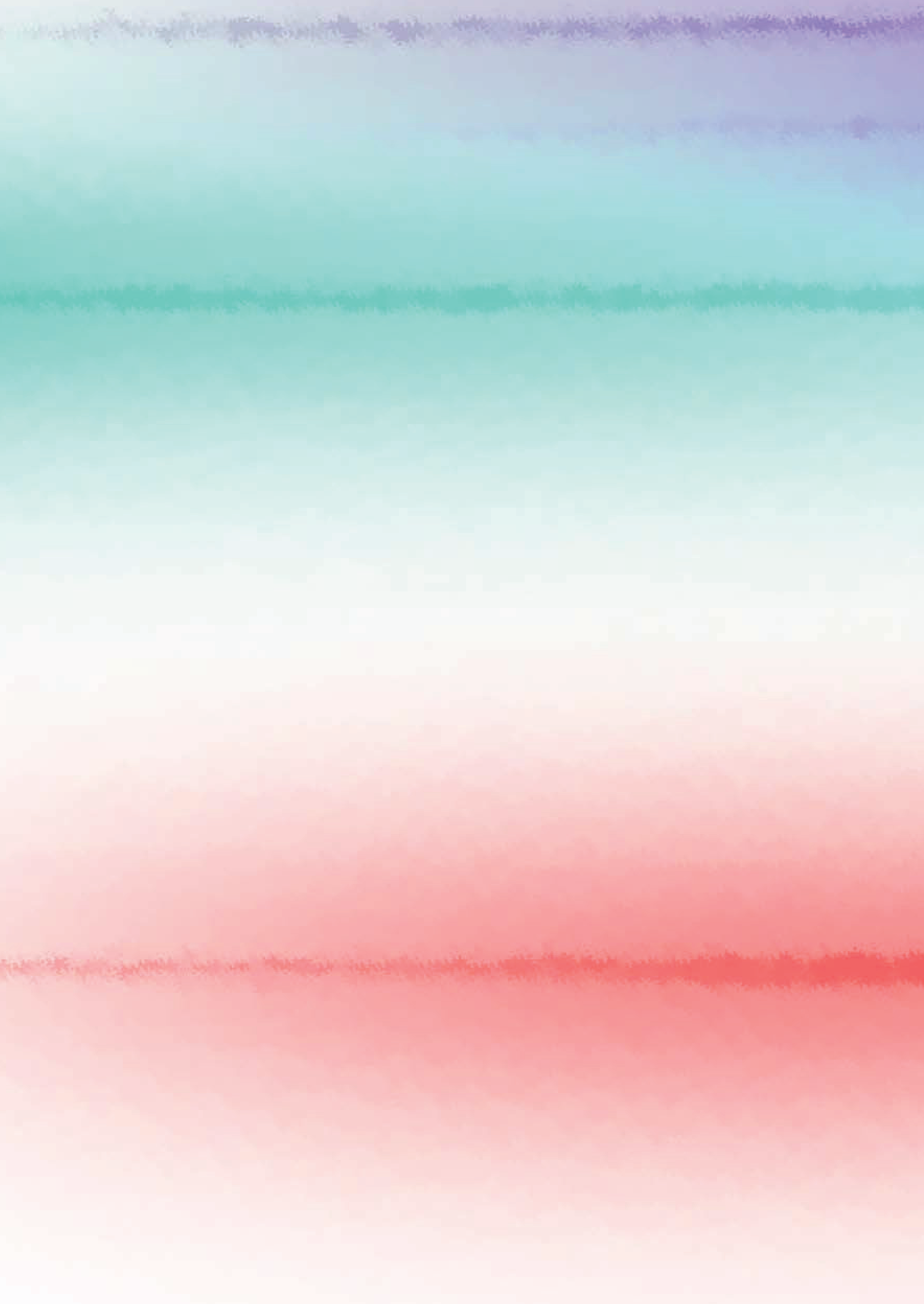


Nederlandse samenvatting

Introductie

Aan het eind van de 19^{e} eeuw dachten natuurkundigen dat alles verklaard kon worden met behulp van de mechanica van Newton en het elektromagnetisme van Maxwell. Rond de eeuwwisseling bleek echter dat waarnemingen op (sub)atomaire schaal niet verklaard konden worden met het toenmalige beeld van de materie. Natuurkundigen gingen op zoek naar een theorie die het gedag van microscopische deeltjes kon beschrijven. Gedurende deze zoektocht kwamen ze tot de ontdekking dat het gedrag van microscopische deeltjes kon worden beschreven door deeltjes ook op te vatten als golven en golven (licht) als deeltjes: de quantummechanica was 'geboren'. Deze nieuwe theorie leidde tot een grote verandering in de manier waarop natuurkunde de microscopische wereld beschrijft. Deze verandering in het begrip van (sub)atomaire deeltjes veroorzaakte in de 20^{e} eeuw de ontwikkeling van laser- en halfgeleiderfysica: de eerste quantumrevolutie. Op dit moment is er een tweede quantumrevolutie gaande: quantummechanische principes worden nu gebruikt om nieuwe materialen en technologieën te ontwikkelen. Quantummechanica heeft een enorme impact op de maatschappij en deze impact zal alleen maar groter worden. Daarom is quantummechanica in veel landen onderdeel geworden van het middelbare schoolcurriculum.

Onderzoeksvragen

Quantummechanica is gebaseerd op geavanceerde wiskunde, die geen onderdeel uitmaakt van het curriculum van de middelbare school. Daarnaast heeft quantummechanica geleid tot een nieuwe manier van denken die conflicteert met het klassieke denken van scholieren. Het is daarom nodig om te onderzoeken op welke wijze quantummechanica op een toereikende manier kan worden onderwezen aan middelbare scholieren. Hierbij is het van belang te onderzoeken welke onderwerpen van quantummechanica van belang zijn, welke moeilijkheden middelbare scholieren ondervinden bij het leren van quantummechanica en op welke wijze we leerlingen kunnen helpen om quantummechanica beter te begrijpen. In dit proefschrift presenteren we daarom ons onderzoek naar de volgende onderzoeksvragen:

(1) Wat is de huidige stand van zaken in het onderzoek naar begripsproblemen, lesstrategieën en onderzoeksinstrumenten voor quantummechanica gericht op het middelbare schoolniveau?
