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Opportunities to learn offered by primary school mathematics textbooks in the Netherlands

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OPPORTUNITIES TO LEARN OFFERED BY PRIMARY SCHOOL MATHEMATICS TEXTBOOKS IN THE NETHERLANDS

LEERAANBOD EN LEERONDERSTEUNING IN NEDERLANDSE REKEN-WISKUNDEMETHODES

(met een samenvatting in het Nederlands)

Proefschrift

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door
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Voor Désirée

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Chapter 1

Introduction

Introduction

1. Textbooks are important

In mathematics education, textbooks have a decisive role in shaping learning opportunities for students and supporting teachers' work (Rezat, Fan, Hattermann, Schumacher, & Wuschke, 2019). For many teachers, textbooks are the main resource for realizing their mathematics teaching (Stein, Remillard, & Smith, 2007), even in times of digitalization (Pepin, Gueudet, & Trouche, 2013). Of course, what is in the textbook is not necessarily the same as what is taught in class (e.g., Stein & Smith, 2010). Nevertheless, in multiple countries textbooks have a substantial impact on teachers' instructional decision-making and strong relations have been found between textbooks and classroom instruction (Valverde, Bianchi, Wolfe, Schmidt, & Houang, 2002).

In the Netherlands too, textbooks are of great importance. TIMSS research (Meelissen et al., 2012; Mullis, Martin, Foy, & Arora, 2012; see also Hiebert et al., 2003) and the large-scale Periodic Assessments of the Educational Level carried out by Cito, the Dutch institute for educational measurement (Hop 2012; Scheltens, Hemker, & Vermeulen, 2013), showed that a vast majority of Dutch primary school teachers rely heavily on the textbook they use. Most teachers involved in these studies indicate that they follow almost all the content in their textbook and only a minority uses additional resources, mainly materials for repetition. In two of the Cito studies (Janssen, Van der Schoot, & Hemker, 2005; Kraemer, Janssen, Van der Schoot, & Hemker, 2005), a majority of teachers reported to use such additional learning materials. Furthermore, several studies revealed that Dutch teachers, in addition to following the content of the textbook, also adhere, to a certain extent, to the instructional directions provided by the guidelines (Blok & Elshof, 2012; De Vos, 1998; Gravemeijer, Van den Heuvel-Panhuizen et al., 1993; Harskamp & Suhre, 1988; Van Putten, Van den Brom-Snijders, & Beishuizen, 2005). Thus, in the Netherlands, textbooks have a determining influence on the daily teaching practice and consequently on the learning opportunities offered to Dutch students. That the latter is indeed the case is confirmed by the Cito studies, which repeatedly showed significant differences between the effects of different textbooks (corrected for relevant co-variables) on the learning performances of the students taught with them (Bokhove, Van der Schoot, & Eggen, 1996a, 1996b; Hop, 2012; Janssen, Van der Schoot, Hemker,

& Verhelst, 1999; Kraemer et al., 2005; Noteboom, Van der Schoot, Janssen, & Veldhuijzen, 2000; Scheltens et al., 2013; Wijnstra, 1988).

2. Textbooks are mediators of the curriculum

Textbooks are a main part of the curriculum. Curriculum itself is a comprehensive concept that includes multiple elements and levels (see Goodlad, 1979). A common distinction made is that between the intended, the implemented and the attained curriculum (e.g., Travers & Westbury, 1989; Van den Akker, 2003). The intended curriculum refers to the intentions, aims and objectives of education, for example the statutory goals of education in a particular country. The implemented curriculum concerns the teaching-learning processes that actually take place in schools and classrooms. The attained curriculum refers to the results of these processes, in the form of learning outcomes and experiences.

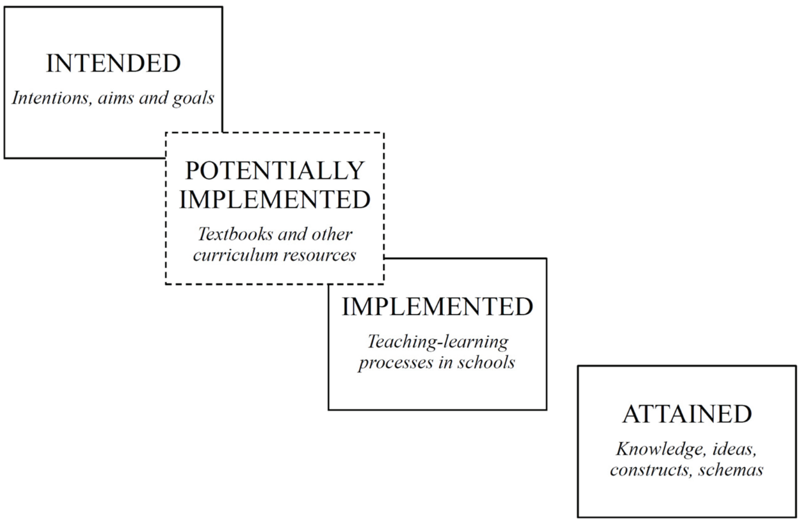

Figure 1. Levels of curriculum (adapted version from Valverde et al., 2002)

Textbooks can be regarded as an intermediate layer between the intended curriculum and the implemented curriculum, and, as such, are referred to as the potentially implemented curriculum (Figure 1) (Valverde et al., 2002). This means that textbooks can be seen as a separate curriculum level mediating between curricular intentions on the one hand and concrete teaching approaches on the other hand. This applies to both *formal* intentions, such as statutory prescribed goals (the formal intended curriculum), and *ideal* intentions as ideas about didactics and pedagogy of teaching and learning (the ideal intended curriculum) (cf., Van den Akker, 2003; see also Pepin, Gueudet, & Trouche, 2013). Thus, textbooks are not only important for being a main resource for teachers, but also as a means to provide concrete elaborations for daily teaching practice of both formal and ideal intentions of education.

3. Textbooks in the Dutch educational context

3.1 Freedom of education

In the Netherlands, freedom of education is a constitutional right (Constitution, 2008). It originates from legal arrangements established in the mid-19th and early 20th century stating that schools can be founded based on a religious denomination or on a specific pedagogical approach, and that such schools receive the same financial resources from the government as public schools (Bakker, Noordman, & Rietveld-van Wingerden, 2010; Boekholt & De Booy, 1987).

Freedom of education functions as a main principle regarding educational guidelines and practice and because of it, the government has a restrained policy towards the realization of education. For example, regarding the intended curriculum, before the first Core Goals for primary education (Ministry of Education, 1993) could be established, eight years of debate took place whether or not governmental prescription of goals was compatible with the freedom of education (Letschert, 1998). Due to the educational freedom, the government strives to make a strict distinction between the *what* of education (the learning goals and content to be taught) and the *how* of it (the way in which this content is taught) (see Committee Parliamentary Research Education, 2008; Ministry of Education, 2008). This implies that schools are free to choose their own textbooks and other curriculum materials (Constitution, 2008), and that the government does not judge the quality or the content thereof (e.g., Ministry of Education, 2019). Until 2012, the Netherlands Institute for Curriculum Development checked very globally whether the Core Goals were recognizably present in textbooks (e.g., SLO, 1997, 2012), but nowadays these checks are no longer carried out. Since the government never interfered with textbooks, and there is no other authority that checks or approves them, there are hardly any to no restrictions regarding developing and releasing textbooks.

3.2 Realistic Mathematics Education

In the Netherlands, starting in the late 1960s, a reform has taken place that is currently known under the name *Realistic Mathematics Education* (RME) (Van den Heuvel-Panhuizen & Drijvers, 2020). The term *realistic* refers to reality, which is seen as a source for learning processes, as well as to the Dutch verb *zich realiseren*, which means to realize and to imagine what is going on (e.g., Van den Brink, 1973, 1989; Wijdeveld, 1980; Van den Heuvel-Panhuizen, 1996). RME was developed as an alternative for the mechanistic approach to mathematics education, which focused on teaching fixed procedures in a step-by-step manner and in which real-world problems were only used for the application of previously learned calculation procedures (e.g., De Jong, 1986; Treffers, 1978, 1987).

Characteristic for RME is that mathematics is not seen as ready-made knowledge to be transmitted to students, but as a human activity of mathematizing (e.g., Freudenthal, 1973, 1991). The latter is distinguished into horizontal and vertical mathematizing. Horizontal mathematizing refers to transforming real-world problems into mathematical terms, whereas vertical mathematizing refers to using mathematical means to solve the problems, generalizing the solution process and achieving a higher level of formalization (Treffers, 1987). Another big idea of RME is guided reinvention, which refers to a balance in the teaching-learning processes between, on the one hand, letting students think and invent things, and on the other hand providing guidance that provokes reflective thinking (Freudenthal, 1991). For this, students' own input, especially in the form of own productions of problems and own constructions of solution strategies, is crucial (Treffers, 1987, see also Van den Heuvel-Panhuizen, 2019).

RME has had a considerable impact on Dutch textbooks. The Cito Periodic Assessments indicate an increasing market share of RME-oriented textbooks from around 15% in the mid-1980s to 75% in the 1990s (Janssen et al., 1999) and further to 100% around 2004 (Janssen et al., 2005). However, as a result of a severe debate that took place halfway through the first decade of the 21st century (Van den Heuvel-Panhuizen, 2010), the textbook market became more diverse again. A lot of voices emerged that argued for abandoning RME in favor of a return to a traditional approach to mathematics education. A committee of the Royal Netherlands Academy of Sciences (KNAW) that was commissioned to make an inventory of the claims about the effectiveness of the traditional approach and RME, concluded that there was no convincing empirical evidence that one approach was more effective than the other (KNAW, 2009).

4. Focus of this PhD project

An important recommendation of the KNAW committee (2009) that was in line with this new diversity in textbooks, is that textbooks should be subjected to an in-depth and objective mathematics-didactical analysis, so that schools can make a justified choice and parents also have well-founded information. According to the committee, "this defines a field of activity for the new generation of researchers in mathematics and mathematics education" (KNAW, 2009, p. 80).[1] This call did not fall on deaf ears and led to this PhD study.

4.1 Earlier research on textbooks in the Netherlands

A first category of textbook research carried out in the Netherlands consists of historical studies. This category includes studies on mathematics curriculum and textbooks in the 15th and 16th century (Kool, 1999) and from the 19th century on (Leen, 1961; Treffers, 2015, see also Treffers & Van den Heuvel-Panhuizen, 2020). These studies describe developments concerning, for example, the mathematical content covered, the nature of the exercises, and pedagogical and didactical ideas on education in general and on mathematics education in particular. Similar descriptive studies were conducted specifically regarding geometry in the 19th and 20th century (De Moor, 1999), and on mathematics in vocational and higher education from the 17th century on (Krüger, 2014).

The main category of research on Dutch textbooks concerns comparative studies. From the mid-1980s to the early 1990s, three such studies were done focusing on the impact of the RME reform on textbooks and textbook use.

De Jong (1986) investigated to what degree the RME reform was incorporated in textbooks from 1970 to 1985. He classified textbooks based upon their content and didactical features as belonging to different approaches to mathematics education, including RME-oriented, mechanistic and hybrid textbooks. De Jong's study clearly illustrated the growing impact of RME in terms of decreasing market share of mechanistic textbooks and increasing market share of RME-oriented textbooks. However, it also showed that there were great differences in the way the respective approaches were elaborated in the textbooks. Within the category of mechanistic textbooks, De Jong distinguished two main variations.

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[1] Translation by author.

He also found that RME-oriented textbooks were rather varied in their alignment with the intentions and characteristics of RME. Furthermore, he identified a textbook that was presented by its publisher as based on RME, while it could not actually be classified as such.

Harskamp and Suhre (1988) researched the use in Grade 6 of three textbooks classified as ‘modern’ (*Operatoir Rekenen*, *Getal in Beeld* and *Taltaal,* released in respectively 1980, 1975 and 1976) and five textbooks marked as ‘traditional’ (*Op Veilig Spoor*, *Naar Aanleg en Tempo* 2nd edition, *Naar Zelfstandig Rekenen* 2nd edition, *Nieuw Rekenen* and *Niveau Cursus Rekenen*, published in respectively 1970, 1983, 1978, 1969 and 1970). The classification into modern and traditional textbooks corresponded with the classification of these textbooks by De Jong (1986) into respectively RME-oriented and mechanistic. Harskamp and Suhre (1988) established clear correspondences between the textbooks and the way in which the teachers used them. Teachers participating in the study varied the mathematical content in their lessons and applied differentiation in accordance with the directions provided in the textbooks they used.

Gravemeijer, Van den Heuvel-Panhuizen et al. (1993) investigated in the MORE study (Mathematics Textbooks Research) the use of the RME-oriented textbook *De Wereld in Getallen* (1st edition, 1980) and the mechanistic textbook *Naar Zelfstandig Rekenen* (edition of 1978). It was found that the nature of the teaching in Grade 1, 2 and 3 of the participating schools generally corresponded with the didactical approach the respective textbooks were attributed to, although this applied to a higher degree to the mechanistic textbook than to the RME-oriented textbook. Furthermore, a clear relationship was found between the content included in the respective textbooks and the learning outcomes regarding that content of students taught with them.

The Cito Periodic Assessments (Bokhove et al., 1996a, 1996b; Hop, 2012; Janssen, et al., 1999, 2005; Kraemer et al., 2005; Noteboom et al., 2000; Scheltens et al., 2013; Wijnstra, 1988) were conducted from 1987 to 2011 to investigate the achievements of students halfway through and at the end of primary school. Moreover, by including the textbooks used in the analyses, these studies also could reveal differences in achievements between students who were taught with different textbooks.

In the 1990s, two studies were carried out in which two RME-oriented textbooks were compared. Both studies included *De Wereld in Getallen* (the same edition as included in the MORE study) and *Rekenen & Wiskunde* (published in 1984). De Vos (1998) investigated the use of these textbooks in Grade 5 in ten schools. He found that the differences between the textbooks, especially regarding the directions for teachers, were reflected in the teaching of the teachers involved in the study. Although these teachers differed in the degree to which they followed their textbook, overall, noticeable relations were found between the textbook and the teaching, both regarding content and didactics. Another finding by De Vos (1998) was that teachers from 433 other schools using these and other textbooks, including mechanistic ones, also indicated that they were following the didactical directions of their textbooks.

Van Putten, Van den Brom-Snijders and Beishuizen (2005) compared strategy development on long division of students in Grade 4 taught with *De Wereld in Getallen* or *Rekenen & Wiskunde*. They did not research teachers' use of the textbook directly but analyzed the strategies the students used for carrying out long division compared with the directions the textbooks provided for it. Two significant differences were established between the strategies used by the two groups of students, corresponding with differences found in the two textbooks. Students taught with *De Wereld in Getallen* used more number relations solving long divisions, and students taught with *Rekenen & Wiskunde* made more progress in the use of schematic solutions. These findings indicate that most teachers in this study followed the approach on long division provided by their textbook.

After the turn of the century, four more studies on mathematics textbooks were carried out. Bruin-Muurling (2010) researched textbooks for primary school (Grade 6) and secondary education (Grade 7) regarding fractions. The textbooks included for primary school were *De Wereld in Getallen* (3rd edition, published in 2000), *Pluspunt* (2nd edition, 2000), *Alles Telt* (1st edition, 2000) and *Rekenrijk* (2nd edition, 2002). The secondary school textbooks were *Getal en Ruimte* (edition of 2004) and *Moderne Wiskunde* (edition of 2007). A finding of this study was that the didactical approaches towards fractions in the primary school textbooks and in those for secondary school, at first sight show similarities, but actually differ significantly. An analysis regarding multiplication of fractions revealed that the primary school textbooks aimed at number-specific and context-based procedures, while the secondary school textbooks immediately started with a formal mathematical procedure. As a result, a gap occurred in the teaching-learning trajectory, which was found to be reflected in the performance of secondary school students.

Van Stiphout (2011) analyzed the same two textbooks for secondary school, *Getal en Ruimte* and *Moderne Wiskunde*, on the approach to linear relationships and linear equations. It was found that both textbooks did not have a consistent instructional sequence at their disposal, and therefore did not support students' development of conceptual algebraic proficiency. This, again, was found to be reflected in students' performance.

Kolovou, Van den Heuvel-Panhuizen and Bakker (2009) analyzed primary school textbooks on the presence of non-routine problem-solving tasks. This is the first Dutch study which refers to the concept of opportunity-to-learn (OTL) as a reason for analyzing textbooks. For Grade 4, all textbooks on the market at the time were analyzed: *De Wereld in Getallen* (3rd edition, 2000), *Pluspunt* (2nd edition, 2000), *Alles Telt* (1st edition, 2000), *Rekenrijk* (2nd edition, 2002), *Wis en Reken* (2nd edition) and *Talrijk* (n.d.). It was found that the learning opportunities offered for genuine problem-solving were very limited and were mostly restricted to parts of the textbooks meant for the more able students.

Finally, Blok and Elshof (2012) studied the contemporary RME-oriented textbook *Wizwijs* (published in 2010). In this study, a majority of teachers of Grade 2, 4 and 6 indicated that they followed closely both the content and the teaching directions of this particular textbook.

Most textbooks studies carried out so far (apart from the historical studies) focused on the relationship between the potentially implemented curriculum (the textbooks) on the one hand and the implemented curriculum (teachers' use of textbooks) and the attained curriculum (the achievements of the students) on the other hand. Only the study by De Jong (1986) explicitly addressed the relationship between textbooks and the then prevailing ideas of RME, which can be regarded as an ideal intended curriculum. Regarding the relationships between textbooks and the formal intended curriculum, i.e., statutory prescribed goals, no research has been carried out yet.

Of all the previously mentioned studies, only the one by Kolovou et al. (2009) explicitly addressed OTL for researching textbooks. This is remarkable, because OTL is an important concept in educational research (see for an overview, e.g., Elliott & Bartlett, 2016; Scheerens, 2017). Central elements which determine the OTL are the time spent on learning in relation to the time needed to learn (e.g., Cogan & Schmidt, 2015), the content coverage and exposure, and the quality of instruction (e.g., Elliot & Bartlett, 2016). Textbooks provide directions for these elements and therefore influence the OTL, especially in the Dutch educational context where textbooks play such an important role in teaching. Also, in international research, the textbook in itself is seen as a key determinant of OTL (Valverde et al., 2002, see also, e.g., Hadar, 2017; Hagarty & Pepin, 2002, Törnroos, 2005; Van den Ham & Heinze, 2018).

4.2 Textbook research carried out in this PhD project

In this PhD project, analyses were carried out of Dutch mathematics textbooks regarding their content and didactics. More specifically, the research aimed to gain a better understanding of contemporary textbooks for primary school to provide solid information to the educational field. To do so, the concept of OTL was used as a lens to analyze textbooks. This means that textbooks were analyzed on features that contribute to OTL.

Three aspects of OTL offered by textbooks were distinguished in this research: the *learning content* included, the *performance expectations* articulated, and the *learning facilitators* incorporated. Content and performance expectations were adapted from Valverde et al. (2002). Content refers to the subject matter to be taught, and performance expectations refers to what students are expected to do with that content. A requirement of both content and performance expectations included in textbooks is that they are well aligned with the content and performance expectations prescribed in the formal intended curriculum (cf., Schmidt, Houang, & Cogan, 2002; Schmidt, Wang, & McKnight, 2005). Furthermore, how content and performance expectations are presented is also of importance (e.g., Valverde et al., 2002). In their presentation of content and performance expectations, textbooks provide several features that are meant to enable and facilitate learning, such as a particular sequence in which learning content is presented or a certain didactical model that is provided with it. The term learning facilitators refers to such features that are of influence on the quality of instruction, including features that belong to a certain view on mathematics education or a didactical approach.

The project as a whole had three foci. First, the research focused on textbooks with different didactical approaches to show the consequences of such approaches for the OTL. Second, historical comparisons were carried out of textbooks published in different time periods, with the aim of gaining a better understanding of RME, including the relationship between its intentions and the implementation in textbooks. The third focus was the coherence between the formal intended curriculum and the potentially implemented curriculum (see Table 1).

5. Structure of this thesis

This thesis consists of seven chapters, including five publications on the studies that have been carried out. Table 1 gives an overview of the chapters and the studies with their foci, the mathematical topics dealt with, and the grade levels of the textbook materials that were investigated.

Table 1
Structure of this thesis

Chapter		Study		
		Focus	Mathematical focus	Grade level
1	Introduction			
2	Freedom of design: The multiple faces of subtraction in Dutch primary school textbooks	Didactical approaches	Subtraction up to 100	2
3	Opportunity to learn problem solving in Dutch primary school mathematics textbooks	Didactical approaches	Problem solving	4 & 6
4	Past and current approaches to decimal numbers in Dutch primary school mathematics textbooks	Historical comparison	Decimal numbers	4, 5 & 6
5	Mathematics curriculum reform and its implementation in textbooks: Early addition and subtraction in Realistic Mathematics Education	Historical comparison	Early addition and subtraction	1, 2 & 3
6	Primary school mathematics in the Netherlands: The perspective of the curriculum documents	Curricular coherence	Multiple topics	1 to 6
7	Summing up and conclusion			

Chapter 2 addresses the consequences for the OTL of a specific didactical approach. For this, two Dutch textbook series based on different views on mathematics education were analyzed: the RME-oriented textbook *Rekenrijk* (3rd edition, 2009) and a textbook that was released as an alternative for the RME

approach: *Reken Zeker* (2010). The study focuses on subtraction up to 100 in Grade 2. The following research questions were addressed:

- Do Dutch mathematics textbooks reflect the content of the Dutch intended curriculum concerning subtraction up to 100?
- Do Dutch mathematics textbooks reflect the performance expectations of the Dutch intended curriculum concerning subtraction up to 100?
- What learning facilitators for learning subtraction up to 100 are incorporated in Dutch mathematics textbooks?

Chapter 3 addresses the OTL regarding problem solving in four textbooks. Three common Dutch textbooks were included that together are used in a vast majority of Dutch schools: *De Wereld in Getallen* (4^{th} edition, 2009), *Pluspunt* (3^{rd} edition, 2009), and *Alles Telt* (2^{nd} edition, 2009). In addition, a textbook with a contrasting approach was included: *Rekenwonders* (2011), the adapted Dutch version of the Singaporean textbook *My Pals Are Here! Maths*. The research questions addressed were:

- To what degree do current Dutch primary school textbooks contain mathematical problem-solving tasks?
- In what other ways do these textbooks facilitate the opportunity to learn problem solving?
- How inclusive are these textbooks with respect to offering opportunities to learn problem solving for students with varying mathematical abilities?

Chapter 4 describes a historical study which investigated what has changed in Dutch textbooks since the start of RME regarding content, performance expectations and learning facilitators for decimal numbers in Grade 4, 5 and 6. Three textbooks were analyzed: *De Wereld in Getallen* (4^{th} edition, 2009), which is the RME-oriented textbook that has the longest history with the most editions, and two pre-RME textbooks, namely *Functioneel Rekenen* (published in 1958) and *Nieuw Rekenen* (published in 1969). The research questions were:

- What RME characteristics of teaching decimal numbers can be identified in a typical contemporary RME-oriented textbook?
- To what degree are onsets of RME characteristics of teaching decimal numbers already present in pre-RME textbooks and in what way does the approach to teaching decimal numbers in a contemporary RME textbook differ from the approach in pre-RME textbooks?

In *Chapter 5*, two historical studies are described regarding the reform history of RME at two curriculum levels: the ideal intended level and the textbook level. The focus is on learning facilitators for early addition and subtraction in Grade 1, 2 and 3. In the first study on the intentions of RME, three core curriculum documents were studied: *Wiskobas* (1975), *Proeve* (1990) and *TAL* (1999). In the second study thirteen textbooks were analyzed: five consecutive editions of *De Wereld in Getallen* (published in 1981, 1991, 2001, 2009 and 2019), four editions of *Pluspunt* (published in 1991, 2000, 2009 and 2019), the textbook series *Rekenen & Wiskunde* (published in 1983) and its successor *Wis en Reken* (edition of 2000), and two editions of *Rekenrijk* (editions of 2000 and 2009). The research questions for the respective studies were:

- How did the RME approach on early addition and subtraction evolve?
- How was the RME approach implemented in consecutive generations of RME-oriented textbooks?

Chapter 6 addresses the coherence between the different mathematics curriculum levels. For this, the formal intended curriculum and the potentially implemented curriculum were studied and compared regarding multiple mathematical learning topics, including differentiated content and learning facilitators for different groups of students. In this study, the four most used textbooks were included: *De Wereld in Getallen* (4th edition 2009), *Pluspunt* (3rd edition, 2009), *Alles Telt* (2nd edition, 2009) and *Rekenrij*k (3rd edition, 2009). In this study, the coherence of the Dutch primary school mathematics curriculum was investigated, as expressed in the similarities and differences between the documents of the intended and the potentially implemented curriculum.

Finally, in Chapter 7, the findings from the five studies are summarized and discussed.

References

Bakker, N., Noordman, J., & Rietveld-van Wingerden, M. (2010). *Vijf eeuwen opvoeden in Nederland* [Five centuries of education in the Netherlands] (2nd edition). Assen, the Netherlands: Van Gorcum.

Blok, H., & Elshof, D. (2012). *Gebruik, waardering en leeropbrengsten bij Wizwijs, een rekenmethode voor het basisonderwijs* [Use, appreciation and learning outcomes of *Wizwijs*, a mathematics textbook for primary education]. Amsterdam, the Netherlands: Kohnstamm Institute, University of Amsterdam.

Boekholt, P., & De Booy, E. (1987). *Geschiedenis van de school in Nederland* [A history of schools in the Netherlands]. Assen, the Netherlands: Van Gorcum.

Bokhove, J., Van der Schoot, F., & Eggen, T. (1996a). *Balans van het rekenonderwijs halverwege de basisschool 2. Periodieke Peiling van het Onderwijsniveau* [Balance of arithmetic education halfway primary school 2. Periodic Assessment of the Education Level]. Arnhem, the Netherlands: Cito.

Bokhove, J., Van der Schoot, F., & Eggen, T. (1996a). *Balans van het rekenonderwijs aan het einde van de basisschool 2. Periodieke Peiling van het Onderwijsniveau* [Balance of arithmetic education at the end of primary school 2. Periodic Assessment of the Education Level]. Arnhem, the Netherlands: Cito.

Bruin-Muurling, G. (2010). *The development of proficiency in the fraction domain. Affordances and constraints in the curriculum.* Eindhoven, the Netherlands: Eindhoven School of Education, University of Eindhoven.

Cogan, L., & Schmidt, W. (2015). The concept of opportunity to learn (OTL) in international comparisons of education. In K. Stacey & R. Turner (Eds.), *Assessing Mathematical Literacy*. doi:10.1007/978-3-319-10121-7_10

Committee Parliamentary Research Education. (2008). *Tijd voor onderwijs* [Time for education]. Den Haag, the Netherlands: Sdu Uitgevers.

Constitution. Artikel 23: Het openbaar en bijzonder onderwijs [Article 23: Public and denominational education] (2008).

De Jong, R. (1986). *Wiskobas in methoden* [Wiskobas in textbook series]. Utrecht, the Netherlands: Utrecht University.

De Moor, E. (1999). *Van vormleer naar realistische meetkunde. Een historisch-didactisch onderzoek van het meetkundeonderwijs aan kinderen van vier tot veertien jaar in Nederland gedurende de negentiende en twintigste eeuw* [From ‘vormleer’ to realistic geometry. A historical-didactic research into geometry teaching to children of four to fourteen years of age during the nineteenth and twentieth centuries in the Netherlands]. Utrecht, the Netherlands: CD-β Press/Freudenthal Institute, Utrecht University.

De Vos, W. (1998). *Het methodegebruik op basisscholen* [The use of textbooks at primary schools]. Maastricht, the Netherlands: Shaker Publishing.

Elliott, S., & Bartlett, B. (2016). *Opportunity to learn. Oxford online handbooks*. doi:0.1093/oxfordhb/9780199935291.013.70

Freudenthal. H. (1973). *Mathematics as an Educational Task*. Dordrecht, the Netherlands: Reidel Publishing Company.

Freudenthal, H. (1991). *Revisiting mathematics education.* Dordrecht/Boston/London: Kluwer Academic Publishers.

Goodlad, J. (1979). *Curriculum Inquiry. The study of curriculum practice.* New York, NY: McGraw-Hill.

Gravemeijer, K., Van den Heuvel-Panhuizen, M., Van Donselaar, G., Ruesink, N., Streefland, L., Vermeulen, W., Te Woerd, D., & Van der Ploeg, D. (1993). *Methoden in het reken-wiskundeonderwijs, een rijke context voor vergelijkend onderzoek* [Textbooks in mathematics education, a rich context for comparative research]. Utrecht, the Netherlands: CD-β Press/Freudenthal Institute, Utrecht University.

Hadar, L. (2017). Opportunities to learn: Mathematics textbooks and students' achievements. *Studies in Educational Evaluation, 55,* 153–166.

Haggarty, L., & Pepin, B. (2002). An investigation of mathematics textbooks and their use in English, French and German Classrooms: Who gets an opportunity to learn what? *British Educational Research Journal, 28*(4), 567–590.

Harskamp, E., & Suhre, C. (1988). Rekenmethoden vergeleken: een effectonderzoek aan het einde van de basisschool [Arithmetic textbooks compared: an effect research at the end of primary school]. *Pedagogische Studiën, 65*, p. 208–219.

Hiebert, J., Gallimore, R., Garnier, H., Givven, K., Hollingsworth, H., Jacobs, J., et al. (2003). *Teaching mathematics in seven countries: Results from the TIMSS 1999 Video Study.* Washington, DC: U.S. Department of Education, National Center for Education Statistics.

Hop, M. (Ed.). (2012). *Balans van het reken-wiskundeonderwijs halverwege de basisschool 5. Periodieke Peiling van het Onderwijsniveau* [Balance of mathematics education halfway primary school 5. Periodic Assessment of the Education Level]. Arnhem, the Netherlands: Cito.

Janssen, J., Van der Schoot, F., & Hemker, B. (2005). *Balans van het reken-wiskundeonderwijs aan het einde van de basisschool 4. Periodieke Peiling van het Onderwijsniveau* [Balance of mathematics education at the end of primary school 4. Periodic Assessment of the Education Level]. Arnhem, the Netherlands: Cito.

Janssen, J., Van der Schoot, F., Hemker, B., & Verhelst, N. (1999). *Balans van het reken-wiskundeonderwijs aan het einde van de basisschool 3. Periodieke Peiling van het Onderwijsniveau* [Balance of mathematics education at the end of primary school 3. Periodic Assessment of the Education Level]. Arnhem, the Netherlands: Cito.

KNAW [Royal Netherlands Academy of Sciences]. (2009). *Rekenonderwijs op de basisschool* [Mathematics education in primary school]. Amsterdam: KNAW.

Kolovou, A., Van den Heuvel-Panhuizen, M., & Bakker, A. (2009). Non-routine problem solving tasks in primary school mathematics textbooks – A needle in a haystack. *Mediterranean Journal for Research in Mathematics Education, 8*(2), 29–66.

Kool, M. (1999). *Die conste vanden getale. Een studie over Nederlandstalige rekenboeken uit de vijftiende en zestiende eeuw, met een glossarium van rekenkundige termen* [The art of numbers. A study on fifteenth and sixteenth century Dutch arithmetic books, with a glossary of arithmetical terms]. Hilversum, the Netherlands: Verloren.

Kraemer, J., Janssen, J., Van der Schoot, F., & Hemker, B. (2005). *Balans van het reken-wiskundeonderwijs halverwege de basisschool 4. Periodieke Peiling van het Onderwijsniveau* [Balance of mathematics education halfway primary school 4. Periodic Assessment of the Education Level]. Arnhem, the Netherlands: Cito.

Krüger, J. (2014). *Actoren en factoren achter het wiskundecurriculum sinds 1600* [Actors and factors behind the mathematics curriculum since 1600]. Utrecht, the Netherlands: Freudenthal Institute for Science and Mathematics Education, Utrecht University.

Leen, A. (1961). *De ontwikkeling van het rekenonderwijs op de lagere school in de 19^e^ en het begin van de 20^e^ eeuw* [The development of arithmetic education in primary school in the 19th and early 20th century]. Groningen, the Netherlands: J.B. Wolters.

Letschert, J. (1998). *Wieden in een geheime tuin. Een studie naar kerndoelen in het Nederlandse basisonderwijs* [Weeding in a secret garden. A study of core goals in Dutch primary education]. Enschede, the Netherlands: SLO.

Meelissen, M., Netten, A., Drent, M., Punter, R., Droop, M., & Verhoeven, L. (2012). *PIRLS en TIMSS 2011. Trends in leerprestaties in Lezen, Rekenen en Natuuronderwijs* [PIRLS and TIMSS 2011. Trends in achievement in Reading, Mathematics and Science]. Enschede/Nijmegen, the Netherlands: Twente University/Radboud University.

Ministry of Education. (1993). *Kerndoelen basisonderwijs* [Core Goals Primary Education]. Den Haag, the Netherlands: Ministry of Education.

Ministry of Education. (2008). Beleidsreactie 'Tijd voor onderwijs' - brief aan de Tweede Kamer [Reaction to 'Time for education' - letter to the Parliament]. Den Haag, the Netherlands: Ministry of Education.

Ministry of Education. (2019). Kamervragen (Aanhangsel) 2019-2020, nr. 894 [Answers to Parliamentary questions 2019-2020 nr. 894]. Retrieved from https://zoek.officielebekendmakingen.nl/ah-tk-20192020-894.html

Mullis, I., Martin, M., Foy, P., & Arora, A. (2012). *TIMSS 2011 International results in mathematics*. Chestnut Hill, MA, USA: TIMSS & PIRLS International Study Center / International Association for the Evaluation of Educational Achievement.

Noteboom, A., Van der Schoot, F., Janssen, J., & Veldhuijzen, N. (2000). *Balans van het reken-wiskundeonderwijs halverwege de basisschool 3. Periodieke Peiling van het Onderwijsniveau* [Balance of mathematics education halfway through primary school 3. Periodic Assessment of the Education Level]. Arnhem, the Netherlands: Cito.

Pepin, B., Gueudet, G., & Trouche, L. (2013). Investigating textbooks as crucial interfaces between culture, policy and teacher curricular practice: two contrasted case studies in France and Norway. *ZDM Mathematics Education, 45*(5), 685–698. doi:10.1007/s11858-013-0526-2

Rezat, S., Fan, L., Hattermann, M., Schumacher, J., & Wuschke, H. (2019). *Proceedings of the Third International Conference on Mathematics Textbook Research and Development*. Paderborn, Germany: Paderborn University.

Scheerens, J. (Ed.). *Opportunity to Learn, Curriculum Alignment and Test Preparation.* SpringerBriefs in Education. doi:10.1007/978-3-319-43110-9_2

Scheltens, F., Hemker, B., & Vermeulen, J. (2013). *Balans van het reken-wiskundeonderwijs aan het einde van de basisschool 5. Periodieke Peiling van het Onderwijsniveau* [Balance of mathematics education at the end of primary school 5. Periodic Assessment of the Education Level]. Arnhem, the Netherlands: Cito.

Schmidt, W., Houang, R., & Cogan, L. (2002). A coherent curriculum. The case of mathematics. *America Educator 26*(2), 1–17.

SLO. (1997). *Gids voor onderwijsmethoden voor het basisonderwijs. Rekenen/wiskunde* [Guide on textbooks for primary education. Mathematics]. Enschede, the Netherlands: Netherlands Institute for Curriculum Development.

SLO. (2012). *Kerndoelenanalyse Alles Telt* [Core goal analysis *Alles Telt*]. Enschede, the Netherlands: Netherlands Institute for Curriculum Development.

Stein, M., Remillard, J., & Smith, M. (2007). How curriculum influences student learning. In F. Lester (Ed.), *Second handbook of research on mathematics teaching and learning* (Vol. 1, pp. 319–369). Charlotte, NC: Information Age Publishing.

Stein, M., & Smith, M. (2010). The influence of curriculum on students' learning. In B. Reys, R. Reys & R. Rubenstein (Eds.), *Mathematics curriculum. Issues, trends, and future directions* (pp. 351–362). Reston, VA: National Council of Teachers of Mathematics.

Travers, K., & Westbury, I. (1989). The IEA Study of Mathematics I: Analysis of Mathematics Curricula. Supplement. Urbana-Champaign, IL: University of Illinois at Urbana-Champaign.

Treffers, A. (1978). *Wiskobas doelgericht* [Wiskobas goal-directed]. Utrecht: IOWO.

Treffers, A. (1987). *Three dimensions. A model of goal and theory description in mathematics instruction – The Wiskobas project*. Dordrecht, the Netherlands: Reidel Publishing Company.

Treffers, A. (2015). *Weg van het cijferen. Rekenmethodes vanaf 1800 tot heden* [Love for algorithms / Less emphasis on algorithms. Mathematics textbooks from 1800 until the present]. Utrecht, the Netherlands: Utrecht University.

Treffers, A., & Van den Heuvel-Panhuizen, M. (2020). Dutch didactical approaches in primary school mathematics as reflected in two centuries of textbooks. In M. van den Heuvel-Panhuizen (Ed.), *National reflections on the Netherlands didactics of mathematics. Teaching and learning in the context of Realistic Mathematics Education* ICME-13 Monographs (pp. 77–103). doi:10.1007/978-3-030-33824-4_6

Törnroos, J. (2005). Mathematical textbooks, opportunity to learn and student achievement. *Studies in Educational Evaluation, 31*(4), 315–327.

Valverde, G., Bianchi, L., Wolfe, R., Schmidt, W., & Houang, R. (2002). *According to the book. Using TIMSS to investigate the translation of policy into practice through the world of textbooks*. Dordrecht/Boston/London: Kluwer Academic Publishers.

Van den Akker, J. (2003). Curriculum perspectives: An introduction. In J. van den Akker, W. Kuiper & U. Hameyer (Eds.), *Curriculum landscapes and trends* (pp. 1–10). Dordrecht, the Netherlands: Kluwer Academic Publishers.

Van den Brink, J. (1973). Bijna noemen [Almost mention it]. *Wiskobas-Bulletin 3*, 129–131.

Van den Brink, J. (1989). *Realistisch rekenonderwijs aan jonge kinderen* [Realistic mathematics education for young students]. Utrecht, the Netherlands: Utrecht University.

Van den Ham, A., & Heinze, A. (2018). Does the textbook matter? Longitudinal effects of textbook choice on primary school students' achievement in mathematics. *Studies in Educational Evaluation, 59*, 133–140.

Van den Heuvel-Panhuizen, M. (1996). *Assessment and realistic mathematics education*. Utrecht, the Netherlands: CD-β Press/Freudenthal Institute, Utrecht University.

Van den Heuvel-Panhuizen, M. (2010). Reform under attack – Forty years of working on better mathematics education thrown on the scrapheap? No way! In L. Sparrow, B. Kissane, & C. Hurst (Eds.), *Shaping the future of mathematics education: Proceedings of the 33rd annual conference of the Mathematics Education Research Group of Australasia* (pp. 1–25). Fremantle, Australia: MERGA.

Van den Heuvel-Panhuizen, M. (2019). Didactics of Mathematics in the Netherlands. In W. Blum et al. (Eds.), *European Traditions in Didactics of Mathematics*, ICME-13 Monographs (pp. 57–94). Cham, Switzerland: Springer Nature.

Van den Heuvel-Panhuizen, M., & Drijvers, P. (2020). Realistic Mathematics Education. In S. Lerman (Ed.), *Encyclopedia of mathematics education*. Cham, Switzerland: Springer. doi:10.1007/978-3-030-15789-0_170

Van Putten, C., Van den Brom-Snijders, P., & Beishuizen, M. (2005). Progressive mathematization of long division strategies in Dutch primary schools. *Journal for Research in Mathematics Education, 36*, 44–73. doi:10.2307/30034920

Van Stiphout, I. (2011). *The development of algebraic proficiency.* Eindhoven, the Netherlands: Eindhoven School of Education, University of Eindhoven.

Wijdeveld, E. (1980). Zich realiseren [To realize]. In S. Pieters (Ed.), *De achterkant van de Möbiusband* [The back side of the Möbius band] (pp. 23–26). Utrecht: IOWO.

Wijnstra, J. (Ed.). (1988). *Balans van het rekenonderwijs in de basisschool. Periodieke Peiling van het Onderwijsniveau* [Balance of arithmetic education in primary school. Periodic Assessment of the Education Level]. Arnhem, the Netherlands: Cito.

Chapter 2

Freedom of design: The multiple faces of subtraction in Dutch primary school textbooks

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Freedom of design: The multiple faces of subtraction in Dutch primary school textbooks

Abstract

Mathematics textbook series largely determine what teachers teach and consequently, what students learn. In the Netherlands, publishers have hardly any restrictions in developing and publishing textbooks. The Dutch government only prescribes the content to be taught very broadly and does not provide guidelines on how content has to be taught. In this study, the consequences of this freedom of design are investigated by carrying out a textbook analysis on the topic of subtraction up to 100. To examine the relationship between the intended curriculum and the potentially implemented curriculum, we analyzed the mathematical content and performance expectations of two Dutch textbook series. In order to get a closer view of the learning opportunities offered, the learning facilitators of the textbook series were also analyzed. The results of the analysis show that the investigated textbook series vary in their agreement with the intended curriculum with respect to content and performance expectations. The textbook series reflect divergent views on subtraction up to 100 as a mathematical topic. Furthermore, they differ in the incorporated ideas about mathematics education, as shown in the learning facilitators they provide. Consequently, the examined textbook series provide very different opportunities to students to learn subtraction up to 100.

Keywords: textbook analysis; subtraction up to 100; mathematical content; performance expectations; learning facilitators; intended curriculum; potentially implemented curriculum.

1. Introduction

Textbooks are of great importance in mathematics education. They mediate between the intended curriculum (the statutory goals of education) and the implemented curriculum (the actual teaching in classrooms). Therefore, textbooks are referred to as the potentially implemented curriculum (Valverde, Bianchi, Wolf, Schmidt, & Houang, 2002). Mathematics textbook series largely determine what teachers teach and, consequently, what students learn (Stein & Smith, 2010). Although teachers' teaching is not always in alignment with the textbook they use (Weiss, Knapp, Hollweg, & Burril, 2002), the textbook is for many teachers the decisive source to realize their mathematics teaching. In the Netherlands, textbooks have a determining role in daily teaching practice. In recent studies it was found that 94% of the teachers indicate that a textbook is the main source of their teaching (Meelissen et al., 2012) and at least 80% of primary school teachers are following more than 90% of the textbook content (Hop, 2012).

The intended curriculum and what shows up in a textbook series is not always the same. Textbooks are not only influenced by educational goals, but also by other factors such as commercial considerations, concerns about underprepared teachers (Weiss et al., 2002) and the existence of different ideas about the nature of mathematics that should be emphasized, as well as what instructional approaches should be applied (Reys & Reys, 2006). Differences may appear during the transition from the intended curriculum to the potentially implemented curriculum, particularly in countries where there is no centralized textbook design.

In the Netherlands, there is no authority which recommends, certifies or approves textbook series before they are put on the market. Thus, publishers have hardly any restrictions in developing and designing textbook series. In order to investigate the consequences of this freedom of design, we examined in two textbook series how the Dutch intended curriculum is 'translated' into content in the form of tasks, performance expectations, and learning facilitators. To unambiguously determine the possible consequences of this freedom of design, we chose an apparently simple and straightforward mathematical topic for our analysis: subtraction up to 100.

2. Context and focus of the study

2.1 Textbook development in the Netherlands

Freedom of educational design in a way follows from the Dutch constitutional 'freedom of education'. Originating from an arrangement that gave parents the right to found schools in accordance with their religious views, freedom of education has been laid down in the Constitution since 1917. Nowadays, it also allows schools to be founded based on particular pedagogical and instructional approaches.

Because of the freedom of education, the government is rather restrained in giving instructional prescriptions. This means that the Ministry of Education prescribes only the 'what', the subject matter content to be taught, and not the 'how', the way in which this content is to be taught. Not having guidelines for the 'how' gives textbook authors the opportunity to bring in their own views and ideas on teaching mathematics.

There is another reason why textbook authors can express their own interpretations. For several years the 'what' in the intended mathematics curriculum was only described very broadly in the Core Goals for primary school (OCW, 1993; 2006). It was not until 2009 that the Core Goals were extended with the Reference Standards (OCW, 2009), describing in more detail what students should be able to at the end of primary school. However, there is still room for interpretation. For example, the Reference Standards state that students should learn to calculate using a standard method, but they do not prescribe what standard method should be taught.

There are ten textbook series[1] for teaching primary school mathematics on the market in the Netherlands. The newest have all been released between 2009 and 2012. Several have a history of earlier editions, including two that date back to the 1970s and 1980s,[2] when a reform movement in mathematics education was being enacted in the Netherlands. This reform movement was aimed at developing an alternative for the then prevailing mathematics education, which

[1] *De Wereld in Getallen*, *Pluspunt*, *Rekenrijk*, *Alles Telt*, *Talrijk*, *Wis en Reken*, *Wizwijs*, *Reken Zeker*, *Rekenwonders* and *Het Grote Rekenboek*.

[2] *De Wereld in Getallen*, developed from 1975 on, and *Pluspunt*, the development of which started in 1985.

had a very mechanistic character, and in which teaching began at a formal, symbolic level. To give children a better basis for understanding mathematics, Freudenthal and the Wiskobas group developed a new approach to mathematics education in which, among other things, the use of contexts to encourage insight and understanding played a crucial role. This reform, which was later called ‘Realistic Mathematics Education’ (RME) (e.g., Van den Heuvel-Panhuizen, 2001), was largely supported by reform-oriented textbook series.[3] Until recently all Dutch textbooks series were based more or less on this approach to teaching mathematics and they were all labeled by their publishers as ‘realistic’. However, due to a debate that has taken place in the Netherlands since 2007 criticizing the RME approach in favor of a return to the traditional, mechanistic approach (Van den Heuvel-Panhuizen, 2010) some textbook series have adapted their content (more emphasis on algorithms[4]) and teaching approach (more attention to repetition[5]) in their new editions. Moreover, new textbook series have been released that are presented as an alternative for realistic textbook series, that restore the traditional mechanistic approach with only one calculation method for each operation and a step-by-step approach with a focus on repetition.[6] Furthermore, a new textbook series which is a Dutch version of a textbook series developed in Singapore[7] was published. Thus, as a result of the debate about mathematics education, the corpus of Dutch mathematics textbooks series has become very diverse.

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[3] This underlines the crucial role that mathematics textbooks have in the Netherlands.

[4] A folder released for the textbook series *De Wereld in Getallen* (4th edition) and *Pluspunt* (3rd edition) says “Algorithms get more attention and are gradually built up until the classic long division appears.” (All translations of folders and examples from textbooks are done by the authors of this chapter.)

[5] A folder released for the textbook series *De Wereld in Getallen* (4th edition) and *Pluspunt* (3rd edition) says: “There is much more room for practice, repetition and automatization.”

[6] A folder released for the textbook series *Reken Zeker* says: “Practice, practice and more practice”, “One strategy for all children”. A folder released for the materials of *Het Grote Rekenboek* says: “This textbook series gives an answer to the recent criticism on mathematics education.”

[7] A folder released for the textbook series *Rekenwonders* says: “This is the Dutch edition of an extremely successful and internationally praised Singaporean textbook.”

2.2 Subtraction in the Dutch intended curriculum

According to the current Dutch Core Goals for primary school mathematics, children have to "learn to use mathematical language and have to gain numeracy and mathematical literacy" (OCW, 2006, p. 37). Mathematical language includes arithmetical and mathematical terms and notations. Mathematical literacy and numeracy refer to, among other things, coherent insight in numbers and a repertoire of number facts and calculation methods. Furthermore, the Core Goals indicate that children "learn to ask mathematical questions and formulate and solve mathematical problems [...] and explain the solutions in mathematical language to others" (OCW, 2006, p. 39). Concerning the basic operations, the Core Goals mention that students learn to calculate both in smart ways and using standard methods (OCW, 2006, p. 43). Specifically concerning subtraction up to 100, the Core Goals state that children "learn to quickly carry out the basic calculations in their heads using whole numbers, at least up to 100, with additions and subtractions up to 20 […] known by heart" (OCW, 2006, p. 43).

The Dutch Reference Standards for mathematics (OCW, 2009) distinguish three types of knowing: "knowing-what", "knowing-how" and "knowing-why". With this in mind, the Standards can be considered a description of what Valverde et al. (2002, p. 125) call "expectations of performance" which refers to "what students should be able to do with content." "Knowing-what" relates to knowledge of number facts and calculation methods. Subtraction up to 100 includes mental calculation, both using standard methods and using properties of numbers and operations. Furthermore, students learn to subtract both by taking away and by determining the difference. "Knowing-how" refers to making functional use of particular number facts and calculation methods, including using standard methods with insight in real-life situations and converting context situations to bare number problems. "Knowing-why" refers to understanding. This includes, for example, knowledge about the operations, such as knowing that the commutative property does not apply to subtraction as it does to addition.

2.3 A mathedidactical analysis of subtraction up to 100

2.3.1 Subtraction as a mathematical concept

Relationships between whole numbers can be additive and multiplicative. These relationships ensure that one can think of and reason within an interrelated number system instead of having to deal with an innumerous set of individual loose numbers (Kilpatrick, Swafford, & Findel, 2001). The additive and multiplicative relationships interconnect, combine, and generate numbers.

Addition and subtraction refer to additive number relationships. This implies that the numbers involved reflect a part-whole relationship. Combining parts into a whole can be considered an addition, whereas taking a part from a whole can be considered a subtraction. Furthermore, the operation of subtraction is the inverse of addition: subtraction undoes addition and vice versa (if $a + b = c$, then $c - b = a$).

Although subtraction is mostly associated with removing a part from a whole, it has two phenomenological appearances: taking away and determining the difference (Van den Heuvel-Panhuizen & Treffers, 2009). The two manifestations of subtraction reflect two meanings of subtraction. These two different semantic structures can nevertheless be expressed by the same symbolic representation: $c - b = a$. Written as a minuend minus a subtrahend it can literally stand for taking away b from c, but it can also represent comparing c and b to find the difference, for example, by adding on. So, depending on the semantic structure behind the symbolic representation, the answer to a subtraction problem can have two different meanings: a remainder and a difference (Usiskin, 2008).

Just like the minus symbol in the symbolic representation $c - b = a$ does not always mean taking away, the operation of subtraction is not exclusively restricted to problems in which the minus symbol appears (Freudenthal, 1983). For example, problems with a + symbol in the form of $\ldots + b = c$ and $a + \ldots = c$ can be solved by a subtraction operation. These latter problems are actually subtraction problems in an addition format (Selter, Prediger, Nührenbörger, & Huβmann, 2012).

2.3.2 Calculation methods for subtraction up to 100

The methods that can be applied for carrying out subtractions up to 100 can be described from both the number perspective and the operation perspective (Van den Heuvel-Panhuizen, 2012; Peltenburg, Van den Heuvel-Panhuizen, Robitzsch, 2012) (see Figure 1).