(2) Welke onderwerpen vinden Nederlandse experts op het gebied van quantummechanica en aanverwante onderzoeksgebieden belangrijk om te onderwijzen op middelbare scholen?
(3) Welke begripsproblemen hebben Nederlandse leerlingen na hun lessen over quantummechanica? En wat zijn de onderliggende problemen en oorzaken die leiden tot deze begripsproblemen?
(4) Is het mogelijk om het begrip van quantummechanica te verbeteren door de onderliggende oorzaken en problemen aan te pakken?

In dit hoofdstuk geven we een samenvatting van de vier onderzoeken in de proefschrift en reflecteren we op de resultaten van deze onderzoeken.

De huidige stand van zaken van onderzoek naar het onderwijzen van quantummechanica

In hoofdstuk 2 hebben we door middel van een literatuuronderzoek in kaart gebracht wat de huidige stand van zaken is in het onderzoek naar het onderwijzen van quantummechanica op middelbare school- en bachelor niveau. Uit een analyse van 75 artikelen blijkt dat er veel onderzoek is gedaan naar het begrip van de golf-deeltjesdualiteit en atomen, maar minder naar het begrip van de golffunctie en complex quantumgedrag. Bovendien is het onderzoek naar het begrip van de golffunctie alleen gericht op het bachelor niveau. Uit bestaand onderzoek blijkt dat het feit dat studenten de neiging hebben om vast te houden aan hun klassieke, deterministische manier van denken het belangrijkste probleem is. Voor de golf-deeltjes dualiteit leidt dit tot een vermenging van golf- en deeltjesgedrag, voor golffuncties tot een te letterlijke interpretatie van analogieën, en voor het atoom tot het vasthouden aan semi-klassieke atoommodellen. Studenten hebben moeite om hun nieuwe kennis van quantummechanica te integreren in hun bestaande, klassieke denkstructuren.

Het literatuuronderzoek laat ook zien dat er verschillende onderzoeksinstrumenten en testen zijn, maar deze testen zijn vooral gericht op het niveau van de bachelor-studenten en beslaan maar een deel van de deelonderwerpen van quantummechanica. Slechts één test, de QMCS, heeft betrekking op golf-deeltje dualiteit, golffuncties, atomen en complex quantumgedrag. Maar deze test is niet grondig geëvalueerd voor het middelbare schoolniveau en bevat daarnaast te weinig vragen voor een statistische analyse.

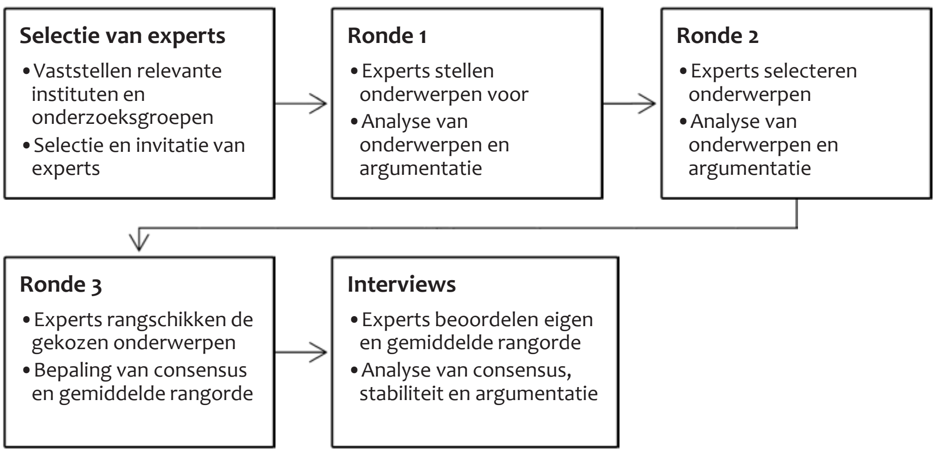
Een analyse van eerder onderzoek toont ook aan dat er verschillende methoden zijn gebruikt om moeilijkheden bij het leren van quantummechanica aan te pakken. Van slechts enkele van deze methoden is de impact op het begrip van de leerlingen geëvalueerd. Deze evaluaties toonden aan dat studenten niet noodzakelijkerwijs een wiskundige aanpak nodig hebben om quantummechanica te begrijpen. Daarnaast zijn er drie benaderingen aangetoond die het begrip van de studenten verbeteren: (1) nadruk op interpretaties; (2) nadruk op de ontwikkeling van, en de verschillen tussen verschillende atoommodellen, en (3) actief leren. Ook zijn er veel interactieve applicaties ontworpen. Deze applicaties zijn voornamelijk geëvalueerd voor praktisch gebruik, daarom is er meer onderzoek nodig naar hun invloed op het begrip van de studenten.

Er is veel geschreven en gepubliceerd over het onderwijzen van een inleiding in de quantummechanica. Maar omdat quantummechanica pas sinds kort op middelbare scholen is ingevoerd, is er niet veel empirisch onderzoek gedaan. Daarom zijn er in het gedane literatuuronderzoek veel niet-empirische studies opgenomen.

Hoofdstuk 2 geeft een overzicht van veel voorkomende problemen, maar er valt nog veel te leren over de onderliggende problemen van leerlingen en de impact van specifieke onderwijsstrategieën. Duidelijk is dat studenten moeite hebben met de niet-klassieke en niet-deterministische manier van denken. Er zijn verschillende veronderstellingen over hoe deze moeilijkheden effectief kunnen worden aangepakt, maar er is behoefte aan meer empirisch onderzoek naar het effect van verschillende onderwijsstrategieën. Het ontbreken van adequate onderzoeksinstrumenten kan een reden zijn voor het gebrek aan empirisch onderzoek. Om het onderzoek met betrekking tot het onderwijzen van quantummechanica op middelbare scholen te bevorderen, is het ontwerpen van een geschikte en goed geëvalueerde concepttest van groot belang.

Belangrijke onderwerpen voor het onderwijzen van quantummechanica op middelbare scholen

Hoofdstuk 3 beschrijft een Delphi-studie waarin is onderzocht welke onderwerpen Nederlandse experts op het gebied van quantummechanica en gerelateerde onderzoeksvelden belangrijk vinden om op middelbare scholen te onderwijzen. De Delphi-studie bestond uit drie rondes en een aanvullend interview (zie Figuur 1). In de eerste ronde werd de experts gevraagd om onderwerpen uit de quantummechanica voor te stellen die zij belangrijk vonden om te onderwijzen op middelbare scholen. Ook werd er gevraagd om te beargumenteren waarom ze deze onderwerpen belangrijk vonden. In de tweede ronde kregen de experts een overzicht van de voorgestelde onderwerpen en de argumentatie van alle experts. Deze keer werd de experts gevraagd om alle onderwerpen te selecteren die zij belangrijk vonden voor het natuurkundecurriculum en dit te beargumenteren. In de derde en laatste ronde kregen de experts een overzicht van de meest gekozen



Figuur 1 Opzet van de Delphi-studie

onderwerpen en de bijbehorende argumentatie. In deze ronde werd de experts gevraagd om de onderwerpen te rangschikken naar belangrijkheid.