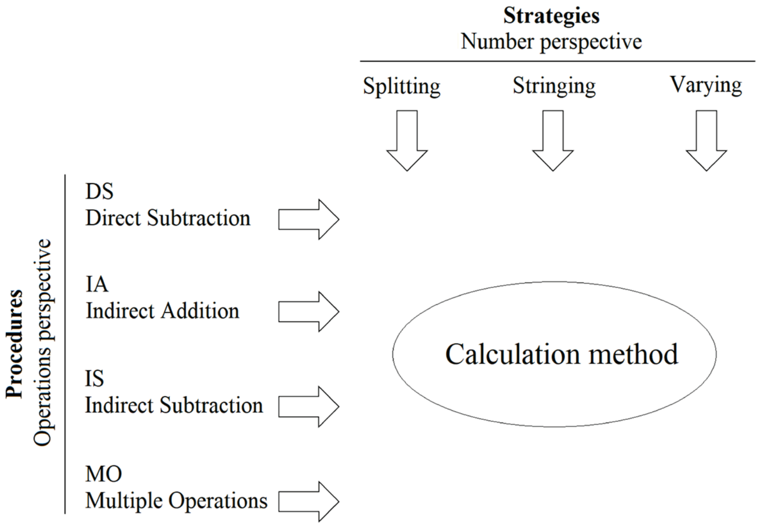

Figure 1. Two perspectives for describing calculation methods for subtraction up to 100

From the operation perspective, subtraction problems up to 100 can be solved by (1) taking the subtrahend away from the minuend, (2) adding on from the subtrahend until the minuend is reached, and (3) taking away from the minuend until the subtrahend is reached. These procedures are respectively called: direct subtraction (DS), indirect addition (IA), and indirect subtraction (IS) (De Corte & Verschaffel, 1987; Torbeyns, De Smedt, Stassens, Ghesquière, & Verschaffel (2009).

The number perspective describes how the numbers involved are dealt with. Roughly speaking, there are three strategies: splitting, stringing, and varying. Although researchers do not always use the same wording—for example, other expressions can be found in Klein, Beishuizen and Treffers (1998) and Torbeyns et al. (2009)—there is broad agreement about the general meaning of these strategies. In the splitting strategy, the minuend and the subtrahend are split into tens and ones and then the tens and ones are processed separately. In the stringing strategy, the minuend is kept intact and the subtrahend is decomposed in suitable parts which are subtracted one after another from the minuend. When a varying strategy is applied, the minuend and/or the subtrahend are changed to get an easier subtraction problem. Although in theory all three strategies can be combined with each of the four procedures, not all combinations are common or suitable (see for a more detailed discussion, Peltenburg et al., 2012).

DS can be applied with both splitting (e.g. 67 – 41 is solved by 60 – 40 = 20 and 7 – 1 = 6, followed by 20 + 6 = 26) and stringing (e.g. 67 – 41 is solved by 67 – 40 = 27 and 27 – 1 = 26). Both the IA and IS procedures can also be combined with splitting and stringing. For example, in the case of 67 – 41, applying IA with a splitting strategy means calculating 40 + 20 = 60 and 1 + 6 = 7, and then 20 + 6 = 26. Combining IA with a stringing strategy means calculation is 41 + 9 = 50 and 50 + 10 = 60 and 60 + 7 = 67, followed by 9 + 10 + 7 = 26. Although this latter method can require more steps (when there is a large difference between minuend and subtrahend), the advantage of the stringing strategy is that the problem is not split into two problems. The starting number is kept as a whole.

For subtraction problems that require crossing the ten, applying a DS procedure combined with splitting easily leads to the mistake of reversing the ones (e.g., in the case of 75 – 38, 70 – 30 is frequently incorrectly followed by 8 – 5). This mistake does not happen when DS is combined with stringing. Even more convenient is applying an IA or IS procedure combined with stringing, for example, when there is a small difference between the minuend and the subtrahend, such as in the case of 62 – 58. Solving these problems by a stringing strategy combined with IA (58 + 2 = 60 and 60 + 2 = 62, followed by 2 + 2 = 4) or with the less common IS procedure (62 – 2 = 60 and 60 – 2 = 58, followed again by 2 + 2 = 4) are easier methods that are less sensitive to errors.

Finally, the varying strategy implies multiple operations. Applying this strategy means that a problem is solved through changing it into another problem by making use of properties of numbers and operations. For example, a problem like 77 – 29 can be solved by first calculating 77 – 30, followed by 47 + 1 = 48.

2.3.3 Learning facilitators for subtraction up to 100

According to Kilpatrick et al. (2001), mathematical proficiency involves five interwoven and interdependent components, including conceptual understanding; procedural fluency; formulating, representing and solving mathematical problems; having the capacity for reflection and justification; and seeing mathematics as useful and worthwhile. Following this interpretation of mathematical proficiency—which is also reflected in the Dutch intended curriculum—implies that performance expectations should not be restricted to carrying out routine procedures, but also include flexible application of calculation methods, strategy choice, and contextual interpretation of outcomes (Verschaffel, Greer, & De Corte, 2007).

Applied to the learning of subtraction up to 100, this means that students should be offered opportunities to build a broad mental constitution of subtraction, including the different semantic structures, symbolic representations, and calculation methods of subtraction. Textbooks can contribute to this broad constitution of subtraction by including didactical support in their exposure to subtraction up to 100, such as sufficient contexts and models.

Contexts First of all, contexts can present students with situations in which subtraction emerges as a mathematical concept in a rather natural manner. The role of contexts is to add meaning to this mathematical concept in order to support the development of understanding. This can happen especially when the contexts that are used are not restricted to word problems in a stereotyped text frame, but instead come in a variety of forms and refer to students' real-life knowledge (De Corte & Verschaffel, 1987). Thus, students can become aware that subtraction can apply to all kinds of situations, reflecting different meanings of subtraction. For example, eating cookies and ascertaining how many are left, filling an album with photos and determining how many can still be included, and figuring out how many centimeters a particular person is taller than another person. These contexts which refer to different semantic structures of subtraction can prompt students to use either the DS or the IA or IS procedure.[8] By manipulating the variety in contexts, textbooks can support students' understanding of the different semantic structures of subtraction and learning various calculation methods to solve subtraction problems (see also Fuson, 1992). We refer to this use of contexts as contexts for supporting understanding, which we distinguish from the use of contexts for just applying subtraction methods. The latter reflects a performance expectation rather than a form of didactical support. To make a clear distinction between these two functions of contexts, in this study we interpreted contexts for supporting understanding as contexts that serve as a source for something new to be learned, such as a new calculation method.

Models Besides contexts, models are also important to support students' learning of subtraction up to 100. This is especially true for carrying out calculation methods and specifically applies to the strategies that are used. A requirement for making this support of models effective is that the models that

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[8] This use of contexts should fade away after some time. After all, even though a context can steer a certain calculation method, in term, in the decision what calculation method will be used, not the context, but the numbers involved play a key role.

are used match the strategies used (Van den Heuvel-Panhuizen, 2008). Models and strategies should be epistemologically consistent. This means that, for example, the splitting strategy and the stringing strategy each have their own supporting models. The splitting strategy, which is strongly related to the cardinal aspect of numbers, can best be supported by a group model that also reflects the cardinal aspect, like base-10 arithmetic blocks. Likewise, the stringing strategy, which is strongly related to the ordinal aspect of number, finds its supportive model equivalent in line models such as a number line. A line model is also suitable for visualizing and supporting a varying strategy. For example, in the case of 78 – 29 this means first making a backward jump of 30, followed by a forward jump of 1. As stated earlier, solving 78 – 29 by a splitting strategy easily leads to the mistake of reversing the ones. A line model would not help to overcome this difficulty, because dealing separately with the 70 and the 20, and the 8 and the 9 on a number line does not make sense. In other words, in teaching calculation methods, strategies and models should match, otherwise models do not have the supportive function they are assumed to have. Consequently, depending on the strategy that is intended, textbooks should give more attention either to group models or to line models.

Symbolic representations Building a broad mental constitution of subtraction also requires that students are offered various symbolic representations of subtraction. Besides the standard representation $c - b = \ldots$, students should also have opportunities to deal with alternative symbolic representations such as $c - \ldots = a$ and $a + \ldots = c$. These problems make it clear that the operation symbol in a problem can have different meanings (Fuson, 1992), and is not per se equivalent to the operation that can be applied to find the solution of that problem. The different symbolic representations reflect the part-whole aspect of additive number relationships and the link between addition and subtraction. Furthermore, it supports the understanding that the = symbol does not only mean ‘results in’ but also ‘is equivalent to’. According to Fuson (1992), textbooks do not always pay much attention to the different meanings of the equal and operation symbols.

2.4 Research questions

The purpose of this study is to reveal the consequences of freedom of design for Dutch textbooks as the potentially implemented curriculum for primary school and for the learning opportunities that students are offered. Focusing on subtraction up to 100, we came up with the following research questions:

1. Do Dutch mathematics textbooks reflect the content of the Dutch intended curriculum concerning subtraction up to 100?
2. Do Dutch mathematics textbooks reflect the performance expectations of the Dutch intended curriculum concerning subtraction up to 100?
3. What learning facilitators for learning subtraction up to 100 are incorporated in Dutch mathematics textbooks?

3. Method

To answer the research questions, a textbook analysis was carried out in which we examined two Dutch textbooks series. The analysis focused on three perspectives: the mathematical content, the performance expectations and the learning facilitators.

3.1 Textbook materials included in the analysis

To include the full scope of didactical approaches in the Netherlands in our analysis we examined two recently developed textbook series that, although from the same publisher, are clearly positioned in two contrasting approaches to mathematics education (see Section 2.1). The first textbook series, called *Rekenrijk* (RR) (Bokhove et al., 2009), is an RME-oriented textbook series. The name *Rekenrijk* means both “realm of arithmetic” and “rich arithmetic”. The second textbook series, called *Reken Zeker* (RZ) (Terpstra & De Vries, 2010), is a new textbook series that is presented as an alternative for RME-oriented textbook series. The name of this textbook series means “arithmetic with certainty”.

Because subtraction up to one hundred is mainly taught in Grade 2, the textbook analysis was carried out with textbook materials from this grade only. We analyzed all materials for Grade 2 that are meant for all students. Textbook materials meant for evaluation, and subsequent optional lessons for repetition or enrichment, were not included in our analysis.

3.2 Framework for textbook analysis

To analyze the textbook materials, we developed a framework containing the perspectives of content, performance expectations, and learning facilitators (see Figure 2). Most categories within these three perspectives were initially formulated on the basis of the Dutch intended curriculum for subtraction (see Section 2.2) and our mathedidactical analysis of subtraction up to 100 (see Section 2.3). Several subcategories were established after an initial round of the analysis, based on what we actually found in the textbook series.

Perspective	Category	Subcategory	
Content	Types of problems	Prerequisite knowledge	Decomposing numbers up to 10
			Backwards counting with tens
		Subtraction up to 10	
		Subtraction up to 20	Without bridging the ten
			Bridging the ten
		Subtraction up to 100	Without bridging a ten
			Bridging a ten
	Format of problems	Bare number problems	
		Context problems	
	Semantic structure of problems	Subtraction as taking away	
		Subtraction as determining the difference	
Performance expectations	Knowing subtraction facts	Knowing subtraction facts up to 10	
		Knowing subtraction facts up to 20	
	Carrying out subtractions	Using standard methods	
		Using alternative methods	
	Applying subtractions	Using subtraction methods in context problems	
	Understanding subtraction	Giving explanations	
		Choosing an appropriate method	
Learning facilitators	Degree of exposure	Number and distribution of tasks	
	Structure of exposure	Sequence in types of problems	
		Sequence in level of abstraction	
	Didactical support in exposure	Use of contexts for supporting understanding	
		Use of models	
		Use of various symbolic representations	
		Use of textual instructions	

Figure 2. Framework for textbook analysis

3.2.1 Content

The perspective of content involves problem types, problem formats, and semantic structures of the problems presented in the textbook materials. Regarding the problem types we made a subdivision based on the number domain involved. We incorporated relevant prerequisite knowledge for subtraction: decomposing numbers up to 10 and counting backwards with tens. For the format of the problems we made a distinction between bare number problems and context problems. The semantic structure of problems refers to the two phenomenological appearances of subtraction.

3.2.2 Performance expectations

Regarding performance expectations, we included knowing subtraction facts, carrying out subtractions, applying subtractions and understanding subtraction. The first category corresponds to “knowing-what”, the second and third to “knowing-how” and the fourth to “knowing-why”, as described in the Dutch Reference Standards. Knowing subtraction facts is subdivided into knowing subtraction facts up to 10 and knowing subtraction facts up to 20. Carrying out subtractions is subdivided into using standard calculation methods (DS combined with splitting or stringing) and alternative calculation methods (e.g., IA combined with stringing or MO combined with a varying strategy). This distinction is in agreement with the Dutch intended curriculum. Applying subtractions refers to using already learnt subtraction facts and calculation methods in context problems. For the category “understanding” we distinguished “giving explanations” and “choosing an appropriate method”, based on performance expectations found in the first round of analysis, that go beyond knowing, carrying out and applying subtractions, and unambiguous apply to understanding.

3.2.3 Learning facilitators

With respect to learning facilitators, we included degree and structure of exposure, based on the importance of the amount and sequencing of content in textbooks (Valverde et. al., 2002). We included didactical support in exposure based on our mathedidactical analysis. The subcategory “use of textual instructions” was added after the first round of the textbook analysis, again based on what we found in the textbook series that can also be considered as supporting learning.

3.3 Unit of analysis

In both textbook series, the content is organized in lessons meant for one mathematics hour. These lessons are subdivided into sets of tasks. In our study, we use the term “task” to refer to the smallest unit that requires an answer from a student. Because the number of tasks vary per set of tasks (see Figure 3), and content and performance expectations may vary per single task, we used the task as unit of analysis.

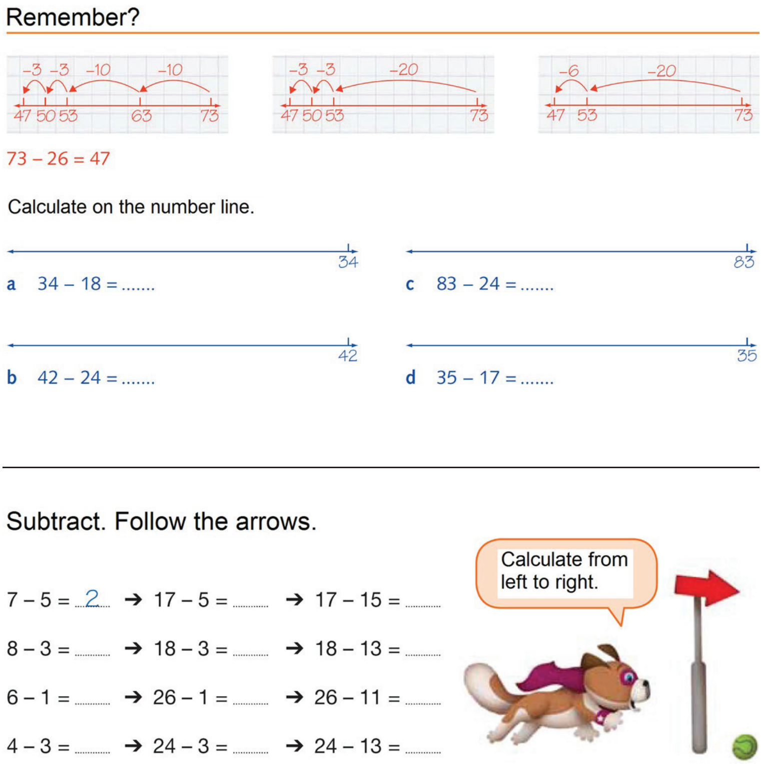

Figure 3. RR set of 4 tasks (above, RR-book 4b-1, p. 30) and RZ set of 11 tasks (below, RZ-book 4c, p. 26)

3.4 Analysis procedure

First, we identified all subtraction-related tasks. After an initial round of analysis was carried out, we added the following subcategories: “giving explanations”, "choosing an appropriate method" and "use of textual instructions." Then, the first author of this chapter coded all subtraction-related tasks according to the final version of our framework. Each task received several codes. For the content, a code was given for the problem type, the problem format, and the semantic structure of the problem. For the performance expectations, each task was first coded as knowing subtractions facts, carrying out subtractions or applying subtractions. If neither of these sub-categories was applicable, no code was given. Next, for each task, if applicable, a code was given for the category understanding of subtractions. For the learning facilitators, the degree of exposure was determined from the number of tasks. Because the tasks were counted in consecutive lessons, we got an overview of the distribution of the subtraction-related tasks. This also made it possible to reveal the structure of exposure, i.e., the sequence in types of tasks and in level of abstraction. Finally, for each task it was checked which subcategories of didactical support were applicable.

A reliability check of the coding was based on an independent coding by two teacher-trainees. To that end we used a selection of about one tenth of all subtraction-related tasks in which all categories of the framework were included. The two teacher-trainees reached a 93% agreement. The agreements between each of the teacher-trainees and the first author were respectively 93% and 95%.

4. Results

4.1 Content

A substantial difference between the two textbook series for Grade 2 is the number of tasks included. The total number of tasks in RR is 5331, whereas RZ has 7051 tasks. However, of these amounts of tasks the proportion of subtraction-related tasks is about the same in both textbooks: RR contains 22% subtraction tasks (1166 tasks) and RZ 20% (1440 tasks).

4.1.1 Types of problems

Both Grade 2 textbook series concentrate more on tasks involving subtraction between 20 and 100, and less on tasks involving subtraction up to 10 and up to 20 (see Table 1). Regarding subtraction up to 20, RR offers more tasks that require bridging the ten than RZ. Within subtraction tasks up to 100, the number of tasks that require bridging a ten is larger in RZ, but relatively RR offers more tasks concerning this type of problem (in RR: 378 out of 572 tasks, about 66%; and in RZ: 480 out of 1096 tasks, about 44%).

The amount of attention to the prerequisite knowledge for these problems differs. Regarding decomposing numbers up to 10, RR has a substantial number of such tasks and RZ almost none. For counting backwards with tens (e.g., 46; 36; 26), RR has very few tasks, while RZ has none. When we checked whether, for example, decomposing numbers up to 10 is already dealt with in Grade 1, we found that both textbook series did indeed put more of an emphasis on this prerequisite knowledge in Grade 1 than in Grade 2. However, the RR booklets for Grade 1 have 418 such tasks, while RZ offers only 167 in its first-grade booklets. So, with respect to providing prerequisite knowledge for subtraction up to 100, there is a large difference between the two textbooks series.

Table 1

Types of problems in subtraction-related tasks in RR and RZ in Grade 2; absolute and relative frequencies of tasks

Types of problems	RR-tasks		RZ-tasks	
Prerequisite knowledge	130	11%	5	0%
Decomposing numbers up to 10	*107*	*9 %*	*4*	*0 %*
Backwards counting with tens	*23*	*2 %*	*1*	*0 %*
Subtraction up to 10	153	13 %	78	5 %
Subtraction up to 20	311	27 %	261	18 %
Without bridging the ten	*79*	*7 %*	*135*	*9 %*
Bridging the ten	*232*	*20 %*	*126*	*9 %*
Subtraction up to 100	572	49 %	1096	76 %
Without bridging a ten	*194*	*17 %*	*616*	*43 %*
Bridging a ten	*378*	*32 %*	*480*	*33 %*
Total number of subtraction related tasks	1166	100 %	1440	100 %

Note. Some percentages do not seem to add up to 100. This is due to rounding off

4.1.2 Format of problems

Both textbook series contain far more bare number problems than context problems (see Table 2). However, RR encloses much more context problems than RZ, both relatively and absolutely, even though in RZ the total number of subtraction tasks is larger than in RR.

Table 2
Format of problems in subtraction-related tasks in RR and RZ in Grade 2; absolute and relative frequencies of tasks

Format of problems	RR-tasks		RZ-tasks	
Bare number problems	1026	88 %	1415	98 %
Context problems	140	12 %	25	2 %
Total number of subtraction related tasks	1166	100 %	1440	100 %

Table 3
Semantic structure of problems in subtraction-related tasks in RR and RZ in Grade 2; absolute and relative frequencies of tasks

Semantic structure of problems	RR-tasks		RZ-tasks	
Taking away	210	18 %	403	28 %
Determing the difference	53	5 %	0	0 %
Both taking away and determining the difference	28	2 %	0	0 %
No distinguishable semantic structure	874	75 %	1037	72 %
Total number of subtraction related tasks	1166	100 %	1440	100 %

4.1.3 Semantic structure of problems

In both textbook series, only a minority of the tasks reflect a clearly distinguishable semantic structure. Both textbook series address subtraction as taking away, but subtraction as determining the difference is only dealt with in RR (see Table 3).

4.2 Performance expectations

Both textbook series contain tasks that clearly focus on certain performances. RR contains 1081 and RZ contains 800 clearly distinguishable performance expectations (see Table 4). In both textbook series, most emphasis lies on performance expectations related to carrying out subtractions, followed by knowing subtraction facts. RR contains more expectations on applying subtractions than RZ. Expectations regarding understanding were only found in RR.

Table 4

Performance expectations reflected in subtraction-related tasks in RR and RZ in Grade 2; absolute and relative frequencies of tasks

Performance expectations	RR-tasks		RZ-tasks	
Knowing subtraction facts	346	32 %	229	29 %
Knowing sybtraction facts up to 10	*258*	*24 %*	*55*	*7 %*
Knowing subtraction facts up to 20	*88*	*8 %*	*174*	*22 %*
Carrying out subtractions	513	47 %	546	68 %
Using standard methods	*413*	*38 %*	*546*	*68 %*
Using alternative methods	*100*	*9 %*	*0*	*0 %*
Applying subtractions	111	10 %	25	3 %
Understanding subtraction	111	10 %	0	0 %
Choosing an appropriate method	*74*	*7 %*	*0*	*0 %*
Giving explanations	*37*	*3 %*	*0*	*0 %*
Total number of performance expectations	1081	100 %	800	100 %

Note. In some tasks we distinguished two performance expectations (e.g., carrying out a subtraction and explaning the calculation method.

Some percentages do not seem to add up to 100. This is due to rounding off

4.2.1 Knowing subtraction facts

RR contains more performance expectations for knowing subtraction facts than RZ. In RR, most emphasis is on knowing subtraction facts up to 10. In RZ, most emphasis is on knowing subtraction facts up to 20.

4.2.2 Carrying out subtractions

Using standard methods In both textbook series students are expected to learn one standard method for carrying out subtractions up to 20 and up to 100, namely DS combined with stringing. However, the textbook series differ in the way that students are supposed to notate their calculations. In the case of tasks that involve bridging a ten, both textbooks suggest the notation of in-between steps or in-between answers. In RR this is done by writing down under the subtrahend how it is decomposed or by keeping track of the taken-away steps on an empty number line (see Figure 4).

In RZ, the students have to notate the first in-between answer directly after the = symbol, which is supposed to be followed by the remaining part that has to be taken away (see Figure 5). Although the symbolic representation that results in the end is mathematically correct (in fact it describes two equivalent subtractions), notating the calculation in this way implies that students have to perform several in-between steps mentally.

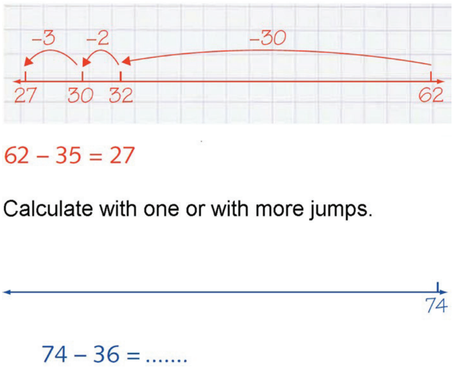

Figure 4. DS combined with stringing in RR (RR-book 4b-1, p. 57)

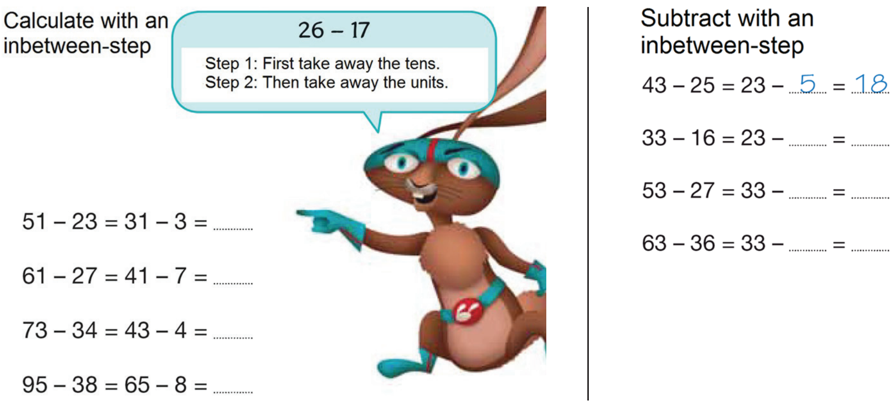

Figure 5. DS combined with stringing in RZ tasks (RZ-book 4c, p. 71; p. 74)

Using alternative methods Only in RR are students expected to learn alternative subtraction methods also, namely, the procedures IA and IS and a varying strategy (see Figure 6). Although RZ contains missing number tasks (e.g., 28 – … = 23) which could prompt IS, this textbook series does not otherwise pay attention to this procedure or to any alternative method.

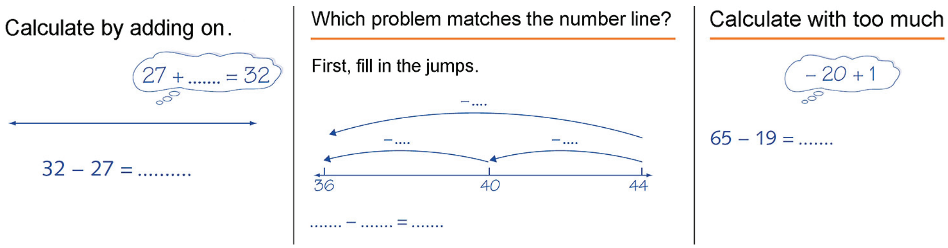

Figure 6. IA (left), IS (middle) and a varying strategy (right) in RR tasks (RR-book 4b-2, p. 61; 4b-1, p. 2; 4b-2, p. 78)

4.2.3 Applying subtractions

In both textbook series, contexts are used for the application of calculation methods that are presented earlier. RR offers such contexts more than four times as often as RZ (see Table 4). Both textbook series use contexts that refer to real life situations. In RZ all contexts concern taking-away situations, presented by a series of similar sentences. RR offers contexts referring both to taking away and determining the difference, presented in various ways (see Figure 7).

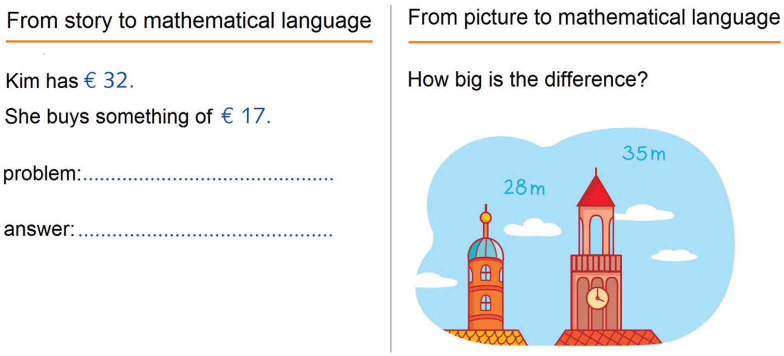

Figure 7. Context problems in RR reflecting taking away (left) and determining the difference (right) (RR-book 4b-2, p. 37; p. 78)

4.2.4 Understanding subtraction

In RR, we found 111 tasks explicitly offering directions or questions to prompt students' reasoning (see Table 4). These tasks include questions for students to explain their thinking (e.g., *Hoe heb je dit uitgerekend?* [How did you calculate this?]), visualize their calculation method or choose an appropriate calculation method for a given subtraction with certain numbers (see Figure 8). In RZ, we did not find clearly distinguishable performance expectations regarding understanding.

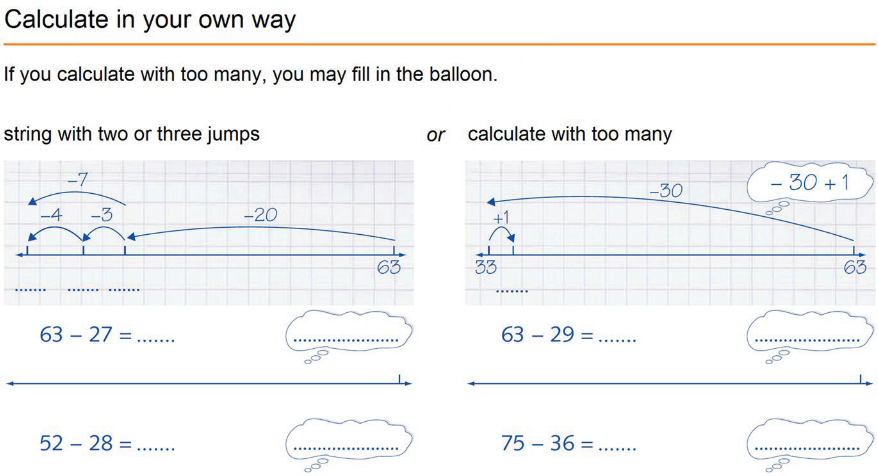

Figure 8. RR tasks that prompt students to choose an appropriate strategy (RR-book 4b-2, p. 64)

4.3. Learning facilitators

4.3.1 Degree of exposure

As mentioned before, RZ provides more subtraction-related tasks (1440) than RR (1166). Figure 9 displays how these tasks are distributed over time (covering the 36 weeks of a school year). Both textbooks provide five mathematics lessons each week. The bars in the diagram indicate the number of subtraction tasks per lesson. Every third week in RR and every fourth week in RZ are not filled in (the gray areas). These weeks are meant for evaluation, followed by repetition or enrichment work, and were not included in our analysis.

In RR, the degree of exposure varies: in weeks 1, 7 and 34 relatively more attention is paid to subtraction than in other weeks. In week 1, this concerns the repetition of prerequisite knowledge presented in Grade 1, namely number decomposing up to 10. In weeks 7 and 34, a new step in the learning of subtraction is taken. Week 7 is the first time that students encounter subtraction up to 100 and week 34 is the first time that IA is applied to subtraction up to 100. RZ has a fixed pattern of weekly lessons in which 50 to 70 subtraction tasks are offered, with the exception of two periods of three weeks in which almost no attention is paid to subtraction.

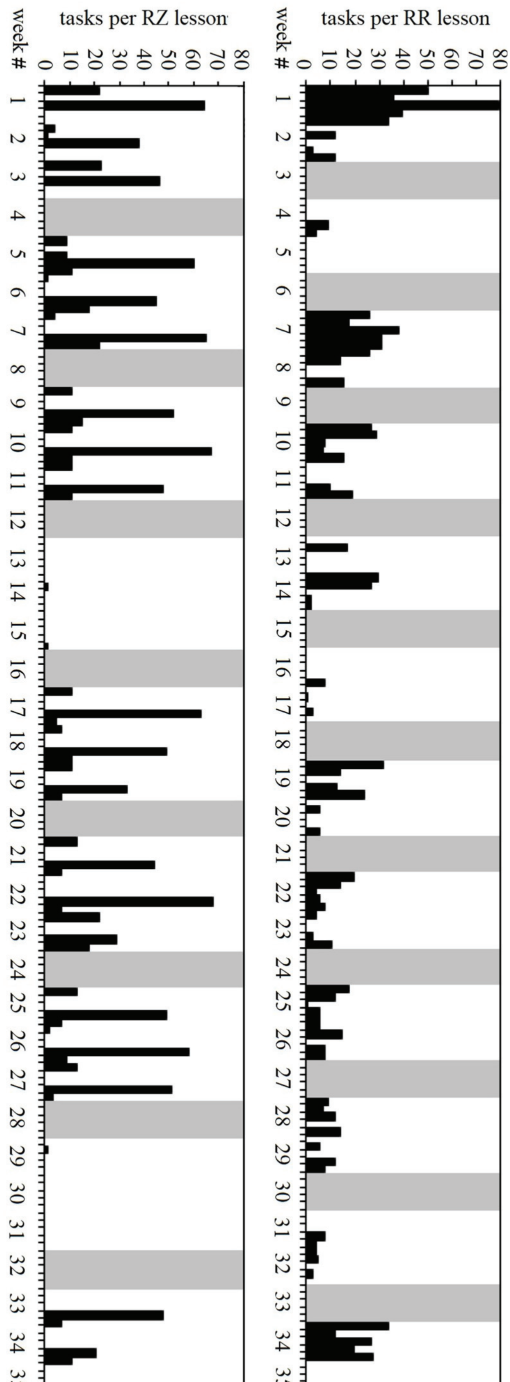

Figure 9. Distribution of subtraction-related tasks over the school year in RR (above) and RZ (below)

4.3.2 Structure of exposure

Sequence in types of problems Table 5a and 5b show how the main types of tasks are distributed over the school year. The gray shading indicates the number of certain types offered: the darker the gray, the larger the number of tasks. The tasks in both textbook series increase in difficulty during the course of the school year. RZ reaches the most difficult types of tasks earlier than RR.

Table 5a
Sequence in types of problems in subtraction-related tasks in RR

Types of problems	Number of RR-tasks								
Subtraction up to 100, bridging a ten					50	54	84	45	145
Subtraction up to 100, without bridging a ten	3		70	49	22	10	31	5	4
Subtraction up to 20	58	173	25	24	23	1	7		
Subtraction up to 10	111	27		12	3				
Prerequisite knowledge	107		21	1	1				
Month #	1	2	3	4	5	6	7	8	9

Table 5b
Sequence in types of problems in subtraction-related tasks in RZ

Types of problems	Number of RZ-tasks								
Subtraction up to 100, bridging a ten	4	3	138		25	97	115		98
Subtraction up to 100, without bridging a ten		166	58	1	150	107	88	1	45
Subtraction up to 20	151	57	29	1	13	3	2		5
Subtraction up to 10	39	18	12		9				
Prerequisite knowledge	4					1			
Month #	1	2	3	4	5	6	7	8	9

Sequence in level of abstraction Both textbook series provide bare number problems, context problems (see Table 2) and tasks with supporting models (see Table 6). However, there is a difference regarding the provided context problems. Both textbook series contain context problems to apply earlier learned subtraction methods (which we consider a performance expectation), but only RR also contains contexts for supporting understanding of subtraction (see Table 6).

To get an image of the sequence in level of abstraction, we zoomed in on one particular type of task, namely subtraction up to 20 bridging 10. Figure 10 shows the sequence in level of abstraction of this type of task in the first ten lessons in which it is included. Every black box represents one set of these tasks. Figure 10 illustrates that the sequence in level of abstraction differs between the two textbook series. RR starts with contexts for supporting understanding, followed by tasks with models and then contexts for application. Only in the sixth lesson are bare number tasks provided for the first time. RZ has a different sequence in which bare number tasks and tasks with models are alternated. In contrast with RR, the textbook series RZ begins with bare number tasks. Another difference is that RR provides students with context problems for application several times, while RZ does this only once within the first ten lessons.

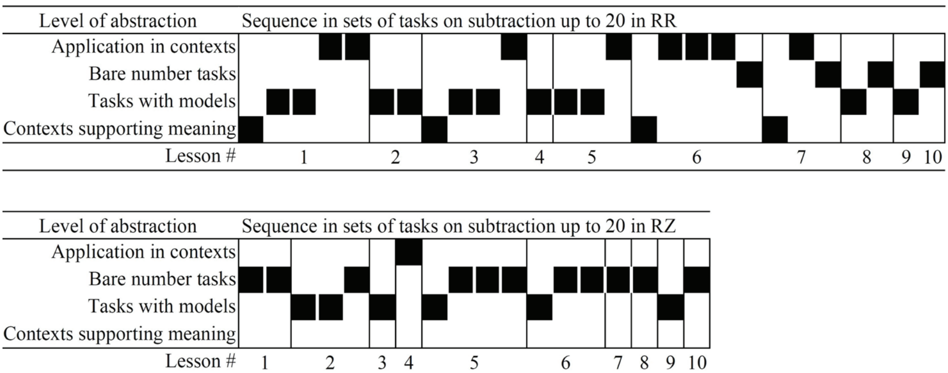

Figure 10. Sequence in level of abstraction regarding subtraction up to 20 bridging 10 in RR (above) and RZ (below)

4.3.3 Didactical support in exposure

Both textbook series offer tasks that provide some form of didactical support. In RR, this is the case in 821 of the total of 1166 subtraction-related tasks (about 70%) and in RZ, this is the case in 280 of the 1440 subtraction-related tasks (about 19%) (see Table 6).[9]

Table 6

Types of didactical support in RR and RZ in Grade 2; absolute and relative frequencies of subtraction-related tasks with this support

Didactical support	RR-tasks		RZ-tasks	
Use of contexts for supporting understanding	29	4 %	0	0 %
Use of models	423	52 %	108	39 %
Artihmetic rack	*102*	*12 %*	*0*	*0 %*
Arithmetic blocks	*0*	*0 %*	*98*	*35 %*
Number line (structured)	*11*	*1 %*	*10*	*4 %*
Number line (empty)	*305*	*37 %*	*0*	*0 %*
Number strip	*5*	*1 %*	*0*	*0 %*
Use of textual instructions	369	45 %	172	61 %
Instructions how to solve tha task	*186*	*23 %*	*108*	*39 %*
Choices offered for solving the task	*146*	*18 %*	*64*	*23 %*
Reflection-eliciting questions	*37*	*5 %*	*0*	*0 %*
Total number of tasks with didactical support	821	100 %	280	100 %

Note. Some percentages do not seem to add up to 100. This is due to rounding off

Use of contexts for supporting understanding Although both textbook series contain context problems, only in RR do some of the provided contexts serve as a source for new topics to be learned, thus supporting understanding of subtraction (see Table 6). An example is shown in Figure 11, in which subtracting as adding on (IA) is introduced and related to taking away (DS).

Use of models RR uses the arithmetic rack as the dominant model for subtraction up to 20 and the empty number line for subtraction up to 100 (see Figure 12). RR uses the empty number line for all calculation methods: stringing combined with DS; IS; IA; and varying (see Figure 4, Figure 6, and Figure 8). In

[9] The use of various symbolic representations of subtractions was not included in this count, because by definition every bare number task has some form of symbolic representation.

the case of IA, the visualization on the empty number line does not always match the symbolic representation (in 18 of 48 tasks), as can be seen in Figure 12 (right). In this example, the students are invited to apply an adding on procedure (IA), but the number line (that refers to 73 – … = 68 or to 68 + … = 73) and the symbolic representation … – 68 = … do not match to this procedure nor to each other.

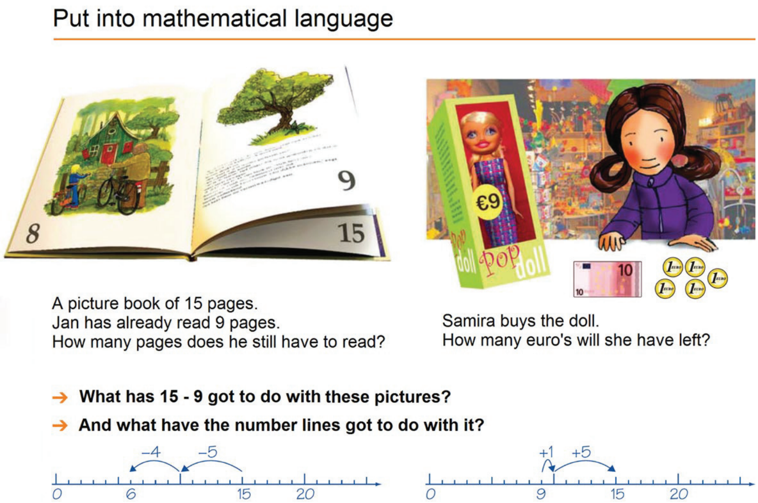

Figure 11. Relating IA and DS in RR (RR-book 4a, p. 24)

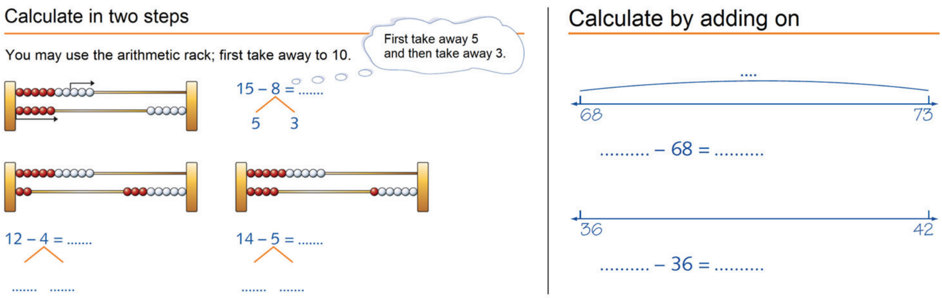

Figure 12. RR use of the arithmetic rack for subtraction up to 20 (left) and the empty number line for subtraction up to 100 (right) (RR-book 4a-1, p. 58; 4b-2, p. 78)

RZ uses (pictures of) base-10 arithmetic blocks as the only model for subtraction up to 100 (see Figure 13). For subtraction up to 20, the structured number line is used also. Although base-10 blocks and the stringing strategy are not epistemologically consistent, RZ uses base-10 blocks as its only supporting model to provide DS combined with stringing, which is the only calculation method that is taught in this textbook series (see Section 4.2.2). Furthermore, RZ does not always use this model consistently; sometimes the base-10 structure is not used for subtracting tens (in 11 of 43 tasks, see Figure 13, middle) while at other times it is (in 32 of 43 tasks, see Figure 13, right).

Use of various symbolic representations Besides the standard representation $c - b = ...$, both textbook series present little alternative symbolic representations of subtractions. Only RR contains subtraction-related tasks in an addition format (12 of 1166 tasks), to relate subtraction and addition and to elicit subtraction as adding on (IA) (see Figure 6, left). On the other hand, missing number subtractions (e.g., 19 = 20 – … and 26 – … = 21) are only dealt with in RZ (44 of 1440 tasks).

Use of textual instructions Both textbooks provide students with textual instructions on how to solve subtractions and offer choices for solving tasks. Reflecting-eliciting questions were only found in RR (see Table 6).

Textual instructions on how to solve subtractions that were found are instructions to use a specific calculation method or how to carry out a specific calculation method. In RR, most of these instructions (120 out of 186) concern subtractions up to 20, and include first subtracting down to 10 and then subtracting the rest (e.g., "First take away to ten", see Figure 12, left). In RZ, most of the instructions (35 out of 108) concern subtractions up to 100, and are about first subtracting the tens and then subtracting the units (e.g. "Step 1: First take away the tens. Step 2: Then take away the units", see Figure 5, left).

Both textbook series offer students choices on how to perform certain tasks. A choice that both offer is whether or not to use a model for solving the task (in RR 53 out of 146 choices offered and in RZ 21 out of 64). The other choices that are offered are rather different in nature. In RR this involves choosing an appropriate calculation method: for instance, to use either a stringing or a varying strategy (see Figure 8) or to take more or less jumps when using the stringing strategy (in the remaining 93 out of 146 choices offered). In RZ, the remaining 43 (out of 64) choices concern whether or not to use scrap paper.

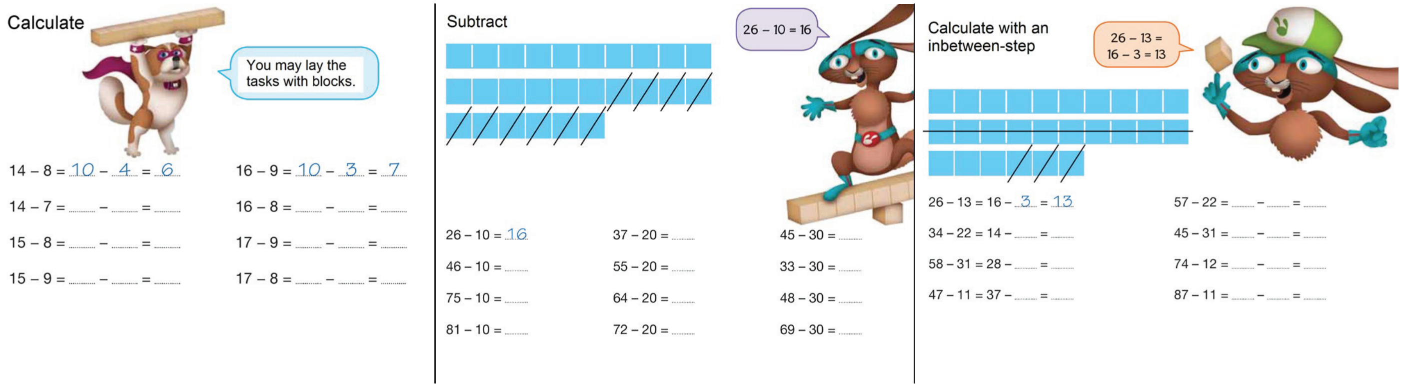

Figure 13. RZ use of base-10 blocks for subtraction up to 20 (left) and up to 100 (middle and right) (RZ-book 4a, p.9; 4c p. 25; p. 39)

Questions that prompt students to think and reason about tasks were only found in RR. Examples are: “How did you calculate this?”; and “What has 15 – 9 got to do with these pictures?” and “And what have the number lines got to do with them?” (see Figure 11).

5. Concluding remarks

Our analysis revealed that freedom of design can result in varying agreement of the potential implemented curriculum with the intended curriculum. In our framework, seven categories—covering content and performance expectations—are related to the intended curriculum. With respect to subtraction up to 100, in three of these categories (types of problems, format of problems and knowing subtraction facts), the textbook series RR and RZ are comparable in their agreement with the Dutch intended curriculum. However, in the other four categories, the fit of RR to the intended curriculum is closer than that of RZ. Figure 14 summarizes our findings.

Regarding the content (research question 1), both textbooks series present subtraction problems up to 100, and both textbook series offer bare number problems as well as context problems. RZ offers more bare number tasks and RR offers more context problems. In deviation of the intended curriculum, RZ only addresses one semantic structure of subtraction. In contrast, RR deals with both.

The degree in which the two textbook series reflect the performance expectations of the intended curriculum (research question 2) also differs. RR offers more tasks on knowing subtractions in total, but RZ presents more tasks on knowing subtractions up to 20. In both textbooks, students are expected to learn the standard calculation method of DS combined with stringing. Only RR expects students to learn alternative calculations methods as well. The way that RZ notates in-between answers can easily lead to incorrect notations (e.g. $12 - 3 = 12 - 2 = 10 - 1 = 9$ instead of $12 - 3 = 10 - 1 = 9$), especially when students interpret the = symbol only as ‘results in’ and not as an equivalence symbol. Both textbook series employ context problems for application of subtraction, but only in RR is this done by presenting various forms of contexts and by including both semantic structures of subtraction. Finally, only RR contains explicit performance expectations regarding understanding of subtraction.

		Intended curriculum	RR	RZ
Content	Types of problems	Subtraction up to 100	Both RR and RZ offer subtraction up to 100	
	Format of problems	Context situations	Both RR and RZ offer context situations and bare number problems	
		Bare number problems	RR offers more context problems than RZ	RZ offers more bare number problems than RR
	Semantic structure of problems	Subtraction as taking away	Both RR and RZ present subtraction as taking away	
		Subtraction as determining the difference	RR presents subtraction as determining the difference	RZ does not present subtraction as determining the difference
Performance expectation	Knowing subtraction facts	Knowing subtractions up to 20 by heart	RR puts most emphasis on knowing subtraction facts up to 10	RZ puts most emphasis on knowing subtractions facts up to 20
	Carrying out subtractions	Subtraction by standard methods	Both RR and RZ expect students to learn one standard method, namely stringing combined with DS	
		Subtraction in smart ways	RR expects students to learn IA, IS, and varying strategies	RZ does not expect students to learn alternative calculation methods
	Applying subtractions	Using standard methods with insight in real-life situations	Both RR and RZ expect students to apply subtraction methods in context problems	
			RR offers context situations with various forms and both semantic structures	RZ offers context situations with one text form and one semantic structure
	Understanding of subtraction	Understanding of the operation subtraction	RR expects students to explain their thinking, visualize their calculation method and choose appropriate calculation methods	RZ does not offer clearly distinguishable performance expectations regarding understanding

Figure 14. Agreement of RR and RZ with the Dutch intended curriculum regarding subtraction up to 100

The two textbook series also differ in the learning facilitators they offer students (research question 3). Figure 15 summarizes our findings on this research question.

RZ offers a larger number of subtraction-related tasks and reaches more difficult types of tasks at an earlier stage. However, RR spends more tasks on prerequisite knowledge and uses contexts for supporting understanding as the first step in the sequence of level of abstraction, resulting in offering a solid base for the learning of subtraction up to 100. Furthermore, RR offers almost three times as much didactical support compared to RZ. This includes forms of didactical support that are absent in RZ, namely contexts for supporting understanding, textual instructions for choosing appropriate calculation methods, and reflection-eliciting questions. Another shortcoming of RZ is that it uses base-10 arithmetic blocks for supporting stringing, which means that model and strategy are not epistemologically consistent. To a certain degree, a similar inadequacy applies also to RR when using a particular symbolic representation of subtraction which does not match the presentation on the empty number line.

<table>
<tr><th></th><th></th><th>RR</th><th>RZ</th></tr>
<tr><td rowspan="2">Degree of exposure</td><td rowspan="2">Number of tasks</td><td colspan="2">In both RR and RZ about 20% of all tasks in Grade 2 addresses subtraction up to 100</td></tr>
<tr><td>In absolute numbers, RR offers considerably less subtraction-related tasks</td><td>In absolute numbers, RZ offers considerably more subtraction-related tasks</td></tr>
<tr><td rowspan="3">Structure of exposure</td><td rowspan="2">Sequence in types of tasks</td><td colspan="2">Both RR and RZ have a structure of increasing difficulty in the course of the school year</td></tr>
<tr><td>RR spends more tasks on prerequisite knowledge</td><td>RZ reaches more difficult types of tasks at an earlier stage</td></tr>
<tr><td>Sequence in level of abstraction</td><td>RR uses contexts for supporting understanding as the first step in the sequence of level of abstraction</td><td>RZ uses bare number tasks as the first step in the sequence of level of abstraction</td></tr>
<tr><td rowspan="7">Didactical support in exposure</td><td>On the whole</td><td>RR offers considerably more tasks with didactical support</td><td>RZ offers considerably less tasks with didactical support</td></tr>
<tr><td>Use of contexts for supporting understanding</td><td>RR uses contexts for supporting understanding of subtraction</td><td>RZ does not use contexts for supporting understanding of subtraction</td></tr>
<tr><td>Use of models</td><td>RR uses the empty number line, both for stringing and for alternative methods
In some tasks, the visualization on the empty number line does not match the symbolic representation</td><td>RZ uses (pictures of) base-10 blocks for stringing, the only calculation method it offers, even though this model is not epistemologically consistent with this strategy</td></tr>
<tr><td rowspan="2">Use of various symbolic representations</td><td colspan="2">Both RR and RZ provide the standard symbolic representation $c - b = \ldots$</td></tr>
<tr><td>RR provides subtractions in addition format ($a + \ldots = c$)</td><td>RZ provides missing number subtractions ($a = c - \ldots$ and $c - \ldots = a$)</td></tr>
<tr><td rowspan="2">Use of textual instructions</td><td colspan="2">Both RR and RZ provide textual instructions on how to carry out a specific calculation method</td></tr>
<tr><td>RR uses textual instructions for choosing an appropriate calculation method and eliciting reflection</td><td>RZ uses textual instructions for choosing whether or not to use scrap paper</td></tr>
</table>

Figure 15. Learning facilitators for subtraction up to 100 in RR and RZ

Both examined textbook series do only provide very few tasks involving various symbolic representations of subtraction. The textbook series differ with respect to the textual instructions they provide. RZ offers instructions on how to proceed, whereas RR provides instructions that prompt students to reflect.