Uit het Delphi-onderzoek blijkt dat er een gematigde tot sterke consensus is met betrekking tot het opnemen van de volgende onderwerpen in het natuurkundecurriculum:

(1) De golf-deeltjes dualiteit;
(2) het deeltjesgedrag van licht;
(3) golffuncties;
(4) de deBroglie-golflengte;
(5) waarschijnlijkheid;
(6) energieniveaus en kwantisatie; en
(7) Heisenbergs onzekerheidsprincipe.

De volgende voorbeelden werden door de meerderheid van de deskundigen als belangrijk beschouwd:

(1) Het dubbelspleet-experiment;
(2) spectraallijnen;
(3) het foto-elektrisch effect;
(4) de atoomstructuur;
(5) de eendimensionale oneindige potentiaalput;
(6) het waterstofatoom; en
(7) het periodiek systeem.

Er was geen consensus over welke toepassingen deel zouden moeten uitmaken van het curriculum. Uit interviews bleek dat de meningen van de experts vooral gebaseerd waren op het idee dat studenten een zeker begrip moeten hebben van belangrijke wetenschappelijke concepten. Onderwerpen die als te complex of abstract werden beschouwd, werden als minder essentieel beschouwd.

Als we kijken naar het Nederlandse natuurkundecurriculum en het internationale kerncurriculum , dan zien we dat deze in belangrijke mate overeenkomen met de onderwerpen die door deskundigen als belangrijk worden beschouwd. Echter, de resultaten uit dit onderzoek zijn gebaseerd op de mening van academici, die voornamelijk redeneerden vanuit welke kennis zij belangrijk achtten. Ook baseerden zij hun keuzes op hun inschatting van de haalbaarheid van het onderwijzen van een onderwerp. De resultaten van dit onderzoek zijn daarom slechts een startpunt voor het ontwerp van een curriculum. Er is nog behoefte aan onderzoek naar de haalbaarheid van de verschillende onderwerpen en naar de invloed van de verschillende onderwerpen op de attitude en vaardigheden van leerlingen.

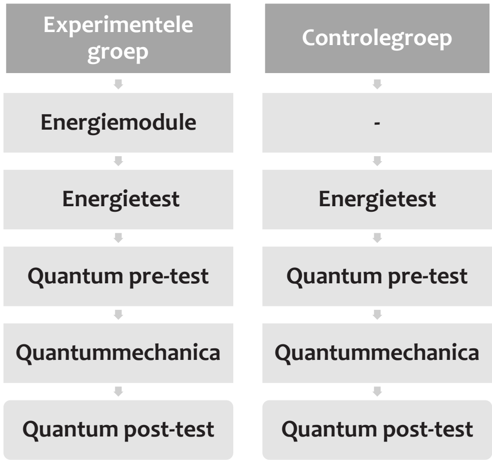
Begripsproblemen van scholieren over de eendimensionale oneindige potentiaalput en tunneling

Hoofdstuk 4 beschrijft de resultaten van een onderzoek naar de begripsproblemen die Nederlandse middelbare scholieren hebben nadat quantummechanica in de klas behandeld is. Hiervoor is een conceptuele begripstest afgenomen, die gebaseerd is op de onderwerpen van het Nederlandse natuurkundecurriculum. Deze test bestond uit open vragen, meerkeuzevragen en het uitleggen van de gemaakte keuze. Ook zijn er enkele leerlingen geïnterviewd.

Uit de kwantitatieve analyse van de meerkeuzevragen blijkt dat Nederlandse middelbare scholieren dezelfde moeilijkheden ondervinden als bachelor-studenten in eerder onderzoek: de leerlingen vermengen klassieke- en quantummodellen en maken oneigenlijk gebruik van klassieke modellen en beschrijvingen. De kwalitatieve analyse van de open vragen, uitleg en interviews toont aan dat Nederlandse scholieren moeite hebben om de kennis van het 1D oneindige potentieel goed te verbinden met hun voorkennis. Bij het redeneren over de eendimensionale oneindige potentiaalput gebruiken de scholieren de golf- en de energierepresentatie vaak in één gecombineerd model. Dit resulteert in creatieve, maar onjuiste modellen. De scholieren verwarren bijvoorbeeld de amplitude of de evenwichtsstand met het energieniveau, of beschrijven een deeltje dat trilt of beweegt over een sinusvormige baan. Bij het omschrijven van tunneling redeneren de studenten vaak deterministisch. Scholieren beschrijven bijvoorbeeld een deeltje dat door of over een barrière beweegt, of gebruiken termen zoals inspanning of afstand. De belangrijkste problemen die in deze studie zijn gevonden, hebben te maken met de moeite die scholieren hebben om hun nieuwe kennis van quantummechanica te integreren in hun bestaande denkstructuren. Het is daarom van belang om onderzoek te doen naar de relatie tussen klassieke voorkennis en het begrip van quantummechanica.

De invloed van begrip van potentiële energie op het begrip van quantummechanica

Hoofdstuk 5 beschrijft een onderzoek naar de invloed van het begrip van de voorkennis over potentiële energie op het begrip van quantummechanica. Hiervoor is een quasi-experimentele interventie uitgevoerd, waarbij de experimentele groep een aanvullend programma kreeg over potentiële energie in klassieke contexten. Met behulp van een begripstest over energie en een quantumtest over de eendimensionale oneindige potentiaalput en tunneling is er onderzocht wat de invloed van dit aanvullende programma was op het begrip van quantummechanica. De energietest is hierbij gebruikt als voorkennistest, de quantumtest als pre- en post-test (zie Figuur 2).



FIGUUR 2 Onderzoeksopzet

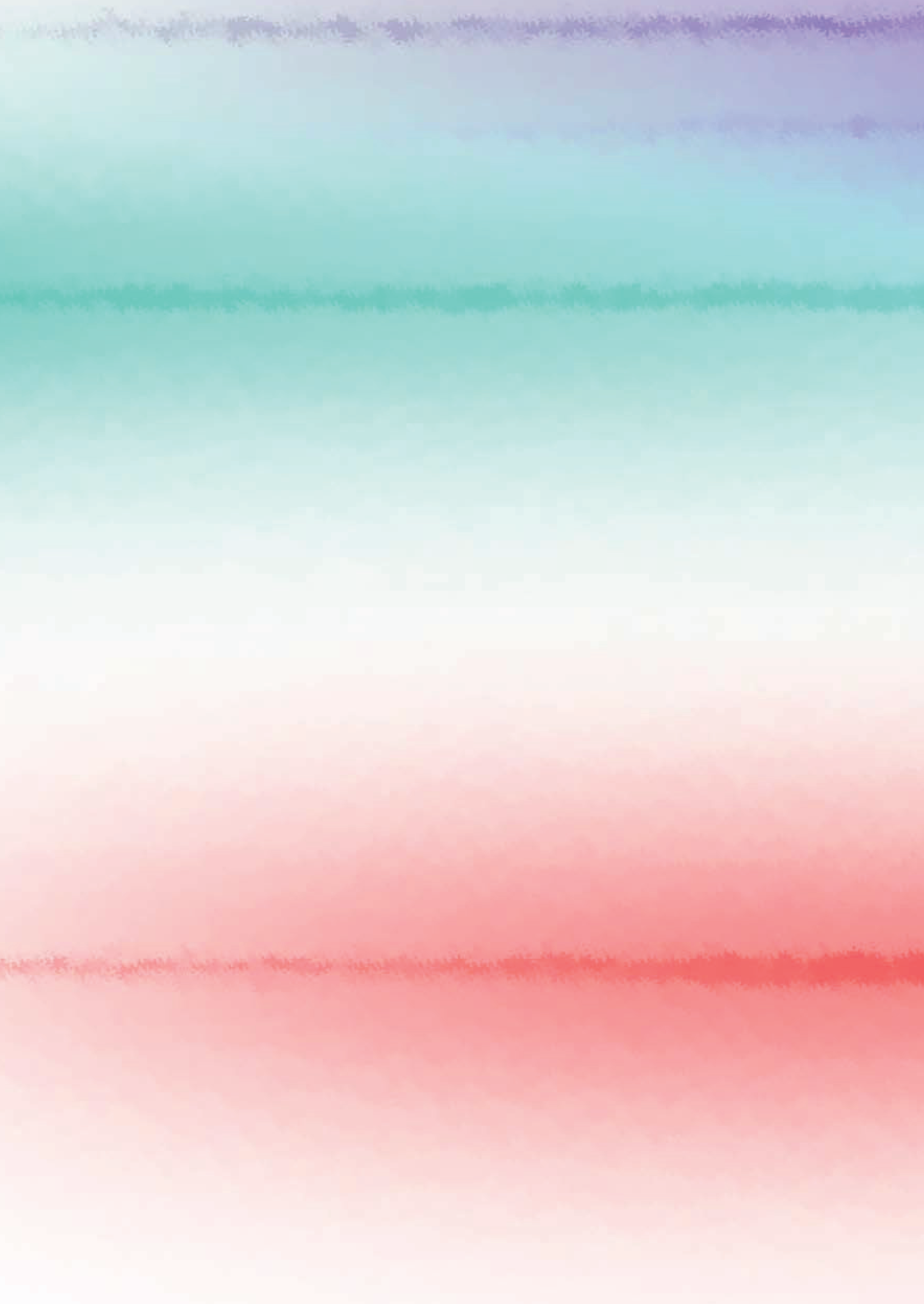
Uit analyse van de testresultaten blijkt dat de experimentele groep niet alleen een significant beter begrip had van potentiële energie, maar ook van quantummechanica, zelfs al voorafgaand aan quantummechanica-lessen. Deze resultaten laten zien dat begrip van quantummechanica wordt ondersteund door een goed begrip van potentiële energie. Statistische analyse van de energietest en de quantum pre- en posttest toont aan dat er een significante correlatie bestaat tussen het begrip van potentiële energie en quantummechanica. De waargenomen correlatie kan vooral worden toegeschreven aan de correlatie tussen begrip van 'de relatie tussen potentiële energie en positie' en het begrip van quantummechanica. Begrip van potentiële energie heeft dus een positieve invloed op het begrip van quantummechanica, maar de resultaten van dit onderzoek roepen ook de vraag op of er andere vaardigheden of natuurkundige concepten zijn die belangrijk zijn voor het begrip van quantummechanica.