Our analysis made it clear that freedom of design can result in a potentially implemented curriculum that may deviate from the intended curriculum. The two examined textbook series differ noticeably in their view on subtraction up to 100 as a mathematical topic. RZ reflects a limited view including one semantic structure, one meaning, and one calculation method. RR supports students' development of a broad mental constitution of subtraction, including both meanings and both semantic structures, as well as various calculation methods. Furthermore, our results show that the incorporated ideas of the two textbook series about mathematics education (RR is presented as an RME-oriented textbook series and RZ as an alternative to this approach) actually result in different learning opportunities for students. It really makes a difference for students whether or not they are offered a broad mental constitution of subtraction, whether or not they are given reflection-eliciting questions, and whether or not there is a match between models and symbolic representations or calculation methods.

Of course, what is in the textbook is not necessarily similar to what is taught in class. However, following Valverde et al. (2002, p. 125), we think that "how content is presented in textbooks (with what expectations for performance) is how it will likely be taught in the classroom." Therefore, textbook analysis can provide an inside view in how a subject might be taught. As such, textbook analyses are a crucial tool that can preserve us from having teaching practices not in agreement with the intended curriculum and that do not offer students the desired learning opportunities. How necessary such analyses are was shown when a textbook analysis disclosed that higher-order problem solving is lacking in Dutch mathematics textbooks (Kolovou, Van den Heuvel-Panhuizen, & Bakker, 2009), even though it is part of the Dutch intended curriculum.

In the present textbook analysis on the topic of subtraction it was again revealed that the textbook matters. The examined textbook series contain different learning opportunities. Disclosing these opportunities is as important as examining the efficacy of textbooks. After all, when students cannot encounter particular content along with sufficient learning facilitators, we cannot expect them to learn this content.

References

De Corte, E., & Verschaffel, L. (1987). The effect of semantic structure on first graders' strategies for solving addition and subtraction word problems. *Journal for Research in Mathematics Education, 18*(5), 363–380.

Freudenthal, H. (1983). *Didactical Phenomenology of Mathematical Structures*. Dordrecht/Boston/Lancaster: Reidel Publishing Company.

Fuson, K. (1992). Research on whole number addition and subtraction. In D. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 243–275). New York, NY: MacMillan.

Hop, M. (Ed.). (2012). *Balans van het reken-wiskundeonderwijs halverwege de basisschool 5. Periodieke Peiling van het Onderwijsniveau* [Balance of mathematics education halfway primary school 5. Periodic Assessment of the Education Level]. Arnhem, the Netherlands: Cito.

Kilpatrick, J., Swafford, J., & Findell. B. (2001). *Adding it up. Helping children learn mathematics.* Washington, DC: National Academy Press.

Klein, A., Beishuizen, M., & Treffers, A. (1998). The empty number line in Dutch second grades: Realistic versus gradual program design. *Journal for Research in Mathematics Education, 29*, 443–464.

Kolovou, A., Van den Heuvel-Panhuizen, M., & Bakker, A. (2009). Non-routine problem solving tasks in primary school mathematics textbooks – A needle in a haystack. *Mediterranean Journal for Research in Mathematics Education, 8*(2), 31–68.

Meelissen, M., Netten, A., Drent, M., Punter, R., Droop, M., & Verhoeven, L. (2012). *PIRLS en TIMSS 2011. Trends in leerprestaties in Lezen, Rekenen en Natuuronderwijs* [PIRLS and TIMSS 2011. Trends in achievement in Reading, Mathematics and Science]. Enschede/Nijmegen: Twente University/Radboud University.

OCW (1993; 2006). *Kerndoelen basisonderwijs* [Core Goals Primary Education]. Den Haag: OCW.

OCW (2009). *Referentiekader taal en rekenen* [Reference Standards language and mathematics]. Den Haag: OCW.

Peltenburg, M., Van den Heuvel-Panhuizen, M., & Robitzsch, A. (2012). Special education students' use of indirect addition in solving subtraction problems up to 100 – A proof of the didactical potential of an ignored procedure. *Educational Studies of Mathematics, 79*, 351–369.

Reys, B., & Reys, R. (2006). The development and publication of elementary mathematics textbooks: let the buyer beware! *Phi Delta Kappan, 87*(5), 377–383.

Selter, C., Prediger, S., Nührenbörger, M., & Hußmann, S. (2012). Taking away and determining the difference – A longitudinal perspective on two models of subtraction and the inverse relation to addition. *Educational Studies of Mathematics, 79*, 389–408.

Stein, M., & Smith, M. (2010). The influence of curriculum on students' learning. In B. Reys, R. Reys & R. Rubenstein (Eds.), *Mathematics curriculum. Issues, trends, and future directions* (pp. 351–362). Reston, VA: National Council of Teachers of Mathematics (NCTM).

Torbeyns, J., De Smedt, B., Stassens, N., Ghesquière, P., & Verschaffel, L. (2009). Solving Subtraction Problems by Means of Indirect Addition. *Mathematical Thinking and Learning, 11*, 79–91.

Usiskin, Z. (2008). The arithmetic curriculum and the real world. In D. de Bock, B. Dahl Søndergaard, B. Gómez Alfonso, & C. Litwin Cheng (Eds.), *Proceedings of ICME-11-Topic Study Group 10: Research and Development in the Teaching and Learning of Number Systems and Arithmetic*, pp. 11–16. Retrieved from https://lirias.kuleuven.be/bitstream/123456789/224765/1/879.pdf

Valverde, G., Bianchi, L., Wolfe, R., Schmidt, W., & Houang, R. (2002). *According to the book. Using TIMSS to investigate the translation of policy into practice through the world of textbooks*. Dordrecht/Boston/London: Kluwer Academic Publishers.

Van den Heuvel-Panhuizen, M. (2001). Realistic Mathematics Education in the Netherlands. In J. Anghileri (Ed.), *Principles and Practices in Arithmetic Teaching: Innovative Approaches for the Primary Classroom* (pp. 49–63). Buckingham/Philadelphia: Open University Press.

Van den Heuvel-Panhuizen, M. (2008). Learning From "Didactikids": An Impetus for Revisiting the Empty Number Line. *Mathematics Education research Journal 20*(3), 6–31.

Van den Heuvel-Panhuizen, M. (2010). Reform under attack – Forty Years of Working on Better Mathematics Education thrown on the Scrapheap? No Way! In L. Sparrow, B. Kissane & C. Hurst (Eds.), *Shaping the future of mathematics education: Proceedings of the 33rd annual conference of the Mathematics Education Research Group of Australasia* (pp. 1–25). Fremantle: MERGA.

Van den Heuvel-Panhuizen, M. (2012). *Mathematics education research should come more often with breaking news*. Lecture on the occasion of receiving the Svend Pedersen Lecture Award 2011. Retrieved from http://www.mnd.su.se/polopoly_fs/1.75423.1328790378!/menu/standard/file/svendPedersenLecture_120205.pdf.

Van den Heuvel-Panhuizen, M., & Treffers, A. (2009). Mathe-didactical reflections on young children's understanding and application of subtraction-related principles. *Mathematical Thinking and Learning, 11*(1–2), 102–112.

Verschaffel, L., Greer, B., & De Corte, E. (2007). Whole number concepts and operations. In F. Lester (Ed.), *Second Handbook of Research on Mathematics Teaching and Learning*. Charlotte, NC: NCTM.

Weiss, I., Knapp, M., Hollweg, K., & Burill, G. (Eds.). (2002). *Investigating the influence of standards.* Washington, DC: National Academic Press.

Textbook series

Bokhove, J., Borghouts, C., Kuipers, K., & Veltman, A. (2009). *Rekenrijk* [Realm of Arithmetic / Rich Arithmetic], 3rd edition. Groningen/Houten: Noordhoff Uitgevers.

Terpstra, P. & De Vries, A. (2010). *Reken Zeker* [Arithmetic with Certainty]. Groningen/Houten: Noordhoff Uitgevers.

Authors' contributions

This paper was a collaborative work of the two authors. Both authors participated in the selection of the textbooks and the development of the analysis framework. MZ carried out the coding and the analysis of the data. The findings were frequently discussed with MH. MZ prepared the first draft of the manuscript. Both authors participated in revising the manuscript. Both authors read and approved the final manuscript.

Chapter 3

Opportunity to learn problem solving in Dutch primary school mathematics textbooks

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Opportunity to learn problem solving in Dutch primary school mathematics textbooks

Abstract

In the Netherlands, mathematics textbooks are a decisive influence on the enacted curriculum. About a decade ago, Dutch primary school mathematics textbooks provided hardly any opportunities to learn problem solving. In this study we investigated whether this provision has changed. In order to do so, we carried out a textbook analysis in which we established to what degree current textbooks provide non-routine problem-solving tasks for which students do not immediately have a particular solution strategy at their disposal. We also analyzed to what degree textbooks provide 'gray-area' tasks, which are not really non-routine problems, but are also not straightforwardly solvable. In addition, we inventoried other ways in which present textbooks facilitate the opportunity to learn problem solving. Finally, we researched how inclusive these textbooks are with respect to offering opportunities to learn problem solving for students with varying mathematical abilities. The results of our study show that the opportunities that the currently most widely used Dutch textbooks offer to learn problem solving are very limited, and these opportunities are mainly offered in materials meant for more able students. In this regard, Dutch mainstream textbooks have not changed compared to the situation a decade ago. A textbook that is the Dutch edition of a Singapore mathematics textbook stands out in offering the highest number of problem-solving tasks, and in offering these in the materials meant for all students. However, in the ways this textbook facilitates the opportunity to learn problem solving, sometimes a tension occurs concerning the creative character of genuine problem solving.

Keywords: non-routine problems; heuristics; learning facilitators; opportunity to learn for all students; textbook analysis; comparing textbooks

1. Introduction

Mathematics is inextricably linked with problem solving. Problem solving is even considered the heart of mathematics (Halmos, 1980; Schoenfeld, 1992; Dossey, 2017). However, despite the long-standing recognition of the importance of problem solving, there are still different interpretations of what is meant by it. The term is used in several ways, with different connotations (e.g., Schoenfeld, 1992; Van Meriënboer, 2013; Xenofontos, 2010). Problem solving can refer to a skill, a process, an educational goal, and a teaching approach. Specifically, in the field of mathematics education, a distinction is made between teaching *of* mathematical problem solving and the teaching of mathematics *through* problem solving (e.g., Liljedahl, Santos-Trigo, Malaspina, & Bruder, 2016). In the current study, the focus is on teaching *of* problem solving.

Several authors (e.g., Burkhardt, 2014; Zhu & Fan, 2006) have indicated that the term 'problem' itself can also be interpreted differently. In the meaning of a mathematical task on which students have to work, the term problem can refer to all types of tasks regardless of their cognitive demands, but it is also used for specific kinds of tasks, such as word problems in which previously learned mathematics has to be applied, or puzzle-like tasks which are new to the students and which they themselves have to figure out how to solve. The latter meaning is used in this study. By problems we mean non-routine mathematical tasks for which students do not immediately have a particular solution strategy at their disposal.

In mathematics education, textbooks largely determine what teachers teach and consequently, what students learn (Stein & Smith, 2010). In the Netherlands, this is very much the case (see Section 2.1). Generally speaking, if certain content is not included in the textbook, it will probably not be covered in the classroom (Stein, Remillard, & Smith, 2007). Thus, what is in the textbooks is of great importance for the learning opportunities students get, including the learning of problem solving. As a consequence, knowledge of the content of textbooks is very important. About a decade ago an analysis of Dutch primary school mathematics textbooks showed that non-routine problem-solving tasks were hardly included in the textbooks (Kolovou, Van den Heuvel-Panhuizen, & Bakker, 2009). The current study was meant as a follow up of this study and investigated to what degree non-routine problem-solving tasks are now included in Dutch textbook series. In addition to this purpose, it was also explored whether apart from offering these tasks, there exist other ways in which Dutch textbooks facilitate the opportunity to learn problem solving.

2. Background of the study and research questions

2.1 Textbooks and opportunity to learn in the Netherlands

The aforementioned research claim, that textbooks have a determining role in the enacted curriculum (Stein & Smith, 2010), applies also to a great extent in the Netherlands. TIMSS research among Grade 4 teachers found that 94% of these teachers say that their textbook is the main source of their teaching (Meelissen et al., 2012). Other studies carried out in the Netherlands found that more than 80% of Grade 2 and Grade 3 teachers indicate that they are following over 90% of the textbook content (Hop, 2012). A minority of these teachers sometimes skip content of their textbook, but they still teach 60% to 90% of the content (ibid.). Another investigation revealed that only a minority of Grade 6 teachers use additional resources next to their textbooks, mainly materials for less able students and software for repetition (Scheltens, Hemker, & Vermeulen, 2013). The results of these studies indicate that a vast majority of Dutch primary school teachers rely heavily in their teaching on the textbook series they use. This means that mathematics textbook series play a decisive role in Dutch daily teaching practice and therefore in the learning opportunities that students are offered. As a result, the textbook used has a significant effect on learning outcomes, as repeatedly shown in Dutch national evaluations of educational progress. For several learning topics, these studies have shown that students taught with different textbook series differ significantly in their mathematics achievement (e.g., Hop, 2012; Kraemer, Janssen, Van der Schoot, & Hemker, 2005; Scheltens et al., 2013).

2.2 Opportunity to learn non-routine mathematical problem solving

In his seminal work *How to Solve it* (1945), Pólya does not use the term ‘non-routine’, but he does define routine problems, namely as tasks that “can be solved either by substituting special data into a formerly solved general problem, or by following step by step, without any trace of originality, some well-worn conspicuous example” (p. 171). This implies that whether a task can be considered routine or not at least partly depends on factors other than the task itself, such as whether an example is given. Schoenfeld (1985, 2013) points out that difficulty alone does not define a task as a problem. Instead, it is not a property of the task itself which makes it a problem, but the “particular relationship between the individual and the task" (Schoenfeld 1985, p. 74). Thus, whether a task can be considered a problem can differ per person: a task that forms a genuine problem for one student may be a routine task for another student. Furthermore, whether a task is a problem may differ over time, after all, "the person who has worked on, and solved, a problem, is not the same person who began working on it. He or she approaches the next problem knowing more than before" (Schoenfeld, 2013, p. 20). In other words, what is at first considered a problem can become a routine task.

The issue of this relative and personal character of problem solving has been raised already very often (e.g., Kantowski, 1977; Lesh & Zawojewski, 2007; Manouchehri, Zhang, & Liu, 2012; Lester, 2013) and is also reflected in a recent OECD publication in which is stated that a mathematical problem "involves a situation, posed in either an abstract or contextual setting, where the individual wrestling with the situation does not immediately know how to proceed or of the existence of an algorithm that will immediately move toward a solution" (Dossey 2017, p. 61). Researchers may use different wording, but they generally agree that the feature that makes a mathematical task a problem is that the person who has to solve the problem does not directly have a solution procedure at his or her disposal. Otherwise this task is what Pólya (1945) calls a "routine problem" or what other authors call, with a more distinctive term, an "exercise" (e.g., Burkhardt, 2014; Manouchehri et al., 2012; Schoenfeld, 1985). Such tasks are solvable by straightforward calculation (Pretz et al., 2003), executing rules or procedures (Lesh & Zawojewski, 2007), applying known algorithms or following worked out examples (Manouchehri et al., 2012), or by following a seen or taught solution pathway (Burkhardt, 2014). In this study, we refer to such tasks as *straightforward*.

Different from straightforward tasks, non-routine problems require more than just executing the required calculations. Non-routine problems ask for more complex processes and set higher cognitive demands. Solving such tasks involves analyzing the problem at hand, relating procedures with their underlying mathematical concepts and making connections between different representations (Stein, Smith, Henningsen, & Silver, 2000). Or, in the words of Lester (2013, p. 255): "For non-routine tasks a different type of perspective is required, one that emphasizes the making of new meanings through construction of new representations." A related distinctive feature of a genuine problem is that it requires modeling (e.g., Lesh & Zawojewski, 2007; English, Lesh, &

Fennewald, 2008), which as Lesh and Zawojewski (2007) point out, in a way is *creating* mathematics. In other words, problem solving requires creative mathematical thinking, which is underlined in the recently published ICME 'state-of-the-art' report on problem solving (Liljedahl et al., 2016).

Another important perspective on problem solving that is also emphasized in this ICME publication is that of a heuristic approach (see Pólya, 1945, 1962; Schoenfeld, 1985, 1992, 2013; Mason, Burton, & Stacey, 2010). This approach involves the conscious use of a number of problem solving strategies that may help to find a solution, including acting the problem out with objects, drawing a diagram, guessing a seemingly reasonable answer and checking it, reasoning logically, making a systematic list or a table, restating the problem, simplifying the problem, solving part of the problem, thinking of a related problem, using a model or an equation, and working backwards (e.g., Fan & Zhu, 2007; Lee, Yeo, & Hong, 2014).

According to Liljedahl et al. (2016), there can occur a certain tension between this approach and the non-routine, creative character of problem solving because the "problem solving heuristics that are based solely on the processes of logical and deductive reasoning distort the true nature of problem solving" (p. 19). Thus, for example, the heuristic "think of a related problem" may lead to the recalling of a known solution procedure for that particular type of problem, which means that in that situation no genuine problem solving occurs. Yet, the heuristic "make a systematic list or a table" can provoke divergent thinking, which is an aspect of genuine problem solving, and the heuristic "draw a diagram" can support the creative process of modeling a problem. However, drawing a diagram may also lead to the use of a well-known procedure, just as thinking of a related problem does not necessarily lead to recalling of a known procedure. Nevertheless, as Schoenfeld (1992) points out, when students are given intensive practice in certain heuristics, these become mere algorithms. Thus, just as problem solving and problems are relative in nature, heuristics can also be characterized as such—they either can or cannot contribute to the opportunity to learn problem solving, which, moreover, is partly due to how instruction takes form (see also English et al., 2008). For example, introducing heuristics in an isolated way could provoke students to see them as rules (Fan & Zhu, 2007).

Although Lester (2013) claims that research does not tell enough yet about problem-solving instruction, he does list important principles that have emerged from research in the last decades. The two most important principles in his view are that students, in order to improve their problem-solving abilities, have to "work on problematic tasks on a regular basis over a prolonged period of time" (p. 272) and have to be "given opportunities to solve a variety of types of problematic tasks" (ibid.). In other words, the learning of problem solving is enhanced by the opportunity to actually work on genuine and varied non-routine problems. As a consequence, a clear way in which mathematics textbooks can contribute to the opportunity to learn problem solving is the inclusion on a regular basis of tasks that potentially can be genuine problems for students. In addition to including problem-solving tasks, Doorman et al. (2007) recommend that textbook series explicitly pay attention to heuristics. They also point out that genuine problem-solving is often believed to be only attainable by the best students (ibid.). However, Stein and Lane (1996) reason that all students, of varying abilities, may benefit from the opportunity to work on tasks with high level cognitive demands such as non-routine problem solving. More recently, Johnsson, Norqvist, Liljekvist, & Lithner (2014) found that cognitively less proficient students also profit from working on tasks that require creative mathematical reasoning. Based on their study, they argue that all students should be given the opportunity to be involved in problem solving (ibid.). That less able students indeed may have the ability to learn problem solving, given the opportunity, was for example demonstrated in a Dutch study with students that attend special education (Peltenburg, Van den Heuvel-Panhuizen, & Robitzsch, 2012). Thus, a final way in which textbooks can enhance the opportunity to learn problem solving is to include problem-solving tasks and heuristics not only in materials that are meant exclusively for more able students, but in materials meant for all students.

2.3 Research questions

About a decade ago, Kolovou et al. (2009) researched to what degree Dutch textbook series contained non-routine problem-solving tasks and so called "gray-area" tasks, which were defined as tasks that fall in between genuine problem-solving tasks and straightforward tasks (see Section 3.3. for a more precise description of these two types of tasks). All then available textbook series were investigated. The analyzed textbook materials were meant for the first half schoolyear of Grade 4. It was found that the proportion of non-routine problems was very low, varying from 0% to 2% of the total number of tasks. When taken together, the proportion of non-routine problem-solving tasks and gray-area tasks was still rather low—varying from 5% to 13% of all tasks. Furthermore, it was found that most non-routine and gray-area tasks were included in additional enrichment materials of the textbook series, meaning that the already limited opportunity to learn problem solving was even lower for students that were not given the chance to work with these materials.

Currently, all textbook series that were investigated then have been replaced by new editions or have been withdrawn from the market. New textbook series have also been published in the meantime. In order to find out whether these present mathematics textbooks have changed with respect to the opportunity to learn problem solving, we carried out a replication study, in which we investigated to what degree current mathematics textbooks offer non-routine problems. Further, bearing in mind the different perspectives on problem solving discussed in the previous section, we additionally researched whether textbooks facilitate the opportunity to learn problem solving in other ways, such as presenting heuristics. Finally, we analyzed to what degree learning opportunities are offered in materials meant for all students and in materials meant only for more able students. So, our research questions were as follows:

1. To what degree do current Dutch primary school textbooks contain mathematical problem-solving tasks?
2. In what other ways do these textbooks facilitate the opportunity to learn problem solving?
3. How inclusive are these textbooks with respect to offering opportunities to learn problem solving for students with varying mathematical abilities?

3. Method

To answer these research questions, we carried out a textbook analysis of primary school mathematics textbook series presently in use in the Netherlands.

3.1 Selection of textbooks and textbook materials

Nowadays in the Netherlands, there are eight different mathematics textbook series for primary school on the market. For selecting textbook series to be included in our study, we first looked at the textbooks' market share. We wanted to include textbook series that together are in use in a majority of schools because this would provide a sound basis for drawing conclusions regarding the Dutch situation in general. This led to the selection of three textbook series that together are used in approximately 90% of all schools. These textbook series are *De Wereld in Getallen* (The World in Numbers) (Huitema et al., 2009-2014), *Pluspunt* (Plus Point) (Van Beusekom, Fourdraine, & Van Gool, 2009-2013) and *Alles Telt* (Everything Counts) (Van den Bosch-Ploegh et al., 2009-2013). Previous editions of these three textbook series were also included in the study of Kolovou et al. (2009). A fourth textbook we included in our textbook analysis was *Rekenwonders* (Wonder Calculators) (Projectgroep Rekenwonders Bazalt Groep, 2011-2015), which is the Dutch version of the Singapore textbook series *My Pals Are Here! Maths* (Kheong, Ramakrishan, & Soon, n.d.). As a result of the high performance of Singapore students as established in international research, a Dutch publisher took the initiative to translate and adapt this textbook series for the Netherlands. Compared to the other textbook series involved in our study, *Rekenwonders* has only a very small percentage of market share. This is not only because this textbook series is not that long on the market, but also because the content and teaching method are quite new for teachers and deviate somewhat from what is traditionally taught in Dutch primary schools. The reason that we nevertheless included this textbook in our study was that this textbook is purposely put in the market to enhance students' problem-solving skills. Thus, for us it is interesting to investigate what opportunities to learn problem solving this textbook series offers.

To make a comparison possible with the Dutch textbooks involved in the study of Kolovou et al. (2009) a decade ago, we included textbook materials for the same school period as was done in this earlier study, namely materials meant for the first half schoolyear of Grade 4. In order to determine possible differences within textbook series between materials meant for different grades, we also included materials for the first half schoolyear of Grade 6.

All four textbook series consist of the following materials: lesson books and work books for students, accompanying teacher guidelines, and additional materials such as work sheets and software. In the analysis we included all student materials that, as indicated in the teacher guidelines, belong to the daily lessons. Materials with no such link, such as software for repetition of basic knowledge and skills (e.g., the multiplication tables) were left out of our analysis. Because directions for instructional approaches, which are often included in teacher guidelines, are also of influence on the opportunity to learn (Remillard, Harris, & Agodini, 2014), we also included these guidelines in our analysis.

3.2 Unit of analysis

Although the four selected textbook series differ in their quantitative features such as number and size of student book pages, they all provide content for five daily mathematics classes per week, for 36 weeks per schoolyear. Also, in all four textbook series lessons are subdivided into numbered segments, mostly consisting of sets of tasks (see Figure 1). With the term ‘task’ we refer to the smallest unit that requires an answer from a student. In this study, we used the *set of tasks* as unit of analysis. This approach corresponds to the approach in the study by Kolovou et al. (2009). A difference between the two studies was that in the earlier study the teacher guidelines were left out of the analysis, while in our study we considered directions given in the teacher guidelines for a set of tasks as belonging to that set of tasks.

Which number belongs to each letter?

a A + B = 7 A – B = 3 A = …… B = ……	**b** C + D = 10 C – D = 4 C = …… D = ……	**c** E + F = 14 E – F = 6 E = …… F = ……	**d** L + M = 11 L – M = 9 L = …… M = ……
e R + S = 18 R – S = 2 R = …… S = ……	**f** T × U = 12 T – U = 4 T = …… U = ……	**g** V × X = 12 V – X = 1 V = …… X = ……	**h** J + K = 14 J – K = 4 J = …… K = ……

Figure 1. A set of tasks from *De Wereld in Getallen* meant for Grade 4, consisting of eight tasks

3.3 Analysis procedure

Because of the relative and personal character of problem solving, it is not easy to decide whether a task has to be classified as a problem or as a straightforward task. As Zhu and Fan (2006) reason, making such a judgement in textbook research is difficult, if not impossible, due to the fact that the features of the students solving the tasks are not known. Therefore, the methodological challenge of this study was to develop an analysis framework that indicates when a task should be classified as a genuine problem-solving task and when not.

We started the development of our analysis framework with several rounds of preliminary classifying tasks based on the theoretical insights as described in Section 2.2 and the analysis framework used by Kolovou et al. (2009). So, based on our judgement to what degree tasks require higher-order thinking skills such as analyzing or creative thinking, they were classified in three categories: straightforward tasks, non-routine problems, and gray-area tasks. If a set of tasks included tasks of more than one category, it was classified according to the highest category.

Figure 2 shows an example of a task that we classified as a non-routine problem. This task, meant for Grade 4, is a magic frame in the form of a triangle that has to be filled in with the numbers 1 to 9 in such a way that each side of the triangle adds up to 17. Students in Grade 4 will most likely have no known solution procedure at their disposal for this task and there are also no directions provided in the textbook on how to solve it. Therefore, we considered this a puzzle-like task that requires analyzing and creative thinking in combining numbers that add up to 17, while taking into account that the three numbers at the corners of the triangle are used twice in a combination of numbers adding up to 17. So, placing higher cognitive demands, requiring creative mathematical thinking and being puzzle-like, classifies this task as non-routine.

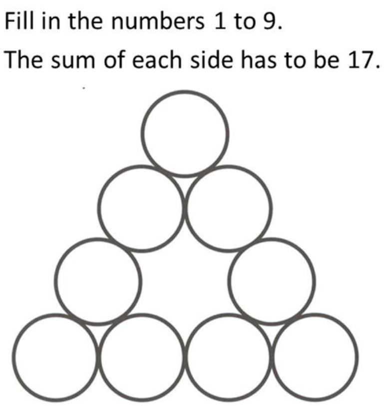

Figure 2. A Grade 4 task from *Alles Telt* classified as a non-routine problem

The task in Figure 3, meant for students in Grade 6, also concerns a magic frame. Again, the textbook does not provide directions on how to solve the task. However, because of the four already filled-in numbers, the cognitive demands of this task differ considerably from the task in Figure 2. Based on the filled-in numbers in the upper row, it can be derived directly what has to be filled in in the upper left empty cell. Also, the number that has to be filled in in the middle cell can be derived directly from the two numbers already given in the middle column. In this way, the solution process can go on—every time an empty cell can be filled in from two given or earlier filled in numbers in the same row, column, or diagonal. Thus, the creative thinking needed and the cognitive demands are limited compared to those of the task in Figure 2. Yet, its solution pathway cannot be qualified as completely straightforward, since solving this task requires finding a suitable starting point, as well as determining throughout the solution process what next step can be taken. Therefore, we classified this task as gray-area.

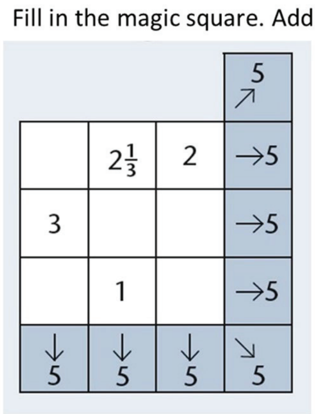

Figure. 3. A Grade 6 task from *Pluspunt* classified as gray-area

Based on the initial rounds of classification and a review of literature describing features of tasks, we developed further indicators to be used for the definitive classification of the tasks. These indicators concern features of tasks that are expected to provoke or require analyzing, modeling and creative thinking and therefore contribute to the opportunity to learn problem solving. Since all preliminary tasks labeled non-routine and gray-area were of the type that Pólya (1945, 1962) calls “problems to find”, we formulated indicators for each of the principal parts he distinguishes for these type of tasks, namely the data provided by the task, the unknown that has to be found, and the conditions that have to be fulfilled linking the unknown to the data. These principal parts are for the task shown in Figure 3, for example, as follows: the unknown consists of five numbers that have to be filled in the empty cells; the data provided are the four already filled in numbers; and the condition is that the numbers in each row, column and diagonal have to add up to 5.

The number of relations between the provided data and the required conditions that have to be processed in parallel while solving a problem may influence the complexity of the problem (Jonassen & Hung, 2008). This means that the more conditions that have to be fulfilled in a task, the more its complexity increases. Another way in which this occurs is when the data provided in the tasks are interdependent, as is the case in the task shown in Figure 4, in which weights that have to be added are expressed in terms of each other. It also makes a difference whether or not data are provided in the same order as is needed for solving the problem (Goldin & McClintock, 1979). Since increasing complexity gives more need for analyzing and modeling, we took these features—the number of conditions, interdependency of data and the order in which the data are provided—into account as indicators for problem solving. Regarding the processing of data while solving a problem, the number of steps that have to be made can affect the nature of the task (e.g., Zhu & Fan, 2006). A task that has no easily determinable starting point, such as the problem in Figure 2, or that involves reasoning back and forwards, requires multiple steps in the solution process. However, we consider multiple steps not a distinctive feature as such—when multiple steps in solving a task involve nothing more than just straightforward calculation, we consider that task still to be a straightforward one.

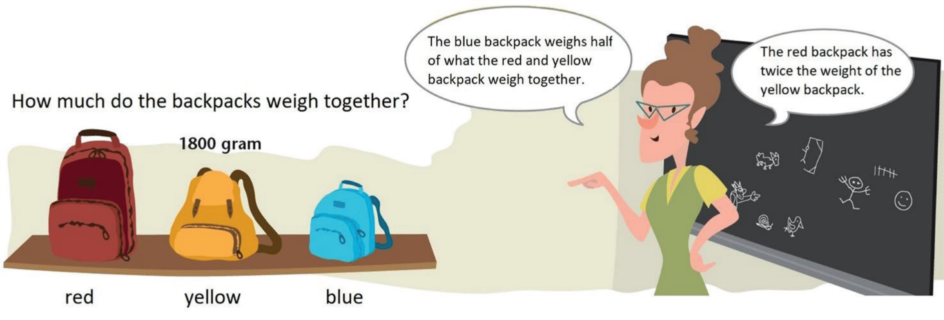

Figure 4. A Grade 6 task from *Pluspunt* with interdependent data: the weights of the backpacks are expressed in terms of each other

Indicators regarding the unknown apply only to tasks that have multiple correct solutions. It makes a difference whether only one or some correct solutions have to be found, or that all correct solutions have to be found (e.g., Pólya 1962; Pretz, Naples, & Sternberg, 2003). For example, in a combinatorics task, finding all correct solutions requires modeling and making a systematic analysis. This type of task can therefore be considered a non-routine problem (as long as a standard procedure for it is not yet known). The cognitive demands of a combinatorics task in which only a few correct solutions have to be given, is considerably lower. For solving such a task, a systematic analysis is not necessary, but providing a solution is still creative in nature. Therefore, we classified such a task as gray-area.

Altogether, we consider a larger number of conditions (below more on this), interdependency of data, and another order in the presentation of data than needed in the solution pathway as features of tasks that may lead to analyzing, modeling and creative thinking. Therefore, these features may serve as general indicators that a task may be a non-routine problem or a gray-area task. However, each of these indicators on its own may be applicable to non-routine problems as well as on gray-area tasks. It is the combination of multiple of these features which qualifies a task as non-routine problem and therefore we needed a quantitative decision rule (see also Goldin & McClintock, 1979) to use for the definitive classification of tasks. Based upon the results of the preliminary classification rounds we decided to classify a task that meets two or all three of these features as a non-routine problem, and a task that meets one of these features (and that cannot be considered a straightforward task) as a gray-area task. Further, regarding the first feature of a larger number of conditions, we specified “larger” as three or more. This, again, is based upon the results of the initial analysis. Next to these general indicators, we added a specific indicator for tasks with multiple correct solutions, namely, when all possible correct solutions have to be given, the task is considered non-routine and when just one or some of the correct solutions is sufficient, the task concerning is classified as gray-area.

Table 1 shows our final analysis framework, including the decision rules. We illustrate how this framework was applied in the final analysis through the task shown in Figure 5. The unknown that has to be found in this task is four whole numbers. These numbers have to meet all the given data, resulting in three conditions, namely (1) three of these numbers have to be half of another one of the numbers; (2) all numbers have to be even; and (3) the numbers have to add up to 30. The provided data are not interdependent (as opposed to the data of the task shown in Figure 4). To combine the given data to find the unknown, they have to be processed in another order than they are given: the first piece of information that limits the possible correct answers—the numbers add up to 30—is given last. So, this task meets two of the features in our framework (there are three conditions; the provided data must be processed in another order), classifying it as non-routine.

Table 1
Analysis framework for classification of tasks

Category	Indicators and decision rules
Non-routine problems	The task meets two or three of the following features: the unknown has to meet three or more conditions the data provided are interdependent the data are provided in another order than needed for solving the task In case the task has multiple correct solutions: all possible correct solutions have to be given *or* the total number of all possible correct solutions has to be given
Gray-area tasks	The task meets one of the following features: the unknown has to meet three or more conditions the data provided are interdependent the data are provided in another order than needed for solving the task In case the task has multiple correct solutions: one possible correct solution has to be given *or* some but not all possible correct solutions have to be given
Straight-forward tasks	The task is solvable by straightforward calculation *or* The task is offered after an explanation or an example which demonstrates how it can be solved

Linde is thinking of four whole numbers. When ordered from small to large, each number is half of the next number. All numbers are even. Together the numbers add up to 30.
Which numbers is Linde thinking of?

Figure 5. A Grade 6 task from *Alles Telt*

Based on our framework, the final classification of tasks was done by the first author. An independent expert on mathematics education who was not involved in the development of the framework performed a reliability check of the classification. For this, we used a selection of tasks (*n* = 100) covering a quarter of the total tasks classified by the first author as non-routine or gray-area. In this selection, all found appearances of non-routine problems and gray-area tasks were included. Moreover, this selection also contained 'similar-looking' straightforward tasks (*n* = 15). The agreement between the classifications of the first author and the external rater was 87.8%. After discussing the differences between the two classifications the agreement was 96.5%.

For answering the second research question, a qualitative analysis was carried out in which we inventoried all directions for problem solving strategies included in the student books as well as in the teacher guidelines. For the latter, along with the final classification of sets of tasks, we systematically checked all the accompanying descriptions included in the directions for the daily lessons in these guidelines. In addition, we checked whether the general texts of the teacher guidelines include directions for the learning of problem solving.

4. Results

4.1 Mathematical problem-solving tasks in Dutch primary school textbooks

Our first research question considered the degree in which current textbooks contain mathematical problem-solving tasks. In all textbook series included in our analysis, the percentage of non-routine problems is low, varying from 0% to 5% in the materials meant for Grade 4 and varying from 2% to 8% for Grade 6 (Table 2). The percentage of gray-area tasks varies from 2% to 4% for Grade 4 and from 1% to 4% for Grade 6. In all textbook series the majority of tasks for both grades is of the straightforward category, varying from 91% to 97%. Compared to a decade ago, when Kolovou et al. (2009) found that the textbooks then in use had a proportion of straightforward tasks varying from 87% to 95%, this number has not changed much. All the textbook series are still mainly filled with straightforward tasks.

Within the low percentage of problem-solving tasks, the part of non-routine problems and gray-area tasks in the current textbooks has changed compared to a decade ago. The average over all textbook series of non-routine problems in the materials meant for Grade 4 was 1% and is now 3% (and 4% for Grade 4 and Grade 6 together). The average of gray-area tasks for Grade 4 has shifted from 9% to 3% (and 2% for Grade 4 and Grade 6 combined). So, the current textbooks include on average relatively more non-routine problems and less gray-area tasks (both relatively and absolutely). The combined average of non-routine problems and gray-area tasks however, dropped from 9% to 6%. This indicates that the current textbooks include even fewer problem-solving tasks than those investigated a decade ago.

Table 2

Absolute and relative frequency of non-routine problems, gray-area tasks and straightforward tasks

	De Wereld in Getallen		Pluspunt		Alles Telt		Rekenwonders	
Grade 4 (first half school year)								
Non-routine problems	30	5 %	3	0 %	25	2 %	43	5 %
Gray-area tasks	24	4 %	28	3 %	23	2 %	25	3 %
Straightforward tasks	592	92 %	926	97 %	960	95 %	832	92 %
Total Grade 4	646	100 %	957	100 %	1008	100 %	900	100 %
Grade 6 (first half school year)								
Non-routine problems	16	2 %	15	2 %	40	4 %	80	8 %
Gray-area tasks	9	1 %	35	4 %	17	2 %	8	1 %
Straightforward tasks	666	96 %	854	94 %	1007	95 %	920	91 %
Total Grade 6	691	100 %	904	100 %	1064	100 %	1008	100 %

Note. Due to rounding off, some percentages do not add up correctly

For the textbook series *De Wereld in Getallen*, *Pluspunt* and *Alles Telt* a more precise comparison regarding Grade 4 can be made between the current editions analyzed in this study and the previous editions included in the study by Kolovou et al. (2009). This analysis shows that for each of these three textbook series on its own, the number of non-routine problems has increased and the number of gray-area tasks has decreased (Table 3). The combined percentage of non-routine problems and gray-area tasks is comparable in the two editions of *Alles Telt* but has decreased in the respective editions of *De Wereld in Getallen* and *Pluspunt*.

Table 3
Distribution of non-routine problems, gray-area tasks and straightforward tasks for Grade 4 in the current and former editions of De Wereld in Getallen, Pluspunt and Alles Telt

	De Wereld in Getallen				Pluspunt				Alles Telt			
	3rd ed.		4th ed.		2nd ed.		3rd ed.		1st ed.		2nd ed.	
Non-routine problems	10	2 %	30	5 %	0	0 %	3	0 %	6	2 %	25	2 %
Gray-area tasks	43	11 %	24	4 %	41	9 %	28	3 %	35	4 %	23	2 %
Non-routine and gray-area tasks together	53	13 %	54	8 %	41	9 %	31	3 %	41	5 %	48	5 %

Note. Due to rounding off, some percentages do not add up correctly

In all the textbook series, except *De Wereld in Getallen* the materials meant for Grade 6 provide more non-routine problems and gray-area tasks than the materials for Grade 4. Yet, the percentage of these tasks in Grade 6 is still low.

Out of all the textbook series, *Rekenwonders* provides the most problem-solving tasks in both grades. *Pluspunt* provides the least number of problem-solving tasks for Grade 4 and *De Wereld in Getallen* for Grade 6.

4.2 Other ways to facilitate the opportunity to learn problem solving

Our second research question concerned other ways, besides the offering of problem-solving tasks, in which Dutch textbooks facilitate the opportunity to learn problem solving. At this point, we found a striking difference between the textbook series *Rekenwonders* and the other three textbook series. Only in *Rekenwonders* did we come across regularly and systematically offered directions for both students and teachers that can be interpreted as facilitators for the learning of problem solving.

In the teacher guidelines of *Rekenwonders* problem solving heuristics are provided such as guess and check, making a systematic list, and working backwards. Learning to use heuristics is explicitly mentioned as a goal. Furthermore, the teacher guidelines provide suggestions for questions to ask students to make them aware of the problem-solving process. These include asking students to sum up the data, conditions and unknowns of problems, and stimulating them to think about suitable representations of problems. However, it must be noted that these suggestions are also given for tasks that we classified as straightforward.

In the student books of *Rekenwonders*, two sorts of learning facilitators are provided. One involves the bar model, which is extensively used for different topics, in the way described by Kho, Yeo, and Fan (2014). The bar model is also explicitly presented as a tool for solving non-routine word problems. This is done by providing partly worked out examples in which the steps for solving a specific word problem are already given and students have only to fill in the numbers (see Figure 6 for an example). Thus, although the bar model is presented as a problem-solving tool, the way in which this is done remarkably requires little more than straightforward calculation.

The bar model is also present in the other three Dutch textbook series, but less so than in *Rekenwonders* and not as a tool for problem solving.

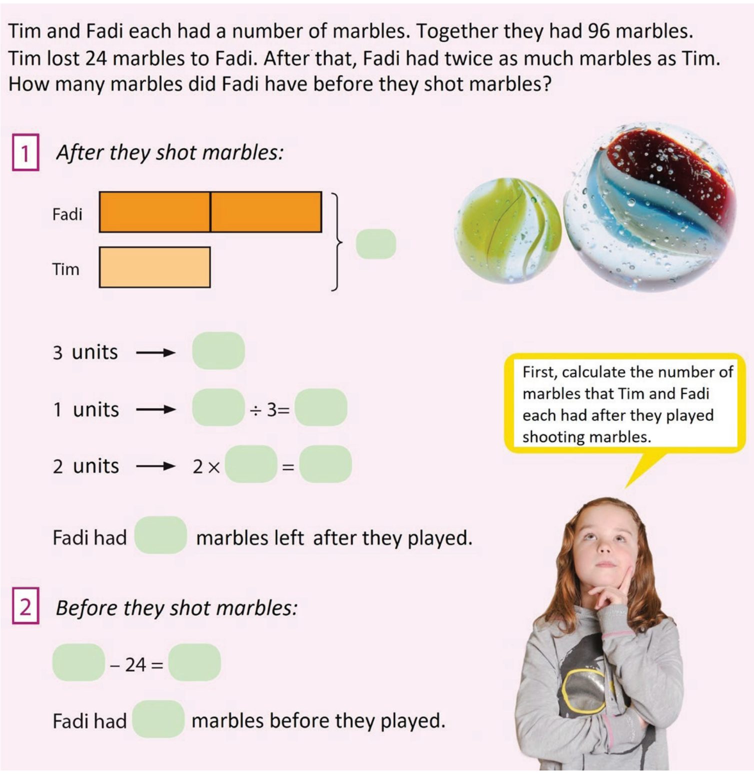

Figure 6. A Grade 4 task from *Rekenwonders* in which is demonstrated how the bar model can be used for solving it

The second learning facilitator that *Rekenwonders* offers in the student books is presenting special text sections including summaries and reflections on particular learning content. For example, students are asked to think of other situations in which a particular way of solving a problem also could be applicable. Similarly to the other facilitators already mentioned above, this one is not exclusively used for problem solving, but for all kinds of learning topics.

The other textbooks provide hardly any learning facilitators for problem solving comparable to those given by *Rekenwonders*. The teacher guidelines of these textbook series do provide suggestions for questions that can be asked of students, but not for the learning of problem solving. Only in a few cases *Alles Telt* provides the suggestion in the student book to draw a table. In *Pluspunt*, sometimes in the teacher guidelines it is emphasized that students should read a problem well and should work systematically. In *De Wereld in Getallen* we found no directions.

4.3 Opportunity to learn problem solving for students with varying mathematical abilities

Our final research question addressed the issue of how inclusive the current Dutch textbooks are with respect to offering opportunities to learn problem solving for students with varying mathematical abilities. All analyzed textbooks aim to a certain extent to be inclusive by having their materials organized in parts meant for different groups of students. In *De Wereld in Getallen*, *Pluspunt* and *Alles Telt* these parts contain differentiated tasks organized in three levels: tasks for almost all students, more cognitively demanding tasks for more able students, and easier tasks especially for less able students. *Rekenwonders* offers two levels of tasks: tasks for all students and more demanding tasks for more able students. Thus, in this textbook series the less able students also get the 'tasks for all', while in the other Dutch textbook series, these students get easier tasks. This means that *Rekenwonders* actually offers less able students more challenging tasks than the other Dutch textbooks do.

Figure 7 shows for the four textbook series the distribution of problem-solving tasks over the different levels. In *De Wereld in Getallen*, *Pluspunt* and *Alles Telt* most non-routine problems and gray-area tasks are included in the materials meant for the more able students. This was also the case in the former editions of *Pluspunt* and *Alles Telt*, as established by Kolovou et al. (2009). In the former edition of *De Wereld in Getallen* most non-routine and gray-area tasks provided were included in the materials meant for all students (ibid.), which in the current edition of this series is no longer the case. The situation in which most problem-solving tasks are meant for all students now applies only to *Rekenwonders*.

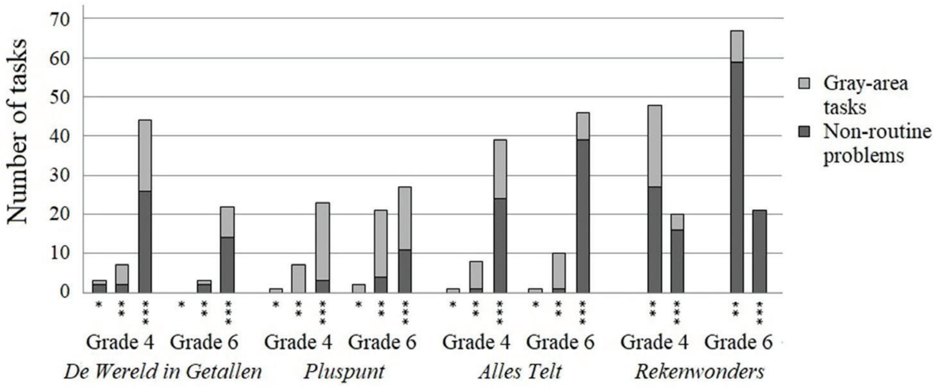

Figure 7. Frequency of non-routine problems and gray-area tasks over materials meant for less able students (*), almost all students (**), and more able students (***)

5. Conclusion and discussion

The importance of problem solving together with the finding from a decade ago that Dutch primary school mathematics textbooks hardly included problem-solving tasks at that time, led us to investigate the opportunity to learn problem solving provided by current Dutch textbooks. We found that in the textbook series *De Wereld in Getallen*, *Pluspunt* and *Alles Telt* this opportunity still turns out to be low. These textbooks provide only a small number of problem-solving tasks, incorporate hardly any other ways to facilitate the learning of problem solving, and the problem-solving tasks that are provided are mainly included in the parts that are meant for the more able students. The textbook *Rekenwonders* offers more opportunities to learn problem solving. This textbook provides the highest number of problem-solving tasks, systematically offers heuristics and other facilitators for learning problem solving, and moreover, includes most of the problem-solving tasks in the materials that are meant for all students.

All in all—also taking into account that *De Wereld in Getallen*, *Pluspunt* and *Alles Telt* together are in use in about 90% of Dutch schools and *Rekenwonders* only in a few schools—the opportunity to learn problem solving provided by current textbooks is for a vast majority of Dutch students very limited, just as was the case a decade ago.

Apart from bringing into view what mathematical content is offered to students directly, textbook analysis can also reveal implicit or hidden choices that are made in textbooks. Especially the comparison with textbooks that originate from different traditions in mathematics education may shine new light on content and teaching approaches that are taken for granted and can show that also other choices can be made. In this way our study can be of interest for a broader audience than only the Dutch mathematics education community. By not only doing an analysis on the content but also on the organizational structure of the textbooks, it was revealed that investigating the opportunity to learn offered in textbooks should also take into account what content is offered to whom. As a result of the organizational structure of *Rekenwonders*, in this textbook also less able students are offered genuine problem-solving tasks. This differs from the structure of the other three Dutch textbooks, in which the less able students obtain easier tasks which do not have a problem-solving character. This approach to problem solving as only an additional learning topic for the more able students is more or less in line with the formal Dutch intended curriculum in which only limited attention is paid to problem solving (Van Zanten et al., 2018). Conversely, in Singapore problem solving plays a central role in the curriculum and is situated in the heart of the Mathematics Curriculum Framework (Ministry of Education of Singapore 2006).

Our study clearly shows how complex the concept of opportunity to learn is from the perspective of the textbook. Just exposure of the content does not tell the whole story. As we have described above, it is also necessary to bear in mind to which students the opportunity to learn applies. A further factor that determines whether an opportunity to learn really can be considered as such, is its quality. Therefore, in our study we did not look only at the exposure of problem-solving tasks but also at the offered learning facilitators and their quality. An example is the presenting of the bar model as is done in *Rekenwonders* as a tool for problem solving, in such a way that it requires little more than straightforward calculation. This use of the bar model is a clear illustration of the tension that can occur between the creative character of genuine problem solving and the use of certain problem-solving heuristics as rules to be followed. Another learning facilitator that also might not be so helpful for learning problem solving is what *Rekenwonders* offers for new types of problems, namely systematically partly worked out examples (such as shown in Figure 6). These examples will not really trigger the creative problem-solving process of modeling, but is rather a systematic exercise in using this particular model. Taking the quality of the opportunity to learn into account we have to put our initial conclusion that *Rekenwonders* offers more opportunities to learn problem solving into perspective. What in any case remains is that students in all four investigated textbooks are offered few opportunities to learn problem solving.

This brings us to our final thought. In our view, problem solving is an important learning topic for *all* students. After all, as Halmos (1980, p. 253) puts it: “The major part of every meaningful life is the solution of problems.” Or in the words of Freudenthal (1973, p. 95): “How can mathematics be a discipline of the mind if people never experience mathematics as an activity of solving problems?” The chance of getting such experiences will be greatly enhanced if future Dutch mathematics textbooks—and this may apply for any mathematics textbooks—will provide more opportunities to learn problem solving—for all students.

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References

Burkhardt, H. (2014). Curriculum design and systematic change. In Y. Li & G. Lappan (Eds.), *Mathematics Curriculum in School Education* (pp. 13–34). Dordrecht/Heidelberg/New York/London: Springer.

Cho, Y., Caleon, I., & Kapur, M. (2015). *Authentic problem solving and learning in the 21st century*. Singapore: Springer.

Csapó, B., & Funke, J. (Eds.). (2017*). The nature of problem solving: Using research to inspire 21st century learning*. Paris, France: OECD Publishing.

Doorman, M., Drijvers, P., Dekker, T., Van den Heuvel-Panhuizen, M., De Lange, J., & Wijers, M. (2007). Problem solving as a challenge for mathematics education in the Netherlands. *ZDM Mathematics Education, 39*, 405–418. doi:10.1007/s11858-007-0043-2

Dossey, J. (2017). Problem solving from a mathematical standpoint. In B. Csapo & J. Funke (Eds.), *The nature of problem solving: Using research to inspire 21st century learning* (pp. 59–72). Paris, France: OECD Publishing.

English, L., Lesh, R., & Fennewald, T. (2008, July). *Future directions and perspectives for problem solving research and curriculum development*. Paper presented at the 11th International Congress on Mathematical Education, Monterrey, Mexico. Retrieved from https://eprints.qut.edu .au /28450/

Fan, L., & Zhu, Y. (2007). Representation of problem-solving procedures: A comparative look at China, Singapore, and US mathematics textbooks. *Educational Studies in Mathematics, 66*, 61–75. doi:10.1007/s10649-006-9069-6

Freudenthal, H. (1973). *Mathematics as an educational task.* Dordrecht, the Netherlands: Reidel Publishing Company.

Goldin, G., & McClintock, C. (Eds.). (1979). *Task variables in mathematical problem solving*. Retrieved from http://files.eric.ed.gov/fulltext/ED 178366.pdf

Halmos, P. (1980). The heart of mathematics. *The American Mathematical Monthly, 87*(7), 519–524.

Hop, M. (Ed.). (2012). *Balans van het reken-wiskundeonderwijs halverwege de basisschool 5. Periodieke Peiling van het Onderwijsniveau* [Balance of mathematics education halfway primary school 5. Periodic Assessment of the Education Level]. Arnhem, the Netherlands: Cito.

Jonassen, D., & Hung, W. (2008). All problems are not equal: Implications for problem-based learning. *The Interdisciplinary Journal of Problem-based Learning, 2*(2), 6–28. doi: 10.7771/1541-5015.1080

Jonsson, B., Norqvist, M., Liljekvist, Y., & Lithner, J. (2014). Learning mathematics through algorithmic and creative reasoning. *The Journal of Mathematical Behavior, 36*, 20–32.

Kantowski, M. (1977). Processes involved in mathematical problem solving. *Journal for Research in Mathematics Education, 8*, 163–180.

Kheong, F. Ramakrishan, C., & Soon, G. (n.d.). *My pals are here! Maths*. Singapore: Marshall Cavendish International.

Kho, T., Yeo, S., & Fan, L. (2014). Model method in Singapore primary mathematics textbooks. In K. Jones, C. Bokhove, G. Howson, & L. Fan (Eds.), *Proceedings of the International Conference on Mathematics Textbook Research and Development* (pp. 275–282).

Kolovou, A., Van den Heuvel-Panhuizen, M., & Bakker, A. (2009). Non-routine problem solving tasks in primary school mathematics textbooks. A needle in a haystack. *Mediterranean Journal for Research in Mathematics Education*, 8(2), 31–68.

Kraemer, J., Janssen, J., Van der Schoot, F., & Hemker, B. (2005). *Balans van het reken-wiskundeonderwijs halverwege de basisschool 4. Periodieke Peiling van het Onderwijsniveau* [Balance of mathematics education halfway primary school 4. Periodic Assessment of the Education Level]. Arnhem, the Netherlands: Cito.

Lee, N., Yeo, D., & Hong, S. (2014). A metacognitive-based instruction for Primary Four students to approach non-routine mathematical word problems. *ZDM Mathematics Education, 46*, 465–480. doi:10.1007/s11858-014-0599-6

Lesh, R., English, L., Riggs, C., & Sevis, S. (2013). Problem solving in the primary school (K-2). *The Mathematics Enthusiast, 10*(1), 35–59.

Lesh, R., & Zawojewski, J. (2007). Problem solving and modeling. In Lester, F. (Ed.), *Second handbook of research on mathematics teaching and learning* (Vol. 2, pp. 763–804). Charlotte, NC: Information Age Publishing.

Lester, F. (2013). Thoughts about research on mathematical problem-solving instruction. *The Mathematics Enthusiast, 10*(1), 245–278.

Liljedahl, P., Santos-Trigo, M., Malaspina, U., & Bruder, R. (2016). *Problem solving in mathematics education*. doi:10.1007/978-3-319-40730-2

Manouchehri, A., Zhang, P., & Liu, Y. (2012, July). *Forces hindering development of mathematical problem solving among school children*. Paper presented at the 12th International Congress on Mathematical Education, Seoul, Korea. Retrieved from http://www.icme12.org/forum/forum.asp

Mason, J., Burton, L., & Stacey, K. (2010). *Thinking mathem*atically (2nd edition). Harlow, England: Pearson Education Limited.

Meelissen, M., Netten, A., Drent, M., Punter, R., Droop, M., & Verhoeven, L. (2012). *PIRLS en TIMSS 2011. Trends in leerprestaties in Lezen, Rekenen en Natuuronderwijs* [PIRLS and TIMSS 2011. Trends in achievement in reading, mathematics and science]. Enschede/Nijmegen, the Netherlands: Twente University/Radboud University.

Ministry of Education of Singapore. (2006). Mathematics syllabus primary. Retrieved from https://www.moe.gov.sg/docs/default-source/document/education/syllabuses/sciences/files/2007-mathematics-%28primary%29-syllabus.pdf

Peltenburg, M., Van den Heuvel-Panhuizen, M., & Robitzsch, A. (2012, August). *Special education students' ability to solve elementary combinatorics problems*. Paper presented at EARLI SIG 15, Utrecht, the Netherlands.

Pólya, G. (1945). *How to solve it* (2nd edition). Princeton, NJ: Princeton University Press.

Pólya, G. (1962). *Mathematical discovery. On understanding, learning, and teaching problem solving*. New York, NY: John Wiley & Sons.

Pretz, J., Naples, A., & Sternberg, R. (2003). Recognizing, defining, and representing problems. In J. Davidson & R. Sternberg (Eds.), *The psychology of problem solving* (pp. 3–30). Cambridge, United Kingdom: Cambridge University Press.

Remillard, J., Harris, B., & Agodini, R. (2014). The influence of curriculum material design on opportunities for student learning. *ZDM Mathematics Education, 46*, 735–749. doi:10.1007/s11858-014-0585-z

Scheltens, F., Hemker, B., & Vermeulen, J. (2013). *Balans van het reken-wiskundeonderwijs aan het einde van de basisschool 5. Periodieke Peiling van het Onderwijsniveau* [Balance of mathematics education at the end of primary school 5. Periodic Assessment of the Education Level]. Arnhem, the Netherlands: Cito.

Schoenfeld, A. (1985). *Mathematical problem solving*. Orlando: Academic Press, Inc.

Schoenfeld, A. (1992). Learning to think mathematically: Problem solving, metacognition, and sense making in mathematics. In D. Grouws (Ed.), *Handbook for research on mathematics teaching and learning* (pp. 334–370). New York, NY: Macmillan.

Schoenfeld, A. (2013). Reflections on problem solving theory and practice. T*he Mathematics Enthusiast, 10*(1), 9–34.

Stein, M., & Lane, S. (1996). Instructional tasks and the development of student capacity to think and reason: An analysis of the relationship between teaching and learning in a reform mathematics project. *Educational Research and Evaluation, 2*(1), 50–80.

Stein, M., & Smith, M. (2010). The influence of curriculum on students' learning. In B. Reys, R. Reys, & R. Rubenstein (Eds.), *Mathematics curriculum. Issues, trends, and future directions* (pp. 351–362). Reston, VA: National Council of Teachers of Mathematics (NCTM).

Stein, M., Remillard, J., & Smith, M. (2007). How curriculum influences student learning. In F. Lester (Ed.), *Second handbook of research on mathematics teaching and learning* (Vol. 1, pp. 319–369). Charlotte, NC: Information Age Publishing.

Stein, M., Smith, S., Henningsen, M., & Silver, E. (2000). *Implementing standards-based mathematics instruction*. New York, NY: Teachers College Press.

Van Meriënboer, J. (2013). Perspectives on problem solving and instruction. *Computers & Education, 64*, 153–160.

Van Zanten, M., & Van den Heuvel-Panhuizen, M. (2018). Primary school mathematics in the Netherlands. The perspective of the curriculum documents. In D. Thompson, M. Huntley, & C. Suurtamm (Eds.), *International perspectives on mathematics curriculum* (pp. 9–39). Charlotte, NC: Information Age Publishing.

Xenofontos, C. (2010). *International comparative research on mathematical problem solving: Suggestions for new research directions.* Paper presented at the 6th Congress of the European Society for Research in Mathematics Education, Lyon, France. Retrieved from http://ife.ens-lyon.fr/publications/edition-electronique/cerme6/wg13-08-xenofontos.pdf

Zhu, Y., & Fan, L. (2006). Focus on the representation of problem types in intended curriculum: A comparison of selected textbooks from mainland China and the United States. *International Journal of Science and Mathematics Education, 4*(4), 609–626.

Textbook series

Huitema, S., Erich, L., Van Hijum, R., Nillesen, C., Osinga, H., Veltman, H., & Van de Wetering, M. (2009-2014). *De Wereld in Getallen* [The World in Numbers], 4th edition. Den Bosch, the Netherlands: Malmberg.

Projectgroep Rekenwonders Bazalt Groep (2011-2015). *Rekenwonders* [Wonder Calculators]. Rotterdam, the Netherlands: Bazalt Publishing.

Van Beusekom, N., Fourdraine, A., & Van Gool, A. (Eds.). (2009-2013). *Pluspunt* [Plus Point], 3rd edition. Den Bosch, the Netherlands: Malmberg.

Van den Bosch-Ploegh, E., Van den Brom-Snijders, P., Hessing, S., Van Kraanen, H., Krol, B., Nijs-van Noort, J., Plomp, A., Sweers, W., & Vuurmans, A. (2009-2013). *Alles Telt* [Everything Counts], 2nd edition. Amersfoort, the Netherlands: ThiemeMeulenhoff.

Authors' contributions

This paper was a collaborative work of the two authors. Both authors participated in the selection of the textbooks and the development of the analysis framework. Both authors participated in the initial data analysis and the development of the final analysis framework. MZ carried out the final coding and data analysis. The findings were frequently discussed with MH. MZ prepared the first draft of the manuscript. Both authors participated in revising the manuscript. Both authors read and approved the final manuscript.

Chapter 4

Past and current approaches to decimal numbers in Dutch primary school mathematics textbooks

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Past and current approaches to decimal numbers in Dutch primary school mathematics textbooks

Abstract

In the Netherlands, most contemporary textbook series for primary school mathematics education are influenced by the so-called *Realistic Mathematics Education* (RME) reform. This reform dates back to the 1970s. In the study described in this paper we investigated what this reform means for the approach to decimal numbers. We analyzed how this content domain is treated in a contemporary RME-oriented textbook series and two pre-RME textbook series. Our study revealed that most, although not all, of the RME characteristics included in our analysis framework were found to be present in the researched contemporary RME-oriented textbook *The World in Numbers* (2009). Furthermore, it was found that onsets of several RME characteristics were already present in the two older textbooks *New Arithmetic* (1958) and *Functional Arithmetic* (1969) that date from before the RME reform started.

Keywords: textbook analysis; decimal numbers; Realistic Mathematics Education; contemporary textbook series; older textbook series

1. Introduction

Approaches to mathematics education change and evolve over time (e.g., Howson, 1982; Walmsley, 2007). This means that current ideas about the learning and teaching of mathematics inevitably trace back to the past. Therefore, knowledge of mathematics education in earlier times may contribute to a better understanding of today's approaches. One way—and maybe the only one—to reveal this knowledge from the past and get to know what mathematics was taught previously and how, is studying the textbooks that were used in those days. Mathematics textbooks can be seen as the potentially implemented curriculum (Valverde, Bianchi, Wolfe, Schmidt, & Houang, 2002). They reflect ideas on mathematics education and translations of these ideas into actual educational approaches. Of course, textbooks are not equivalent to the enacted curriculum, but nevertheless they do bring into view both content and methods of teaching from the period in which they are, or were, in use (e.g., Bjarnadóttir, Christiansen, & Lepik, 2013). Although from the past, no data are available about how strict teachers followed their textbooks, presently in the Netherlands research has shown that for the vast majority of primary school teachers the mathematics textbook is a main source for their teaching (Meelissen et al., 2012). Also, it is found that over three quarters of the Dutch primary school teachers are following more than 90% of the textbook content (Hop, 2012).

In the study described in this paper we looked back at primary school mathematics education of half a century ago with the aim of getting a historically-informed view on the contemporary teaching of mathematics in the Netherlands. The underlying assumption of this study is that reviewing old teaching approaches with the eyes of today and vice versa, may provide indications for improving current mathematics education.

The context of our study is the reform movement in mathematics education that became known under the name 'Realistic Mathematics Education' (RME) (see, e.g., Van den Heuvel-Panhuizen, 2001, 2010; Van den Heuvel-Panhuizen & Drijvers, 2014). The inception of this reform movement was the start, in 1968, of the Wiskobas project with Treffers as one of its leading persons. Wiskobas is an acronym for *Wiskunde op de basisschool*, meaning 'mathematics in primary school'. In 1971 this project became further institutionalized with the establishment of the IOWO (Institute for Research of Mathematics Education) of which Freudenthal was the first director. The main goal of this institute, which in 1991 was renamed as the *Freudenthal Institute*, was to think of an alternative for the then prevailing mechanistic approach to mathematics education. Characteristic of this mechanistic approach (see, e.g., De Jong, 1986) is its focus on learning to calculate with bare numbers and the small amount of attention that is paid to solving real world problems. Students only have to apply mathematics after they have learned and practiced the calculation procedures with bare numbers. Another feature of the mechanistic approach is that mathematics is taught in an atomized way. Students learn the procedures in a step-by-step manner with the teacher demonstrating to them how they have to solve a problem. Contrary to the mechanistic approach, in RME 'realistic' situations are given a major place in the learning process. These so-called contexts are used in two ways. First, they serve as sources for building up mathematical concepts, tools and procedures. Progressively, this knowledge becomes more general and less context-specific. Second, the learned mathematical knowledge can be applied in other problem situations. In RME, to support the shift from context-based strategies to more formal ways of working, didactical models such as the number line play a crucial role. This emphasis on models makes it clear that the differences between the mechanistic approach and the RME approach do not only lie in the procedural aspects of learning mathematics (e.g., learning how to calculate), but that they are even more pronounced with respect to the conceptual aspects of learning mathematics (e.g., learning the decimal structure of the number system as units of units and learning how operations with numbers are related to each other) (see, e.g., Treffers, 1991).

Although after forty-five years of reform much has changed in Dutch mathematics education (Van den Heuvel-Panhuizen, 2010), the current situation is not that every class is precisely taught according to the principles of RME. Also, not every RME-oriented contemporary textbook is exactly designed according to all RME principles. Yet, since the beginning of the development of RME, the nature of textbooks has changed dramatically. Until the 1970s, mathematics textbooks in the Netherlands generally had a mechanistic approach to the teaching of mathematics. In the early 1980s, the market share of RME textbooks was only 5%, whereas in 1987 already 15% of the textbooks had a RME signature. In 1992 this market share had increased to almost 40%, and in 1997 to 75%. In 2004 RME textbooks reached a 100% market share. Recently (see, e.g., Van den Heuvel-Panhuizen, 2010), two new mechanistic textbooks have appeared on the market, but until now based on information we got from communication with publishers, their market share has stayed below 5%.

The purpose of the current study was to get a better understanding of what has changed in primary school mathematics textbooks in the Netherlands since the start of RME. To investigate this, we carried out a comparative textbook analysis in which we took RME characteristics of teaching mathematics as a point of departure for examining how these are currently reflected in textbooks and which of these characteristics could also be traced in textbooks which date from before RME. The focus of our analysis was on the content domain of decimal numbers, since the teaching and learning of this content domain incorporates both conceptual aspects (e.g., understanding what a decimal number is and how it is related to other numbers) and procedural aspects (e.g., how to calculate with decimal numbers). More particularly our research questions were:

1. What RME characteristics of teaching decimal numbers can be identified in a typical contemporary RME-oriented textbook?
2. To what degree are onsets of RME characteristics of teaching decimal numbers already present in pre-RME textbooks and in what way does the approach to teaching decimal numbers in a contemporary RME textbook differ from the approach in pre-RME textbooks?

2. The RME approach to decimal numbers

Characteristic for the mechanistic approach to teaching decimal numbers is that from the very start the teaching takes place at a formal level. Decimal numbers are introduced by definition and presented as another way to write common fractions (Streefland, 1974). Furthermore, in the mechanistic approach there is a strong focus on teaching rules for placing the decimal point (Freudenthal, 1983). RME takes another course. Here the start of teaching decimal numbers is situated at the informal level of students' daily life knowledge. Based on the didactical phenomenology promoted by Freudenthal (1983), RME uses situations in which students can encounter particular phenomena in reality that may contribute to the development of particular mathematical concepts (Van den Heuvel-Panhuizen, 2014).

For decimal numbers this occurs in measurement activities. Through measuring, decimal numbers can emerge in a natural way (Streefland, 1991). Therefore, in RME the interpretation of decimal numbers as measurement numbers related to measuring length or distance forms the basis of introducing students to this content domain (Streefland, 1974; Treffers, Streefland, & De Moor, 1996). This introduction is in agreement with the general RME idea of using context problems not only for the application of earlier learned subject matter, but also as a source for learning new mathematical content. For example, when measuring the length of the classroom with a particular unit of measurement, such as a meter, the result may be seven meters and a little bit more.

When we regard this approach in retrospect, this 'little bit more' and especially how to express this extra length is crucial for developing understanding of decimal numbers. For sure this 'little bit more' is smaller than 1 (meter), but what is it exactly? There are two ways to proceed: refinement of the unit of measurement (after meters continuing with decimeters and centimeters, and so on) or refinement of the number unit (keep the meter as the unit of measurement but continuing with tenths and hundredths of a meter, and so on). Students have to experience both refinements to understand how they are related. Connecting the concrete refinement (moving to a smaller unit of measurement) to the abstract refinement (moving from whole numbers to decimal numbers) can help students to get a better understanding of decimal numbers in several ways. To begin with, the tenths and hundredths can be interpreted as decimeters and centimeters respectively, which gives them a good basis for comparing and ordering decimal numbers grounded in understanding the place value of the digits in decimal numbers.

This connection of decimal numbers to measurement numbers can also prevent or dispel the fallacy that can result from the interference with students' whole numbers knowledge, that a number with more digits always represents a greater value than a number with fewer digits and that, for example, 5.68 is wrongly considered larger than 5.8. In the context of measurement, it is clearer for students that this is not true: 5 meters and 8 decimeters is longer than 5 meters and 68 centimeters and therefore, 5.8 is larger than 5.68 (Treffers et al., 1996). To a certain degree this support can also be found within the context of monetary values notation. There, it is also clear that 5.8 is more than 5.68. However, using money to explain decimal numbers is not so suitable for getting understanding of the continuous character of decimal numbers because banknotes and coins factually represent discrete quantities instead of continuous quantities. With money the refinement factually ends after cents, whereas in the measurement context refinement can go on infinitely. When working with measurement numbers students can experience that between 3.4 meter and 3.5 meter there lie other numbers such as 3.41 meter and 3.412 meter and so on. Also, by writing down the result of a measurement like '7.632… meter', the dots represent the theoretically possible infinite ongoing refinement in tens (Streefland, 1974).

Contrary to the mechanistic approach, in RME the main focus is not on written calculation of decimal number, or, more precisely, on digit-based algorithmic processing of these numbers. Consequently, the focus in RME is also not on accompanying rules for how the decimal point should be put in the right position in each algorithm, which in RME is considered as blocking students' insight (Freudenthal, 1983; Treffers & De Moor, 1984). In RME, the focus on written calculation (either whole-number-based or digit-based algorithmic processing) of decimal numbers is reduced and postponed to later stages of learning decimal numbers. Characteristic for RME is that greater emphasis is put on estimation and mental calculation with decimal numbers. Moreover, within RME attention is also paid to calculating with decimal numbers using a calculator. However, the reason for this is not just to provide students with a device to find an answer. The calculator is also used as a didactical means to investigate the decimal number system and a means to check found answers (Treffers & De Moor, 1984). Also, estimation is not only a goal in itself, but is employed to support students' understanding of where to place the decimal point. Estimating prevents students from giving a too large or too small number that does not make sense as an answer (Treffers & De Moor, 1984; Treffers et al., 1996). In agreement with the RME characteristic of intertwined learning strands, the four different ways of calculating with decimal numbers are highly integrated, and related to each other.

Another characteristic of RME is the use of models. For learning decimal numbers and dealing with them in the context of measuring length and distance, the model that is pre-eminently suitable is the number line. According to Freudenthal (1983), the power of the number line is that it visualizes both natural numbers and measurement numbers. Consequently, through these measurement numbers the number line can be opened up to decimal numbers and whole number knowledge can be extended to knowledge about decimal numbers. Students can use the number line to compare and order decimal numbers. Additionally, the number line is rather appropriate to visualize the continuous character of decimal numbers. By zooming in on a particular section, enlarging it and adding finer units, more and more digits can be added behind the decimal point. Another model that in RME is used for teaching decimal numbers is the place-value chart (Treffers et al., 1996). In addition, also the abacus with a decimal point (De Jong, 1986) and the so-called 'ladder of refinement' (…, 1000, 100, 10, 1, 0.1, 0.01, 0.001, … presented vertically; see Freudenthal, 1983) are mentioned, but not frequently. These models are not primarily meant for carrying out calculations with decimal numbers, but for understanding the place value of the digits in decimal numbers (Treffers et al., 1996). In the case of multiplications with decimal numbers the area (or rectangle) model is used as an alternative for using rules to understand the place of the decimal point. Whereas rules may block students' insight, because the rule for multiplication competes with the one for adding decimal numbers (Freudenthal, 1983), the area model visualizes that when changing a measurement in centimeters into a measurement in millimeters, a refinement of ten times a refinement of ten, results in a refinement of one hundred (Streefland, 1974).

Finally, in line with the RME principle of having students actively involved in the learning process, in RME students are given much room to explore their own calculation methods, and to come up with self-constructed problems, called 'own productions' (Treffers, 1987) or 'free productions' (Streefland, 1990). Both the students' own calculation methods and their self-generated problems are discussed in class to evoke reflection to supports students' understanding of decimal numbers.

3. Method

3.1 Analysis framework

To carry out the textbook analysis we developed a framework in which we incorporated the main RME characteristics of teaching decimal numbers. These characteristics were split into three perspectives (cf. Van Zanten & Van den Heuvel-Panhuizen, 2014): the taught mathematical content, the performance expectations (what performance are students expected to show regarding particular mathematical content), and the learning facilitators (see Table 1).

With respect to the perspective of the content we focused on the types of decimal numbers that textbooks may deal with. For this we distinguished three sub-categories: bare decimal numbers, measurement decimal numbers, and monetary decimal numbers. The most significant type of decimal numbers, in the sense that this type is in line with how decimal numbers are conceptualized in RME is the sub-category of measurement decimal numbers. Alongside with these numbers we also included monetary decimal numbers in our framework. Although decimal numbers expressing monetary values can also be seen as measurement numbers, we considered them as a separate sub-category, because they are not so suitable for gaining an understanding of the continuous character of decimal numbers and consequently do not have a prominent place in the RME approach to decimal numbers.

Table 1
Analysis framework

Perspective	Category	Sub-category
Content	Types of decimal numbers	Bare decimal numbers Measurement decimal numbers Monetary decimal numbers
Performance expectations	Types of calculation with decimal numbers	Mental calculation Estimation Written calculation Calculation with calculator
Learning facilitators	Didactical support	Use of contexts Use of the number line Use of a place value chart| Use of different calculation methods Use of own productions

Following Valverde et al. (2002), we did not regard performance expectations as part of the content perspective, but considered them a separate perspective, which refers to what type of calculations the students should be able to perform: mental calculation, estimation, written calculation (either whole-number-based or digit-based algorithmic processing), and calculation with a calculator. Here, the RME approach is best recognized by a not-so-prominent position of written calculation with decimal numbers to the advantage of a higher emphasis on estimation and mental arithmetic.

For the perspective of the learning facilitators we included five types of didactical support promoted in RME: the use of contexts as a source for learning decimal numbers, the use of the number line as a model, the use of place value charts, the use of different calculation methods and the use of ‘own productions’.

3.2 Textbooks and textbook materials included in the analysis

We included three textbook series in our analysis: *De Wereld in Getallen* [The World in Numbers, WiN] (Huitema et al., 2009), *Nieuw Rekenen* [New Arithmetic, NA] (Bruinsma, Van den Heuvel, Van Lierop, Van Wijk, & Van Zuilekom, 1969), and *Functioneel Rekenen* [Functional Arithmetic, FA] (Reijners & Snijders, 1958). WiN dates from 2009 and was included because it is the most widely used contemporary RME-oriented textbook series. It has a market share of approximately 40% and has a history of three previous editions, dating back to the early days of RME in the 1980s. Because we wanted to investigate to what degree onsets of RME characteristics of teaching decimal numbers were already present in pre-RME textbooks, we found NA and FA to be suitable textbooks. Both date from before the RME reform and despite the fact that in the study of De Jong (1986) they were classified as belonging to the mechanistic approach to mathematics education, De Jong's research team also identified several elements in these textbooks that were considered innovative for their time. NA dates from 1969 and was in use in more than 35% of Dutch schools in the 1970s. It was still in use in the 1990s. FA dates from 1958 and was used in approximately 5% to 10% of schools until the 1970s (see De Jong, 1986; Janssen, Van der Schoot, Hemker, & Verhelst, 1999; Wiskobas-team, 1979). Thus, our selection of textbooks covers over half a century.

In all three textbook series, decimal numbers are dealt with in Grade 4, Grade 5 and Grade 6, so we limited our research to the materials for these grades. We analyzed all materials of the textbooks meant for all students and the accompanying teacher guides. Optional materials meant for assessment, repetition or enrichment were left out of our analysis.

3.3 Unit of analysis

In all three textbook series, the content is organized in sets of tasks. With the term 'task' we mean the smallest unit in the student books that requires an answer from a student. A 'set of tasks' is always indicated by a number and mostly contains a series of tasks (see Figure 1). In our analysis we used the task as the unit of analysis, together with the directions and models belonging to this task as indicated in the set of tasks and the corresponding directions given in the teacher guides.

Calculate.

a There are some lengths of artificial grass on sale. One is 12,5 m long, one is 3 m long, and one is 7,75 m long. How many meters are there in total?

b
0,5 + 0, 44 =
5,03 + 99 =
0,06 + 60,1 =
7,5 + 3,75 =

c
2,55 + 35 + 102 =
60 + 4,89 + 3,01 =
5,49 + 3,21 + 130 =
12,4 + 0,45 + 3,05 =

d From a 12,5 m roll of fabric, 3,75 m is cut off. How many meters are left?

e
10,5 – 3,48 =
15,7 – 8,95 =
22,4 – 7,85 =
34,6 – 15,69 =

f
12,25 – 8,7 =
38,62 – 14,9 =
45,54 – 28,9 =
53,05 – 48,5 =

Figure 1. A set of tasks (WiN, 2009) with nine addition and nine subtraction tasks (one word problem and eight bare number tasks each)

Note. In the Netherlands, decimal numbers are written with a decimal comma instead of a decimal point

3.4 Tasks on decimal numbers included in the analysis

Although tasks in the domain of decimal numbers have an overlap with calculating with monetary values and with measurement tasks, we only included those tasks in the analysis that primarily focus on decimal numbers. For example, tasks in which certain measures have to be converted to another measurement unit (e.g., 2.5 m = … cm) without problematizing the change from decimal numbers to whole numbers, were considered to belong predominantly to the domain of measurement. These tasks were left out of the analysis, whereas tasks that, for example, focus on place value (e.g., 3.42 m = … m + … dm + … cm) were included, because here the students have to interpret the decimal digits.

3.5 Analysis procedure

First, all tasks on decimal numbers meant for all students were identified. Then, each task was coded according to the framework. For all the coded tasks it was checked whether the assigned codes were in agreement with the directions included in the teacher guide, because sometimes the tasks included in the student books do not offer enough indications for how to code the tasks. To be sure that the coding was done correctly, the complete coding was done a second time. When differences between the two codes of a task occurred, the task was reviewed again and the codes were made the same, and if necessary, codes of other tasks were also revised until all coding results were consistent.

4. Results

4.1 Attention paid to decimal numbers

As is shown in Figure 2, all three textbook series introduce decimal numbers in Grade 4 and pay most attention to it from the second half of Grade 4 to the first half of Grade 6. Over the three grades, the two older textbooks FA and NA offer more tasks on decimal numbers that are meant for all students (2333 and 3396 tasks respectively) than the contemporary textbook WiN (1997 tasks). Note, that these frequencies do not reflect the total amount of attention paid to decimal numbers, because additional optional tasks meant for differentiation were left out of our analysis. In WiN, the number of pages with additional tasks for differentiation is larger than the number of pages with tasks meant for all students. This is the other way around in NA, whereas in the oldest textbook, FA, only now and then an additional task meant for differentiation is given. So although it might seem, based on the number of tasks meant for all students, that the contemporary textbook in a quantitative way pays less attention to decimal numbers, this is not the case.

4.2 Types of decimal numbers included in tasks

All three textbook series offer tasks with bare decimal numbers, measurement decimal numbers and monetary decimal numbers, but the relative frequency of these tasks is quite different in the three textbooks series (see Table 2). In FA, the vast majority of the tasks concern bare decimal numbers. Similarly, in NA most tasks are with bare decimal numbers, but NA also provides a substantial number of tasks with monetary decimal numbers. WiN provides about the same number of tasks with bare decimal numbers and tasks with monetary decimal numbers, and a substantial number of tasks with measurement decimal numbers. Compared to the older textbook series, WiN provides a considerable smaller proportion of tasks with bare decimal numbers and a considerable larger proportion of tasks with measurement decimal numbers. In the two older textbook series, measurement decimal numbers are present, but the proportion of tasks with this type of decimal numbers is very small. Regarding the number of tasks with monetary decimal numbers, no clear difference can be determined between WiN and NA. Both NA and WiN do provide substantially more tasks with monetary decimal numbers than the oldest textbook, FA.

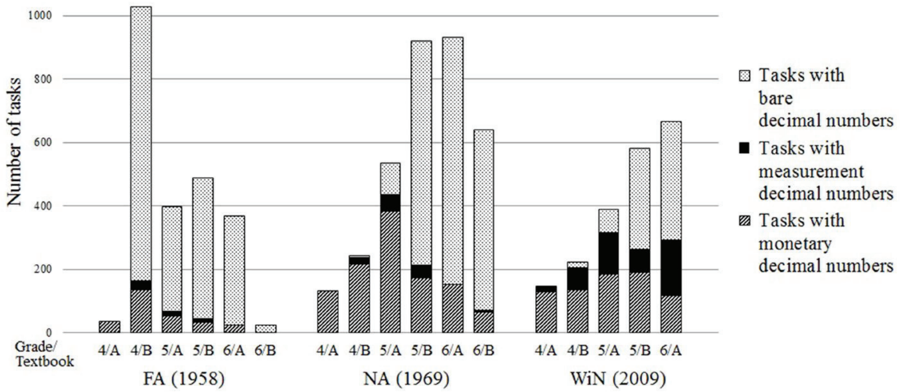

Figure 2. Absolute and relative frequencies of tasks with a particular type of decimal numbers included in the textbooks FA, NA, and WiN; based on the tasks for all students; covering Grade 4 to Grade 6; specified for the respective textbooks for these grades

Table 2

Absolute and relative frequencies of tasks with a particular type of decimal numbers included in the textbooks FA, NA, and WiN; based on the tasks for all students; covering Grade 4 to Grade 6

Type of decimal numbers	FA (1958)		NA (1969)		WiN (2009)	
Bare decimal numbers	2004	86 %	2164	64 %	783	39 %
Measurement decimal numbers	57	2 %	125	4 %	469	23 %
Monetary decimal numbers	272	12 %	1107	33 %	745	37 %
Total of decimal number tasks	2333	100 %	3396	100 %	1997	100%

All three textbook series offer tasks with bare decimal numbers for the first time in the second half of Grade 4. The way in which the three textbook series introduce tasks with bare decimal numbers differs. In FA and NA bare decimal numbers are introduced as another way for writing fractions. In this sense FA and NA clearly reflect the approach to decimal numbers that suits their juncture in time. In FA, bare fractions are used (see Figure 3). In NA, decimal numbers are introduced by making a reference to fractions expressed in monetary values (see Figure 4). In WiN bare decimal numbers are introduced through measurement numbers, without linking decimal numbers to common fractions. Instead, the context of hectometer signs along highways is used to support counting with tenths (see Figure 5).

Write down as decimal numbers:

$\frac{1}{10}=0{,}1$	$\frac{1}{100}=0{,}01$	$\frac{1}{1000}=0{,}001$	$\frac{4}{10}=..$	$\frac{4}{1000}=..$
$\frac{8}{10}=..$	$\frac{9}{100}=..$	$\frac{8}{1000}=..$	$\frac{7}{100}=..$	$\frac{8}{100}=..$
$\frac{7}{10}=..$	$\frac{4}{100}=..$	$\frac{2}{1000}=..$	$\frac{9}{1000}=..$	$\frac{8}{10}=..$
$\frac{9}{10}=..$	$\frac{8}{100}=..$	$\frac{8}{1000}=..$	$\frac{5}{1000}=..$	$\frac{8}{1000}=..$

Figure 3. Introduction of bare decimal numbers in FA (1958), Grade 4, Student book B

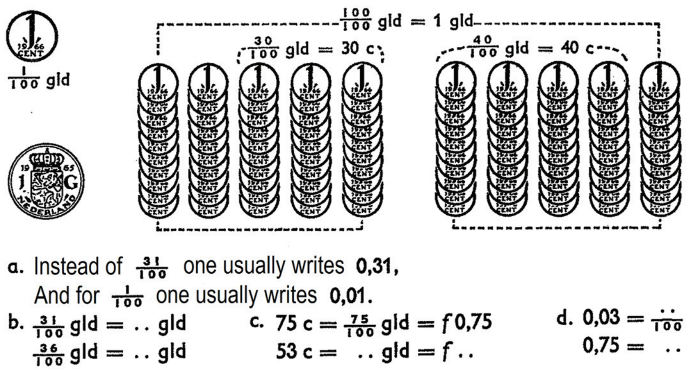

Figure 4. Introduction of bare decimal numbers in NA (1969), Grade 4, Student book B

4.3 Tasks with measurement decimal numbers

Although the proportion of tasks with measurement decimal numbers is considerable smaller in the two older textbooks than in the contemporary textbook, the ways in which measurement decimal numbers are used to teach decimal numbers show certain similarities. In all three textbook series measurement decimal numbers are used to understand the place value of the digits behind the decimal point. Both FA, the oldest textbook, and WiN, the contemporary textbook, provide sets of tasks in which the relationship between the place value in monetary decimal numbers and measurement decimal numbers and the place value in bare decimal numbers is made explicit (see Figures 6 and 7). In FA, tasks on place value with bare decimal numbers are provided before tasks with monetary decimal numbers and measurement decimal numbers. In contrast, in WiN this is done the other way around, which is consistent with the RME approach of using real-life situations as a source for learning.

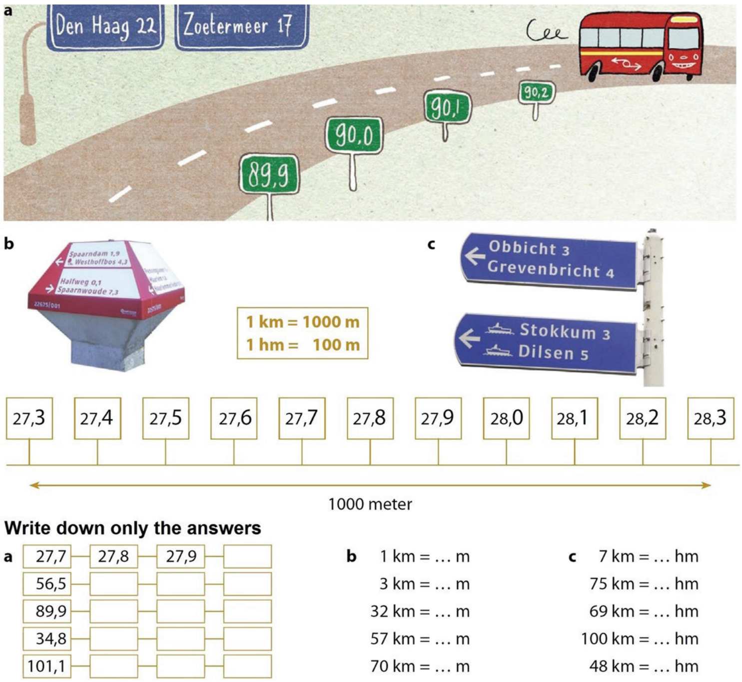

Figure 5. Introduction of bare decimal numbers in WiN (2009), Grade 4, Student book B

8,64 gld. The 8 = …	8,64 m. The 8 = …
8,64 l. The 8 = …	8,64 g. The 8 = …
35,89 m. This 35 + …	*ƒ*35,89 This 35 = …
35,89 kg. This 35 = …	35,89 This 35 = …

When a decimal number has the name ‘gld’ behind it or ‘*ƒ*’ in front of it, the digits before the comma represent guilders. To the right of the comma, you first see the tenths of guilders (ten cent pieces), and then the hundredths of guilders (one cent pieces).
When a decimal number has the name ‘kg’ behind it, the digits before the comma represent … After the comma, you first see tenths of kg (…), and then hundredths of kg (…) and so on.
Try to tell this for a decimal number that has the name ‘m’ behind it. Also for ‘km’. And what happens when no name is given?

Figure 6. A set of tasks on place value in FA (1958), Grade 4, Student book B

In both NA and WiN measurement decimal numbers are used for comparison and ordering of decimal numbers which have a different number of digits or have one or more zeroes behind the decimal point. In contrast, FA only presents such tasks with bare decimal numbers. Another similarity between NA and WiN is that they both use measurement decimal numbers for tasks in which students have to round off decimal numbers, and for tasks in which fractions have to be written as decimal numbers. FA does not use measurement decimal numbers for such tasks. However, the three textbook series were again found to be similar with respect to using measurement decimal numbers for mental arithmetic in contexts.

There are also differences between the two older textbooks and the contemporary one. WiN provides tasks with measurement decimal numbers that are not present in FA nor in NA, including tasks in which students have to continue a counting sequence with decimal numbers (see for an example the tasks in Figure 5 at the bottom on the left) and tasks addressing the continuous character of decimal numbers, such as tasks in which students have to determine a decimal number in between two other decimal numbers (e.g., “Tooske puts her foot exactly between 5.2 meters and 5.3 meters. Where is that?”).

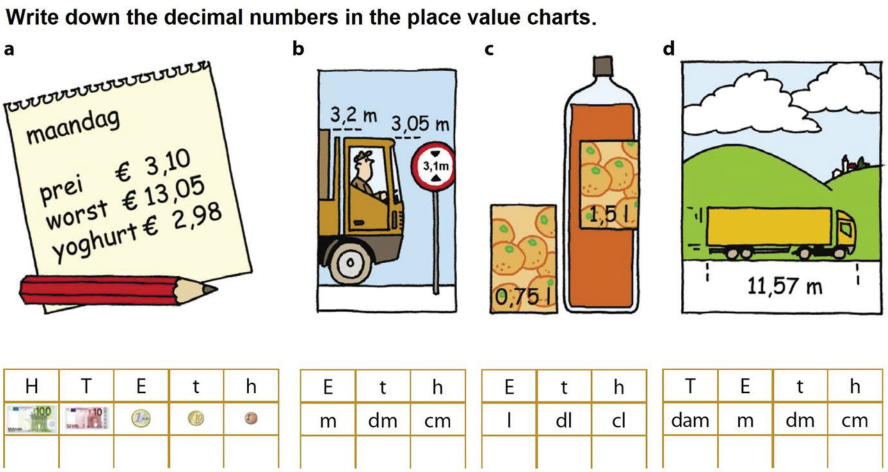

Figure 7. A set of tasks on place value in WiN (2009), Grade 5, Student book A

4.4 Types of calculations with decimal numbers

In all three textbook series, a substantial part of the tasks with decimal numbers is spent on calculations with decimal numbers (see Table 3). In FA, this concerns about 85% (1975 out of 2333 tasks), in NA this is about 80% (2706 out of 3396 tasks), and in WiN about 66% (1322 out of 1997 tasks). The overall trend based on these textbook series is that over the years the relative frequency of estimation tasks increased and that of written calculation tasks decreased. These changes are in line with the RME shift towards more emphasis on estimation, but the relative frequency of estimation tasks in NA makes it clear that the emphasis on estimation had increased already prior to the start of RME. Also the substantial attention to mental calculation has pre-RME roots. This attention was found in both FA and NA. Moreover, the relative frequency of mental calculation tasks is in FA about the same as in WiN. The only type of calculation with decimal numbers that was found in WiN and not in the two older textbook series was that of calculation with a calculator, which is obvious because calculators only became available in schools from the 1980s on. A type of task that is present in FA and NA but not in WiN, is that in which students themselves have to choose between using mental or written calculation. In WiN, a comparable type of task is present in which students have to choose between whether or not to use a calculator, but only very rarely (two tasks).

Table 3
Absolute and relative frequencies of tasks including a particular type of calculation with decimal numbers in the textbooks FA, NA, and WiN; based on the tasks for all students; covering Grade 4 to Grade 6

Type of calculation	FA (1958)		NA (1969)		WiN (2009)	
Mental calculation	474	24 %	346	13 %	321	24 %
Estimation	188	10 %	472	17 %	367	28 %
Written calculation	671	34 %	651	24 %	238	18 %
Calculation with a calculator	0	0 %	0	0 %	81	6 %
Free choice by student	21	1 %	67	2 %	2	0 %
Not indicated in textbook	621	31 %	1170	43 %	313	24 %
Total number of calculation tasks	1975	100 %	2706	100 %	1322	100 %

4.5 Estimation with decimal numbers

In all three textbook series students are expected to estimate the outcome of an operation before calculating the precise answer. This estimation is meant to support the correct placement of the decimal point in the answer. In FA this concerns 92% (173 out of 188) of all estimation tasks with decimals, in NA 66% (310 out of 472) and in WiN 31% (112 out of 367). The relative low percentage in WiN is caused by the fact that in WiN most estimation tasks are not linked to a precise calculation, but estimation is a goal in itself. In all three textbook series most estimation tasks are offered on multiplication.

Regarding the strategy used for estimation, all three textbook series offer tasks in which students have to do the calculation with rounded-off numbers. Another strategy is that of clamping, which means that the students have to determine between which numbers an answer must lie. In the two older textbooks, the students have to apply this strategy with multiplications and divisions with decimal numbers, where they have to indicate between which whole numbers the answer lies (e.g., "2.4 × 7.6 is bigger than … and smaller than …"). In the contemporary textbook WiN this always concerns division tasks where students are asked between which decimal numbers the answer lies (e.g., "Between what decimal numbers does the answer lie? 205 ÷ 15. Choose: between 13.0 and 13.5 or between 13.5 and 14.0").

WiN also offers estimation tasks in which students have to choose what answer can be correct (e.g., "30 × 0.15 = 450 or 4.5 or 0.45") and tasks that concern the recognition and correction of incorrectly placed decimal points (e.g., "3.58 × 52.3 = 1872.34; correct the mistake"). The latter type of estimation task was also found in FA, but not in NA. Although these types of estimation tasks do not involve precise calculation, they nevertheless support the correct placement of the decimal point. The contemporary textbook WiN also offers tasks in which a calculator is used for this purpose. This applies to 73% (59 out of 81) of the tasks on using a calculator. In these 59 tasks the calculator is used to check the estimation.

4.6 Use of contexts as a source for learning decimal numbers

In the contemporary textbook WiN contexts are clearly used as a source for learning decimal numbers. Figure 5, shown earlier, presents an example of this: the context of hectometer signs alongside highways is used to introduce decimal numbers. WiN also uses contexts for dealing with the specific difficulty of ordering and comparing decimal numbers that have a different number of digits behind the decimal point. For this, the context of measuring distances with different precision is used (see Figure 8).

The two older textbook series also offer context problems for dealing with specific difficulties such as comparing decimal numbers with a different number of digits or with zeroes behind the decimal point. However, here contexts are not used as a source for learning. Instead, they only serve for applying what students have been taught earlier. The contexts are offered after the students had to compare the decimal numbers in bare number tasks. Though, if one would consider the coins shown in NA when the bare decimal numbers are introduced (see Figure 4) to be a context, then this set of tasks in NA would be an exception to the previous conclusion that in the two older textbooks contexts are only offered after the students had to solve bare number tasks.

Which distances?

Which odometer gives the most information? What information is that?

Figure 8. A context on measuring distances with different precision in WiN (2009), Grade 5, Student book A

4.7 Use of the number line as a model for dealing with decimal numbers

In WiN, the number line as a model for dealing with decimal numbers is used to visualize the partitioning of the units into smaller and smaller units, to relate decimals numbers to fractions, and to position and order decimal numbers (see Figure 9). For this, 169 tasks (8% of all tasks) with bare decimal numbers and with measurement decimal numbers are accompanied by a number line. In addition, the teacher guide explains how a number line can be used in instruction, for example, to contribute to students' understanding of the infinitely ongoing refinement of decimal numbers (see Figure 10). Using the number line for visualizing an addition with decimal numbers was only found once in a WiN student book.

Choose from: 0,46; 0,6; 0,165.

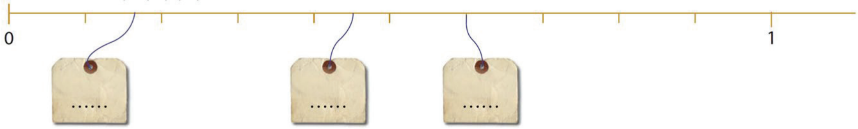

Figure 9. Positioning decimal numbers on a number line in WiN (2009), Grade 5, Student book B

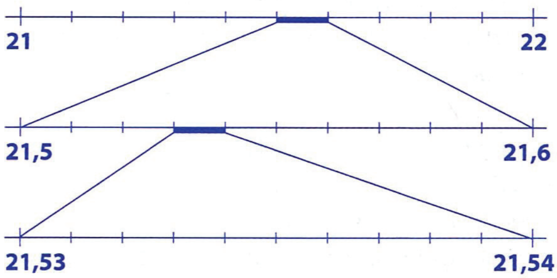

Figure 10. Ongoing refinement of decimal numbers visualized on a number line in WiN (2009), Grade 5, Teacher guide

In contrast with WiN, in FA the number line is not present. In NA, however, the number line was found accompanying 20 tasks (1% of the tasks), but not as a model to support the understanding of the continuous character of decimal numbers. In NA the number line is only used to visualize operations (see Figure 11), to identify what decimal numbers are located in a particular position, and to visualize that multiplying with a number smaller than 1 gives a smaller result (see Figure 12). In the teacher guide, NA also offers examples of how to use the number line during instruction.

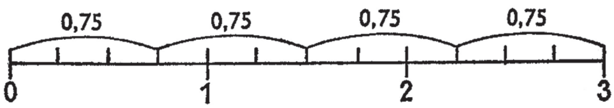

Figure 11. Visualization of a multiplication with decimal numbers (4 × 0.75) on a number line in NA (1969), Grade 5, Student book A

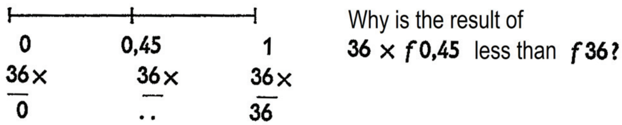

Figure 12. Visualization of multiplying with a number smaller than 1 on a number line in NA (1969), Grade 5, Student book A

4.8 Use of the place value chart

As earlier shown in Figure 7, the contemporary textbook WiN offers a place value chart to relate the place value in monetary decimal numbers and measurement decimal number numbers to the place value in bare decimal numbers. WiN also offers the place value chart to compare decimal numbers with a different number of digits behind the decimal point, but this only is done with measurement decimal numbers, not with bare decimal numbers. Another way in which a place value chart is used in WiN, is to separate the digits of bare decimal numbers that are given in words.

NA also offers a place value chart, which is mostly used to separate the digits of bare decimal numbers (see Figure 13). The oldest textbook series, FA, offers a place value chart only once. The set of tasks that includes this is similar to that of NA as shown in Figure 13, with the difference that the FA tasks deal with place values from millions to ten thousandths.

1. *Say out loud:* 40,004 87,95 326,08 3,507 0,75
4000,4 30,007 32,608 3507 0,009

2. *Write down the numbers of exercise 1 in the place value chart. Do not forget the decimal comma.*

D	H	T	E	t	h	d
		4	0,	0	0	4

Figure 13. Use of place value chart in NA (1969), Grade 5, Student book

4.9 Use of different calculation methods

The contemporary textbook WiN offers directions in the teacher guide on how to give room to students to use their own ways of solving tasks with decimal numbers. The teacher guide also provides examples of possible calculation methods that students may use. However, WiN does not pay attention to different calculation methods with decimal numbers in the student books with the exception of two tasks in which students have to choose whether or not to use a calculator (see Table 3).

The older textbook NA does offer different calculation methods with decimals in the student books. This involves 11 sets of tasks with in total 96 tasks (7% of all calculation tasks) in which students are offered examples of different ways of calculating with decimal numbers, followed by tasks where they are invited to choose the easiest way (see Figure 14). In addition, NA offers in 67 tasks (2% of all calculation tasks) a choice between mental calculation and written calculation (see Table 3).

1. ***Is this allowed?***

15 : 2,5 = 30 : 5 = .. of 3 × (5 : 2,5) = 3 × 2 = ..
of (10 : 2,5) + (5 : 2,5) = .. + ..
21 : 3,5 = 42 : .. = ..

2. ***Calculate in your head, and use the easiest way.***

35 : 2,5 =	**31,5 : 3,5 =**	**45 : 2,5 =**	**38,5 : 3,5 =**
20 : 2,5 =	**28 : 3,5 =**	**32,5 : 2,5 =**	**56 : 3,5 =**

Figure 14. Use of different calculation methods in NA (1969), Grade 6, Student book B

In the oldest textbook FA, both the student books and the teacher guide pay attention to different calculation methods. In the student books, in 19 tasks (1% of the calculation tasks) students are explicitly asked to solve the task in different ways. In the teacher guide, possible different ways of calculating tasks are given as well. FA also offers in 21 tasks (1% of the calculation tasks) a choice between mental calculation and written calculation (see Table 3).

4.10 Use of own productions

All three textbook series provide tasks in which students are asked to come up with own productions. In all cases this applies to decimal numbers in contexts, and with only one exception, these own productions are related to monetary decimal numbers. Examples from the two older textbooks are: “Come up with five subtraction tasks. The answer is always *f* 32.76” (NA, 1969) and “Come up with a problem that has a gain of *f* 4.75 as an answer” (FA, 1958). An example from the contemporary textbook WiN is a set of tasks where several products are on display in a shop window with their prices and the students have to decide what products they will buy for a certain amount of money. In this way they themselves can choose which decimal numbers they use to calculate with. The set of own production tasks that is not related to monetary decimal numbers is shown in Figure 15. Here, the students may locate measurement decimal number of their own choice, which gives them a lot of freedom in making the tasks more or less difficult.

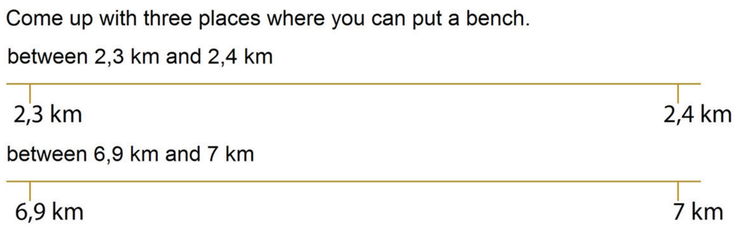

Figure 15. Use of own productions in WiN (2009), Grade 5, Student book B

5. Conclusions

Regarding our first research question, as could be expected, most of the RME characteristics included in our analysis framework were found to be present in the researched contemporary RME-oriented textbook WiN. The results concerning our second research question are more noteworthy: onsets of several RME characteristics were already found in the two older textbook series that date from before the RME reform started. Table 4 summarizes our findings.