CONCLUSIES

Uit het literatuuronderzoek blijkt dat er behoefte is aan meer empirisch onderzoek naar de begripsproblemen van middelbare scholieren bij het leren van quantummechanica, vooral wat betreft het begrip van de golffunctie, de oneindige potentiaalput en tunneling. Om meer inzicht te krijgen in onderliggende problemen van scholieren en systematisch te onderzoeken op welke wijze deze problemen voorkomen of bestreden kunnen worden, is het ontwikkelen van een gevalideerde begripstest gericht op middelbare scholieren van groot belang.

Uit de Delphi-studie blijkt dat de experts de golf-deeltjesdualiteit, golffuncties en atomen essentiële onderwerpen vinden om te onderwijzen binnen het natuurkundecurriculum op middelbare scholen. De deskundigen in dit onderzoek redeneren echter vooral vanuit kennis. Voor het ontwerp van een curriculum is het ook van belang om te onderzoeken wat de invloed van de verschillende deelonderwerpen is op de houding van scholieren ten opzichte van wetenschappelijk onderzoek en hun inzicht in het maatschappelijke belang van quantummechanica.

In het onderzoek naar begripsproblemen komt naar voor dat veel begripsproblemen te maken hebben met een incorrecte koppeling van quantummechanica met de (klassieke) voorkennis die de scholieren hebben. De resultaten van deze studie impliceren daarom dat het belangrijk is om de scholieren te helpen om de nieuwe kennis van quantummechanica in hun bestaande denkstructuren te integreren. In aanvulling hierop, laat het resultaat van de vierde studie zien dat een grotere voorkennis van potentiële energie het begrip van quantummechanica positief beïnvloedt. De uitkomst van dit onderzoek roept de vraag op of er andere onderwerpen uit de natuurkunde van belang zijn voor het begrijpen van quantummechanica. Daarom is er behoefte aan meer onderzoek naar de invloed van het begrip van klassieke natuurkundeconcepten op het begrip van quantummechanica en naar hoe we deze concepten op een goede manier een plaats kunnen geven binnen het natuurkundecurriculum.



Dankwoord

Dankwoord

Na een periode als deze is het goed om terug te kijken. Wat heb ik veel geleerd! Wat heb ik veel leuke dingen mogen doen! Het begon allemaal met een idee; ik wilde extra verdieping naast het lesgeven, iets op het grensvlak van onderwijs en actuele natuurkunde, iets waardoor leerlingen meer onder de indruk zouden raken door de wereld om hen heen. Ik ben dankbaar dat ik de afgelopen jaren bij ELAN bezig mocht zijn met het vormgeven en uitwerken van dit idee. Maar ik had dit natuurlijk niet kunnen doen zonder de ondersteuning van de mensen om mij heen.

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