Regarding the content perspective our analysis revealed that in the contemporary textbook WiN, a substantial part of all tasks with decimal numbers concerns measurement decimal numbers. This type of decimal numbers is offered to support students' understanding of place value, comparing and ordering decimal numbers, rounding off decimal numbers, relating decimal numbers to fractions, continuing a counting sequence of decimal numbers, and understanding the continuous character of decimal numbers. The two older textbooks, FA and NA, also offer tasks with measurement decimal numbers, but the proportion of these tasks is very low. In FA, the oldest textbook, measurement decimal numbers are only meant for understanding place value, whereas in NA more aspects of understanding are addressed.

With respect to the perspective of the performance expectations, our textbook analysis revealed that in the contemporary textbook WiN a substantial proportion of decimal numbers tasks is about either mental calculation or estimation. The attention to mental calculation was also found to be present in both older textbooks, FA and NA, whereas substantial attention for estimation was only found in NA. Yet the didactical use of estimation to support the correct placement of the decimal point in precise calculations, is present in all three textbook series.

Table 4

RME Characteristics in WiN and onsets of RME characteristics in FA and NA

Content	FA (1958)	NA (1969)	WiN (2009)
Tasks with measurement decimal numbers	Proportion of all tasks with decimal numbers:		
	2 %	4 %	23 %
	Used for supporting understanding of:		
	Place value	Place value Comparing and ordering decimal numbers Rounding off decimal numbers Relating decimal numbers to fractions	Place value Comparing and ordering decimal numbers Rounding off decimal numbers Relating decimal numbers to fractions Continuing a counting sequence of decimal numbers Continuous character of decimal numbers
Performance expectations	**FA (1958)**	**NA (1969)**	**WiN (2009)**
Tasks on:	Proportion of all calculation tasks with decimal numbers:		
mental calculation	24 %	13 %	24 %
estimation	10 %	17 %	28 %
Learning facilitators	**FA (1958)**	**NA (1969)**	**WiN (2009)**
Use of contexts	For application	For application	As a source for learning For application
Use of number line	No	Yes, to: Fill in decimal numbers Visualize operations with decimal numbers	Yes, to: Locate and determine decimal numbers Visualize operations with decimal numbers (only once) Visualize the continuous character of decimal numbers Relate decimal numbers to fractions
Use of place value chart	Only once	Yes	Yes
Use of different calculation methods	Yes, present in: Student books Teacher guides	Yes, present in: Student books	Yes, present in: Teacher guides
Use of own productions	Yes, for tasks with: Monetary decimal numbers	Yes, for tasks with: Monetary decimal numbers	Yes, for tasks with: Monetary decimal numbers Measurement decimal numbers (only once)

Our textbook analysis gave mixed results for the perspective of learning facilitators. The RME characteristic of using contexts as a source for learning is present in the contemporary textbook WiN, but absent in the two older textbooks FA and NA. The number line as a model is used in WiN and in NA, but not in the oldest textbook FA. In the contemporary textbook WiN the number line is applied for multiple purposes: locating and determining decimal numbers, visualizing operations and the continuous character of decimal numbers, and for relating decimal numbers to fractions. The place value chart is utilized in all three textbook series, although in the oldest textbook FA only once. The way the place value chart is used is the same in all three textbook series, namely for writing down decimal numbers and determining the place value of the digits. All three textbook series use different calculation methods as a learning facilitator. In the contemporary textbook WiN, directions are given for this only in the teacher guide, whereas in the older textbook NA different calculation methods are included in the student books. Remarkably, in the oldest textbook, FA, both the student books and the teacher guide support the use of different calculation methods. The use of own productions is addressed in all three textbook series.

By including only one contemporary textbook and two old ones in our textbook analysis, the results of our study of course do not give a complete picture of past and current approaches to teaching decimal numbers in Dutch primary school textbooks. Nevertheless, we feel our study contributes to a deeper knowledge of the RME characteristics included in the currently most used textbook WiN. Our study reveals what makes WiN belong to the RME approach and what RME characteristics are not so prominent in WiN. Different from what we expected, our study brought into view that students in WiN are hardly invited to choose by themselves what type of calculation to use to solve a problem with decimal numbers. Also, we did not anticipate that the use of different calculation methods with decimal numbers with the purpose of finding relations between these methods, is not supported in WiN, whereas this is done in FA and NA, the two older textbooks involved in our analysis.

The fact that the use of different calculation methods was found in FA and NA, also indicates another finding of our study, namely that particular RME characteristics were already present in the two textbook series dating from the time before RME came into being. This finding suggests that the RME reform was not a complete break with the past and that the roots of RME go back farther than its start in the late 1960s. In this way, our historical excursion has opened our eyes (again) to using old textbook series as a source to revisit and improve our contemporary textbooks series.

References

Bjarnadóttir, K., Christiansen, A., & Lepik, M. (2013). Arithmetic textbooks in Estonia, Iceland and Norway – similarities and differences during the nineteenth century. *Nordic Studies in Mathematics Education,18*(3), 27–58.

Bokhove, J., Van der Schoot, F., & Eggen, T. (1996). *Balans van het rekenonderwijs halverwege de basisschool 2. Periodieke Peiling van het Onderwijsniveau* [Balance of arithmetic education halfway primary school 2. Periodic Assessment of the Education Level]. Arnhem, the Netherlands: Cito.

De Jong, R. (1986). *Wiskobas in methoden* [Wiskobas in textbook series]. Utrecht, the Netherlands: Utrecht University.

Freudenthal, H. (1983). *Didactical phenomenology of mathematical structures*. Dordrecht, the Netherlands: Reidel Publishing Company.

Hop, M. (Ed.). (2012). *Balans van het reken-wiskundeonderwijs halverwege de basisschool 5. Periodieke Peiling van het Onderwijsniveau* [Balance of mathematics education halfway primary school 5. Periodic Assessment of the Education Level]. Arnhem, the Netherlands: Cito.

Howson, G. (1982). *A history of mathematics education in England.* Cambridge/London/New York/New Rochelle/Melbourne: Cambridge University Press.

Janssen, J., Van der Schoot, F., Hemker, B., & Verhelst, N. (1999). *Balans van het reken-wiskundeonderwijs aan het einde van de basisschool 3. Periodieke Peiling van het Onderwijsniveau* [Balance of mathematics education at the end of primary school 3. Periodic Assessment of the Education Level]. Arnhem, the Netherlands: Cito.

Janssen, J., Van der Schoot, F., & Hemker, B. (2005). *Balans van het reken-wiskundeonderwijs aan het einde van de basisschool 4. Periodieke Peiling van het Onderwijsniveau* [Balance of mathematics education at the end of primary school 4. Periodic Assessment of the Education Level]. Arnhem, the Netherlands: Cito.

Meelissen, M., Netten, A., Drent, M., Punter, R., Droop, M., & Verhoeven, L. (2012). *PIRLS en TIMSS 2011. Trends in leerprestaties in Lezen, Rekenen en Natuuronderwijs* [PIRLS and TIMSS 2011. Trends in achievement in Reading, Mathematics and Science]. Enschede/Nijmegen, the Netherlands: Twente University/Radboud University.

Streefland, L. (1974). Introductie decimale getallen [Introduction decimal numbers]. *Wiskobas bulletin, 3*(2), 166–190.

Streefland, L. (1990). Free productions in teaching and learning mathematics. In K. Gravemeijer, M. van den Heuvel-Panhuizen, & L. Streefland, *Contexts, free productions, test, and geometry in Realistic Mathematics Education* (pp. 33–52). Utrecht, the Netherlands: OW&OC, Utrecht University.

Streefland, L. (1991). *Fractions in Realistic Mathematics Education. A paradigm of developmental research*. Dordrecht/Boston/London: Kluwer Academic Publishers.

Treffers, A. (1978). *Wiskobas doelgericht*. [Wiskobas goal-directed]. Utrecht, the Netherlands: IOWO.

Treffers, A. (1987). *Three dimensions. A model of goal and theory description in mathematics instruction – the Wiskobas project*. Dordrecht, the Netherlands: Reidel Publishing Company.

Treffers, A. (1991). Didactical background of a mathematics program for primary education. In L. Streefland (Ed.), *Realistic Mathematics Education in primary school* (pp. 21–56). Utrecht, the Netherlands: CD-β Press, Utrecht University.

Treffers, A., & De Moor, E. (1984). *10 voor de basisvorming rekenen/wiskunde* [10 for elementary education in mathematics]. Utrecht, the Netherlands: Utrecht University.

Treffers, A., Streefland, L., & De Moor, E. (1996). *Proeve van een nationaal programma voor het reken-wiskundeonderwijs op de basisschool 3B: Kommagetallen* [Sample of a national program for mathematics education in primary school 3B: Decimal numbers]. Tilburg, the Netherlands: Zwijsen.

Valverde, G., Bianchi, L., Wolfe, R., Schmidt, W., & Houang, R. (2002). *According to the book. Using TIMSS to investigate the translation of policy into practice through the world of textbooks*. Dordrecht/Boston/London: Kluwer Academic Publishers.

Van den Heuvel-Panhuizen, M. (2001). Realistic Mathematics Education in the Netherlands. In J. Anghileri (Ed.), *Principles and Practices in Arithmetic Teaching: Innovative Approaches for the Primary Classroom* (pp. 49–63). Buckingham/Philadelphia: Open University Press.

Van den Heuvel-Panhuizen, M. (2010). Reform under attack – Forty years of working on better mathematics education thrown on the scrapheap? No way! In L. Sparrow, B. Kissane, & C. Hurst (Eds.), *Shaping the future of mathematics education: Proceedings of the 33rd annual conference of the Mathematics Education Research Group of Australasia* (pp. 1–25). Fremantle: MERGA.

Van den Heuvel-Panhuizen, M. (2014). Didactical phenomenology. In S. Lerman (Ed.), *Encyclopedia of Mathematics Education* (pp. 174–176). Dordrecht/Heidelberg/New York/London: Springer.

Van den Heuvel-Panhuizen, M., & Drijvers, P. (2014). Realistic Mathematics Education. In S. Lerman (Ed.), *Encyclopedia of Mathematics Education* (pp. 521–525). Dordrecht/Heidelberg/New York/London: Springer.

Van Zanten, M., & Van den Heuvel-Panhuizen, M. (2014). Freedom of design: The multiple faces of subtraction in Dutch primary school textbooks. In Y. Li & G. Lappan (Eds.), *Mathematics curriculum in school education* (pp. 231–259). Heidelberg/Dordrecht/London/NewYork: Springer.

Walmsley, A. L. E. (2007). *A history of mathematics education during the twentieth century.* Lanham/Boulder/New York/Toronto: University Press of America.

Wiskobas-team (1979). *Rapportboekje 3. Overzicht rekenmetoden anno 1979* [Rapport booklet 3. Overview arithmetic textbooks anno 1979]. Utrecht, the Netherlands: IOWO.

Textbook series

Bruinsma, B., Van den Heuvel, H., Van Lierop, A., Van Wijk, G. & Van Zuilekom, D. (1969). *Nieuw Rekenen* [New Arithmetic]. Baarn, the Netherlands: Bosch & Keuning.

Huitema, S., Erich, L., Van Hijum, R., Nilissen, C., Osinga, H., Veltman, H., & Van de Wetering, M. (2009). *De Wereld in Getallen* [The World in Numbers], 4th edition. ‘s Hertogenbosch, the Netherlands: Malmberg.

Reijnders, J. & Snijders, J. (1958). *Functioneel Rekenen* [Functional Arithmetic]. Amsterdam/Antwerpen, the Netherlands/Belgium: Versluys.

Authors’ contributions

This paper was a collaborative work of the two authors. Both authors participated in the selection of the textbooks and the development of the analysis framework. MZ carried out the coding and the analysis of the data. The findings were frequently discussed with MH. MZ prepared the first draft of the manuscript. Both authors participated in revising the manuscript. Both authors read and approved the final manuscript.

Chapter 5

Mathematics curriculum reform and its implementation in textbooks: Early addition and subtraction in Realistic Mathematics Education

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Mathematics curriculum reform and its implementation in textbooks: Early addition and subtraction in Realistic Mathematics Education

Abstract

Since the late 1960s, a reform in mathematics education, which is currently known under the name *Realistic Mathematics Education* (RME), has been taking place in the Netherlands. Characteristic for this approach to mathematics education is that mathematics is not seen as ready-made knowledge but as an activity of the learner. Although much has been written about the big ideas and intentions of RME, and multiple RME-oriented textbooks have been published, up to now the development of this approach to mathematics education has not been thoroughly investigated. In the research reported in this article, we traced how RME has evolved over the years. The focus in our study was on early addition and subtraction in primary school. For this, we studied RME core curriculum documents and analyzed RME-oriented textbooks that have been published between the onset of RME and the present. We found that the big ideas and teaching principles of RME were clearly reflected in the learning facilitators for learning early addition and subtraction and were steadily present in curriculum documents over the years, although some were made concrete in further detail. Furthermore, we found all RME learning facilitators also to be present in all RME-oriented textbooks, though in some cases in other ways than originally intended. Our research shows the complexity of a curriculum reform process and its implementation in textbooks.

Keywords: mathematics curriculum reform; Realistic Mathematics Education; primary school; textbooks; early addition and subtraction

1. Introduction

Making decisions about what and how to teach forms the core of teaching mathematics. At a general level, the leading resource for making these decisions is the intended curriculum, which includes the aims, goals and intentions of education (e.g., Valverde, Bianchi, Wolfe, Schmidt, & Houang, 2002; Van den Akker, 2003). At school level, guidelines for making decisions about these activities and eliciting the required learning processes can in particular be found in textbooks. These often serve as the main resource for the daily lessons in classrooms (e.g., Stein, Remillard, & Smith, 2007). Since textbooks form an intermediate layer between the intended curriculum and the implemented curriculum, they are considered the potentially implemented curriculum (Valverde et al., 2002). Ideally, the intended and the potentially implemented curriculum are closely aligned with each other, but this is not self-evident. Several studies have shown examples of lacking coherence between these curriculum levels in mathematics education (Dingman, 2007; Johansson, 2003; Van Zanten & Van den Heuvel-Panhuizen, 2018a). There are numerous factors that can cause differences between the intended and the potentially implemented curriculum, such as interpretations of goals, aims and intentions by textbook authors, and commercial considerations from publishers.

Also, the intended curriculum itself can be subject to change. Decisions regarding what is intended are largely based on what is believed to be good education and these beliefs are not a fixed given. Ideas on what mathematics should be taught and how this could be taught best, can differ significantly between countries (see, e.g., Leung, Graf, & Lopez-Real, 2006; Schmidt, McKnight, Valverde, Houang, & Wiley, 1997). Also, within countries, seen over time, new views on mathematics education can emerge and approaches to teaching mathematics can change (e.g., Furinghetti & Karp, 2018; Karp & Schubring, 2014; Stanic & Kilpatrick, 2003).

The Netherlands too, has a history of mathematics education reform. Starting in the late 1960s, under guidance of the mathematician Hans Freudenthal, an approach to mathematics education has been developed, which over time became known as *Realistic Mathematics Education* (RME) (Van den Heuvel-Panhuizen & Drijvers, 2014). Although much has been written about the intentions and ideas of RME, and publishers have released multiple RME-oriented textbooks, up to now the development of this approach to mathematics education has not been thoroughly researched. In fact, little is known about how particular RME ideas on the what and how of teaching mathematics have evolved in the past half century. The same can be said about how the RME reform can be found in textbooks. After the study *Wiskobas in textbooks* in the 1980s (De Jong, 1986), no other systematic research has been carried out to investigate this.

The purpose of the current research was to contribute to closing this knowledge gap, to trace how the RME reform evolved, and to unravel the relationship between the intentions of the RME reform and its implementation in textbooks. To this end, we studied RME designs and their underlying ideas from the onset of RME up to the current day, and we examined how these designs and ideas ended up in textbooks over the years. To clearly bring into view the in-depth characteristics of RME as they are laid down in instructional materials, we focused on one key primary school mathematics domain: early addition and subtraction.

2. Background of the study and research questions

2.1 The origin of RME

In the late 1950s, a need was felt in many western countries for "radical changes and improvements in the teaching of mathematics" (OEEC, 1961, p. 11). There were many reasons for this, varying from the increasing importance of mathematics and its applications for society to new insights on learning and teaching mathematics (e.g., Bjarnadóttir, 2014, 2018). The case and proposals for reform were addressed at the conference held at the *Centre Culturel de Royaumont* in Asnière-sur-Oise (France) in 1959 (OEEC, 1961). This conference was the commencement of the world-wide impact of the New Math reform movement (e.g., Bjarnadóttir, 2014; Kilpatrick, 2012). In the Netherlands however, under guidance of the "leading dissident" Freudenthal (Robitaille & Travers 2003, p. 1495), New Math would not gain a foothold. Instead, another direction was chosen, which eventually led to RME (Treffers, 1993).

As a consequence of the Royaumont conference, in 1961 the Dutch government installed the *Commissie Modernisering Leerplan Wiskunde* [Commission Modernization Mathematics Curriculum] (CMLW), of which Freudenthal was a member and of which, in 1969, he became the chairman. The CMLW was commissioned to investigate what reform of content and didactics was needed in the Netherlands (CMLW, 1961). Originally, the focus was only on secondary education, but in 1968, with the establishment of the Wiskobas project as part of the CMLW, primary education also came into the picture. Wiskobas is an acronym for *Wiskunde op de Basisschool*, which means 'mathematics in primary school'. The inception of the Wiskobas project marks the start of the development of RME. From 1971 on, the work of CMLW and Wiskobas continued in the newly-founded *Instituut voor Ontwikkeling van het Wiskundeonderwijs* [Institute for Development of Mathematics Education], with Freudenthal as its first director. In 1991, the successor of this institute was renamed the Freudenthal Institute.

2.2 The big ideas and principles of RME

Having a project named *Wiskobas* indicated a break with the past. Traditionally, in Dutch primary schools, mathematics as a subject was called *rekenen*, i.e., arithmetic. Wiskobas wanted to bring a more mathematical perspective into primary education. Around 1970, the prevailing approach to arithmetic education in the Netherlands was rather mechanistic (Treffers, 1978, 1987). It had a characteristic focus on teaching fixed procedures in a step-by-step manner, with the teacher demonstrating how to proceed in each step. Real-world problems were only used for the application of previously learned calculation procedures, and little or no attention was paid to developing insight into the underlying mathematics of these procedures (e.g., De Jong, 1986; Van den Heuvel-Panhuizen & Drijvers, 2020). New Math was not considered a suitable alternative. In the words of Freudenthal (1981, p. 141), New Math took "the wrong perspective [...] of replacing the learner's insight by the adult mathematician's insight." The mathematics that Wiskobas had in mind was of another nature, namely mathematics as an activity of the learner. Freudenthal (1968, 1973) considered mathematics *as an activity* rather than as ready-made knowledge. The latter refers to the already created system of mathematics of which Freudenthal (1973) said that students, when it is offered to them, can only reproduce it. For him, in the first place, mathematics means mathematizing reality. Therefore, in his view, mathematics education should not be about transmitting ready-made mathematics to students, but about the activity of mathematizing (Freudenthal, 1981).

This big idea of mathematics as the *human activity of mathematizing* became an important notion underlying the work of Wiskobas and the further development of RME. Treffers, one of the leading persons in the development of RME, later on made the distinction between *horizontal* and *vertical mathematizing*. Horizontal mathematizing refers to transforming real-world problems into mathematical terms, whereas vertical mathematizing refers to using mathematical means to solve the problems, generalizing the solution process and achieving a higher level of formalization (Treffers, 1987).

In RME, reality is seen as a starting point for learning processes. This important role of reality is expressed in the term *realistic*. However, in RME, realistic also has a broader connotation. As well as using reality as a source for mathematics education, it refers to the Dutch verb *zich realiseren*, which means to realize and to imagine what is going on (Van den Brink, 1973, 1989; Wijdeveld, 1980; Van den Heuvel-Panhuizen, 1996). In RME, students are offered problems which they can imagine, including problems from the fantasy world of fairy tales and the formal world of mathematics. Providing students with such problems implies a proactive role of the teacher, which is expressed in Freudenthal's other big idea (1991), *guided reinvention*. According to Freudenthal the "re" in reinvention refers to the steps in learning processes, while the adjective "guided" points to the instructional environment of the learning processes. Guiding reinvention implies a balance between, on the one hand, letting students think and invent things, and on the other hand providing guidance that provokes reflective thinking. Treffers (1987) underlined this view by emphasizing that students' *own constructions* and *own productions* have a decisive influence on the learning process. According to him, stimulating students to come up with self-made solutions when solving problems or making up problems themselves is a crucial element of RME and "the basis of everything" (Treffers in Van Zanten, 2019, p. 72).

These big ideas of RME ("the human activity of mathematizing" in connection with "horizontal and vertical mathematization" and "guided reinvention" in connection with "own constructions and productions") that stem from the work of Wiskobas have been repeatedly formulated and specified in different ways. Treffers (1978, 1987) characterized the Wiskobas approach in *eight starting points*, which included three didactical principles (activity, differentiation, and vertical planning) and five mathematical principles (structure, language, applicability, dynamics, and a specific approach), which he summarized in his 1987 publication as the *four starting points of realistically oriented mathematics education*: paying much attention to reinvention, various levels of concreteness and abstraction, historical-genetic (vertical) planning, and reality-bound, meaningful (mathematically rich) instruction. Additionally, Treffers (1987) defined *five instruction principles that guide progressive mathematizing*: phenomenological exploration, bridging by vertical instruments, self-reliance: students' own constructions and productions, interactivity, and intertwining. In a later publication (Treffers, De Moor, & Feijs, 1989), these instruction principles were reformulated as the *five fundamental learning principles of the reconstruction didactics*: constructing and concretizing, levels and models, reflection and own production, social context and interaction, structuring and intertwining.

The beginning years of RME were clearly ones of continuously reconceptualizing what the reformed mathematics education stood for. Several starting points and principles had much in common, but were reworded from time to time. Van den Heuvel-Panhuizen (2001a) summarized these RME characteristics and identified the following six principles: the activity, the reality, the level, the intertwinement, the interactivity, and the guidance principle. We follow these principles here. The *activity principle* refers to the concept of mathematics as a human activity. Students, instead of being the receivers of ready-made mathematics, are treated as active participants in the learning process in which they are stimulated to develop mathematical tools and insights. The *reality principle* emphasizes that learning mathematics is considered as originating in mathematizing reality. This means starting from meaningful, mathematically rich context situations, which offers opportunities to attach meaning to mathematical concepts. The *level principle* underlines that learning mathematics implies that students pass through various levels of understanding: from starting at the level of using informal context-related solutions, via making various shortcuts and schematizations, to being able to use formal procedures. Models serve as an important device for bridging between concrete situations and formal mathematics. The *intertwinement principle* means that related learning strands are taught in integration. The *interactivity principle* signifies that learning mathematics is not only an individual activity but also a social one. Through whole-class discussions and group work, students can learn from each other's strategies and insights. The *guidance principle* signifies the importance of offering students a guided opportunity to re-invent mathematics. In RME, the teachers, supported by long-term teaching-learning trajectories and textbooks, have a crucial role in steering the learning process by providing the students with a learning environment which enables that they can develop mathematical understanding.

2.3 RME and textbooks

Over the years, RME has made a considerable impact on the textbook market in the Netherlands. Periodic studies by Cito, the Dutch institute for educational measurement, indicate an increasing market share of RME-oriented textbooks from around 15% in the mid-1980s to 75% in the 1990s (Janssen, Van der Schoot, Hemker, & Verhelst, 1999) and further to 100% around 2003 (Janssen, Van der Schoot, & Hemker, 2005).

After 2007, due to a debate criticizing the RME approach in favor of a return to the traditional mechanistic approach (KNAW, 2009; Van den Heuvel-Panhuizen, 2010), the textbook market became more diverse again (see for details Van Zanten & Van den Heuvel-Panhuizen, 2014). Also, new editions of textbooks that were originally presented by their publishers as RME-oriented, were no longer labeled as such. Of course, the approach to mathematics education that is attributed to a textbook does not tell the whole story. Textbooks that are no longer presented as RME-oriented may still include RME characteristics, just as textbooks positioned as RME-oriented do not necessarily include all RME features.

The latter was demonstrated by De Jong (1986) in his study about Wiskobas in textbooks. He analyzed textbooks in use from 1970 to 1985 and found that not all textbooks that were presented by their authors or publishers as based on Wiskobas could actually be classified as such. His study showed a steadily growing market share of textbooks that included RME characteristics, but also revealed that these characteristics rather varied in their degree of alignment with the intentions as articulated by Wiskobas at that time. Since the De Jong's study, no more research has been carried out to investigate how RME-oriented textbooks are in line with the original ideas of RME.

2.4 Research questions

Besides what is known from De Jong's study, no other knowledge is available regarding the relationship between the intentions of RME and its implementation in textbooks. The aim of the current research was to get a more complete and up-to-date view on this relationship, taking into account both the original ideas of Wiskobas and the evolvement of RME ideas since that time period. This implied that we had to carry out two studies: a study into the intentions in Wiskobas and RME and their evolvement over time, and a study into how, from the Wiskobas years to the present, these intentions ended up in RME-oriented textbooks.

To investigate this on a detailed level, we focused on one domain of primary school mathematics: early addition and subtraction. We chose this domain for two reasons. First, it is a core domain of mathematics that serves as a basis for all other primary school mathematical domains. Second, in the Wiskobas time, this domain was worked on from the very beginning, which gave us the opportunity to investigate how the approach to this domain evolved within RME.

Our research questions for the respective studies were:

1. How did the RME approach on early addition and subtraction evolve?
2. How was this approach implemented in consecutive generations of RME-oriented textbooks?

We specified “early addition and subtraction” as: addition and subtraction in the lower grades of primary school, from the moment that these operations are introduced, and before written algorithmic calculation is introduced.

3. Study 1: The RME way of teaching early addition and subtraction as intended

3.1 Method

3.1.1 Selection of RME documents

To answer our first research question, we analyzed the core RME curriculum documents that address early addition and subtraction. These documents are:

- the first *Wiskobas* overview of primary school mathematics education (De Jong, Treffers, & Wijdeveld, 1975; *Part 1: Scenes of mathematics education in the lower grades*, pp. 13–120);
- the publication *Proeve van een nationaal programma voor het reken-wiskundeonderwijs op de basisschool* [Design of a national program for mathematics education in primary school] (hereafter called *Proeve*) (Treffers & De Moor, 1990; *Part 1: Basic operations* pp. 9–173);
- the *TAL* [Teaching and learning trajectory] document meant for teaching mathematics in the lower grades of primary school (Treffers, Van den Heuvel-Panhuizen, & Buys, 1999) which has also been published in English (Van den Heuvel-Panhuizen, 2001b; *Calculation up to twenty* and *Calculation up to one hundred*, pp. 43–74).

3.1.2 Analysis procedure

In our analysis, we first detected all suggestions that these documents offer for facilitating the learning of early addition and subtraction. We classified these suggestions into categories of learning facilitators based upon the big ideas and principles of RME. These categories are (1) the use of reality, (2) the use of models, (3) the use of students' own input, and (4) the use of non-routine problems. Finally, we compared and related the suggested learning facilitators and their intended use to each other and created a chronological overview.

3.2 Results

3.2.1 Use of reality

In the *Wiskobas* document, addition and subtraction are introduced by using the bus context as the reality-related starting point. This means that the students play a bus game. One student acts as the bus driver and the other students are the passengers. At bus stops situated throughout the classroom students get on and off the bus, which gives them a first meaning for addition and subtraction. The students keep track of how many passengers get on and off the bus and how many are on the bus after a bus stop. From the beginning on, one situation is used for carrying out multiple calculations. Also, addition and subtraction are introduced simultaneously. This arises naturally from the fact that people can get both into and out of a bus. Figuring out what happens gives students opportunities for mathematical reasoning:

> What may have happened, when the bus drives away from the bus stop with three passengers more than it arrived with? Did three passengers get on or did perhaps five people get on and two off the bus?[1] (De Jong et al., 1975, p. 36).

After the physical experiences of the bus game, similar situations are also presented on worksheets (Figure 1).

[1] All translations by the authors.

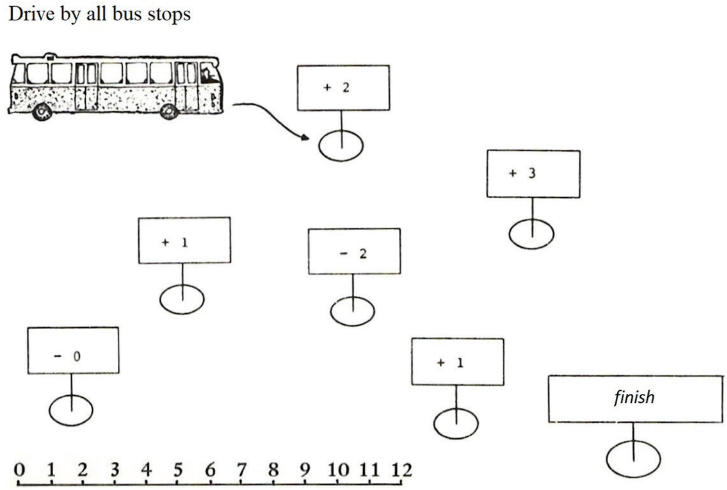

Figure 1. The bus context in *Wiskobas*. De Jong et al. (1975, p. 41)

Directly at the start, arrow language is used as a symbolic representation that refers to reality. It is used to let students describe that "something is happening" (De Jong et al., 1975, p. 39). They have to put into words what is happening in a situation and write this down in arrow language. They do so in all kinds of situations, which provides them with multiple meanings of addition and subtraction. Moreover, interpreting particular situations in different ways underlines the relationship between addition and subtraction (Figure 2).

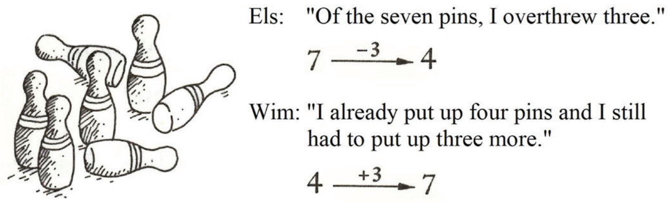

Figure 2. Arrow language in Wiskobas (De Jong et al., 1975, p. 40)

The *Proeve* document does not address the initial introduction of addition and subtraction and does not mention the bus context. However, *Proeve* makes it clear that to develop understanding of addition and subtraction, students have to be provided with varied context situations. Arrow language is used to express what is happening in these context situations, but *Proeve* does not suggest letting students write down arrow language, nor to use it to relate addition and subtraction.

In *TAL*, various context situations are used for giving meaning to addition and subtraction, including the bus context. Addition and subtraction are not jointly introduced in one situation. Carrying out multiple calculations in one particular situation is also not described. Arrow language is mentioned for expressing what is happening in a situation.

3.2.2 Use of models

The core documents describe several models. The most prominently present are the number line, the one hundred square and the arithmetic rack.

The number line that is used in *Wiskobas* is segmented. This number line is introduced for calculations beyond ten, when students experience that counting on their fingers is no longer sufficient. The segmented number line is used to support calculating by counting and moving forward (for addition) and backward (for subtraction). It is also used for positioning numbers, which means identifying where particular numbers are located (Figure 3). Furthermore, in combination with bars, it is used for laying the relationship between addition and subtraction (Figure 4).

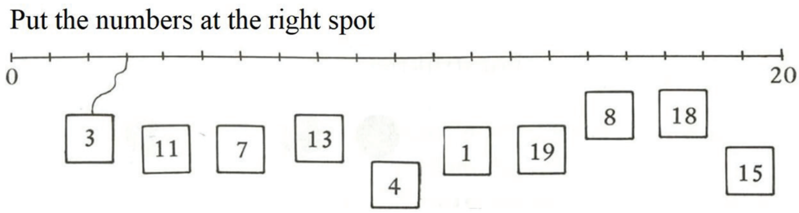

Figure 3. Positioning numbers on a segmented number line in *Wiskobas* (De Jong et al., 1975, p. 40)

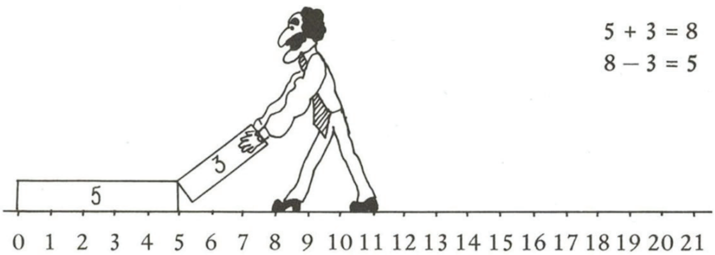

Figure 4. Relating addition and subtraction on a segmented number line in *Wiskobas* (De Jong et al., 1975, p. 88)

In *Proeve* and *TAL*, the number line has a different appearance: it is now an empty number line. It is emphasized that a number line with no or only a few marked points stimulates the use of number relations (e.g., 19 is near 20 and 10 is in the middle between 0 and 20). To introduce this number line, both documents suggest a string of beads. The alternating pattern of colors (Figure 5, above) provides students with a ten-structure, which helps them to apply shortened counting with tens. A pin placed on the bead string indicates the number of beads before that particular point. This representation of numbers is also used for the empty number line (Figure 5, below).

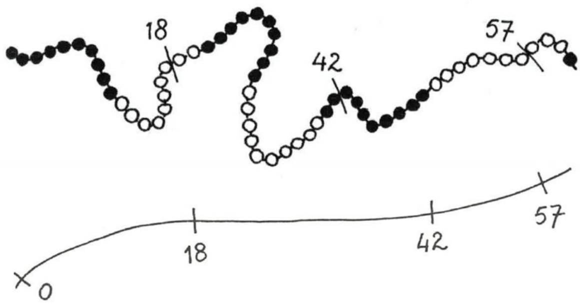

Figure 5. The bead string and the empty number line in Proeve (Treffers & De Moor, 1990, p. 51-52)

In *Proeve* and *TAL,* the empty number line is also applied for acting out variously-structured calculation procedures to solve addition and subtraction problems. For example, the subtraction 65 – 38 can be calculated by stringing, first by making jumps of ten, (65 – 10 – 10 – 10 – 8, Figure 6a) and later by making fewer jumps (e.g., 65 – 30 – 5 – 3, Figure 6b). Also, varying strategies are possible, such as a shortened approach via a nearby round number (65 – 40 + 2, Figure 6c) or an adding on strategy (38 + … = 65, Figure 6d). This latter strategy is the only case in *Proeve* and *TAL* in which the relationship between addition and subtraction is brought to the fore.

Proeve also describes the splitting strategy in which the tens and ones are processed separately (e.g., 27 + 38 calculating as 20 + 30 and 7 + 8). For this strategy, not the number line, but 10-blocks and 1-blocks are suggested. *TAL* turns away from the decimal splitting strategy and suggests not to emphasize it because it can easily lead to mistakes in the case of subtraction.

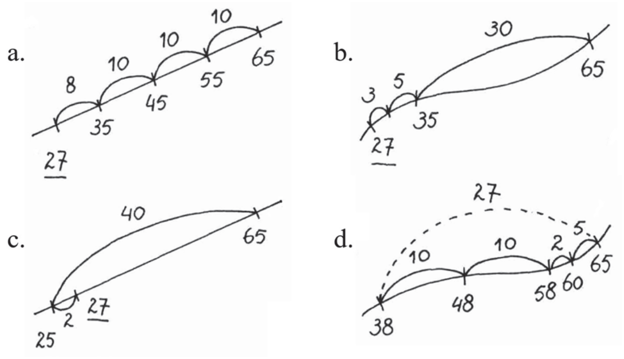

Figure 6. Four ways of calculating 65 – 38 on the empty number line in *Proeve* (Treffers & De Moor, 1990, p. 54-55)

The second model is the one hundred square. This is a ten-by-ten frame in which the numbers 1 to 100 are placed. In *Wiskobas*, this model is a kind of extension of the segmented number line up to 100 (Figure 7). The one hundred square can be used completely or partly filled with numbers, and as both the whole square and fragments. It is employed for all kinds of exploratory activities (Figure 8) and for practicing addition and subtraction by moving on the square. For example, three steps to the left mean subtracting three and two steps downward represent adding twenty.

In *Proeve,* it is said that the one hundred square is hard to understand and to use for many children. One reason that is mentioned is that jumps of one (to the right or left) and jumps of ten (downward or upward) on the one hundred square look as if they have the same value (each is a move of one cell), while in fact they have not.

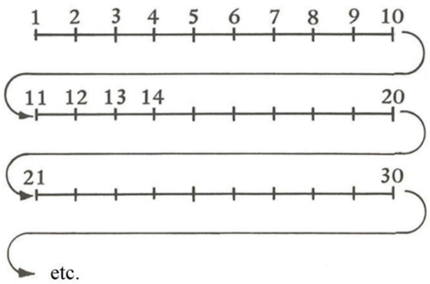

Figure 7. The one hundred square as an extension of the number line in Wiskobas (De Jong et al., p. 60)

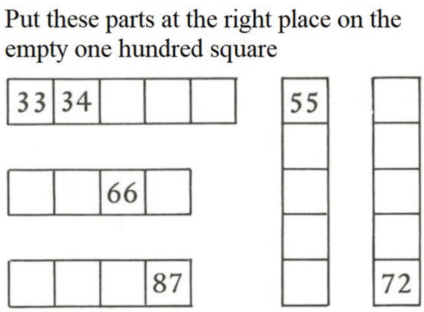

Figure 8. An exploratory activity with the one hundred square in Wiskobas (De Jong et al., p. 61)

In *TAL*, the one hundred square is largely absent. It is mentioned only once as a model to represent numbers, and it is argued to be not very suitable to use for addition and subtraction.

The third model is the arithmetic rack for addition and subtraction up to twenty. It has two lines of ten beads, each divided in two sets of five beads. This model is mentioned for the first time in *Proeve* (Figure 9). It is meant for structuring numbers using the five-structure, ten-structure and the doubles. These number representations can help students to shift from calculation by counting to calculation by structuring. *Proeve* describes the example of $6 + 7$, which for example can be put on the arithmetic rack as shown in Figure 9. Students can derive the answer from seeing the five and five, which makes ten, and adding one and two, making thirteen ($5 + 5 = 10$ and then $10 + 1 + 2 = 13$). Another way is seeing the double six and adding one ($6 + 6 = 12$ and then $12 + 1 = 13$). Furthermore, they can calculate via ten ($6 + 4 = 10$ and then $10 + 3 = 13$). The arithmetic rack is considered an important model because "it allows flexible use of multiple solution procedures, while at the same time offering fixed number images that facilitate memorizing calculations up to 20" (Treffers & De Moor, 1990, p. 48).

In *TAL*, the arithmetic rack is used for structuring numbers and for addition and subtraction up to twenty in the same way as in *Proeve*.

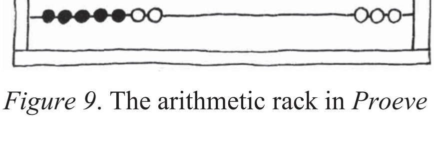

Figure 9. The arithmetic rack in *Proeve* (Treffers & De Moor, 1990, p. 45)

3.2.3 Use of students' own input

In *Wiskobas*, the use of students' input in the teaching-learning process comes to the fore in two ways. The first one is the already mentioned emphasis on letting students explain in their own words what is happening in context situations and writing this down with arrow language. The second way is to ask students to produce their own problems, such as thinking of a route for the bus that results in a specific number of passengers in the end.

Proeve also emphasizes having students putting into words what is happening in context situations. In addition, the students are asked to explain their acting in words while carrying out calculations. Moreover, it is emphasized to give students ample opportunities to come up with their own calculation procedures, and to make use of these own constructions as a basis for shortened and structured solution procedures. This can be done, for example, by using the arithmetic rack and the empty number line that allow various solution strategies, as described in the section about models. Inviting students to come up with own productions (i.e., thinking of problems with a particular answer) is also present in *Proeve*.

TAL emphasizes that letting students use their own wordings is important in all phases of the teaching-learning process, including letting students explain and compare their own constructed and used solution procedures. Furthermore, using students' own productions is stretched into a form of practicing addition and subtraction, for which the term "productive practice" (Van den Heuvel-Panhuizen, 2001b, p. 52) is used.

3.2.4 Use of non-routine problems

Wiskobas suggests asking students questions that require more than only straightforward calculation but instead evoke reflection and reasoning. The earlier mentioned question about what may have happened when the bus drives away from the bus stop with three passengers more than it arrived with is an example of this. Another example concerns the context of a double-decker bus, in which some passengers take a seat on the lower level of the bus and others on the top level. This context is used for splitting numbers (a prerequisite skill for addition and subtraction) and it is suggested to ask students to figure out whether they have found all possible splittings for a particular number. Answering this question requires a systematic approach and mathematical reasoning.

In *Proeve,* it is stated that providing non-stereotype problems from time to time is important to prevent students from routinely guessing what operation to apply instead of consciously reasoning for themselves. An example is:

> Two friends celebrate their birthday together. Without consulting each other, they each invite their friends. One invites five friends and the other invites six. How many friends are invited? (Treffers & De Moor, 1990, p. 157).

Finally, *TAL* provides examples of non-routine, puzzle-like problems that combine problem-solving with practicing calculations and emphasizes to use such problems on a regular basis.

3.3 Conclusion Study 1

Study 1 revealed both differences and similarities in the intended RME approaches on early addition and subtraction as laid down in the analyzed core curriculum documents. A summarizing visualization of our findings is shown in Figure 10.

The three curriculum documents correspond on the use of context situations to provide meaning to addition and subtraction. In *Wiskobas* and *TAL,* this includes the bus context. In *Wiskobas,* this context is used for the simultaneous introduction of addition and subtraction and for carrying out multiple calculations in one situation. Arrow language is suggested in all three documents for symbolizing what is happening, and in *Wiskobas* also for letting students write it down and for relating addition and subtraction.

The documents differ remarkably regarding the models suggested. The one hundred square is explicitly promoted in *Wiskobas*, but no longer recommended in *Proeve,* and finally, *TAL* suggests not to use it. The number line evolved from a segmented line in *Wiskobas* to an empty line in *Proeve* and *TAL*. This has significant implications. On the segmented number line, positioning numbers and making calculations can be done by simply counting the tick marks, while the empty number line evokes the use of number relations and of shortened and structured calculation procedures. The arithmetic rack is not yet present in *Wiskobas*, but it has a prominent role in *Proeve* and *TAL*. This model and the empty number line are both used for level-raising by supporting structured calculation. Both also allow a variation of calculation procedures which is helpful for the idea of using students' own constructions of solution procedures

RME learning facilitators and intended use for early addition and subtraction	W 1975	P 1990	T 1999
Use of reality			
Bus context			
• Providing meaning to both operations	■	□	■
• Introducing both operations in one situation	■	□	□
• Carrying out multiple calculations in one situation	■	□	□
Other context situations			
• Providing meaning to both operations	■	■	■
Arrow language			
• Symbolizing what happens	■	■	■
• Relating both operations	■	□	□
• Students using arrow language	■	□	□
Use of models			
Segmented number line			
• Positioning numbers by counting	■	□	□
• Supporting calculations	■	□	□
• Relating both operations	■	□	□
Empty number line			
• Positioning numbers using number relations	□	■	■
• Supporting structured and varied calculations	□	■	■
One hundred square			
• Offering exploratory activities	■	□	□
• Practicing both operations	■	□	□
Arithmetic rack			
• Providing number images	□	■	■
• Supporting structured and varied calculations	□	■	■
Use of students' own input			
• Students putting into words what is happening	■	■	■
• Students' own productions of problems	■	■	■
• Students' own constructions of solution procedures	□	■	■
Use of non-routine problems			
• Evoking reflection and mathematical reasoning	■	■	■

■ = present, □ = absent

Figure 10. RME learning facilitators and their intended use for early addition and subtraction as indicated in *Wiskobas* (W), *Proeve* (P), and *TAL* (T)

as introduced in *Proeve*. The other ways of using students' own input—letting students put into words what is happening and come up with their own productions of problems—are present in all three documents. The same applies to the use of non-routine problems that evoke reflection and reasoning.

A final finding concerns the relationship between addition and subtraction, which reflects the RME intertwinement principle. Only *Wiskobas* suggests ways to connect the two operations, namely by use of the bus context, arrow language and the segmented number line. The relationship between addition and subtraction is not entirely absent in *Proeve* and *TAL*. They both discuss that some subtraction situations can easily be solved by an adding-on strategy, but they do not put an explicit emphasis on the relationship between addition and subtraction.

To conclude Study 1, the answer to our research question on the evolvement of the RME approach on early addition and subtraction as intended is threefold. First, the categories of learning facilitators based upon the big ideas and teaching principles of RME are steadily present in all analyzed documents. Second, some of the learning facilitators were made concrete in further detail over the years. This especially comes to the fore in the use of the arithmetic rack and the empty number line which both provide opportunities for structured solution procedures and make way for using students' own constructions of varied calculation strategies. Third, another remarkable change concerns the intertwinement of addition and subtraction, which is only emphasized in *Wiskobas* and no longer in *Proeve* and *TAL*.

4. Study 2: The RME way of teaching early addition and subtraction as implemented in textbooks

To answer the second research question, we investigated how the intended RME approach to teaching early addition and subtraction—as established in Study 1—has been implemented in textbooks over the years. To this end, we carried out an analysis of RME-oriented textbook series.

4.1 Method

4.1.1 Selection of textbook series

The selection of textbook series to be included in our study was based on two criteria. First, the design of a textbook should be intentionally RME-oriented, evidenced by making explicit references to Wiskobas or RME in teacher guidelines or information brochures. Textbooks that are adapted versions of textbooks from other countries or that are based upon other didactical approaches may have some RME characteristics, but were not included in our study. The second criterion concerns the market share. Since a larger market share is an indication of a greater acceptance of a textbook by teachers, we only included textbooks that reached a 15% market share or more and their successive editions. For determining the market share, we used information from the evaluation studies that Cito carried out between 1986 and 2013 and oral information from publishers and sellers of textbooks. The two criteria combined resulted in a collection of thirteen (editions of) textbook series in use from the 1980s on until today (Table 1): five consecutive editions of *De Wereld in Getallen* [The World in Numbers], four editions of *Pluspunt* [Plus Point], the textbook series *Rekenen & Wiskunde* [Arithmetic & Mathematics] and its successor *Wis en Reken* [Certainty and Calculate], and two editions of *Rekenrijk* [Rich Arithmetic / Realm of Arithmetic]. The two 1980s textbooks were also included in the previously mentioned study *Wiskobas in textbooks* (De Jong, 1986), in which it was concluded that these textbooks were highly aligned to the ideas of Wiskobas.

4.1.2 Selection of textbook materials

We selected from each textbook series the materials for the grade levels in which early addition and subtraction are taught. These are Grade 1 and Grade 2 in the 1980s textbooks and Grade 1, Grade 2 and Grade 3 in the textbooks from the later time periods. The two textbooks published in 2019 introduce addition and subtraction already in K2, but these parts of the textbooks are not yet on the market and could therefore not be included in our analysis.

All the textbook series consist of student material, including lesson books, work sheets and (from the 2000s on) software. They also all provide teacher guidelines, sometimes with additional user brochures, including daily lesson directions and background information. In our analysis, we incorporated all parts of these materials that provide information about the intended instructional approaches, from both the student and the teacher materials. These include the parts in which, according to the teacher guidelines, instruction and whole-class teaching are supposed to take place in the daily lessons, and information about teaching sequences and didactical directions provided in the guidelines and user brochures. Materials such as tests, materials for repetition and enrichment materials were left out of our analysis.

Table 1
Textbook series included in the study

Time	Textbook series		Publication	Market
period [a]	Abbr.	Name	year	share [b]
1980s	WiG1	De Wereld in Getallen, 1st edition	1981	28%
	R&W	Rekenen & Wiskunde	1983	29%
1990s	WiG2	De Wereld in Getallen, 2nd edition	1991	19%
	PP1	Pluspunt, 1st edition	1991	29%
2000s	WiG3	De Wereld in Getallen, 3rd edition	2001	30%
	PP2	Pluspunt, 2nd edition	2000	55%
	W&R	Wis en Reken	2000	7%
	RR2	Rekenrijk, 2nd edition	2000	16%
2010s	WiG4	De Wereld in Getallen, 4th edition	2009	50%
	PP3	Pluspunt 3rd edition	2009	25%
	RR3	Rekenrijk, 3rd edition	2009	5%
current	WiG5	De Wereld in Getallen, 5th edition	2019	…
	PP4	Pluspunt, 4th edition	2019	…

Note. [a] Decade in which the textbook was or is most used. [b] Highest known or estimated percentage of schools that at some point in time used the textbook. Market share is not yet known for the two textbooks published in 2019

4.1.3 Analysis procedure

We used the findings of Study 1 (Figure 10) as a framework for the analysis of the textbooks. The focus was on use of reality, use of models, and use of students' own input. The use of non-routine problems was not included in our second analysis, because this is already extensively reported upon in other studies (see Kolovou, Van den Heuvel-Panhuizen, & Bakker, 2009; Van Zanten & Van den Heuvel-Panhuizen, 2018b).

The analysis contained two steps. First, we searched in the selected textbook materials for the presence of the RME learning facilitators. Next, we checked whether the use as indicated in the textbooks corresponds to the intended use according to the RME core documents. To ensure that no relevant information was overlooked and the correspondence with the RME approach was judged correctly, the search and the checking were done multiple times. The analysis was mainly carried out by the first author and findings were frequently discussed with the second author.

4.2 Results

The RME learning facilitators were found to be present in all the analyzed textbooks, but we also discovered multiple differences. For all analyzed textbook series, the findings are visualized in Figure 11.

4.2.1 Use of reality

The eleven textbooks released from the 1980s to the 2010s all use the bus context to introduce addition and subtraction. In none of these textbooks do the operations take place simultaneously in one situation as suggested by *Wiskobas*. Some teacher guidelines state explicitly that only one thing happens at each bus stop: passengers are either getting on or off the bus. Regarding offering multiple calculations in one bus situation, the textbooks differ. The two 1980s textbooks include this, as well some textbooks in the following decades (WiG2, WiG3, RR2 and RR3), but the others do not. We were not able to establish whether the bus context is still used in the two textbooks published in 2019, since the kindergarten materials in which addition and subtraction are introduced in these textbook series were not yet available at the time of our analysis.

All thirteen textbooks offer varying other context situations, providing multiple meanings of addition and subtraction. In the textbooks released from the 1980s to the 2010s, this is also combined with arrow language. Regarding the use of arrow language, the interpretation in the textbooks differs from *Wiskobas*. In all these textbooks, arrow language is mainly offered as a ready-made fill-in exercise (Figure 12) in which students only have to add the missing numbers, and sometimes missing operation signs as well. Only very occasionally do students have to describe a problem situation by using arrow language, and when this is asked, the guidelines state that the teacher has to demonstrate each step. Furthermore, in only one textbook (W&R) is arrow language used to relate addition and subtraction to each other. In the two textbooks published in 2019, arrow language is no longer used.

RME learning facilitators and intended use for early addition and subtraction	1980s WiG1	1980s R&W	1990s WiG2	1990s PP1	2000s WiG3	2000s PP2	2000s W&R	2000s RR2	2010s WiG4	2010s PP3	2010s RR3	2019 WiG5	2019 PP4
Use of reality													
Bus context													
• Providing meaning to both operations	■	■	■	■	■	■	■	■	■	■	■		
• Introducing both operations in one situation	□	□	□	□	□	□	□	□	□	□	□		
• Carrying out multiple calculations in one situation	■	■	■	□	■	□	□	■	□	□	■		
Other context situations													
• Providing meaning to both operations	■	■	■	■	■	■	■	■	■	■	■	■	■
Arrow language													
• Symbolizing what happens	■	■	■	■	■	■	■	■	■	■	■	□	□
• Relating both operations	□	□	□	□	□	□	■	□	□	□	□	□	□
• Students using arrow language	□	□	□	□	□	□	□	□	□	□	□	□	□
Use of models													
Segmented number line													
• Positioning numbers by counting	■	■	■	■	■	■	■	■	■	■	■	■	■
• Supporting calculations	■	■	□	■	□	■	■	■	■	□	□	■	■
• Relating both operations	■	■	□	□	□	□	□	■	■	□	■	■	■
Empty number line													
• Positioning numbers using number relations	□	□	■	■	■	■	□	■	■	■	■	■	■
• Supporting structured and varied calculations	□	□	■	■	■	■	■	■	■	■	■	■	■
One hundred square													
• Offering exploratory activities	■	■	□	■	□	■	■	□	□	□	□	□	□
• Practicing both operations	■	■	□	■	□	■	■	□	□	□	□	□	□
Arithmetic rack													
• Providing number images	□	□	■	■	■	■	■	■	■	■	■	■	■
• Supporting structured and varied calculations	□	□	■	■	■	■	■	■	■	■	■	■	■
Use of students' own input													
• Students putting into words what is happening	■	■	■	■	■	■	■	■	■	■	■	■	■
• Students' own productions of problems	■	■	■	■	■	■	■	■	■	■	■	■	■
• Students' own constructions of solution procedures	□	□	■	■	■	■	■	■	■	■	■	■	■

■ = present, □ = absent

Figure 11. RME learning facilitators and their intended use for early addition and subtraction as included in RME-oriented textbooks

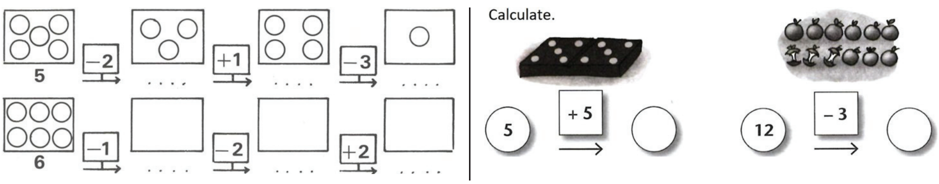

Figure 12. Arrow language in R&W (1980s, left, Grade 1, student book 1-3, p. 50) and PP3 (2010s, right, Grade 1, student book 7-8, p. 46)

4.2.2 Use of models

All thirteen textbooks provide the model of the segmented number line and use it for positioning numbers. The textbooks differ on other ways of using the segmented number line. Using it to relate addition and subtraction to each other is done in all the 1980s and the 2019 textbooks, and in three textbooks (RR2, RR3 and WiG4) from the intervening period. In the majority of textbooks, the segmented number line is used for supporting calculation. Some of these textbooks (W&R, WiG4, WiG5 and PP4) provide the segmented number line only for calculations up to 20 and not to support calculations with larger numbers.

The empty number line is present in all eleven textbooks published from the 1990s on. All these textbooks but one (W&R) use it to position numbers. This means that in most textbooks the positioning of numbers is supported both by the segmented and the empty number line. The bead string, which was introduced in *Proeve* (1990) together with the empty number line, is present in all but one (WiG2) of the textbooks that were published since then. In six textbooks (PP1, PP2, PP3, RR2, WiG3 and WiG4), the bead string is used to attach meaning to the empty number line, and in four textbooks (RR2, RR3, WiG5 and PP4) the bead string is (also) related to the segmented number line (Figure 13).

All eleven textbooks published from the 1990s on in which the empty number line is present use it to support structured and varied calculation procedures (Figure 14). They all offer the empty number line to support calculation by stringing (e.g., calculating 36 + 29 by doing 36 + 10 + 10 + 4 + 5) and by using varying strategies (e.g., 36 + 29 by doing 36 + 30 – 1). This use of the empty number line is consistent with *Proeve* and *TAL*.

For splitting procedures (e.g., 27 + 18 by doing 20 + 10 and 7 + 8), the first three editions of WiG and the first two editions of PP use visualizations with 10-blocks and 1-blocks, which is also consistent with *Proeve*. Since the 2010s, this use of blocks is no longer present in textbooks.

The one hundred square is provided in both 1980s textbooks. In line with *Wiskobas*, this model is used for exploratory activities and for practicing calculations. This is also the case in one 1990s textbook (PP1) and two 2000s textbooks (PP2 and W&R). From the 2010 textbooks on, the use of the one hundred square for early addition and subtraction has disappeared, which corresponds with *Proeve* and *TAL*.

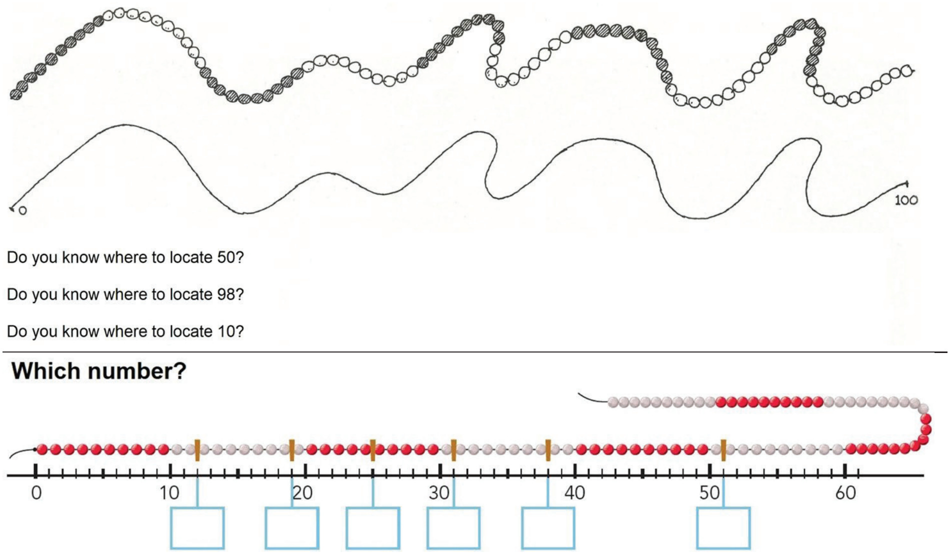

Figure 13. The number bead combined with the empty number line in PP1 (1990s, above, Grade 2, lesson book p. 57) and with the segmented number line in WiG5 (2019, below, Grade 1, student book 8, p. 4)

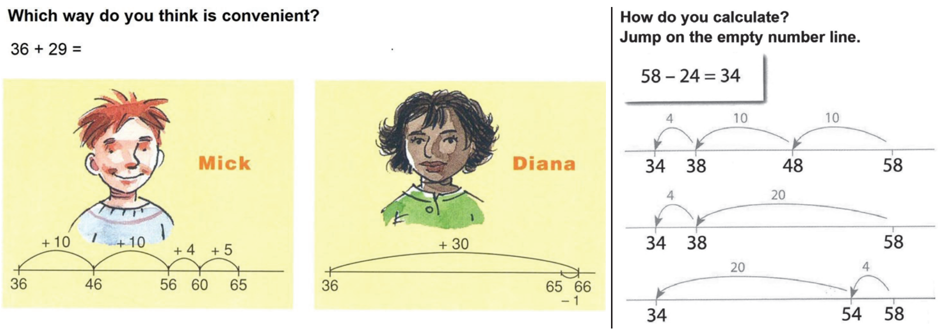

Figure. 14. Stringing and varying calculation strategies on the empty number line in PP2 (2000s, left, Grade 3, lesson book p. 9) and WiG4 (2010s, right, Grade 2, student book p. 19)

The arithmetic rack is used in all eleven textbooks published from the 1990s on. All these textbooks use it for providing multiple number images and for replacing counting by structured addition and subtraction (Figure 15), which is both in line with *Proeve* and *TAL*.

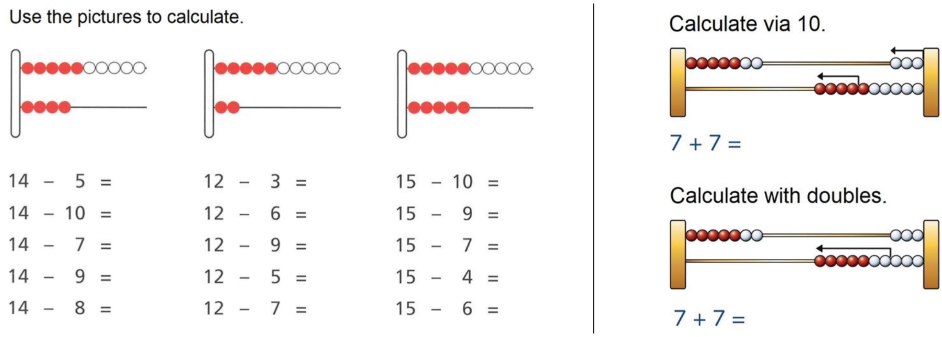

Figure 15. Calculating with the arithmetic rack in W&R (2000s, left, Grade 2, student book 4, p. 14) and RR3 (2010s, Grade 2, student book 4a, p. 16)

4.2.3 Use of students' own input

All thirteen textbooks included in our analysis emphasize in their teacher guidelines to let students explain in their own words what is happening in context situations, but (as already mentioned) none of them invite students to express what is happening by using arrow language. All textbooks provide students with various assignments in the student books to produce problems themselves, the so-called "own productions" (Figure 16). Making use of students' own input by letting them come up with their "own constructions" of solution procedures is to a certain extent recognizable in all textbooks published from the 1990s on. These textbooks regularly present worked examples of different calculation strategies for particular addition or subtraction tasks, such as shown in Figure 14. These worked examples provide the opportunity to discuss multiple calculation procedures with students, yet this is not precisely the same as comparing and making use of students' own solution procedures—as emphasized in *Proeve* and *TAL*.

Figure 16. Assignments for own productions in WiG1 (1980s, left, Grade 1, student book 1b, p. 60) and RR2 (2000s, right, Grade 1, Student book 3b, p. 29)

4.3 Conclusion Study 2

Study 2 showed that RME learning facilitators mentioned in the core documents including use of reality, models, and students' own input, are also present in the analyzed RME-oriented textbooks. However, regarding the *use* of particular learning facilitators, our findings are mixed—we found correspondences of RME-oriented textbooks with the RME core documents, as well as deviations. To provide an overview of our results from Study 2, we aggregated the textbooks findings per time period and placed them alongside the findings from Study 1 (Figure 17).

In most cases, the use of learning facilitators provided by all or most RME-oriented textbooks corresponds with the intended use of these facilitators as indicated in the RME core documents. This applies to the use of the bus context and other context situations for providing meaning, the use of arrow language for symbolizing what is happening, and the use of the arithmetic rack and the empty number line. Also, the use and the disappearance of the one hundred square are in line with the core documents. Furthermore, there is a correspondence between the textbooks and core documents regarding the use of students' own input in the teaching-learning process in the form of their own wordings, their own productions of problems, and—although only to a certain extent—their own constructions of solution procedures.

In several other cases, most or all textbooks deviate from the original intentions articulated by *Wiskobas*. This applies for the use of the bus context to introduce addition and subtraction in one situation, for the use of arrow language to relate addition and subtraction, and for letting students use arrow language to describe what is happening. These ways of using these learning facilitators, which are no longer mentioned in *Proeve* and *TAL*, are missing in the textbooks published after *Proeve* as well. In some textbooks, the use of the bus context to carry out multiple calculations in one situation is also not included.

A learning facilitator on which textbooks differ from *Proeve* and *TAL* is the segmented number line. While there is a change in the core documents from the segmented number line (in *Wiskobas*) to the empty number line (in *Proeve* and *TAL*), the textbooks released from the 1990s on (thus after the publication of *Proeve*) do not make this change. Besides the empty number line, the segmented number line is still in use as well, mostly for positioning numbers and supporting calculations. Regarding relating addition and subtraction on the segmented number line, there is no overall pattern—some textbooks do this and others not.

RME learning facilitators and intended use for early addition and subtraction	W 1975	P 1990	T 1999	Textbooks 80s	90s	00s	10s	2019
Use of reality								
Bus context								
• Providing meaning to both operations	■	□	■	■	■	■	■	
• Introducing both operations in one situation	■	□	□	□	□	□	□	
• Carrying out multiple calculations in one situation	■	□	□	■	⬓	⬓	⬓	
Other context situations								
• Providing meaning to both operations	■	■	■	■	■	■	■	■
Arrow language								
• Symbolizing what happens	■	■	■	■	■	■	■	□
• Relating both operations	■	□	□	□	□	⬓	□	□
• Students using arrow language	■	□	□	□	□	□	□	□
Use of models								
Segmented number line								
• Positioning numbers by counting	■	□	□	■	■	■	■	■
• Supporting calculations	■	□	□	■	⬓	⬓	⬓	■
• Relating both operations	■	□	□	■	□	⬓	⬓	■
Empty number line								
• Positioning numbers using number relations	□	■	■	□	■	⬓	■	■
• Supporting structured and varied calculations	□	■	■	□	■	■	■	■
One hundred square								
• Offering exploratory activities	■	□	□	■	⬓	⬓	□	□
• Practicing both operations	■	□	□	■	⬓	⬓	□	□
Arithmetic rack								
• Providing number images	□	■	■	□	■	■	■	■
• Supporting structured and varied calculations	□	■	■	□	■	■	■	■
Use of students' own input								
• Students putting into words what is happening	■	■	■	■	■	■	■	■
• Students' own productions of problems	■	■	■	■	■	■	■	■
• Students' own constructions of solution procedures	□	■	■	□	■	■	■	■

■ = present, □ = absent, ⬓ = present in some, absent in other textbooks

Figure 17. RME learning facilitators and their intended use for early addition and subtraction in *Wiskobas* (W), *Proeve* (P), *TAL* (T), and RME-oriented textbooks per time period

5. Overall conclusion and discussion

The aim of this research was to trace how the RME reform evolved, and to unravel the relationship between the intentions of the RME reform and its implementation in textbooks. Our focus was on early addition and subtraction. We carried out two studies—one on the intended curriculum level as laid down in RME core documents, and one on the potentially implemented curriculum level in the form of RME-oriented textbooks.

In the study on RME as intended, we found that RME ideas on using reality, models, students' own input, and non-routine problems are steadily present in the RME core documents published over time. In these documents, we also found changes over time regarding particular learning facilitators and their intended use. Some of these changes can clearly be characterized as further refined concretizations of RME ideas. An example of this is the evolvement of the number line from a segmented to an empty one, which enables structured and varied calculation. These changes imply that reconceptualization of RME ideas not only took place on the overarching level of big ideas and teaching principles of RME, but also on the more detailed level of particular learning topics.

We also detected that some ideas that were emphasized in *Wiskobas*, the oldest RME core document, are surprisingly not present in the more recent ones *Proeve* and *TAL*. The most outspoken example of this is the use of learning facilitators for relating addition and subtraction to each other, which reflects the RME principle of intertwining related learning strands. This leaving out of ideas that earlier were present may be unintentional. We could not determine why these changes actually occurred, since the RME core documents did not provide arguments for these changes.

Our study on RME-oriented textbooks also led to mixed findings. We found that the RME-oriented textbooks generally correspond with the core RME documents on the point of the *presence* of learning facilitators, but at the same time we could not detect an overall pattern of alignment regarding the *use* of these learning facilitators. Thus, on the one hand, based on our findings we can conclude that over the course of fifty years the RME reform was and is still clearly present in Dutch primary school textbooks. On the other hand, this does not mean that these textbooks are fully in consonance with RME's intentions. We found several differences between the *intended* use of particular learning facilitators as indicated in the RME core documents, and the *actual* use of these facilitators as described in the textbooks. Something similar was already found in the study *Wiskobas in textbooks* (De Jong, 1986). In our study, we again found, over a long period of time, deviations from the original RME intentions in RME-oriented textbooks. A striking example of this is the use of arrow language. All textbooks that suggest its use apply it as a filling-in exercise, which is almost the opposite of using arrow language to describe what is happening, which is what was originally intended. Such findings clearly indicate that the mere presence of a particular RME feature in a textbook does not tell the whole story. To get a broader picture of how the RME reform was implemented in textbooks, the way in which it is interpreted in the textbooks has also to be taken into account.

This brings us to the limitations of our research. Our design only covered the level of intended curriculum (the RME core documents) and the potentially implemented curriculum (the textbooks), but not the level of the implemented curriculum (the teaching and learning processes in school). This was beyond our scope and needs to be investigated as well to achieve a full understanding of the development and implementation process of RME. Of course, such a study should also not be restricted to early addition and subtraction but should include the full spectrum of mathematical content domains.

Although we are aware of the constraints of our research, we think we can conclude that it clearly shows how complicated the process is of generating and further developing ideas for how to teach mathematics and consequently getting them implemented in materials that teachers can use for their teaching. Our research has disclosed this process, which is in no way a straightforward route.

References

Bjarnadóttir, K. (2014). History of teaching arithmetic. In A. Karp & G. Schubring (Eds.), *Handbook on the history of mathematics education* (pp. 431–457). doi:10.1007/978-1-4614-9155-2

Bjarnadóttir, K. (2018). Influences from the 1959 Royaumont seminar. Proposals om arithmetic and algebra teaching at lower-secondary level in Iceland. In F. Furinghetti & A. Karp (Eds.), *Researching the history of mathematics education. An international overview* (pp. 1–21). doi:10.1007/978-3-319-68294-5

CMLW. (1961). *Commissie Modernisering Leerplan Wiskunde* [Commission Modernization Mathematics Curriculum]. Euclides, 37(5), 144–151.

De Jong, R. (1986). *Wiskobas in methoden* [Wiskobas in textbook series]. Utrecht, the Netherlands: Utrecht University.

De Jong, R., Treffers, A., & Wijdeveld, E. (Eds.). (1975). *Overzicht van wiskundeonderwijs op de basisschool. Leerplanpublikatie 2* [Overview of mathematics education in primary school. Curriculum document 2]. Wiskobas Bulletin, 5(2/3).

Dingman, S. (2007). Mathematics textbooks and state curriculum standards: An analysis of the alignment between the written and intended curricula. Columbia, MO: University of Missouri.

Freudenthal, H. (1968). Why to teach mathematics so as to be useful. *Educational Studies in Mathematics, 1*(1-2), 3–8.

Freudenthal. H. (1973). *Mathematics as an Educational Task*. Dordrecht, the Netherlands: Reidel Publishing Company.

Freudenthal, H. (1981). Major problems of mathematics education. *Educational Studies in Mathematics, 12*, 133–150.

Freudenthal, H. (1991). *Revisiting Mathematics Education*. Dordrecht/Boston/ London: Kluwer Academic Publishers.

Furinghetti, F., & Karp, A. (2018). *Researching the History of Mathematics Education. An International Overview*. Springer. doi:10.1007/978-3-319-68294-5

Janssen, J., Van der Schoot, F., & Hemker, B. (2005). *Balans van het reken-wiskundeonderwijs aan het einde van de basisschool 4. Periodieke Peiling van het Onderwijsniveau* [Balance of mathematics education at the end of primary school 4. Periodic Assessment of the Education Level]. Arnhem, the Netherlands: Cito.

Janssen, J., Van der Schoot, F., Hemker, B., & Verhelst, N. (1999). *Balans van het reken-wiskundeonderwijs aan het einde van de basisschool 3. Periodieke Peiling van het Onderwijsniveau* [Balance of mathematics education at the end of primary school 3. Periodic Assessment of the Education Level]. Arnhem, the Netherlands: Cito.

Johansson, M. (2003). *Textbooks in mathematics education. A study of textbooks as the potentially implemented curriculum.* Luleå, Sweden: Luleå University of Technology.

Karp. A., & Schubring, G. (Eds.). (2014). *Handbook on the History of Mathematics Education.* New York/Heidelberg/Dordrecht/London: Springer.

Kilpatrick, J. (2012). The New Math as an international phenomenon. *ZDM Mathematics Education, 44*(4), 563–571. doi:10.1007/s11858-012-0393-2

KNAW [Royal Netherlands Academy of Sciences]. (2009). *Rekenonderwijs op de basisschool* [Mathematics education in primary school]. Amsterdam, the Netherlands: KNAW.

Kolovou, A., Van den Heuvel-Panhuizen, M., & Bakker, A. (2009). Non-routine problem solving tasks in primary school mathematics textbooks. A needle in a haystack. *Mediterranean Journal for Research in Mathematics Education, 8*(2), 31–68.

Leung, F., Graf, K., & Lopez-Real, F. (2006). *Mathematics Education in Different Cultural Traditions-A Comparative Study of East Asia and the West.* The 13th ICMI Study. New York/Heidelberg/Dordrecht/London: Springer. doi:10.1007/0-387-29723-5

OEEC. (1961). *New Thinking in School Mathematics*. Paris, France: Organisation for European Economic Co-operation.

Robitaille, D. & Travers, K. (2003). International Connections in Mathematics Education. In G. Stanic & J. Kilpatrick (Eds.), *A History of School Mathematics* (pp. 1491–1508). Reston, VA: NCTM.

Schmidt, W., McKnight, C., Valverde, G., Houang, R., & Wiley, D. (1997). *Many Visions, Many Aims. A Cross-National Investigation of Curricular Intentions in School Mathematics.* Dordrecht/Boston/London: Kluwer Academic Publishers.

Stanic, G., & Kilpatrick, J. (2003). *A History of School Mathematics*. Reston, VA: NCTM.

Stein, M., Remillard, J., & Smith, M. (2007). How curriculum influences student learning. In F. Lester (Ed.), *Second handbook of research on mathematics teaching and learning* (vol 1, pp. 319–369). Charlotte, NC: Information Age Publishing.

Treffers, A. (1978). *Wiskobas doelgericht* [Wiskobas goal-directed]. Utrecht: IOWO.

Treffers, A. (1987). *Three dimensions. A model of goal and theory description in mathematics instruction. The Wiskobas project*. Dordrecht, the Netherlands: Reidel Publishing Company.

Treffers, A. (1993). Wiskobas and Freudenthal. Realistic mathematics education. *Educational Studies in Mathematics, 25*(1-2), 89–108.

Treffers, A., & De Moor, E. (1990). *Proeve van een nationaal programma voor het reken-wiskundeonderwijs op de basisschool 2. Basisvaardigheden en* *cijferen* [Design of a national program for mathematics education in primary school 2. Basic operations and algorithmic calculation]. Tilburg, the Netherlands: Zwijsen.

Treffers, A., De Moor, E., & Feijs, E. (1989). *Proeve van een nationaal programma voor het reken-wiskundeonderwijs op de basisschool 1. Overzicht einddoelen* [Design of a national program for mathematics education in primary school 1. Overview of goals]. Tilburg, the Netherlands: Zwijsen.

Treffers, A., Van den-Heuvel-Panhuizen, M., & Buys, K. (1999*). Jonge kinderen leren rekenen. Tussendoelen annex leerlijnen* [Young children learn mathematics. Intermediate attainment targets and teaching-learning trajectories]. Groningen, the Netherlands: Wolters-Noordhoff.

Valverde, G., Bianchi, L., Wolfe, R., Schmidt, W., & Houang, R. (2002). *According to the book. Using TIMSS to investigate the translation of policy into practice through the world of textbooks*. Dordrecht/Boston/London: Kluwer Academic Publishers.

Van den Akker, J., (2003). Curriculum perspectives: An introduction. In J. van den Akker, W. Kuiper & U. Hameyer (Eds.). *Curriculum landscapes and trends* (pp. 1–10). Dordrecht/Boston/London: Kluwer Academic Publishers.

Van den Brink, J. (1973). Bijna noemen [Almost mention it]. *Wiskobas-Bulletin 3*, 129–131.

Van den Brink, J. (1989). *Realistisch rekenonderwijs aan jonge kinderen* [Realistic mathematics education for young students]. Utrecht, the Netherlands: Utrecht University.

Van den Heuvel-Panhuizen, M. (1996). *Assessment and realistic mathematics education.* Utrecht: CD-β Press/Freudenthal Institute, Utrecht University.

Van den Heuvel-Panhuizen, M. (2001a). Realistic mathematics education in the Netherlands. In J. Anghileri (Ed.), *Principles and practices in arithmetic teaching. Innovative approaches for the primary classroom* (pp. 49–63). Buckingham, Philadelphia: Open University Press.

Van den Heuvel-Panhuizen, M. (Ed.). (2001b). *Children learn mathematics. A teaching-learning trajectory with intermediate attainment targets for calculation with whole numbers in primary school.* Utrecht, the Netherlands: Freudenthal Institute, Utrecht University.

Van den Heuvel-Panhuizen, M. (2010). Reform under attack – Forty years of working on better mathematics education thrown on the scrapheap? No way! In L. Sparrow, B. Kissane, & C. Hurst (Eds.), *Shaping the future of* *mathematics education: Proceedings of the 33rd annual conference of the Mathematics Education Research Group of Australasia* (pp. 1–25). Fremantle, Australia: MERGA.

Van den Heuvel-Panhuizen, M., & Drijvers, P. (2020). Realistic Mathematics Education. In S. Lerman (Ed.), *Encyclopedia of mathematics education*). Cham, Switzerland: Springer. doi:10.1007/978-3-030-15789-0_170

Van Zanten, M. (2019). Students' own productions and own constructions. Adri Treffers' contributions to Realistic Mathematics Education. In W. Blum, M. Artigue, M. Mariotti. R. Sträβer, & M. van den Heuvel-Panhuizen (Eds.), *European Traditions in Didactics of Mathematics, ICME-13 Monographs* (pp. 67–73). Cham, Switzerland: Springer Nature.

Van Zanten, M., & Van den Heuvel-Panhuizen, M. (2014). Freedom of design: The multiple faces of subtraction in Dutch primary school textbooks. In Y. Li & G. Lappan (Eds.), *Mathematics curriculum in school education* (pp. 231–259). Dordrecht/Heidelberg/New York/London: Springer. doi:10.1007/978-94-007-7560-2_12

Van Zanten, M., & Van den Heuvel-Panhuizen, M. (2018a). Primary school mathematics in the Netherlands. The perspective of the curriculum documents. In D. Thompson, M. Huntley, & C. Suurtamm (Eds.), *International perspectives on mathematics curriculum* (pp. 9–30). Charlotte, NC: Information Age Publishing.

Van Zanten, M. & Van den Heuvel-Panhuizen, M. (2018b). Opportunity to learn problem solving in Dutch primary school mathematics textbooks. *ZDM Mathematics Education, 50*(5), 827–838. doi:10.1007/s11858-018-0973-x

Wijdeveld, E. (1980). Zich realiseren [To realize]. In: S. Pieters (Ed.), *De achterkant van de Möbiusband* [The back side of the Möbiusband] (pp. 23–26). Utrecht, the Netherlands: IOWO.

Textbook series

Bazan, K., Bokhove, J., Borghouts, C., Buter, A., Kuipers, K., & Veltman, A. (2009). *Rekenrijk* [Realm of Arithmetic / Rich Arithmetic], 3rd edition. Groningen/Houten: Noordhoff Uitgevers.

Bokhove, J., Kuipers, K., Postema, J., & Borghouts, C. (2000). *Rekenrijk* [Realm of Arithmetic / Rich Arithmetic], 2nd edition. Groningen/Houten: Noordhoff Uitgevers.

Buys, K. (Ed.). (2000). *Wis en Reken* [Certainty and Calculate]. Baarn, the Netherlands: Bekadidact.

Faessen, S., Hut, M., Jongeling, M., Kroon, S., De Kruyf, D., & Wichgers, A. (Eds.). (2019). *De Wereld in Getallen* [The world in numbers], 5th edition. Den Bosch, the Netherlands: Malmberg

Gilissen, L. & Beemster, J. (Eds.). (1991). *Pluspunt* [Plus Point], 1st edition. Den Bosch, the Netherlands: Malmberg.

Gravemeijer, K. (Ed.). (1983). *Rekenen & Wiskunde* [Arithmetic & Mathematics]. Baarn, the Netherlands: Bekadidact.

Huitema, S., Erich, L., Van Hijum, R., Nillesen, C., Osinga, H., Veltman, H., & Van de Wetering, M. (2009). *De Wereld in Getallen* [The world in numbers], 4th edition. Den Bosch, the Netherlands: Malmberg.

Huitema, S., Van der Klis, A., & Timmermans, M. (2001). *De Wereld in Getallen* [The world in numbers], 3rd edition. Den Bosch, the Netherlands: Malmberg.

Huitema, S., Van der Klis, A., Van de Molengraaf, F., Timmermans, M., & Erich L. (1991). *De Wereld in Getallen* [The world in numbers], 2nd edition. Den Bosch, the Netherlands: Malmberg.

Kroon, S., De Kruyf, D., & Wichgers, A. (Eds). (2019). *Pluspunt* [Plus Point], 4th edition. Den Bosch, the Netherlands: Malmberg.

Van Beusekom, N., Fourdraine, A., & Van Gool, A. (Eds.). (2009). *Pluspunt* [Plus Point], 3rd edition. Den Bosch, the Netherlands: Malmberg.

Van Beusekom, N., & Schuffelers, L. (Eds.). (2000). *Pluspunt* [Plus Point], 2nd edition. Den Bosch, the Netherlands: Malmberg.

Werkgroep G. van de Molengraaf (1981). *De Wereld in Getallen* [The world in numbers], 1st edition. Den Bosch, the Netherlands: Malmberg.

Authors' contributions

This paper was a collaborative work of the two authors. Both authors participated in the selection of the curriculum documents and the textbooks, and in designing the research. MZ carried out the document analysis and the textbook analysis, of which the findings were frequently discussed with MH. MZ prepared the first draft of the manuscript. Both authors participated in revising the manuscript. Both authors read and approved the final manuscript.

Chapter 6

Primary school mathematics in the Netherlands: The perspective of the curriculum documents

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Primary school mathematics in the Netherlands: The perspective of the curriculum documents

1. Introduction

In the Netherlands, the school system consists of three stages: primary education; secondary education; and higher education (see Figure 1). Primary school is for students in the age range from 4 to 12 years and starts with two kindergarten grades (Grades K1 and K2), which are followed by six primary school grades (Grades 1 to 6). Secondary education is divided into three different levels with several sub-levels and for these three levels the number of grades differs. Higher education includes vocational education and university education. Although each level of secondary education is meant to prepare students for a particular form of higher education, it is also possible for students to switch between levels. For example, a student who has attained a diploma in HAVO (higher general secondary education) can then go to the fifth and sixth grade of VWO (pre-university secondary education), and after that can go to university.

Children can go to school when they are 4 years old, but education is compulsory from the age of 5 until 16. After this age, education is partly compulsory, which means that students have to continue school until their 18th birthday or until they acquire a diploma (of HAVO, VWO or intermediate vocational education), whichever comes first.

In this chapter, we discuss the mathematics curriculum for the primary school stage. The reason for this choice is that, in the Netherlands, primary education has a longer history than secondary education in thinking about the goals to be achieved by the students. In primary education, the first goal prescriptions were released in 1993, while for secondary education they came only in 2009 and only for the first years of secondary school. For the remaining years, the curriculum is determined by the topics included in the final secondary school examinations. Moreover, the primary school mathematics curriculum is laid down in various curriculum documents which makes it interesting to investigate how these documents together form the curriculum.

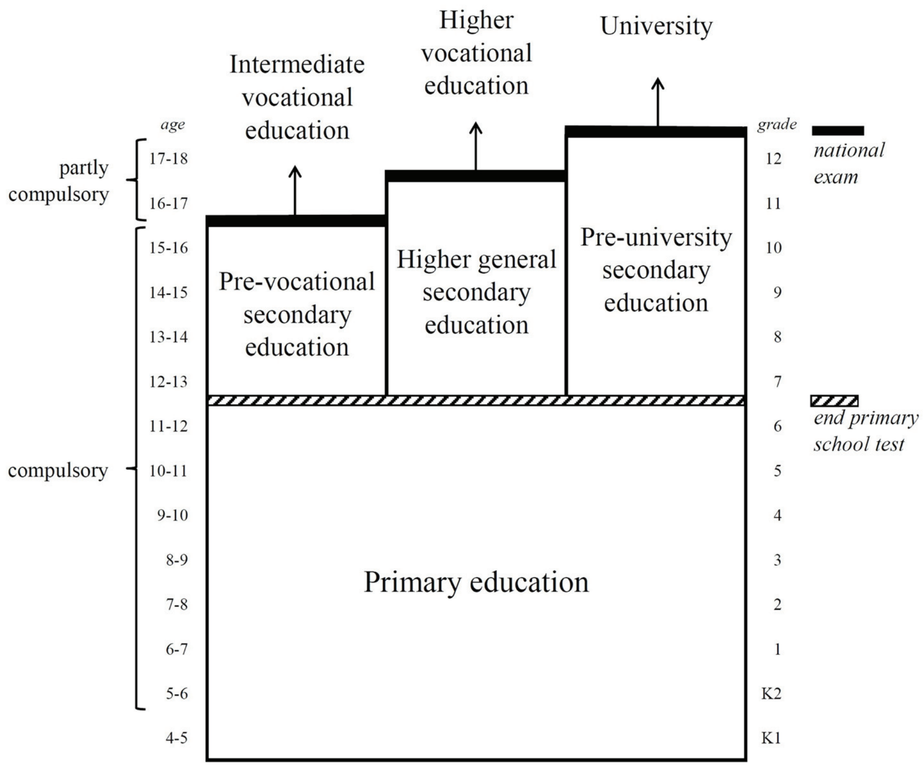

Figure. 1. The Dutch educational system

2. Curriculum documents for primary school mathematics education

Mathematics education starts in the kindergarten years with doing playful mathematics-related activities. In the grade years, mathematics is taught systematically in daily lessons for about five hours per week. The mathematical content that is taught in primary school is mainly defined in four types of curriculum documents:

- the legally prescribed standards;
- resources describing teaching-learning trajectories;
- textbooks; and
- assessment materials, especially compulsory tests at the end of primary school.

These documents represent different curriculum levels (e.g., Goodlad, 1979; Thijs & Van den Akker, 2009). The legally prescribed standards can be regarded as the intended curriculum, that is, the curriculum that describes the desired learning outcomes at a particular time in students' school career. Following Valverde, Bianchi, Wolfe, Schmidt, and Houang (2002), we consider textbooks as a separate level, the potentially implemented curriculum, intermediating between the intended curriculum and the implemented curriculum, which refers to the actual teaching and learning processes taking place in school (see Figure 2).

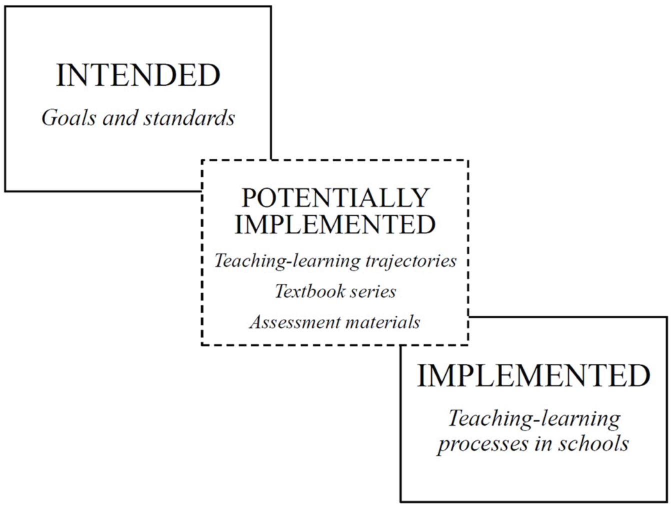

Figure. 2. Levels of curriculum in the Netherlands (adapted version from Valverde et al., 2002)

The teaching-learning trajectories are a mediating layer between the intended and the implemented curriculum and, therefore, belong to the potentially implemented curriculum. These trajectories sketch learning pathways through which students can achieve the standards that have been determined for the end of primary school. Although the development of these teaching-learning trajectories was initiated and financed by the Ministry of Education, they do not have a statutory status and, thus, they are not part of the formal intended curriculum. Finally, assessment materials influence the implemented curriculum because in these materials the mathematical knowledge, skills, and insights students are supposed to achieve over the school grades are operationalized.

The aforementioned curriculum documents each have their own role in supporting mathematics education that is realized in primary school and determined by different actors, including the Ministry of Education, SLO (Netherlands Institute for Curriculum Development), CvtE (College for Tests and Examinations), Cito (National Institute for Educational Measurement), textbook authors and publishers, and developers and researchers of the Freudenthal Institute. Our aim with this chapter is to illustrate the primary mathematics curriculum in these documents and to discuss their coherence. However, to understand the role these curriculum documents play in Dutch mathematics education, we first pay attention to the constitutionally established freedom of education in the Netherlands.

3. Freedom of eduction

In the Netherlands, freedom of education implies that the government is rather restrained in being involved in how education is realized. The origin of this policy dates to the Dutch Constitution of 1848 that permitted the founding of schools based on a religious denomination (Bakker, Noordman, & Rietveld-van Wingerden, 2010). In 1917, this was followed by a law that regulated that such denominational schools from then on were to receive the same financial resources from the government as public schools (ibid.). A few years later, in 1920, it was decided that this regulation also applied to schools with specific pedagogical approaches (Boekholt & De Booy, 1987).

As a consequence of the restrained policy, before the first Dutch standards could be established in 1993 (Ministry of Education, 1993/1998), eight years of debate occurred around the central question of whether or not governmental prescription of goals was compatible with the freedom of education (Letschert, 1998). Since 2008, after a parliamentary inquiry of educational innovations that had taken place, the government has strived more explicitly than before to make a strict distinction between the *what* (the learning goals and content to be taught) and the *how* (the way in which this content is to be taught) of education. In that year, the parliament stated that the government only prescribes the 'what', and not the 'how' (Committee Parliamentary Research Education, 2008; Ministry of Education, 2008). In line with this, the government presently sees freedom of education as grounds for the founding of schools based on specific ideas about educational and didactical approaches (Education Council, 2012; Ministry of Education, 2013). Currently, the Ministry of Education (2015) is working on a law amendment for having a renewed interpretation of the freedom of education in this spirit.

As a result of the freedom of education, the Dutch government does not interfere with textbook development and there is no authority that recommends, certifies, or approves textbooks before they are put on the market. This means that there are few restrictions in developing and publishing textbooks. Schools are free to choose a textbook that they think fits most closely to their view on teaching. Regarding the compulsory student test at the end of primary education, schools have limited choice. Schools may use the test that is developed by Cito and commissioned by the government, or may use a test developed by another company but also approved by the Ministry of Education.

4. The mathematics curriculum as reflected in standards

The standards for mathematics education in primary school are described in two ways. The current *Core Goals* document (Ministry of Education, 2006) describes eleven, globally formulated goals, which leave much room for interpretation about what mathematics students should learn in primary school. In 2010, the Core Goals document was extended with the *Reference Framework* (Ministry of Education, 2009), which describes in more detail what students should have achieved at the end of primary school (and at the end of secondary education and at the end of intermediate vocational education).

4.1 The Core Goals for mathematics

Figure 3 shows the complete list of goals for mathematics as included in the Core Goals document published in 2006. For example, for basic number operations, students have to learn to calculate in practical situations, and should be able to calculate mentally and in clever ways, and should be competent to carry out standardized calculation methods in a more or less curtailed way. What “practical situations” include and what these different methods imply is not specified. Regarding the number range, it is only mentioned that mental calculation should at least cover whole numbers to one hundred and that additions and subtractions up to twenty should be known by heart.

In addition to the goals, the Core Goals document also gives a so-called *characteristic of mathematics* which describes what is valued in mathematics education. Next to the basic mathematical skills and knowledge regarding the relationships and operations that apply to numbers, measurements and structures, more overarching competences should be valued in mathematics education, such as asking mathematical questions and problem solving. Further, it is emphasized that students should develop mathematical understanding and acquire mathematical literacy. By teachers keeping in mind students' knowledge, competences, and interests, students "will feel challenged to carry out mathematical activity and […] will be able to do mathematics at their own level, with satisfaction and pleasure" (Ministry of Education, 2006, p. 39). Students should also learn to respect each other's ways of thinking. Mathematics is, thus, seen as a social activity: in addition to working individually, students have to work in groups and should "learn to use explaining, formulating, notating, and giving and receiving criticism as a specific mathematical method to organize and ground their thinking and to prevent mistakes" (ibid., p. 39). A further guideline is that students should learn mathematics in the context of situations that are meaningful to them.

By including these directions in the characteristic of mathematics education, the Core Goals go, in a way, beyond prescribing just the *what* of mathematics education. They also provide a view on the learning of mathematics, which is reflected in the preamble of the Core Goals document. Although it is clearly stated that the given goals do not comment about didactics, which is in line with the freedom of education, the preamble does provide some indications about the ways in which teachers can stimulate students' development, for example, that education should be structured, interactive, and make connections to daily life (Ministry of Education, 2006, pp. 7-9).

4.2 The Reference Framework for mathematics

The Reference Framework was developed as a result of increasing concerns about the mathematical skills of students in secondary and vocational education (Ministry of Education, 2007). This Reference Framework prescribes standards regarding the attainment targets that students should reach at specified points in their schooling, starting from the end of primary school. These attainment targets concern the domains of number, rational numbers and ratios, measurement and geometry, and data handling. For each domain, three competences are distinguished: using mathematical language, making connections between procedures and concepts, and carrying out applications in contextual situations and bare number problems. Furthermore, for each of these competences, three performance expectations are formulated: knowing by heart, being able to use, and understanding.

The standards are formulated for three age-related target levels (1S, 2S, 3S), and three minimum levels (1F, 2F, 3F) for students who cannot achieve the S-levels. The levels 1S and 1F are meant for the end of primary school and the beginning of secondary education, in which 1S is meant for the majority of students (Expertgroep Doorlopende Leerlijnen, 2008). The 2F, 2S, 3F, and 3S levels are meant for older students. Table 1 shows some examples of the intended content and performance expectations for 1F and 1S in the domain of number.

Mathematical understanding and skills

1. Students learn to use mathematical language.
2. Students learn to solve practical and formal mathematical problems and present their reasoning clearly.
3. Students learn to justify and judge solution strategies for mathematical problems.

Numbers and operations

4. Students learn to understand the structure and interconnectedness of numbers, whole numbers, decimal numbers, fractions, percentages and ratios, and are able to calculate with these in practical situations.
5. Students learn to carry out mentally and quickly the basic operations with whole numbers at least up to 100, whereby the additions and subtractions up to 20 and the multiplication tables are known by heart.
6. Students learn to count and calculate by estimation.
7. Students learn to add, subtract, multiply and divide in clever ways.
8. Students learn written addition, subtraction, multiplication and division in more or less curtailed standardized ways.
9. Students learn to use the calculator with insight.

Measurement and geometry

10. Students learn to solve simple geometry problems.
11. Students learn to measure and calculate with measurement units and measures related to time, money, length, perimeter, area, volume, weight, speed and temperature.

Figure 3. The goals in the Core Goals document for mathematics (from Ministry of Education, 2006, pp. 40-45)

Table 1

Examples of the intended content and performance expectations for 1F and 1S in the domain of number

Level 1F	Level 1S (which also includes level 1F)
Translating a simple problem situation into a number sentence	Translating a complicated problem situation into a number sentence
Rounding off whole numbers to round numbers	Rounding off decimal numbers to whole numbers
Mental calculation: addition, subtraction, multiplication, and division 'with zeroes,' also with simple decimal numbers: $30 + 50$ $1200 - 800$ 65×10 $3600 \div 100$ 1000×2.5 0.25×100	Mental calculation: addition, subtraction, multiplication, and division 'with zeroes,' also with more difficult numbers, including larger numbers and more complicated fractions and decimal numbers: $18 \div 100$ 1.8×1000
Efficient calculation (+, –, ×, ÷) using the properties of numbers and operations, with simple numbers	Efficient calculation with larger numbers Division with a remainder or a (rounded off) decimal number: $122 \div 5$
Addition and subtraction (including determining the difference) with whole numbers and simple decimal numbers: $235 + 349$ $1268 - 38$ €2.50 + €1.25	
Multiplication of a one-digit number with a two-digit or three-digit number: 7×165 5 hours work for €5.75 an hour	
Multiplication of a two-digit number with a two-digit number: 35×67	
Division of a three-digit number with a two-digit number, with or without a remainder: $132 \div 16$	

Note: From Ministry of Education (2009, pp. 23-26)

As compared with the 1F-level, the 1S-level generally involves handling more complex problem situations, dealing with more difficult numbers including larger numbers and complicated fractions and decimal numbers, and a higher level of understanding. For example, students have to understand the difference between a digit and a number, the importance of the number zero, and reasoning about questions like: "Does there exist a smallest fraction?" (Ministry of Education, 2009, p. 25).

The way in which the standards in the Reference Framework are formulated is more specific than in the Core Goals document. For example, in the latter document it is just stated that students have to "learn to add, subtract, multiply, and divide in clever ways" (see Figure 3). The Reference Framework is more specific about what these "clever ways" imply, namely that students should learn "efficient calculation using the properties of numbers and operations" (Ministry of Education, 2009, p. 24). In addition, compared to the Core Goals, in the Reference Framework more directions are given regarding the number range. For example, concerning multiplication, students should learn a standard procedure to multiply a three-digit number by a one-digit number, and a two-digit number by a two-digit number. Similar to the Core Goals, the Reference Framework gives no specifications or examples of efficient calculation methods or standard procedures. The same goes for descriptions as "meaningful", "simple", and "more complex" context situations. Thus, the Reference Framework, like the Core Goals, leaves much room for interpretation.

5. The mathematics curriculum as reflected in teaching-learning trajectories

In the years after 1993 when the first Core Goals were published, there was discussion about whether these end-of-primary-school standards were sufficient to ensure that these goals would be achieved (see De Wit, 1997). In particular, there was a plea for having longitudinal teaching-learning trajectories with intermediate attainment targets. In 1997, this plea for such trajectories, which were a new educational phenomenon at that time, was honored. The Ministry of Education commissioned the Freudenthal Institute to develop *TAL teaching-learning trajectories*. The acronym TAL stands for *Tussendoelen annex leerlijnen* [Intermediate attainment targets annex teaching-learning trajectories].

The first TAL trajectory (see Treffers, Van den Heuvel-Panhuizen, & Buys, 1999) was on whole-number arithmetic in the lower grades of primary school and was followed by a trajectory on whole-number arithmetic in the upper grades of primary school (see Van den Heuvel-Panhuizen, Buys, & Treffers, 2001). For the upper grades, a trajectory for rational numbers was also developed (see Van Galen et al., 2005). For the domain of measurement and geometry, a teaching-learning trajectory was developed for both the lower grades (see Van den Heuvel-Panhuizen & Buys, 2004) and for the upper grades of primary school (Gravemeijer et al., 2007).[1] Later, SLO developed online *TULE*[2] teaching-learning trajectories for all subjects. For mathematics, this TULE document was based on TAL. Because there are only slight differences in content between the TAL and the TULE trajectories, we confine ourselves here to a description of the TAL trajectories and, in particular, to the two on whole-number arithmetic.

In the view of the TAL developers, the term teaching-learning trajectory

> has three interwoven meanings: a learning trajectory that gives a general overview of the learning process of the students; a teaching trajectory, consisting of didactical indications that describe how the teaching can most effectively link up with and stimulate the learning process; and a subject matter outline, indicating which of the core elements of the mathematics curriculum should be taught. (Van den Heuvel-Panhuizen, 2008, p. 13)

To make the interconnectedness of learning content and didactical approach concrete, in the TAL trajectories on whole-number arithmetic there are intermediate attainment targets to serve as landmarks towards achieving the goals as included in the Core Goals document, together with teaching frameworks. These teaching frameworks are descriptions of the teaching-learning processes that are considered to contribute to achieving these targets. For example, regarding addition and subtraction, an intermediate attainment target says that, by the end of Grade 2, students should know how to solve addition and subtraction problems to one hundred, both in context and in a bare number format (see Table 2). The corresponding teaching framework indicates that, in order to reach

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[1] Successively, these TAL trajectories have also been published in English (Gravemeijer et al., 2016; Van den Heuvel-Panhuizen, 2008; Van den Heuvel-Panhuizen & Buijs, 2008; Van Galen et al., 2008).

[2] See http://TULE.slo.nl/. TULE stands for *Tussendoelen en Leerlijnen* [Intermediate goals and teaching-learning trajectories]. Two of the three authors of TULE mathematics (Buijs, Klep, & Noteboom, 2008) were also involved in the development of TAL.

this intermediate attainment target, the teacher should have a good understanding of the nature and function of line and group models to shift students' performance level from applying a counting strategy to a more flexible way of mental calculation and a formal way of operating with numbers.

Table 2
TAL intermediate attainment target and teaching framework for addition and subtraction to one hundred

Addition and Subtraction to 100 at the end of Grade 2	
Intermediate Attainment Target	Teaching Framework
By the end of Grade 2, the students have memorized additions and subtractions to ten and have automatized them to twenty. They should then also be able to solve addition and subtraction problems to one hundred, both in context and in a bare number format. The children may use the empty number line, write down intermediate steps, or do it entirely in their heads.	Necessary for the students to reach these attainment targets is that the teacher takes into account the different levels of the students' understanding and adapts the teaching accordingly. The teacher has to have good insight into the nature and function of line and group models. Both models facilitate the transition from the initial calculation by counting to the later, more flexible, formal operation.

Note: From Van den Heuvel-Panhuizen (2008, p. 74), based on Treffers, Van den Heuvel-Panhuizen, & Buys (1999)

The intermediate attainment targets and teaching frameworks form the essence of the intended teaching-learning processes. In addition, the TAL trajectories describe in full detail sequences of activities to be done, problems to be solved, strategies to be used, and the models that support these strategies. Thus, TAL provides specifications that are absent in the Core Goals and the Reference Framework. For example, for standard calculation methods to one hundred (and beyond), TAL explains both the use of the stringing strategy (e.g., calculating 48 + 29 by doing 48 + 20 → 68 + 2 → 70 + 7 → 77) and the splitting strategy (e.g., calculating 48 + 29 by doing 40 + 20 = 60 and 8 + 9 = 17 followed by 60 + 17 = 77). Also, for efficient calculation methods, several varying strategies are described, such as making use of nearby round numbers (e.g., calculating 48 + 29 by doing 48 + 30 → 78 – 1 → 77) and raising both terms by 1 (e.g., calculating 77 – 29 by doing 78 – 30). Furthermore, examples are given of the way in which models can be used to support specific calculation methods, such as how an empty number line can be used to solve 48 + 29 by applying a stringing strategy (see Figure 4a) and applying a varying strategy (see Figure 4b).

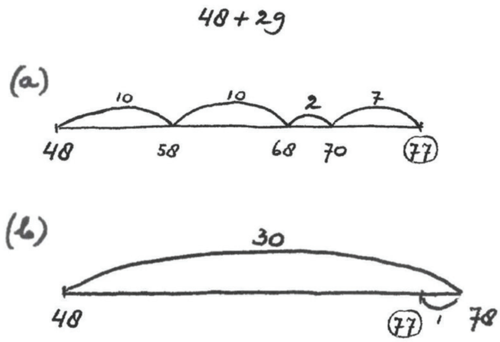

Figure 4. The use of the empty number line to support different calculation strategies for solving 48 + 29 (from Van den Heuvel-Panhuizen, 2008, p. 67-68; based on Treffers, Van den Heuvel-Panhuizen, & Buijs, 1999)

Another example of the specifications that TAL provides concerns two forms of written calculation procedures and their interrelatedness for the upper primary grades: whole-number-based calculation and digit-based algorithmic calculation. In the case of a whole-number-based calculation[3] of 463 + 382 (Figure 5a), the calculation is carried out with whole-number values working from large to small, that is from left to right (400 + 300 = 700; 60 + 80 = 140; 3 + 2 = 5; followed by 700 + 140 + 5). This calculation can also be carried out in the opposite direction working from small to large, that is from right to left (3 + 2 = 5; 60 + 80 = 140; 400 + 300 = 700; followed by 5 + 140 + 700) (Figure 5b). By working from right to left, the procedure can be used as an introduction to digit-based algorithmic calculation[4] involving calculating with digits (3 + 2 = 5; 6 + 8 = 14, write down the 4 and carry the 1; 1 + 4 + 3 = 8) (Figure 5c).

Similar to addition, for subtraction whole-number-based calculation and digit-based algorithmic calculation belong in TAL to the common attainment targets for all students. In the case of multiplication, the most curtailed digit-based algorithmic calculation is not considered an attainment target for the lesser able students. For division, the traditional long division (the digit-based algorithmic calculation), is not considered to be an attainment target in the TAL trajectory for primary school.

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[3] In the TAL teaching-learning trajectory (see Van den Heuvel-Panhuizen, 2008), this whole-number-based calculation is called *column calculation*.

[4] In the TAL teaching-learning trajectory (see Van den Heuvel-Panhuizen, 2008), this digit-based algorithmic calculation is called *algorithmic calculation*.

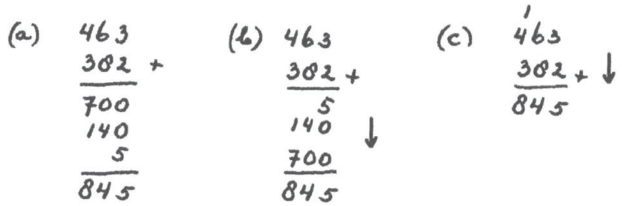


Figure 5. The addition 463 + 382 by (a) whole-number-based calculation from large to small, (b) by whole-number-based calculation from small to large, and (c) by digit-based algorithmic calculation. (from Van den Heuvel-Panhuizen, 2008, p. 147; based on Treffers, Van den Heuvel-Panhuizen, & Buijs, 1999)

Despite the detailed descriptions of the teaching-learning process for the primary school grades, the TAL trajectories are not meant to offer teachers guidance for their teaching on a day-to-day basis. The main purpose of the TAL trajectories was to bring coherence in primary school mathematics curriculum by providing a longitudinal overview of how children's mathematical understanding develops from K1 and K2 to Grade 6, and how the different stages in this development are connected and are built on each other. An example of this structure is apparent in the three levels that are distinguished in the elementary process of learning to calculate: calculating by counting (e.g., solving number problems by counting on fingers), calculating by structuring (e.g., solving number problems by using the empty number line, see Figure 4), and formal calculation (solving number problems by using symbolic notation). The idea is that students can solve problems at different levels, which is also recognizable in the distinction of whole-number-based calculation and digit-based algorithmic calculation. This idea reflects a concentric or spiral approach to teaching, in which a basic foundation is first laid, which later is filled with more complexity and depth. In other words, what is learned in one stage is understood in a later stage at a higher level.

Alongside the domain specific descriptions, TAL explicitly pays attention to the overarching competence of problem solving, emphasizing that students have to work on non-routine problems. For example, for the lower grades of primary school, the problem "Try to make 24 using the following randomly chosen numbers under 10: 3, 4, 7 and 8" is suggested (Van den Heuvel-Panhuizen, 2008, p. 81). In the higher grades, letter problems such as shown in Figure 6, can help students to deepen their understanding of digit-based algorithmic calculation.

Find the correct digit for each letter.
The problem must match the answer.

```
F O R T Y
    T E N
    T E N +
---------
S I X T Y
```

Figure 6. Forty and ten and ten is sixty (Van den Heuvel-Panhuizen, 2008, p. 167; see also Gardner, 1985, p. 18)

6. The mathematics curriculum as reflected in textbooks

Because a vast majority of Dutch primary school teachers rely heavily in their teaching on the textbook they use (Hop, 2012; Meelissen et al., 2012), mathematics textbook series have a determining role in daily teaching practice (Van Zanten & Van den Heuvel-Panhuizen, 2014). Currently, there are seven mathematics textbook series on the Dutch market, all published by independent, commercial publishers. We focus here on the four most frequently used textbook series as identified by Scheltens, Hemker, & Vermeulen (2013): *De Wereld in Getallen* (WiG) (Huitema et al., 2009-2014); *Pluspunt* (PP) (Van Beusekom, Fourdraine, & Van Gool, 2009-2013); *Alles Telt* (AT) (Van den Bosch-Ploegh et al., 2009-2013); and *Rekenrijk* (RR) (Bazen et al., 2009-2013).

All these textbook series provide materials for both students and teachers. Apart from the main books for students, the textbook series also have booklets with additional exercises and software for repetition. For Grades 1 to 6, the textbooks for students are accompanied by extensive teacher guidelines providing detailed information for each daily lesson, including directions for didactical approaches and differentiation. Moreover, these guidelines also provide, for each (sub)domain, grade overviews of the content to be addressed, and the learning goals to be achieved. For the kindergarten years, the textbook series do not have student books but only have source books for the teachers.

All textbook series offer content for numbers and operations (including whole numbers, decimal numbers, fractions, ratios and percentages, and the use of a calculator), measurement (including dealing with length, area, volume, weight, time, speed, temperature and money), geometry (including activities that can be labeled as orienting, constructing and operating with shapes and figures), and data handling (including dealing with graphs and tables, and calculating the average of values).

Table 3
Overview of content and performance expectations regarding addition and subtraction for Grade 1 to 6 in WiG

Grade	Content and Performance Expectations
1	Addition and subtraction situations are offered for the first time. At the end of Grade 1, students have started with solving addition and subtraction to 20, both in context situations and with bare numbers, and have started automatizing splitting, adding, and subtracting with numbers to 10.
2	Students continue automatizing splitting, adding and subtracting to 10, and start automatizing addition and subtraction to 20 and later to 100. One of the strategies students learn is making use of analogous problems (4 + 3 → 74 + 3; 8 – 5 → 48 – 5).
3	Students continue automatizing adding and subtracting to 20. Students add and subtract to 1000, by which they make use of the decimal structure of numbers (300 + 40; 560 – 500) and analogous problems to 100 (65 +… = 100 → 165 + … = 200). All addition and subtraction problems are presented as horizontal number sentences and are calculated mentally in which the use of scrap paper and an empty number line are allowed. A start is made with using clever calculation ways (30 + 30 → 30 + 28) and addition by estimation (205 + 398 ≈).
4	Students add and subtract to 1000 by mental calculation, also in clever ways and by estimation. Hereafter, this is extended to numbers to 10,000 and 100,000, in which students split the numbers, for example, in so many thousands, hundreds, tens, and ones. Students learn whole-number-based written addition and subtraction; after that, they learn digit-based algorithmic addition and subtraction to 1000. A start is made with digit-based algorithmic addition and subtraction with decimal numbers in the context of money.
5	Students add and subtract to 10,000 by mental calculation, also in clever ways and by estimation. Hereafter, this is extended to numbers to 1,000,000, in which students make use of decimally splitting the numbers. A start is made with adding and subtracting bare decimal numbers (3.5 + 0.8; 9.45 – 3.4). Digit-based algorithmic addition and subtraction with whole numbers is done to 10,000 and with decimal numbers in the context of money up to €10,000.
6	Students add and subtract to 1,000,000 by mental calculation, also in clever ways and by estimation. Students add and subtract with decimal numbers (2.55 + 3.5 + 102; 7.85 – 5.4). Digit-based algorithmic addition and subtraction with whole numbers is done to 100,000 and with decimal numbers in the context of money up to €10,000.

Within these (sub)domains, the content and performance expectations included in the textbook series are quite similar. For example, for the domain of numbers and operations, all textbook series contain the automatizing and memorizing of addition and subtraction facts to twenty and the multiplication tables to ten; mental calculation with standard strategies and with varying strategies; estimation; written calculation in one or two standard ways (whole-number-based and digit-based-algorithmic); and making reasoned choices between mental calculation, written calculation, and using a calculator. As an example, Table 3 provides an overview of content and performance expectations regarding addition and subtraction in the textbook series WiG.

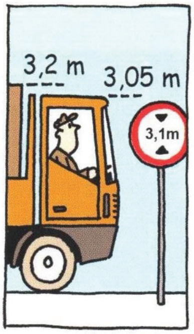
Although there are many similarities among the four textbook series, there are also differences, mostly related to the sequencing of the content and performance expectations over the grades (Table 4). For example, for estimation and written calculation, the sequencing differs among the four textbook series.

Table 4

Sequencing of the content related to addition and subtraction (whole numbers and decimal numbers) over the grades in the four most widely used Dutch textbook series

Content and performance expectations	Textbook series			
	WiG	PP	AT	RR
Addition and subtraction facts up to 20	Grades 1-3	Grades 1-3	Grades 1-3	Grades 1-3
Mental addition and subtraction in standard ways	Grades 2-6	Grades 2-6	Grades 2-6	Grades 2-6
Mental addition and subtraction in varying ways	Grades 2-6	Grades 2-6	Grades 2-6	Grades 2-6
Addition and subtraction by estimation	Grades 3-6	Grades 4-6	Grades 2-6	Grades 2-6
Whole-number-based written addition and subtraction	Grades 4	Grades 3-4	Grades 3-5	Grades 4-6
Digit-based algorithmic written addition and subtraction	Grades 4-6	Grades 4-6	Grades 3-6	Grades 4-6

The performance expectations are also mostly similar across the four textbook series. For example, they all start the automatization of adding and subtracting to 10 in Grade 1 and to 20 in Grade 2. They all also continue the process of memorizing addition and subtraction facts in Grade 3. Furthermore, all textbook series offer context situations for addition and subtraction from Grade 1 to Grade 6, first with whole numbers and later with decimal numbers in the context of money, and bare decimal numbers. Another similarity is that all textbook series provide directions on how to stimulate understanding. An example is that all series explicitly offer ways to encourage students' understanding of place value, for example by using a place value chart and making references to measurement numbers (Figure 7).



E	t	h
m	dm	cm

Figure 7. A place-value chart in WiG (from Huitema et al., 2009-2014; students' book Grade 5, p. 8) (E = eenheden [U = units], t = tienden [t = tenths], h = honderdsten [h = hundredths]. In Dutch, decimal numbers have a decimal comma instead of a decimal point.)

An example of a difference in performance expectations concerns students' understanding of the relationship between whole-number-based and digit-based written calculation. For example, in WiG, AT, and RR, digit-based algorithmic written multiplication is derived from whole-number-based written multiplication, whereas in PP no relationship is explicitly made between the two forms of written multiplication. Another example concerns written addition and subtraction. RR is the only textbook series that offers whole-number-based addition and subtraction to Grade 6 (Table 4), which is related to what this textbook takes as a performance expectation for the lesser able students. In RR, these students may choose to apply a whole-number-based or a digit-based calculation form. Regarding multiplication, WiG and PP have digit-based multiplication as a goal for all students, AT has whole-number-based multiplication as a goal for lesser able students, and RR again lets lesser able students choose between whole-number-based or digit-based multiplication.

Finally, differences occur regarding the goals that textbooks set for the end of primary school. For example, the number range within which the students have to solve written multiplication problems differs among the textbooks. The textbook series WiG, AT, and RR have as a goal that students learn to multiply two-digit numbers with three-digit numbers in a digit-based algorithmic way, whereas PP does not go further than multiplying one-digit numbers with three-digit numbers and two-digit numbers with two-digit numbers.

Besides the exercises that are meant for all students, the four textbooks all provide tasks at mostly three levels. For example, WiG distinguishes so-called *one-star*, *two-stars*, and *three-stars* level tasks. Differences between these levels involve, among other things, the number range used and the complexity of the questioning. Moreover, at the one-star level, more opportunity for repetition is offered and more concrete tasks are given for a longer period of time. For example, in the final lessons in Grade 6 about multiplication by estimation, the one-star level tasks comprise estimation with decimal measurement numbers (Figure 8), whereas the two-star level tasks also include estimation with bare decimal numbers (Figure 9). The three-star level tasks require more insight, and often provide puzzle-like tasks, such as the task shown in Figure 10 (also from the aforementioned lesson), in which students have to use their knowledge of place value in a creative way.

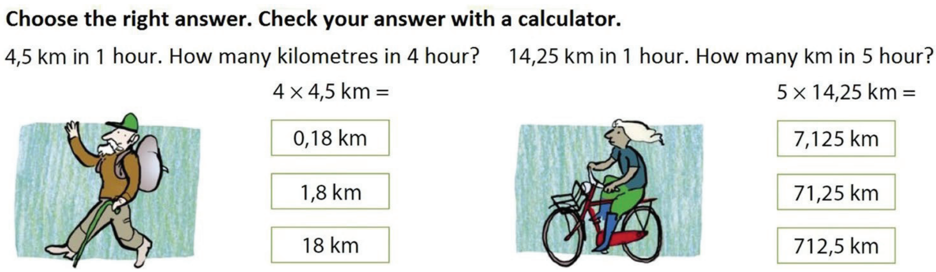


Figure 8. A Grade 6 one-star level task on multiplication by estimation (from Huitema et al., 2009-2014, students' book Grade 6, p. 56)

Estimate and choose the right answer.

32 × 1,9 ≈	6,4	64	640	6400
2,9 × 40,3 ≈	1,2	12	120	1200
7 × 349,98 ≈	2,45	24,5	245	2450
98,67 × 30 ≈	20	300	200	3000
3,05 × 5,97 ≈	1,8	18	180	1800

Figure 9. A Grade 6 two-stars level task on multiplication by estimation (from Huitema et al., 2009-2014, students' book Grade 6, p. 58)

Come up with two multiplications that look like this:

Use these digits. Use each digit only once in each multiplication.
The answer must be as large as possible.

5 1 9 4 3 2

Do it again, but now make sure that the answers are as small as possible.

Figure 10. A Grade 6 three-stars level task on multiplication by estimation (from Huitema et al., 2009-2014, students' book Grade 6, p. 59)

What is lacking in the four textbooks is an overview of the domain-overarching competence of problem solving. This does not mean that the textbook series do not provide assignments that include problem solving. They do, but only a few assignments are included and not in a systematic way. Furthermore, problem solving tasks are mostly offered in the sections meant for the best students. In contrast, the application of mathematical knowledge and skills in solving straightforward context situations is dealt with in almost every lesson in each textbook series. Regarding another domain-overarching competence, namely using mathematical language, only AT provides an overview of mathematical words per grade.

7. The mathematics curriculum as reflected in the end of primary school test

The compulsory test at the end of primary school serves three purposes. First, the test provides objective information used in making a decision about what level of secondary education a student will attend. Second, the test results are used to know what reference level (1F or 1S) a student has mastered. Third, the test results function for the school inspectorate, next to other indicators, as a measure to assess the quality of a school. So, the end of primary school test can be considered a high-stakes test, both for students and schools.

The test that is developed by Cito and commissioned by the government is called *Centrale Eindtoets* [Central End of Primary School Test]. It is used by a majority of schools in the Netherlands (e.g., Hemker, 2016). Currently, there are several other tests developed by commercial testing companies that are approved by the government. The criteria for approval, which are also the criteria for the Central End of Primary School Test, are described in the *Toetswijzer Eindtoets PO* [Directions for End of Primary School Tests] (CvTE, 2014).

7.1 Mathematics in the Directions for End of Primary School Tests

End of primary school tests must meet a number of demands with respect to validity, reliability, and content. Concerning the content, to which we confine ourselves here, end of primary school tests must cover levels 1F and 1S for all domains included in the Reference Framework (number, rational numbers and ratios, measurement and geometry, and data handling). For each domain, a minimum and maximum proportion of test items is prescribed. Also, the competences (using mathematical language, making connections between procedures and concepts, and carrying out applications in context situations and bare number problems) named in the Reference Framework must be dealt with in an end of primary school test. The same applies to the performance expectations (knowing by heart, being able to use, and understanding).

There are also three additional specific demands. The first is that a test must contain both context problems and bare number problems, with a minimum proportion of 30% and 20% of all items, respectively. This demand is a direct outcome of a debate about whether mathematics education at primary school should include context situations or focus on bare number calculation. The second demand is that end of primary school tests should allow the use of scrap paper in at least 80% of all items, adhering to research that indicates using scrap paper was of more influence on getting a correct answer than use of a particular calculation procedure (Hickendorff, 2011). The last demand is that a test should measure whether students are able to use a calculator in a reasonable way.

7.2 Mathematics in the Central End of Primary School Test

Because a majority of schools use the Central End of Primary School Test (hereafter called the 'Central Test'), we limit ourselves here to this test. The Central Test covers all the domains of the Reference Framework (Table 5), but not all performance expectations mentioned in the Reference Framework. This test does not (yet) contain test items assessing the ability to use a calculator, partly because this would require too many test items (CvTE, 2015a). Furthermore, the ability to make use of measurement devices is not assessed, due to the fact that the Central Test used now has a multiple-choice format.

Table 5

Content included in the Central End of Primary School Test for mathematics

Domain mentioned in Reference Framework	Content included in Central End of Primary School Test
Numbers	Number sense
	Operations with whole numbers and decimal numbers
	Operations with fractions
Ratios	Identifying ratios and expressing them as part-whole, fractions, percentages
	Solving problems with ratios (e.g., recipes)
	...
Measurement and geometry	Measurement: length and circumference, area, volume, weight, time and speed, money
	Geometry: shapes and figures, orientation and localization, symmetry and patterns
Data handling	Tables
	Graphs

Note: From CvTE (2015a)

For the Central Test, the Directions for End of Primary School Tests are extended with detailed specifications regarding the content and performance expectations. For example, for basic operations with whole numbers and decimal numbers, these include the following (CvTE, 2015a, p. 53, 55):

- adding and subtracting using properties of numbers and operations, including calculation with numbers with zeroes (e.g. 4000 + 60,000; 180,000 – 2000);
- using standard procedures for addition and subtraction with large whole numbers and decimal numbers with multiple digits;
- adding and subtracting by estimation with large whole numbers and with decimal numbers (49.95 + 128.95 + 32.35 is about 50 + 130 + 30);
- multiplying and dividing by using properties of numbers and operations, including multiplying and dividing whole numbers and decimal numbers by 10, 100, 1000 (1.8 × 100), and multiplying and dividing whole numbers by other numbers with zeroes (60 × 400; 3200 ÷ 40);
- using standard procedures for multiplication and division with large whole numbers and decimal numbers;
- interpreting the remainder of a division problem (e.g., transporting 349 children in buses; each bus can transport 45 children; 659 ÷ 45 = 14 remainder 21, so there are 15 buses needed); and
- multiplying and dividing by estimation with large whole numbers and decimal numbers (49 × 198.97 is about 50 × 200).

Because of the amount of content included in the Central Test, for language and mathematics together, it takes three mornings, including breaks, to administer the test. The 2015 version of the Central Test included 85 items for mathematics. In all items, the use of scrap paper was allowed. Figure 11 shows four items of the 2015 Central Test.

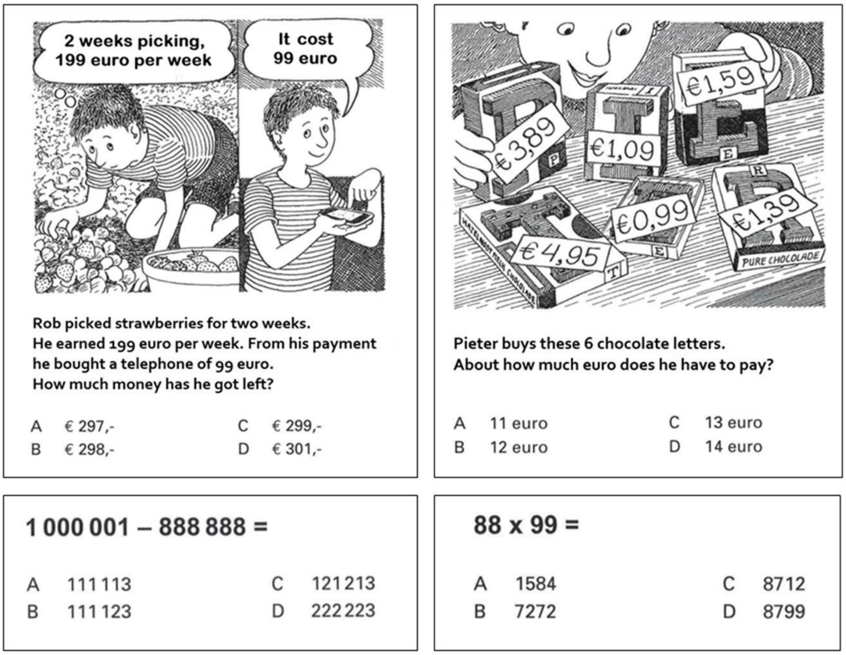


Figure 11. Four items on basic operations from the 2015 Central Test (from CvTE, 2015b)

8. The coherence of the mathematics curriculum

The coherence of a curriculum is of decisive influence on students' opportunities to learn (Schmidt, Houang, & Cogan, 2002). Curricular coherence can be considered in different ways, of which the alignment of different curriculum resources, referring to the degree in which resources accord to one another, can be seen one of the most elementary forms (Schmidt, Wang, & McKnight, 2005). We use the term in this way, which is visualized in the Dutch curricular spider web model (Van den Akker, 2003) (see Figure 12). This model illustrates the coherence of the several elements of a curriculum, but at the same time it also makes clear how vulnerable a curriculum is. When it is pulled too hard at the ends, the spider web can break. For example, if learning materials do not fit the content to be learned, then learning goals probably will not be achieved.

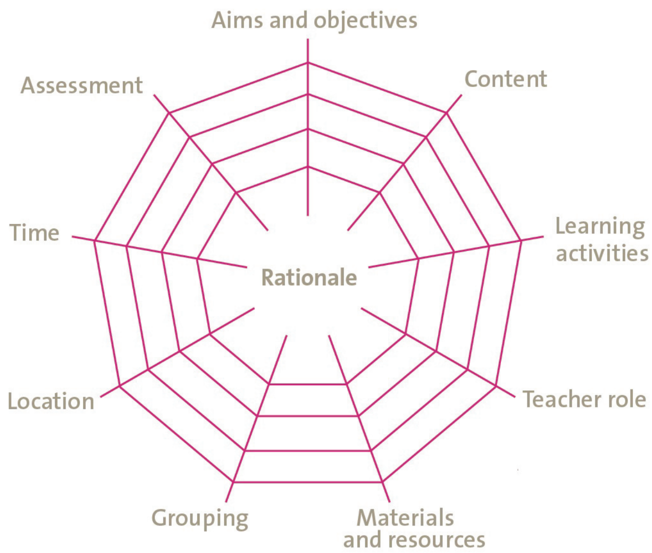


Figure 12. The curricular spider web (Van den Akker, 2003)

The situation in which decisions regarding the curriculum are made by different actors—a government, textbook publishers, testing organizations—who each may have their own goals and visions, can be considered a threat to curricular coherence (Schmidt, Wang, & McKnight, 2005). This situation specifically applies to the Netherlands with its policy of freedom of education. Therefore, in this section, we address whether the documents that describe the intended curriculum (the Core Goals and the Reference Framework) are in alignment with each other, and whether the documents that we consider as the potentially implemented curriculum (the TAL teaching-learning trajectories, the textbook series, and the end of primary school test) correspond with the intended curriculum.

8.1 Coherence within the intended curriculum

The Core Goals document and the Reference Framework give descriptions of the same content and performance expectations. All domains, content, and performance expectations included in the Core Goals document are also mentioned in the Reference Framework. The same goes for the overarching competences of using mathematical language and problem solving, although the latter has a less prominent place in the Reference Framework than in the Core Goals document.

As noted earlier, the Reference Framework is elaborated in more detail than the Core Goals document and the Reference Framework distinguishes two levels in the attainment targets for the end of primary school. There are two other significant differences between the two documents. The first is that the Core Goals document indicates "what primary schools should be aiming for regarding the development of their students" (Ministry of Education, 2006, p. 1), whereas the Reference Framework describes "what students should know and be able to regarding Dutch language and mathematics" (Ministry of Education, 2009, p. 5). Because of the latter, in 2015 the end of primary school test became mandatory, which was not previously the case. Second, although the Reference Framework contains the same content and performance expectations as the Core Goals document (CvTE, 2014), several overarching competences emphasized in the Core Goals document are not included in the Reference Framework. This is, for example, the case for asking mathematical questions, using mathematical literacy, and giving and receiving criticism as a mathematical method. Furthermore, issues regarding attitudes mentioned in the Core Goals, such as feeling challenged and doing mathematics with satisfaction and pleasure, are also not referred to in the Reference Framework. Thus, compared to the Core Goals document, albeit the Reference Framework is more detailed in its descriptions, it is more limited with respect to mathematical attitude and overarching competence foci.

8.2 Coherence within the potentially implemented curriculum

The TAL teaching-learning trajectories, which were developed between 1996 and 2007, are based on the 1993/1998 version of the Core Goals document. Because the 2006 Core Goals document (Figure 3) is far more global than the 1993/1998 version was, it was expected "that the TAL teaching-learning trajectories and the included intermediate attainment targets, will play a large role in guiding decisions about mathematical content" (Van den Heuvel-Panhuizen & Wijers, 2005, p. 294). Currently, indeed, in all four most frequently used textbooks series it is explicitly stated in the accompanying teacher guidelines that the textbooks are based – next to the Core Goals and the Reference Framework – on the TAL teaching-learning trajectories (Bazen et al., 2009-2013, teacher guidelines, p. 4.[5] Huitema et al., 2009-2014, teacher guidelines, p. 2; Van Beusekom, Fourdraine,

[5] In the RR guidelines, it only says 'teaching-learning trajectories', but one of the authors of this textbook series confirmed that here the TAL teaching-learning trajectories are meant.

& Van Gool, 2009-2013, teacher guidelines, p. 5; Van den Bosch-Ploegh et al., 2009-2013, teacher guidelines, p. 12, p. 14). That this indeed is the case is evidenced by the corresponding ways in which content and performance expectations are aligned in the textbook series with the TAL trajectories. This is also true for the use of certain learning facilitators as suggested by TAL, such as the empty number line, which is present in all four textbooks series (see Figure 13 for an example).

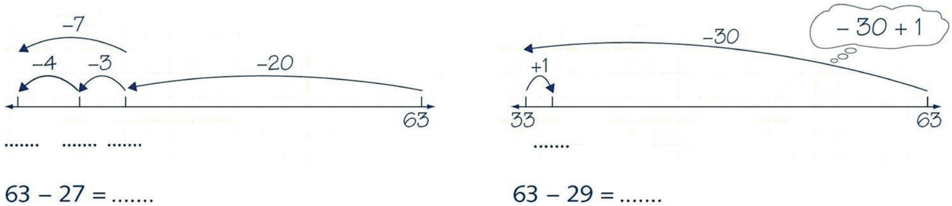


Figure 13. Use of the empty number line in RR (from Bazen et al., 2009-2013; students' book Grade 2, p. 45)

Despite the fact that all four textbook series have a connection with TAL, there are several differences in their elaborations of content and performance expectations (some of which were discussed in the section about textbooks) and the provision of learning facilitators, such as models. Furthermore, not everything emphasized in TAL is also present in all four textbook series. We discuss more about this in the following section.

8.3 Coherence between the intended and the potentially implemented curriculum

The documents of the potentially implemented curriculum – the TAL teaching-learning trajectories, the four most frequently used textbook series, and the Central Test – include all the domains prescribed in the intended curriculum. They all comprise numbers and operations, ratios, measurement, geometry, and data handling. With respect to the four textbook series, analyses carried out by SLO (2012a, 2012b, 2012c, 2012d) have established that the textbooks meet the standards as described in the Core Goals document. However, it should be noted that these analyses were done very broadly and the Reference Framework was not included in these analyses.

Table 6
Similarities and differences among the curriculum documents for written multiplication

Curriculum Document	How this document deals with written multiplication
Core Goals	Students learn written multiplication in more or less curtailed standardized ways
Reference Framework (level 1F and 1S)	Multiplication of a one-digit number with a two-digit or three-digit number
	Multiplication of a two-digit number with a two-digit number
TAL teaching-learning trajectory	The most curtailed digit-based algorithmic multiplication is not considered an attainment target for lesser able students
Textbook series	WiG, AT, and RR have as a goal that students learn to multiply two-digit numbers with three-digit numbers. Digit-based algorithmic multiplication is derived from whole-number-based written multiplication
	PP has as a goal that students learn to multiply one-digit numbers with three-digit numbers and two-digit numbers with two-digit numbers. No relationship is made between whole-number-based and digit-based multiplication
	WiG and PP have digit-based multiplication as a goal for all students. AT has whole-number-based multiplication as a goal for lesser able students. RR lets lesser able students choose between whole-number-based or digit-based multiplication
Central Test	Using standard procedures for multiplication with large whole numbers and decimal numbers

Although the intended curriculum documents are global in nature, the potentially implemented curriculum documents provide detailed elaborations of content and performance expectations. As an example of the similarities and differences that currently exist among the curriculum documents, Table 6 contains a list of the ways in which written multiplication is dealt with in the Core Goals document, the Reference Framework, the TAL teaching-learning trajectories, the four textbook series, and the Central Test.

The Core Goals document and the Reference Framework prescribe that students should learn a form of written multiplication, but do not indicate what specific form (algorithmic digit-based or whole-number-based) that should be. Also, the Directions for the End of Primary School Tests document do not prescribe which multiplication form should be used. The same goes for the Central Test. TAL, however, does provide an indication of the form that students should learn: the most curtailed form of digit-based multiplication is not considered an attainment target for lesser able students. The approach to written multiplication in the four textbook series varies. In agreement with TAL, in AT and RR, digit-based multiplication is not considered an attainment target for lesser able students. In WiG and PP, however, digit-based multiplication is an attainment target for all students, including the less able ones. Another difference between the textbook series is the attainment target regarding the number range in which students should be able to work. The textbook series PP has as a goal that students learn to multiply one-digit numbers with three-digit numbers and two-digit numbers with two-digit numbers, which precisely corresponds with the number range prescribed in the Reference Framework. The other textbook series aim for all students learning to multiply two-digit numbers with three-digit numbers. Finally, in WiG, AT, and RR, digit-based multiplication is derived from whole-number-based multiplication, whereas PP does not make a connection between the two forms.

Regarding the coherence between the intended curriculum and the end of primary school tests, we must say, there is a weak point. According to the Directions for the End of Primary School Tests, these tests have to “test students on their knowledge and skills regarding the Reference Framework” (CvTE, 2014, p.17); the same document also indicates that this automatically means that the content and performance expectations of the Core Goals document are covered (ibid., p. 7). However, the latter is not necessarily true, because some overarching competences included in the Core Goals are missing in the Reference Framework. Furthermore, some performance expectations (such as being able to use a calculator and measuring devices) are not included (yet) in the Central Test.

9. Final remarks

As discussed earlier, freedom of education in the Netherlands implies that there are few restrictions in developing textbooks, and that schools may choose whatever textbook series they want to use. However, because different textbooks may provide different opportunities to learn (Van Zanten & Van den Heuvel-

Panhuizen, 2014) and because textbooks have a determining role for daily teaching practice in the Netherlands, we conclude this chapter with some remarks considering textbook series.

The examples provided in this chapter suggest that different elaborations within the four most frequently used textbook series fall within the boundaries of the globally described intended curriculum. However, we raise two issues.

The first issue is about the differentiated attainment targets as provided by the Reference Framework in which the levels 1F and 1S are distinguished. All four textbook series have incorporated these levels by including differentiated tasks. For example, the learning route following the *one-star* tasks in WiG is supposed to lead to mastery of the 1F-level, and the route of the *two-stars* tasks should lead to the mastery of the 1S-level. However, whether such differentiated learning routes within textbooks indeed lead to the mastery of the levels aimed at is not known. The fact that currently only about 45% of students at the end of primary school master the 1S-level (Educational Inspectorate, 2016[6]), which is meant for a majority of the students, raises the question of whether the 1S-level is well enough incorporated in the textbooks, and also how teachers deal with the differentiated routes provided by the textbooks.

The second issue concerns the domain overarching competences, especially problem solving. Although problem solving is mentioned in both the Core Goals document and the Reference Framework, and the TAL teaching-learning trajectories explicitly emphasize the importance of it, there is only limited attention on problem solving in the four textbook series, and mainly only for the best students. This means that most students have only few opportunities to develop this mathematical competence.

Both issues—having a structure in the textbooks that clearly leads to the 1S-level and offering students the opportunity to develop problem solving competences—are are definitely tasks for textbook developers to address, but to improve textbook series at this point requires that all curriculum levels be involved. Only then can the coherence of the curriculum be secured and the curriculum fulfill its role as a steering tool for high quality education.

[6] The same study shows that 90% of students master the 1F-level at the end of primary school.

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References

Bakker, N., Noordman, J., & Rietveld-van Wingerden, M. (2010). *Vijf eeuwen opvoeden in Nederland* [Five centuries of education in the Netherlands]. (2nd Edition). Assen, the Netherlands: Van Gorcum.

Boekholt, P., & De Booy, E. (1987). *Geschiedenis van de school in Nederland* [A history of schools in the Netherlands]. Assen, the Netherlands: Van Gorcum.

Buijs, K., Klep, J., & Noteboom, A. (2008). *TULE rekenen/wiskunde. Inhouden en activiteiten bij de kerndoelen van 2006*. [TULE Mathematics. Content and activities for the 2006 Core Goals]. Enschede, the Netherlands: Netherlands Institute for Curriculum Development.

Committee Parliamentary Research Education. (2008). *Tijd voor onderwijs* [Time for education]. Den Haag, the Netherlands: Sdu Uitgevers.

CvTE. (2014). *Toetswijzer eindtoets PO* [Directions for end of primary school tests]. Utrecht, the Netherlands: College voor Toetsen en Examens.

CvTE. (2015a). *Toetswijzer bij de Centrale eindtoets PO taal en rekenen* [Directions for the Central End of primary school test for language and mathematics]. Utrecht, the Netherlands: College voor Toetsen en Examens.

CvTE. (2015b). *Central Test 2015*. Utrecht, the Netherlands: College voor Toetsen en Examens.

De Wit, C. (1997). *Over tussendoelen gesproken. Tussendoelen als component van leerlijnen* [Talking about intermediate goals. Intermediate goals as a component of teaching-learning trajectories]. 's Hertogenbosch, the Netherlands: KPC Onderwijs Innovatie Centrum.

Education Council. (2012). *Artikel 23 Grondwet in maatschappelijk perspectief* [Article 23 of the Constitution in societal perspective]. Den Haag, the Netherlands: Education Council of the Netherlands.

Educational Inspectorate. (2016). *Taal en rekenen aan het einde van het basisonderwijs* [Language and mathematics at the end of primary education]. Utrecht, the Netherlands: Educational Inspectorate.

Expertgroep Doorlopende Leerlijnen. (2008). *Over de drempels met taal en rekenen. Hoofdrapport van de Expertgroep Doorlopende Leerlijnen Taal en*

Rekenen [Crossing the thresholds with language and mathematics. Main report of the expert group continuous learning trajectories]. Enschede: Author.

Gardner, M. (1985). *The magic numbers of Dr. Matrix*. New York, NY: Prometeus Books.

Goodlad, J. (Ed.). (1979). *Curriculum inquiry*. New York, NY: McGraw-Hill.

Gravemeijer, K., Figueiredo, N., Feijs, E., Van Galen, F., Keijzer, R., & Munk, F. (2007). *Meten en meetkunde in de bovenbouw. Tussendoelen annex leerlijnen bovenbouw basisschool* [Measurement and geometry in the upper grades. Teaching-learning trajectory for the upper grades of primary school]. Groningen, the Netherlands: Wolters-Noordhoff.

Gravemeijer, K., Figueiredo, N., Feijs, E., Van Galen, F., Keijzer, R., & Munk, F. (2016). *Measurement and geometry in the upper primary school.* Rotterdam, the Netherlands/Tapei, Taiwan: Sense Publishers.

Hemker, B. (2016). *Peiling van de rekenvaardigheid, de taalvaardigheid en wereldoriëntatievaardigheden in jaargroep 8 van het basisonderwijs in 2015* [Survey on skills in mathematical, language and 'world orientation' in Grade 6 in 2015]. Arnhem, the Netherlands: Cito.

Hickendorff, M. (2011). *Explanatory latent variable modeling of mathematical ability in primary school.* Leiden, the Netherlands: Leiden University.

Hop, M. (Ed.). (2012). *Balans van het reken-wiskundeonderwijs halverwege de basisschool 5. Periodieke Peiling van het Onderwijsniveau* [Balance of mathematics education halfway primary school 5. Periodic Assessment of the Education Level]. Arnhem, the Netherlands: Cito.

Letschert, J. (1998). *Wieden in een geheime tuin. Een studie naar kerndoelen in het Nederlandse basisonderwijs* [Weeding in a secret garden. A study of core goals in Dutch primary education]. Enschede, the Netherlands: SLO.

Meelissen, M., Netten, A., Drent, M., Punter, R., Droop, M., & Verhoeven, L. (2012). *PIRLS en TIMSS 2011. Trends in leerprestaties in Lezen, Rekenen en Natuuronderwijs*. [PIRLS and TIMSS 2011. Trends in achievement in Reading, Mathematics and Science.] Enschede/Nijmegen: Twente University/Radboud University.

Ministry of Education. (1993/1998). *Kerndoelen basisonderwijs* [Core Goals Primary Education]. Den Haag, the Netherlands: Ministry of Education.

Ministry of Education. (2006). *Kerndoelen basisonderwijs* [Core Goals Primary Education]. Den Haag, the Netherlands: Ministry of Education.

Ministry of Education. (2007). Reken- en taalvaardigheid en doorlopende leerlijnen. Brief aan de Tweede Kamer [Mathematical and language skills and continual learning trajectories. Letter to the Parliament]. Den Haag, the Netherlands: Ministry of Education.

Ministry of Education. (2008). Beleidsreactie 'Tijd voor onderwijs' - brief aan de Tweede Kamer [Reaction to 'Time for education' Letter to the Parliament]. Den Haag, the Netherlands: Ministry of Education.

Ministry of Education. (2009). *Referentiekader taal en rekenen* [Reference Standards language and mathematics]. Den Haag, the Netherlands: Ministry of Education.

Ministry of Education. (2013). Artikel 23 Grondwet in maatschappelijk perspectief - brief aan de Tweede Kamer [Article 23 of the Constitution in a societal perspective - letter to the Parliament]. Den Haag, the Netherlands: Ministry of Education.

Ministry of Education. (2015). Meer ruimte voor nieuwe scholen: Naar een moderne interpretatie van artikel 23. Brief aan de Tweede Kamer [More room for new schools: Towards a modern interpretation of artikel 23. Letter to the Parliament]. Den Haag, the Netherlands: Ministry of Education.

Scheltens, F., Hemker, B., & Vermeulen, J. (2013). *Balans van het reken-wiskundeonderwijs aan het einde van de basisschool 5. Periodieke Peiling van het Onderwijsniveau* [Balance of mathematics education at the end of primary school 5. Periodic Assessment of the Education Level]. Arnhem, the Netherlands: Cito.

Schmidt, W., Houang, R., & Cogan, L. (2002). A coherent curriculum. The case of mathematics. *American Educator, 26*(2), 1–17.

Schmidt, W., Wang, H., & McKnight, C. (2005). Curriculum coherence: An examination of U.S. mathematics and science content standards from an international perspective. *Journal of Curriculum Studies, 35*, 525–559.

SLO. (2012a). *Kerndoelenanalyse Alles Telt* [Core goal analysis *Alles Telt*]. Enschede, the Netherlands: Netherlands Institute for Curriculum Development.

SLO. (2012b). *Kerndoelenanalyse De Wereld in Getallen* [Core goal analysis *De Wereld in Getallen*]. Enschede, the Netherlands: Netherlands Institute for Curriculum Development.

SLO. (2012c). *Kerndoelenanalyse Pluspunt* [Core goal analysis *Pluspunt*]. Enschede, the Netherlands: Netherlands Institute for Curriculum Development.

SLO. (2012d). *Kerndoelenanalyse Rekenrijk* [Core goal analysis *Rekenrijk*]. Enschede, the Netherlands: Netherlands Institute for Curriculum Development.

Thijs, A., & Van den Akker, J. (2009). *Curriculum in development*. Enschede, the Netherlands: Netherlands Institute for Curriculum Development.

Treffers, A., Van den Heuvel-Panhuizen, M., & Buys, K. (1999). *Jonge kinderen leren rekenen. Tussendoelen Annex Leerlijnen Hele Getallen Onderbouw Basisschool* [Young children learn mathematics. A learning-teaching trajectory with intermediate attainment targets for calculation with whole numbers for the lower grades of primary school]. Groningen, the Netherlands: Wolters-Noordhoff.

Valverde, G., Bianchi, L., Wolfe, R., Schmidt, W., & Houang, R. (2002). *According to the Book. Using TIMSS to investigate the translation of policy into practice through the world of textbooks*. Dordrecht, the Netherlands: Kluwer Academic Publishers.

Van den Akker, J. (2003). Curriculum perspectives: An introduction. In J. Van den Akker, W. Kuiper, & U. Hameyer (Eds.), *Curriculum landscapes and trends.* (pp. 1-10). Dordrecht, the Netherlands: Kluwer Academic Publishers.

Van den Heuvel-Panhuizen, M. (Ed.). (2008). *Children learn mathematics. A learning-teaching trajectory with intermediate attainment targets for calculation with whole numbers in primary school*. Rotterdam, the Netherlands / Tapei, Taiwan: Sense Publishers.

Van den Heuvel-Panhuizen, M., & Buys, K. (Eds.). (2004). *Jonge kinderen leren meten en meetkunde. Tussendoelen annex leerlijnen onderbouw basisschool* [Young children learn measurement and geometry. Teaching-learning trajectory for the lower grades of primary school]. Groningen, the Netherlands: Wolters-Noordhoff.

Van den Heuvel-Panhuizen, M., & Buys, K. (Eds.). (2008). *Young children learn measurement and geometry. A learning-teaching trajectory with intermediate attainment targets for the lower grades in primary school*. Rotterdam, the Netherlands/Tapei, Taiwan: Sense Publishers.

Van den Heuvel-Panhuizen, M., Buys, K., & Treffers, A. (Eds.). (2001). *Kinderen leren rekenen. Tussendoelen annex leerlijnen. Hele getallen bovenbouw basisschool* [Children learn mathematics. A teaching-learning trajectory for whole numbers in the upper grades of primary school]. Groningen, the Netherlands: Wolters-Noordhoff.

Van den Heuvel-Panhuizen, M., & Wijers, M. (2005). Mathematics standards and curricula in the Netherlands. *ZDM The International Journal on Mathematics Education, 37*(4), 287–307.

Van Galen, F., Feijs, E., Figueiredo, N., Gravemeijer, K., Van Herpen, E., & Keijzer, R. (2005). *Breuken, procenten, kommagetallen en verhoudingen. Tussendoelen annex leerlijnen bovenbouw basisschool* [Fractions, percentages, decimal numbers and ratio. Teaching-learning trajectory for the upper grades of primary school]. Groningen, the Netherlands: Wolters-Noordhoff.

Van Galen, F., Feijs, E., Figueiredo, N., Gravemeijer, K., Van Herpen, E., & Keijzer, R. (2008). *Fractions, percentages, decimal numbers and ratio. A learning-teaching trajectory for Grade 4, 5 and 6*. Rotterdam, the Netherlands /Tapei, Taiwan: Sense Publishers.

Van Zanten, M., & Van den Heuvel-Panhuizen, M. (2014). Freedom of design: the multiple faces of subtraction in Dutch primary school textbooks. In Y. Li & G. Lappan (Eds.), *Mathematics curriculum in school education*. (pp. 231–259). Dordrecht/Heidelberg/New York/London: Springer.

Textbook series

Bazen, K., Bokhove, J., Borghouts, C., Buter, A., Kuipers, K., & Veltman, A. (2009-2013). *Rekenrijk* [Realm of arithmetic / Rich arithmetic], 3rd edition. Groningen/Houten, the Netherlands: Noordhoff Uitgevers.

Huitema, S., Erich, L., Van Hijum, R., Nillesen, C., Osinga, H., Veltman, H., & Van de Wetering, M. (2009-2014). *De Wereld in Getallen* [The World in Numbers], 4th edition. Den Bosch, the Netherlands: Malmberg.

Van Beusekom, N., Fourdraine, A., & Van Gool, A. (Eds.). (2009-2013). *Pluspunt* [Plus Point], 3rd edition. Den Bosch, the Netherlands: Malmberg.

Van den Bosch-Ploegh, E., Van den Brom-Snijders, P., Hessing, S., Van Kraanen, H., Krol, B., Nijs-van Noort, J., Plomp, A., Sweers, W., & Vuurmans, A. (2009-2013). *Alles Telt* [Everything Counts], 2nd edition. Amersfoort, the Netherlands: ThiemeMeulenhoff.

Authors' contributions

This paper was a collaborative work of the two authors. Both authors participated in the selection of the curriculum documents and the textbooks, and in designing the research. MZ carried out the document analysis and the textbook analysis, of which the findings were frequently discussed with MH. MZ prepared the first draft of the manuscript. Both authors participated in revising the manuscript. Both authors read and approved the final manuscript.

Chapter 7

Summing up and conclusion

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1. The five studies in short

The aim of this PhD study was to get a better understanding of Dutch primary school mathematics textbooks regarding their contribution to the opportunity-to-learn (OTL). The reason to conduct it was that solid information was needed regarding content and didactics of textbooks, as indicated by the Royal Netherlands Academy of Sciences (KNAW, 2009). Thus, a practical goal of this study was to provide sound data on textbooks to the educational field, especially teachers, as the other crucial determinant of the OTL that is realized in schools and classrooms.

The studies that were carried out focused on the effect of different didactical approaches on the OTL, and historical developments regarding textbooks. Furthermore, the coherence of the mathematics curriculum as a whole was investigated. Contemporary textbooks and textbooks from the past were analyzed on the learning content included, the performance expectations articulated, and the learning facilitators incorporated. The whole PhD project consisted of five studies that are reported on in the chapters 2-6 of this thesis.

In the study included in *Chapter 2*, the consequences for the OTL of choosing a specific didactical approach were investigated. The study included two textbooks, based on different views on mathematics education: the RME-oriented textbook *Rekenrijk* (3rd edition, 2009) and the textbook *Reken Zeker* (2010), which was released as an alternative for the RME approach. The focus in this study was on subtraction up to 100 in Grade 2. Based on the content and performance expectations regarding this topic included in the formal intended curriculum, and on a mathedidactical analysis of it, an analysis framework was developed. We found more differences than similarities in the OTL the textbooks offer, not only in the learning facilitators provided, as could be expected, but also in content and performance expectations. Consequently, the textbooks appear to differ in their agreement with the intended curriculum.

Regarding content, in a deviation from the intended curriculum, *Reken Zeker* only addresses one semantic structure of subtraction: taking away. In contrast, *Rekenrijk* deals with both semantic structures of subtraction: determining the difference as well as taking away. With respect to performance expectations, in both textbooks, students are expected to learn the standard calculation method of direct subtraction combined with stringing. Only *Rekenrijk* expects students to also learn alternative calculations methods as required by the intended curriculum, such as indirect addition. Furthermore, only *Rekenrijk* contains explicit performance expectations regarding understanding of subtraction. Regarding learning facilitators, the textbooks differ in the degree and structure of exposure on subtraction up to 100. They also differ in the didactical support they provide, both quantitative and qualitative. The findings of this study make it clear that the freedom of design that results from the Dutch educational freedom can lead to textbooks that deviate from the intended curriculum, and that different didactical approaches as incorporated in textbooks may result in differences in OTL offered by those textbooks.

Chapter 3 addresses the OTL regarding problem solving in four textbooks in Grade 4 and 6. Three common Dutch textbooks were included: *De Wereld in Getallen* (4^{th} edition, 2009), *Pluspunt* (3^{rd} edition, 2009), and Alles Telt (2^{nd} edition, 2009). In addition, a textbook with a contrasting approach was included in this study: *Rekenwonders* (published in 2011), the adapted Dutch version of the Singaporean textbook *My Pals Are Here! Maths*. This textbook was purposely put on the Dutch market because of its emphasis on problem solving together with its different approach to mathematics education. In the study, the term 'problem' was defined as a "non-routine mathematical task for which students do not immediately have a particular solution strategy at their disposal." Thus, it was aimed for taking into account the relative and personal character of genuine problem solving. Because of this, a methodological challenge was to develop an analysis framework that would indicate when a task could be classified as a genuine problem-solving task and when not. To develop such a framework, several rounds of preliminary classifying tasks based on theoretical insights on problem solving were needed, in which tasks were classified in three categories: straightforward solvable tasks, non-routine problems, and—as an in between category—gray-area tasks. Eventually, based on these initial rounds and further theoretical insights, indicators were developed to be used for the definitive classification of the tasks. These indicators concerned features of tasks that can be expected to provoke or require analyzing, modeling and creative thinking and therefore contribute to the OTL regarding problem solving. Furthermore, a quantitative decision rule was developed and used for the definitive classification of tasks.

It was found that OTL regarding problem solving in the textbook series *De Wereld in Getallen*, *Pluspunt* and *Alles Telt* is low. These textbooks provide only a small number of problem-solving tasks, incorporate hardly any other ways to facilitate the learning of problem solving, and the problem-solving tasks that are provided are mainly included in the parts that are meant for the more able students. The textbook *Rekenwonders* offers more OTL regarding problem solving. This textbook provides the highest number of problem-solving tasks, systematically offers heuristics and other learning facilitators for problem solving, and moreover, includes most of the problem-solving tasks in the materials that are meant for all students. However, *Rekenwonders* also offers learning facilitators meant for problem solving in such a way that a tension occurs concerning the creative character of genuine problem solving. All in all, the findings of this study show that OTL regarding problem solving as provided by textbooks is for a vast majority of Dutch students very limited. Furthermore, the findings show that in textbook analysis, the organizational structure of the textbook has to be taken into account, and the quality of the incorporated learning facilitators has to be evaluated. Finally, similar to the study in Chapter 2, this study again shows that textbooks based upon different approaches, may also differ in the OTL they provide.

Chapter 4 describes a historical study in which it was investigated what has changed in Dutch textbooks since the start of RME regarding for the approach to decimal numbers in Grade 4, 5 and 6. Three textbooks were analyzed: the RME-oriented textbook *De Wereld in Getallen* (4th edition, 2009), and two pre-RME textbooks, namely *Functioneel Rekenen* (published in 1958) and *Nieuw Rekenen* (published in 1969). The analysis framework developed in this study is based on a characterization of the RME approach to decimal numbers. This characterization, in its turn, is based upon RME publications from Freudenthal and other researchers within the RME tradition. These publications show that the RME approach is not only reflected in the use of particular learning facilitators, but also in the inclusion of particular content (i.e., measurement decimal numbers) and performance expectations (i.e., mental calculation and estimation with decimal numbers).

Regarding content, the study revealed that in the contemporary textbook *De Wereld in Getallen* measurement decimal numbers are used in multiple ways to support students' understanding of decimal numbers. The two older textbooks also offer tasks with measurement decimal numbers, but the proportion of these tasks is very low. With respect to performance expectations, it was found that in the contemporary textbook *De Wereld in Getallen* a substantial proportion of decimal numbers tasks is about either mental calculation or estimation. The attention to mental calculation was also found to be present in both older textbooks, *Functioneel Rekenen* and *Nieuw Rekenen*, whereas substantial attention for estimation was only found in *Nieuw Rekenen*. Concerning learning facilitators, the RME characteristic of using contexts as a source for learning is present in the contemporary textbook *De Wereld in Getallen*, but absent in the two older textbooks. A similarity in all three textbooks is the use of different calculation methods and students' own productions as learning facilitators. From the findings, it was concluded that the contemporary RME-oriented textbook *De Wereld in Getallen* surprisingly does not include all RME characteristics in its approach to decimal numbers. Another remarkable finding is that onsets to RME characteristics were clearly present in the two textbooks dating from the time before RME came into being, which suggests that the roots of RME go back farther than its start in the late 1960s.

The history of RME was further investigated in two studies reported on in *Chapter 5*. The studies concerned two curriculum levels and their mutual relationship: the ideal intended level and the textbook level. The focus was on learning facilitators for early addition and subtraction in Grade 1, 2 and 3.

In the study on the intentions of RME, three core curriculum documents were studied: the first Wiskobas overview of primary school mathematics education (De Jong, Treffers & Wijdeveld, 1975), the publication *Proeve van een nationaal programma voor het reken-wiskundeonderwijs op de basisschool* (Design of a national program for mathematics education in primary school) (Treffers & De Moor, 1990), and the TAL Teaching and learning trajectory document meant for teaching mathematics in the lower grades of primary school (Treffers, Van den Heuvel-Panhuizen & Buys, 1999). Both differences and similarities were found between these documents. The documents differ especially regarding the models suggested. The main changes include that the one hundred square is explicitly promoted in Wiskobas, but no longer recommended in the Proeve-document, and finally, TAL suggests not to use it. The number line evolved from a segmented line in Wiskobas to an empty line in Proeve and TAL, which provides opportunities for structured solution procedures and makes way for using students' own constructions of varied calculation strategies. Another remarkable change concerns the intertwinement of addition and subtraction, which is only emphasized in the oldest document Wiskobas and no longer in the more recent ones.

In the second study in this chapter, thirteen RME-oriented textbooks released from 1981 to 2019 were analyzed. It was found that RME learning facilitators mentioned in the core documents generally are also present in these textbooks. In most cases, the use of these learning facilitators in all or most RME-oriented textbooks corresponds with their intended use as indicated in the RME core documents. This applies for example to the use of context situations for providing meaning, the use of the empty number line model, and the use of students' own productions of problems. In several other cases, deviations of textbooks from original RME intentions were found. A striking example of this is the use of arrow language. All textbooks that suggest its use apply it as a filling-in exercise, which is almost the opposite of the original intention of letting students use arrow language to describe what is mathematically happening in a certain situation. From the findings of the second study in this chapter it is concluded that on the one hand, over the course of fifty years, RME was and is still clearly present in Dutch primary school textbooks. On the other hand, this does not mean that these textbooks are fully in consonance with RME's intentions. The mere presence of an RME learning facilitator does not tell the whole story—also the way in which it is used has to be taken into account. This conclusion is in line with the findings regarding learning facilitators for problem solving described in chapter 3.

Chapter 6 reports on a descriptive study on the coherence within the mathematics curriculum as a whole. For this, again, documents regarding the intended curriculum were included in the research; in this study, documents from the formal intended curriculum. These are the legally established *Kerndoelen* (Core Goals) (Ministry of Education, 2006) and *Referentiekader* (Reference Framework) (Ministry of Education, 2009). The textbooks included in this study were the four most used ones: *De Wereld in Getallen* (4^{th} edition 2009), *Pluspunt* (3^{rd} edition, 2009), *Alles Telt* (2^{nd} edition, 2009) and *Rekenrijk* (3^{rd} edition, 2009). Furthermore, next to these textbooks, other documents were included that can be regarded as part of the potentially implemented curriculum. The intended and the potential implemented curriculum were studied and compared regarding multiple mathematical learning topics, including differentiated content and, because of the findings described in Chapter 3 on problem solving, OTL for different groups of students.

Regarding the intended curriculum, the Core Goals and the Reference Framework generally describe the same content and performance expectations. Both documents leave room for interpretation. There are several overarching competences emphasized in the Core Goals, which are also included in the Reference Framework. However, problem solving has a less prominent place in the Reference Framework than in the Core Goals, and other competences mentioned in the Core Goals are absent in the Reference Framework. This is the case for asking mathematical questions, using mathematical literacy, and giving and receiving criticism as a mathematical method. Furthermore, issues regarding attitudes mentioned in the Core Goals, such as feeling challenged and doing mathematics with satisfaction and pleasure, are also not referred to in the Reference Framework. Thus, albeit that, compared to the Core Goals document, the Reference Framework is more detailed in its descriptions, it is more limited with respect to mathematical attitude and overarching competence foci.

With respect to the coherence of the two included curriculum levels, all documents of the potentially implemented curriculum generally correspond with the intended curriculum. On a detailed level, differences are found. Textbooks differ not only in the learning facilitators they provide, but also in content and performance expectations they include, which again shows that textbooks differ in the OTL they offer.

An important issue raised in this study is about the fact that the Reference Framework distinguishes two different levels to be achieved at the end of primary school by different groups of students. The “1S- level” (target level) is meant for the majority of students and the “1F- level” (fundamental or minimum level) for students for which the 1S-level appears to be too difficult. All four textbooks have incorporated these levels by including differentiated tasks. However, whether such differentiated learning routes within textbooks do indeed lead to the mastery of the targeted levels is not known.

2. Main findings and practical implications

The findings of the studies in this thesis show that Dutch textbooks differ greatly in their contribution to the OTL that students are offered. On all three researched features of textbooks—content, performance expectations and learning facilitators—differences were established, both quantitative and qualitative. This applies to the relatively straightforward learning topic of subtraction up to 100 (Chapter 2), as well as to the overarching mathematical competence of non-routine problem solving (Chapter 3). These findings show that important differences in OTL provided by textbooks not only appear in textbooks from different countries and different educational traditions (e.g., Haggarty & Pepin, 2002; Pepin & Haggarty, 2001), but also between textbooks within a country. That said, the study regarding problem solving (Chapter 3) also confirms the existence of differences in OTL in textbooks from different educational traditions, since one textbook in this study was adapted from Singapore, that has problem solving positioned as the center of its intended curriculum, which is in sharp contrast with the intended curriculum of the Netherlands, that pays only limited attention to problem solving.

Another important difference in OTL that comes to the fore in this thesis appears at the national level. Not all students are exposed to the same content and performance expectations. Differences in these occur at the potentially implemented level of the textbook (Chapter 3 and Chapter 6) as well as the formal intended curriculum level (Chapter 6).

Regarding the latter, no distinction was made in differential goals for primary school before the legal establishment of the Reference Framework in 2010 (see Ministry of Education, 1993, 1998, 2006). The reason that differentiated goals were introduced in the intended curriculum was that the Reference Framework was meant as a measure to improve learning outcomes. Therefore, not only a minimum level (the 1F-level) was defined, but also a higher level to aim for (the 1S-level) (Expertgroep Doorlopende Leerlijnen, 2008). However, this differentiation between goals at different levels meant for different students, implies that attainment goals to aim for are not meant for all students.

Another measure that was considered at that time to enhance learning outcomes was to also establish attainment goals for halfway through primary school, or for the end of the kindergarten years. This was eventually not implemented, because it would increase the chance that students would be predetermined as lesser-able or more-able at a very young age (ibid.). Unfortunately, the organization in textbooks of content and performance expectations into different levels of difficulty from Grade 1 on (Chapter 6) readily facilitates such a predetermination of students, resulting in different OTL. Moreover, letting students work in textbook materials that are visually recognizable as meant for lesser-able students from a young age on may well function as a self-fulfilling prophecy.

There are more correspondences between the textbooks and the formal intended curriculum level than the lack of attention for problem solving and the differentiated goals for different students. At a general level, textbooks appear to be well-aligned with the formal intended curriculum (Chapter 6). Textbooks include content and performance expectations from all mathematical domains described in the statutory Core Goals and Reference Framework. However, on a more detailed level, textbooks turn out to differ in their agreement with the intended curriculum (Chapter 2 and Chapter 6). A factor of influence on this appears to be the didactical approach incorporated in the textbook. Textbooks based upon different approaches do not only differ regarding learning facilitators (Chapter 2 and Chapter 3), but also on content and performance expectations (Chapter 2 and Chapter 4). Apparently, the didactical approach incorporated in a textbook may affect the offered learning content. As a consequence, the strict distinction between the what and how of education that the Dutch government strives for (e.g., Committee Parliamentary Research Education, 2008; Ministry of Education, 2008) may not always be so easy to make in mathematics education.

Goodlad (1979) already stated that relationships between different representations of curriculum are interpretations. Thus, different levels of curriculum are not necessarily in agreement with each other. In this light, the finding that textbooks differ in the OTL they offer regarding particular learning content included in the formal intended curriculum is in itself not so remarkable. However, the implication of that finding is important. Since it cannot be blindly taken on trust that textbooks offer all the statutorily intended content and performance expectations, textbook analysis is and remains a necessity, especially in the Dutch context of freedom of education.

To conclude, a similar conclusion as the one regarding the formal intended curriculum can be drawn regarding the ideal intended curriculum. Globally seen, there are clear correspondences between RME-oriented textbooks and curricular intentions of RME (Chapter 5), but looking into more detail, these textbooks show several deviations from RME intentions (Chapter 4 and 5). Also, mutual differences appear between RME-oriented textbooks regarding particular RME learning facilitators (Chapter 5). In the 1980s, it was already established that RME-oriented textbooks at the time showed both similarities with the RME-approach as well as deviations from it (De Jong, 1986; Gravemeijer, Van den Heuvel-Panhuizen, et al., 1993). The findings presented in this thesis show that the conclusion that RME-oriented textbooks are not fully aligned with RME intentions can be drawn generally. This means that based upon only a particular RME-oriented textbook, no general statements can be made regarding RME—which is unfortunately enough frequently done in the Netherlands.

3. Suggestions for further research

Studies on mathematics textbooks can be classifies in several categories (e.g., Fan, Zhu & Miao, 2013, Veilande, 2017). The research presented in this thesis falls into the category of textbook analysis and comparison. The relevance of this type of research is that it reveals learning opportunities, but it also has its limitations (Fan, Zhu, & Miao, 2013). It addresses the textbook in itself, while in the educational context a textbook does not stand on its own (see, e.g., Grevholm, 2012; Gueudet, Pepin, & Trouche, 2012; Rezat & Sträβer, 2012; Van den Ham & Heinze, 2018). Especially the way in which the teacher uses the textbook determines whether the OTL as offered by the textbook is realized into an actual OTL in the mathematics lesson. The same applies to lacking OTL in textbooks, which may be reinforced or compensated for by the teacher. For the latter it seems necessary that the teacher is aware of the shortcomings of a textbook. This raises the question whether teachers are equipped to establish these shortcomings, such as the inconsistencies that may occur between calculation method and didactical model as incorporated in certain textbooks (Chapter 2). Another question is to what degree differences in OTL between textbooks as established in this thesis lead to different learning outcomes between students taught with these textbooks.

A second question on teachers' use of the textbook concerns the organizational structure of most Dutch textbooks into different levels of difficulty (Chapter 3 and Chapter 6), which may result in ability grouping and withholding particular OTL from students. Whether or not this actually occurs, depends on how the teacher handles this structure of textbooks. Fixed grouping throughout the grade years could likely result in predetermination of students, while flexible and temporarily grouping may actually lead to enhanced OTL. Therefore, an important follow-up research question concerns how teachers actually handle this organizational feature of textbooks.

Regarding the attained curriculum, in Chapter 6, it was mentioned that students' performance on the End of Primary School Test (Educational Inspectorate, 2016) suggests that, although a vast majority of students masters the minimum 1F-level as established in the Reference Framework, only a minority also masters the 1S-level that is supposed to be aimed for. This finding has been reproduced since then (Educational Inspectorate, 2018, 2020). This confirms the importance of the abovementioned follow-up research question. Furthermore, the question raised in Chapter 6 whether the 1S-level is incorporated well enough in the textbooks certainly remains to be answered.

A general limitation of textbook analysis is that the findings are not applicable to other textbooks than the ones analyzed. To determine the OTL of newly published textbooks, new analysis will have to be carried out. The findings of the studies in this thesis suggest that the distinction made into the three main textbook features of content, performance expectations and learning facilitators, is sufficient to determine OTL regarding varying learning topics and all Grades 1 to 6.

A final suggestion for further research concerns the deviations in textbooks from the intended curriculum—both the formally intended and the ideally intended curriculum. It could be interesting to investigate how decisions on textbooks regarding the inclusion or exclusion of particular content, performance expectations and learning facilitators are realized. This could provide useful information in order to accomplish a better alignment of textbooks with curricular intentions.

4. Final remarks

As mentioned in the introduction of this thesis, due to the freedom of education, the government does not judge the quality or the content of textbooks. This judgement is up to the schools (e.g., Ministry of Education, 2019). However, it can be questioned whether schools and teachers are up to this task. It is already difficult for schools to choose textbooks and other curriculum materials that are in alignment with their own vision and ideas on education (Kennisnet & SLO, 2017). The process of choosing these resources is mostly gone through in an "unconsciously incompetent way" (ibid., p. 9). Bottlenecks in judging textbooks that teachers themselves experience, as recently indicated in a professional journal (Van Nieuwstadt, 2019), include an overload of commercial information, a lack of objective information, a lack of time, and workload.

The studies in this thesis provide valid information on textbooks, but time is unstoppable and new textbooks and other curriculum resources will be released. As shown in this thesis, looking into a textbook in detail may reveal insights that otherwise remain hidden. A superficial check on a textbook is not sufficient to get a good impression of the OTL it offers. To achieve a valid judgement of the quality of a textbook, thorough research is needed. Schools and teachers would be helped if such research was carried out more regularly.

References

Committee Parliamentary Research Education. (2008). *Tijd voor onderwijs* [Time for education]. Den Haag, the Netherlands: Sdu Uitgevers.

De Jong, R. (1986). *Wiskobas in methoden* [Wiskobas in textbook series]. Utrecht, the Netherlands: Utrecht University.

Educational Inspectorate. (2016). *Taal en rekenen aan het einde van het basisonderwijs* [Language and mathematics at the end of primary education]. Utrecht, the Netherlands: Educational Inspectorate.

Educational Inspectorate. (2018). *Taal en rekenen aan het einde van de basisschool 2016-2017* [Language and mathematics at the end of primary education 2016-2017]. Utrecht, the Netherlands: Educational Inspectorate.

Educational Inspectorate. (2020). *Peil taal en rekenen einde basisonderwijs 2018-2019* [Language and mathematics at the end of primary education 2018-2019]. Utrecht, the Netherlands: Educational Inspectorate.

Expertgroep Doorlopende Leerlijnen. (2008). *Over de drempels met taal en rekenen. Hoofdrapport van de Expertgroep Doorlopende Leerlijnen Taal en Rekenen* [Crossing the thresholds with language and mathematics. Main report of the expert group continuous learning trajectories]. Enschede, the Netherlands: Author.

Fan, L., Zhu, Y., & Miao, Z. (2013). Textbook research in mathematics education: development status and directions. *ZDM Mathematics Education, 45*(5), 633–646.

Goodlad, J. (1979). Curriculum Inquiry. The study of curriculum practice. New York, NY: McGraw-Hill.

Grevholm, B. (2012). Theoretical framework for research on textbooks. Retrieved from https://textbookstudy.files.wordpress.com/2013/05/theoreticalframework.pdf

Gueudet, G., Pepin, B., & Trouche, L. (2012). (Eds.). *From text to 'lived' resources. Mathematics curriculum materials and teacher development.* doi:10.1007/978-94-007-1966-8

Haggarty, L., & Pepin, B. (2002). An investigation of mathematics textbooks and their use in English, French and German Classrooms: Who gets an opportunity to learn what? *British Educational Research Journal, 28*(4), 567–590.

Kennisnet & SLO. (2017). *Eindrapport Verkenning proces leermiddelenkeuze* [Final report on the process of choosing curriculum resources]. Retrieved from https://www.slimmerlerenmetict.nl/document/eindrapport-verkenning-proces-leermiddelenkeuze

Ministry of Education. (1993). *Kerndoelen basisonderwijs* [Core Goals Primary Education]. Den Haag, the Netherlands: Ministry of Education.

Ministry of Education. (1998). *Kerndoelen basisonderwijs* [Core Goals Primary Education]. Den Haag, the Netherlands: Ministry of Education.

Ministry of Education. (2006). *Kerndoelen basisonderwijs* [Core Goals Primary Education]. Den Haag, the Netherlands: Ministry of Education.

Ministry of Education. (2008). Beleidsreactie 'Tijd voor onderwijs' - brief aan de Tweede Kamer [Reaction to 'Time for education' - letter to the Parliament]. Den Haag, the Netherlands: Ministry of Education.

Ministry of Education. (2009). *Referentiekader taal en rekenen* [Rerence Standards language and mathematics]. Den Haag, the Netherlands: Ministry of Education.

Ministry of Education. (2019). Kamervragen (Aanhangsel) 2019-2020, nr. 894 [Answers to Parliamentary questions 2019-2020 nr. 894]. Retrieved from https://zoek.officielebekendmakingen.nl/ah-tk-20192020-894.html

Pepin, B., & Haggarty, L. (2001). Mathematics textbooks and their use in English, French and German classrooms: a way to understand teaching and learning cultures. *ZDM Mathematics Education, 33*(5), 158–175.

Rezat, S., & Sträβer, S. (2012). From the didactical triangle to the socio-didactical tetrahedron: Artifacts as fundamental constituents of the didactical situation. *ZDM Mathematics Education 44*, 641–651.

Stein, M., Remillard, J., & Smith, M. (2007). How curriculum influences student learning. In F. Lester (Ed.). *Second handbook of research on mathematics teaching and learning* (Vol. 1, pp. 319–369). Charlotte, NC: Information Age Publishing.

Treffers, A., & De Moor, E. (1990). *Proeve van een nationaal programma voor het reken-wiskundeonderwijs op de basisschool 2. Basisvaardigheden en cijferen* [Design of a national program for mathematics education in primary school 2. Basic operations and algorithmic calculation]. Tilburg, the Netherlands: Zwijsen.

Treffers, A., Van den-Heuvel-Panhuizen, M., & Buys, K. (1999). *Jonge kinderen leren rekenen. Tussendoelen annex leerlijnen* [Young children learn mathematics. Intermediate attainment targets and teaching-learning trajectories]. Groningen, the Netherlands: Wolters-Noordhoff.

Van den Ham, A., & Heinze, A. (2018). Does the textbook matter? Longitudinal effects of textbook choice on primary school students' achievement in mathematics. *Studies in Educational Evaluation, 59*, 133–140.

Van Nieuwstadt, M. (2019). Keuzestress op de schoolboekenmarkt [Choice stress on the textbook market]. *Onderwijsblad, 2019-02*, 32–36.

Veilande, I. (2017). The characteristics of mathematics textbook research: A meta-study of papers from ICME-10, ICME-11, and ICME-12. In B. Grevholm (Ed.), *Mathematics textbooks, their content, use and influences. Research in Nordic and Baltic countries* (pp. 471-494. Oslo, Norway: Cappelen Damm Akademisk.

Samenvatting

Methodes vervullen een belangrijke rol in het reken-wiskundeonderwijs. Ze slaan als het ware de brug tussen het beoogde curriculum—de intenties en doelen van onderwijs—en het geïmplementeerde curriculum—het daadwerkelijk gerealiseerde onderwijs. Vanwege deze bemiddelende rol tussen intenties en dagelijkse schoolpraktijk worden methodes wel aangeduid als het *potentieel geïmplementeerde curriculum*. Doordat methodes bovendien door een meerderheid van de Nederlandse basisschoolleraren getrouw worden gevolgd, zijn ze van grote invloed op de *opportunity-to-learn* (OTL)[1] die wordt geboden in de reken-wiskundeles. Het doel van dit promotieonderzoek was inzicht te verschaffen in de bijdrage die Nederlandse methodes leveren aan die geboden OTL. Daarbij werden de relaties met het beoogde curriculum betrokken, zowel met het *formele* beoogde curriculum in de vorm van wettelijk vastgestelde doelen, als met andere intenties van reken-wiskundeonderwijs. Dat laatste werd betrokken in de vorm van de bedoelingen en ideeën van realistisch reken-wiskundeonderwijs (RME)[2], hetgeen kan worden beschouwd als een verschijningsvorm van een *ideaal* beoogd curriculum.

Een belangrijk deel van het onderzoek betrof methodeanalyses. Deze richtten zich op de eigenschappen van methodes die hun bijdrage aan de OTL in de reken-wiskundeles bepalen. Drie categorieën eigenschappen werden onderscheiden. De eerste categorie betrof de inhoud die in methodes worden aangeboden. Ten tweede werd geanalyseerd op prestatieverwachtingen, oftewel wat leerlingen volgens de methode geacht worden te doen met de aangeboden inhoud. De derde categorie omvatte alles wat de methode biedt het leren van de leerlingen te faciliteren, zoals een bepaalde opbouw van de leerstof of specifieke didactische ondersteuning. Deze categorie werd aangeduid met *learning facilitators*.[3] Het gehele project besloeg vijf studies, opgenomen in de hoofdstukken 2 tot en met 6 van dit proefschrift.

[1] Opportunity-to-learn kan worden vertaald als gelegenheid om te leren. In deze samenvatting wordt de internationaal gebruikelijke afkorting OTL aangehouden.

[2] RME is de afkorting van de internationale aanduiding Realistic Mathematics Education.

[3] Bij gebrek aan een Nederlandse term wordt in deze samenvatting de Engelse term gebruikt.

In de studie beschreven in *Hoofdstuk 2* zijn de consequenties onderzocht van een specifieke didactische aanpak voor de OTL. Twee methodes, gebaseerd op verschillende opvattingen over reken-wiskundeonderwijs, werden geanalyseerd en vergeleken: *Rekenrijk* (3e editie, 2009), die uitgaat van RME en *Reken Zeker* (2010), die werd uitgegeven als een alternatief voor RME. Van beide methodes werd de aanpak voor aftrekken in groep 4 onderzocht.

De analyseresultaten lieten meer verschillen dan overeenkomsten zien in de OTL die de methodes bieden. Niet alleen, zoals kon worden verwacht, betreffende learning facilitators, maar ook qua inhoud en prestatieverwachtingen. *Rekenrijk* bleek meer overeen te komen met het beoogde curriculum dan *Reken Zeker*. Qua inhoud bijvoorbeeld, biedt alleen *Rekenrijk* zowel aftrekken als wegnemen en aftrekken als het verschil bepalen aan. Een voorbeeld van prestatieverwachtingen is dat weliswaar beide methodes de standaardaanpak direct aftrekken middels de rijgstrategie aanbieden, maar dat alleen *Rekenrijk* ook andere aanpakken aanbiedt die vermeld zijn in het beoogd curriculum, zoals indirect optellen.

Verschillen tussen de methodes in de learning facilitators die ze aanbieden betreffen de hoeveelheid aftrekopgaven, de opbouw die daarin zit, en de didactische ondersteuning die daarbij geboden wordt. In bijvoorbeeld de modellen die de methodes aanbieden zit een opvallend verschil: *Rekenrijk* biedt de lege getallenlijn aan om het leren van de rijgstrategie te ondersteunen, terwijl *Reken Zeker* voor diezelfde strategie (tekeningen van) M.A.B.-materiaal basis 10 aanbiedt. Dat laatste model is echter niet consistent met de rijgstrategie: M.A.B.-materiaal basis 10 heeft een kardinale structuur, terwijl bij de rijgstrategie, die neerkomt op het (verkort) bewegen in de getallenrij, gebruik wordt gemaakt van het ordinale getalsaspect. Een soortgelijke inconsistentie komt in *Rekenrijk* overigens voor bij opgaven over indirect optellen als aftrekprocedure; in sommige gevallen correspondeert de daarbij geboden symbolische representatie en/of de presentatie op de lege getallenlijn niet met deze procedure.

Al met al laten de bevindingen van deze studie zien dat verschillen in visie op reken-wiskundeonderwijs kunnen resulteren in méér dan enkel een verschillende didactische uitwerking in methodes. De twee methodes betrokken in deze studie verschillen op álle onderscheiden eigenschappen die van invloed zijn op de OTL en verschillen bovendien in hun overeenstemming met het formele beoogde curriculum.

Hoofdstuk 3 handelt over de OTL betreffende wiskundig probleemoplossen. Drie veelgebruikte methodes werden onderzocht: *De Wereld in Getallen* (4e editie, 2009), *Pluspunt* (3e editie, 2009), and *Alles Telt* (2e editie, 2009). Daarnaast werd een methode met een contrasterende benadering van rekenen-wiskunde onderzocht: *Rekenwonders* (gepubliceerd in 2011), dat een aangepaste versie is van de Singaporese methode *My Pals Are Here! Maths*. Deze methode was geïntroduceerd in Nederland vanwege de nadruk op probleemoplossen en de andere benadering van reken-wiskundeonderwijs.

Een 'probleem' werd in deze studie opgevat als een niet-routinematig oplosbare opgave, waar de leerling die gesteld wordt voor deze opgave, niet direct een aanpak voor ter beschikking heeft. Gebaseerd op theoretische inzichten over probleemoplossen en kenmerken van probleemopgaven, en op meerdere initiële analyserondes, werden indicatoren en beslisregels ontwikkeld voor classificatie van opgaven in drie categorieën: rechttoe-rechtaan oplosbare opgaven, niet-routinematig oplosbare probleemopgaven en opgaven die daar tussenin zitten, aangeduid als *gray-area* opgaven.

Het bleek dat de OTL ten aanzien van probleemoplossen gering is in *De Wereld in Getallen*, *Pluspunt* en *Alles Telt*. Deze methodes bieden weinig niet-routinematige probleemopgaven aan, bieden nauwelijks learning facilitators aan voor probleemoplossen, en de probleemopgaven díe worden aangeboden, zijn voornamelijk opgenomen in de gedeeltes die enkel bedoeld zijn voor de betere leerlingen. De methode *Rekenwonders* biedt de meeste niet-routinematige probleemopgaven aan, biedt systematisch heuristieken aan en andere learning facilitators voor probleemoplossen, en de meeste aangeboden probleemopgaven zijn bovendien opgenomen in de gedeeltes die bedoeld zijn voor álle leerlingen. Echter, de manier waarop *Rekenwonders* learning facilitators aanbiedt voor probleemoplossen staat op gespannen voet met het creatieve karakter van echt wiskundig probleemoplossen.

De resultaten van deze studie tonen aan dat voor een grote meerderheid van de Nederlandse basisschoolleerlingen de OTL ten aanzien van probleemoplossen zeer beperkt is. Een opbrengst van methodologische aard is de constatering dat bij methodeanalyses rekening moet worden gehouden met de organisatorische structuur van de methodes, om te kunnen beoordelen voor welke leerlingen de aangeboden OTL van toepassing is. Ten slotte laten de bevindingen zien, net als die beschreven in Hoofdstuk 2, dat methodes gebaseerd op verschillende benaderingen van reken-wiskundeonderwijs, ook kunnen verschillen in de OTL die ze bieden.

Hoofdstuk 4 gaat over veranderingen in methodes met de komst van RME. De door RME beïnvloede methode *De Wereld in Getallen* (4^e editie, 2009) werd vergeleken met twee methodes van voor de start van RME: *Functioneel Rekenen* (gepubliceerd in 1958) en *Nieuw Rekenen* (gepubliceerd in 1969). De focus lag op decimale getallen in groep 6, 7 en 8. De methodes werden geanalyseerd op RME kenmerken specifiek voor dit onderwerp, inclusief de aanwezigheid van inhoud en prestatieverwachtingen die binnen RME meer nadruk kregen dan voorheen, namelijk meetgetallen als verschijningsvorm van decimale getallen, en hoofdrekenen en schatten met decimale getallen.

De methodes bleken sterk te verschillen in de aanwezigheid van meetgetallen als decimale getallen. In *De Wereld in Getallen* zijn decimale meetgetallen ruim aanwezig en worden, in overeenstemming met RME, op veel manieren gebruikt, bijvoorbeeld om het continue karakter van decimale getallen te illustreren. De twee oudere methodes bieden decimale meetgetallen maar weinig aan. Het enige gebruik ervan dat in alle drie de methodes voorkomt, is het gebruik van meetgetallen om de positiewaarde van decimalen toe te lichten.

De overeenstemming van *De Wereld in Getallen* met RME komt ook tot uitdrukking in de ruime aandacht voor hoofdrekenen en schatten met decimale getallen. Hoofdrekenen met decimale getallen is ook aanwezig in de twee oudere methodes, maar substantiële aandacht voor schatten met decimale getallen is er verder alleen in *Nieuw Rekenen*. Het didactische gebruik van schatten om bij precieze berekeningen met decimale getallen de komma op de juiste plek in het antwoord te zetten, werd overigens aangetroffen in alle drie de methodes.

Betreffende RME learning facilitators lieten de bevindingen een wisselend beeld zien. Het gebruik van contexten als uitgangspunt voor het leren van decimale getallen bijvoorbeeld, is alleen aanwezig in *De Wereld in Getallen*, en niet in de twee oudere methodes. De getallenlijn als model daarentegen wordt niet alleen in *De Wereld in Getallen* gebruikt, maar ook al in *Nieuw Rekenen*. Er werden ook RME learning facilitators in alle drie de methodes aangetroffen, bijvoorbeeld het laten maken van eigen producties door leerlingen.

De resultaten van deze studie laten zien dat ten aanzien van decimale getallen, enerzijds niet alle RME kenmerken aanwezig zijn in *De Wereld in Getallen*, en anderzijds dat bepaalde RME kenmerken al duidelijk aanwezig zijn in de twee methodes die dateren uit de tijd voordat RME ontstond. Dit laatste suggereert dat RME niet in alle opzichten een breuk met het verleden was.

De geschiedenis van RME werd verder onderzocht in twee studies beschreven in *Hoofdstuk 5*, waarbij de focus lag op learning facilitators voor optellen en aftrekken in groep 3, 4 en 5. In de eerste studie werden drie RME-curriculumdocumenten bestudeerd: een *Wiskobas* leerplanpublicatie uit 1975, de *Proeve van een nationaal programma voor het rekenonderwijs op de basisschool* uit 1990, en de *Tussendoelen annex Leerlijnen* (TAL) publicatie over hele getallen in de onderbouw uit 1999. Deze documenten beschrijven originele intenties en ideeën van RME, en kunnen als zodanig worden beschouwd als een representatie van het ideale beoogde RME-curriculum. Alle suggesties in deze documenten voor learning facilitators voor optellen en aftrekken werden in kaart gebracht en gecategoriseerd. Er werden vier categorieën learning facilitators aangetroffen, die in elk van de documenten aanwezig bleken: het gebruik van de realiteit als bron voor het reken-wiskundeonderwijs, het gebruik van modellen, het gebruik van de eigen inbreng van leerlingen, en het aanbieden van niet-routinematige problemen. Binnen deze categorieën werden naast overeenkomsten ook verschillen tussen de documenten aangetroffen. De verschillen betroffen met name de voorgestelde modellen. Zo wordt het honderdveld als model expliciet gepromoot in het *Wiskobas* document, niet langer aanbevolen in de *Proeve*, en ten slotte afgeraden in *TAL*. Een ander opvallend verschil is de verandering van de streepjesgetallenlijn in *Wiskobas* naar de lege getallenlijn in de *Proeve* en in *TAL*. Met de lege getallenlijn kunnen leerlingen onder meer worden ondersteund in het gebruiken van hun eigen constructies van oplossingsprocedures, een RME-kenmerk dat voor het eerst in de *Proeve* wordt genoemd.

In de tweede studie werd geanalyseerd op welke wijze de RME-intenties in methodes tot uitdrukking kwamen. Hiervoor werden dertien door RME beïnvloede methodes geanalyseerd die verschenen van 1981 tot en met 2019: vijf opeenvolgende edities van *De Wereld in Getallen* (gepubliceerd in 1981, 1991, 2001, 2009 en 2019), vier edities van *Pluspunt* (uit 1991, 2000, 2009 en 2019), de methode *Rekenen & Wiskunde* (gepubliceerd in 1983) en diens opvolger *Wis en Reken* (de editie van 2000), en twee edities van *Rekenrijk* (uit 2000 en 2009). De RME learning facilitators uit *Wiskobas*, de *Proeve* en *TAL* bleken over het algemeen ook aanwezig te zijn in deze methodes. Ook de wijze waarop deze learning facilitators worden gebruikt in de methodes komt meestal overeen met het beoogde gebruik zoals beschreven in de RME-curriculumdocumenten. Voorbeelden hiervan zijn het gebruik van contextsituaties, het gebruik van de lege getallenlijn, en het laten maken van eigen producties door leerlingen. In verschillende andere gevallen echter, bleken de methodes af te wijken van de originele RME-intenties. Een treffend voorbeeld hiervan is het gebruik van pijlentaal. Alle methodes die pijlentaal aanbieden, doen dit als invuloefening. Dat is vrijwel het tegenovergestelde van de oorspronkelijke bedoeling uit *Wiskobas*, namelijk dat leerlingen zelf actief pijlentaal gebruiken om te noteren wat er wiskundig gezien gebeurt in een situatie.

Uit de bevindingen beschreven in dit hoofdstuk blijkt dat RME vijftig jaar lang duidelijk herkenbaar aanwezig was in reken-wiskundemethodes en dat nog steeds is. Dat betekent echter niet dat deze methodes in alle opzichten overeenstemmen met de bedoelingen van RME. Dat een RME learning facilitator aanwezig is in een methode zegt niet alles—het gebruik ervan kan op gespannen voet staan met het beoogde gebruik ervan volgens de oorspronkelijke RME-intenties.

Omdat curriculaire coherentie van doorslaggevende betekenis is voor de OTL, werd de coherentie van het reken-wiskundecurriculum onderzocht in de studie die wordt beschreven in *Hoofdstuk 6*. In deze studie werden documenten betrokken die samen het formele beoogde curriculum en het potentieel beoogde curriculum breed beslaan. Dit waren de Kerndoelen uit 2006 en het Referentiekader uit 2009, de Toetswijzer voor eindtoetsen voor het basisonderwijs, de Centrale Eindtoets, de TAL-publicaties, en de vier meest gebruikte methodes *De Wereld in Getallen* (4^e^ editie 2009), *Pluspunt* (3^e^ editie, 2009), *Alles Telt* (2^e^ editie, 2009) en *Rekenrijk* (3^e^ editie, 2009).

In grote lijnen bleken de curriculumdocumenten met elkaar overeen te komen. Er werden echter ook duidelijke verschillen vastgesteld. Op het niveau van het beoogde curriculum heeft bijvoorbeeld probleemoplossen een minder prominente plek in het Referentiekader dan in de Kerndoelen. Andere wiskundige competenties uit de Kerndoelen, zoals het stellen van wiskundige vragen, ontbreken zelfs helemaal in het Referentiekader. Hetzelfde geldt voor wiskundige attitudes, zoals het beoefenen van wiskunde met tevredenheid en plezier. De verschillen die optreden op het niveau van het potentieel geïmplementeerde curriculum zijn met name verschillen tussen methodes onderling en tussen methodes en de TAL-documenten, bijvoorbeeld in de rekenprocedures die ze aanbieden en het getallenbereik waarmee die procedures moeten worden uitgevoerd. Deze verschillende uitwerkingen vallen wel allemaal binnen de globale richtlijnen van het beoogde curriculum. Dat geldt niet voor de Centrale Eindtoets, waarin bepaalde prestatieverwachtingen uit het beoogde curriculum niet worden getoetst, zoals het kunnen gebruiken van de rekenmachine en meetinstrumenten. Verder hoeven eindtoetsen niet alle wiskundige competenties uit de Kerndoelen te toetsen, doordat de Toetswijzer enkel is gebaseerd op het Referentiekader.

Een belangrijke kwestie die in deze studie aan de orde wordt gesteld, is dat het Referentiekader onderscheid maakt tussen twee niveaus, bedoeld voor verschillende groepen leerlingen. Het 'streefniveau' (1S) is bedoeld voor de meerderheid van de leerlingen, en het 'fundamentele niveau' (1F) is bedoeld voor leerlingen waarvoor het 1S-niveau te moeilijk blijkt te zijn. Alle vier de methodes bieden opgaven en taken aan op verschillende niveaus van moeilijkheid, die in de loop der leerjaren toewerken naar verschillende eindniveaus. Bijvoorbeeld in *De Wereld in Getallen* wordt de 'één ster'-leerroute verondersteld te leiden tot beheersing van het 1F-niveau, en de 'twee sterren'-route tot beheersing van het 1S-niveau. Echter, of zulke gedifferentieerde leerroutes daadwerkelijk leiden tot beheersing van de respectievelijke beoogde niveaus is niet bekend.

Een tweede kwestie betreft de wiskundige competenties, met name probleemoplossen. In de TAL-publicaties wordt het belang van probleemoplossen expliciet benadrukt, maar juist op dit punt wijken de methodes af van TAL. Zoals vastgesteld in Hoofdstuk 3, is er in de methodes maar weinig aandacht voor, wat overigens overeenstemt met de beperkte aandacht voor probleemoplossen in het beoogde curriculum.

Belangrijkste bevindingen, implicaties, en vragen voor verder onderzoek

De onderzoeksresultaten beschreven in dit proefschrift tonen aan dat Nederlandse reken-wiskundemethodes verschillen in hun bijdrage aan de OTL die basisschoolleerlingen wordt geboden. Dat geldt zowel voor een relatief eenvoudig onderwerp als aftrekken tot 100 (Hoofdstuk 2) als voor een complexe vaardigheid als niet-routinematig probleemoplossen (Hoofdstuk 3). Een belangrijke bevinding is dat niet alle Nederlandse basisschoolleerlingen dezelfde OTL wordt geboden. Verschillen in inhoud en prestatieverwachtingen komen voor tussen en in methodes (Hoofdstuk 3 en Hoofdstuk 6), maar ook op het niveau van het formele beoogde curriculum (Hoofdstuk 6). Dat laatste was niet het geval voor de wettelijke vaststelling van het Referentiekader in 2010. De reden dat toen gedifferentieerde doelen werden ingevoerd, was dat het

Referentiekader tot doel had de leerresultaten te verbeteren. Daarom werd niet alleen het minimumniveau 1F voor (bijna) alle leerlingen gedefinieerd, maar ook, voor een meerderheid van de leerlingen, het hogere na te streven 1S-niveau. Deze tweedeling impliceert echter dat niet alle nastrevenswaardige doelen bedoeld zijn voor álle leerlingen.

Een andere maatregel die destijds werd overwogen was om ook een extra niveau vast te stellen voor halverwege de basisschool of aan het eind van groep 2. Daar werd uiteindelijk niet voor gekozen, omdat dit de kans zou vergroten dat leerlingen al op jonge leeftijd als het ware zouden worden voorgesorteerd naar niveau. Paradoxaal genoeg faciliteert de indeling in methodes van opgaven en leerroutes op verschillende niveaus toewerkend naar 1F dan wel 1S (Hoofdstuk 6) juist zo'n vroegtijdige determinatie. Deze indeling kan immers resulteren in verschillende OTL voor verschillende leerlingen en werken als een *selffulfilling prophecy*. Dit levert een belangrijke vraag op voor vervolgonderzoek, namelijk hoe leraren omgaan met die gedifferentieerde leerroutes. De vraag is of die routes worden gebruikt om vaste niveaugroepen te realiseren, wat kan leiden tot vroege determinatie van jonge leerlingen, of dat hier juist flexibel mee wordt omgegaan, wat kan leiden tot juist betere OTL voor bepaalde leerlingen.

Er zijn meer overeenkomsten tussen methodes en het formele beoogde curriculum dan enkel gedifferentieerde doelen en weinig aandacht voor probleemoplossen. Over het algemeen stemmen methodes overeen met het beoogde curriculum (Hoofdstuk 6), maar niet altijd in alle opzichten (Hoofdstuk 2 en Hoofdstuk 6). Van invloed op dit laatste blijkt de didactische benadering in een methode. Methodes gebaseerd op verschillende benaderingen van reken-wiskunde-onderwijs verschillen niet alleen wat betreft learning facilitators (Hoofdstuk 2 en Hoofdstuk 3), maar ook qua inhoud en prestatieverwachtingen (Hoofdstuk 2 en Hoofdstuk 4). Dat betekent dat het strikte onderscheid tussen het 'hoe' en het 'wat' waar de Nederlandse overheid naar streeft, bij reken-wiskundeonderwijs wel eens niet zo gemakkelijk te maken zou kunnen zijn.

Ten aanzien van het ideale beoogde (RME-)curriculum, kan een soortgelijke conclusie worden getrokken als bij het formele beoogde curriculum. Globaal gezien zijn er duidelijke overeenkomsten tussen intenties en uitwerkingen daarvan in methodes (Hoofdstuk 5), maar er zijn ook duidelijke verschillen (Hoofdstuk 4 en Hoofdstuk 5). RME-methodes verschillen in hun overeenstemming met RME-intenties en in bepaalde opzichten wijken alle RME-methodes af van oorspronkelijke intenties. Dat betekent dat op basis van een enkele methode geen algemene uitspraken over RME kunnen worden gedaan—hetgeen helaas regelmatig gebeurt.

Het onderzoek beschreven in dit proefschrift richtte zich op reken-wiskunde-methodes op zich, maar methodes staan in de educatieve context niet op zichzelf. Vooral de manier waarop de leraar de methode gebruikt bepaalt of de bijdrage aan de OTL van de methode ook resulteert in een daadwerkelijke OTL in de reken-wiskundeles, en omissies in methodes kunnen door de leraar worden gecompenseerd. Voor dat laatste is het wel noodzakelijk dat de leraar zich bewust is van tekortkomingen in een methode. Echter, het constateren daarvan vraagt in sommige gevallen veel van de vakspecifieke en vakdidactische kennis van leraren, zoals bijvoorbeeld bij de inconsistenties tussen strategie en model of tussen strategie en representatie (Hoofdstuk 2). Het is de vraag of leraren hiervoor voldoende zijn toegerust. Zo liet journalistiek onderzoek zien dat leraren zelf diverse knelpunten aangeven bij het beoordelen van methodes, zoals te weinig tijd, werkdruk, te veel commerciële informatie en een gebrek aan objectieve informatie.

Voor wat betreft dat laatste, bieden de bevindingen beschreven in dit proefschrift valide informatie over reken-wiskundemethodes. De algemene bevinding dat methodes verschillen in de OTL die ze bieden en in hun overeenstemming met het beoogde curriculum lijkt op zichzelf niet zo opmerkelijk, maar de implicatie hiervan is wel belangrijk: als er niet blind op kan worden vertrouwd dat methodes alle wettelijk beoogde inhouden en prestatieverwachtingen aanbieden, blijven methodeanalyses van nieuwe methodes noodzakelijk. De onderzoeken beschreven in dit proefschrift laten zien dat methodeanalyse een zekere gedetailleerdheid vereist. Een oppervlakkige controle van een methode volstaat niet om zicht te krijgen op de geboden bijdrage aan de OTL. Om een valide oordeel te kunnen vellen over de kwaliteit van een methode is grondig onderzoek nodig. Scholen en leraren zouden geholpen worden als zulk onderzoek vaker zou worden uitgevoerd.

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Curriculum vitae

Marc van Zanten (1967) obtained his teaching degree in 1991 at the Marnix Academie in Utrecht, where he specialized in mathematics education. That same year he started to work as a primary school teacher at De Baanbreker in Utrecht and enrolled in a master study in Educational Sciences at Utrecht University. In 1994, he graduated on the topic of decimal numbers in primary school mathematics education. He was appointed lecturer of mathematics education at the Edith Stein University of Applied Sciences in Hengelo in 1997. From 2002 until 2007, he combined this position with an appointment as a lecturer of special educational needs in mathematics at Fontys University of Applied Sciences. In 2006, Marc became involved in the PANAMA-project (Teacher Education Mathematical Activities) of the Freudenthal Institute, Utrecht University. From 2007 until 2019 he was chairman of the annual PANAMA-conference for teacher educators and school advisors. Marc was a member of the committee Teacher Training Colleges of the Expert Group Continuous Learning Trajectories for Language and Mathematics in 2007 and he was chairman of the committee which established the Dutch Knowledge Base on Mathematics for primary school teachers in 2009. In 2013, he was appointed as a curriculum developer of mathematics at the Netherlands Institute for Curriculum Development SLO. Marc is an author of several textbooks on mathematics education for prospective primary school teachers and regularly publishes articles in professional teaching journals. Alongside his daytime job, he carried out his PhD research at the Freudenthal Institute, under supervision of Prof. dr. Marja van den Heuvel-Panhuizen. The findings of this research have been published in journal articles and handbooks, and have been presented at national and international conferences.

List of publications related to this thesis

Van Zanten, M. & M. van den Heuvel-Panhuizen (2013). Opportunity-to-learn van aftrekken tot 100 in twee reken-wiskundemethodes. *Reken-wiskundeonderwijs: Onderzoek, ontwikkeling, praktijk, 32*, 1–13.

Van Zanten, M., & Van den Heuvel-Panhuizen, M. (2014). Reflections from the past: A contemporary Dutch primary school mathematics textbook in a historical perspective. In K. Jones, C. Bokhove, G. Howson & L. Fan (Eds.), *Proceedings of the International Conference on Mathematics Textbook Research and Development* (ICMT) (pp. 83–88). Southampton, UK: The University of Southampton.

Van Zanten, M., & Van den Heuvel-Panhuizen, M. (2014). Freedom of design: The multiple faces of subtraction in Dutch primary school textbooks. In Y. Li, & G. Lappan (Eds.), *Mathematics Curriculum in School Education* (pp. 231–259). Dordrecht/Heidelberg/New York/London: Springer.

Van Zanten, M., & Van den Heuvel-Panhuizen, M. (2015). Past and current approaches to decimal numbers in Dutch primary school mathematics textbooks. *Nordic Studies in Mathematics Education, 20*(3–4), 57–82.

Van Zanten, M. (2017). Students' own productions and own constructions—Adri Treffers' contributions to Realistic Mathematics Education. In W. Blum, M. Artigue, M. Mariotti, R. Sträβer, & M. van den Heuvel-Panhuizen (Eds.), *European Traditions in Didactics of Mathematics* (pp. 67–73). ICME-13 Monographs. SpringerOpen. doi:10.1007/978-3-030-05514-1

Van Zanten, M., & Van den Heuvel-Panhuizen, M. (2018). Opportunity to learn problem solving in Dutch primary school mathematics textbooks. *ZDM Mathematics Education*, *50*(5), 827–838.

Van Zanten, M., & Van den Heuvel-Panhuizen, M. (2018). Primary school mathematics in the Netherlands. The perspective of the curriculum documents. In D. Thompson, M. Huntley, & C. Suurtamm (Eds.), International Perspectives on Mathematics Curriculum (pp. 9–30). Charlotte, NC: Information Age Publishing.

Van Zanten, M., Van den Heuvel-Panhuizen, M., & Veldhuis, M. (2018). Realistic Mathematics Education in the Netherlands: Textbooks as carriers and barriers for reform. In Y. Shimizu & R. Vithal (Eds.), *ICMI Study 24 Conference Proceedings. School mathematics curriculum reforms:*

Challenges, changes and opportunities (pp. 157–164). Tsukuba, Japan: University of Tsukuba.

Van Zanten, M., & Van den Heuvel-Panhuizen, M. (2019). Probleemoplossen in reken-wiskundemethodes. Het gereedschap van de vakman/vakvrouw. *Volgens Bartjens 38*(3), 22–27.

Van Zanten, M., & Van den Heuvel-Panhuizen, M. (2019). 50 years of Realistic Mathematics Education ideas and their implementation in textbooks—The case of addition and subtraction. In S. Rezat, L. Fan, M. Hattermann, J. Schumacher & H. Wuschke (Eds.), *Proceedings of the Third International Conference on Mathematics Textbook Research and Development (ICMT-3)* (pp. 341–346). Paderborn, Germany: Paderborn University.

Van den Heuvel-Panhuizen, M. & Van Zanten, M. (2020). Realistic Mathematics Education: A brief history of a longstanding reform movement. *Mediterranean Journal for Research in Mathematics Education*, 17, 65–73.

Van Zanten, M., & Van den Heuvel-Panhuizen, M. *Mathematics curriculum reform and its implementation in textbooks: Early addition and subtraction in Realistic Mathematics Education*. Submitted.

Selection of presentations related to this thesis

Van Zanten, M., Treffers, A., & Van den Heuvel-Panhuizen, M. (2011, January). *De basisbewerking aftrekken in twee nieuwe reken-wiskundemethodes* [The basic operation of subtraction in two new mathematics textbooks]. Presentation at the 29th Panama Conference, Noordwijkerhout, the Netherlands.

Van Zanten, M. (2012, March). *Leerlijnen in reken-wiskundemethodes. De basisbewerking aftrekken als voorbeeld* [Learning trajectories in mathematics textbooks. The basic operation of subtraction as an example]. Presentation at the "School Aan Zet" [It is up to Schools] conference, Lunteren, the Netherlands.

Van Zanten, M., & Van den Heuvel-Panhuizen, M. (2013, January). *Reken-wiskundemethodes onderzocht. Gelegenheid tot leren van aftrekken tot 100 in twee reken-wiskundemethodes* [Mathematics textbooks researched. Opportunity to learn subtraction in two mathematics textbooks]. Presentation at the 31st Panama Conference, Utrecht, the Netherlands.

Van Zanten, M., & Van den Heuvel-Panhuizen, M. (2013, April). Freedom of design. The multiple faces of subtraction up to 100 in Dutch primary school textbooks. In Y. Li (Chair), *Mathematics Curriculum Design and Development in the East and West.* Symposium conducted at the Research Pre-session of the NCTM Annual Meeting & Exposition, Denver, CO, USA.

Van Zanten, M., & Van den Heuvel-Panhuizen, M. (2014, July). *Reflections from the past. A contemporary Dutch primary school mathematics textbook in a historical perspective.* Paper presented at the International Conference on Mathematics Textbook Research and Development (ICMT), Southampton, UK.

Van Zanten, M., & Van den Heuvel-Panhuizen, M. (2015, January). *Reflecties uit het verleden. Een historisch perspectief op een hedendaagse reken-wiskundemethode* [Reflections from the past. A historical perspective on a contemporary mathematics textbook]. Presentation at the 33rd Panama Conference, Veldhoven, the Netherlands.

Van Zanten, M., & Van den Heuvel-Panhuizen, M. (2016, January). *Gelegenheid tot leren probleemoplossen in huidige reken-wiskundemethodes* [Opportunity to learn problem solving in current mathematics textbooks]. Presentation at the 34th Panama Conference, Veldhoven, the Netherlands.

Van Zanten, M., & Van den Heuvel-Panhuizen, M. (2017, May). *Opportunity to learn problem solving in Dutch primary school mathematics textbooks.*

Presentation at the Second International Conference on Mathematics Textbook Research and Development (ICMT-2), Rio de Janeiro, Brazil.

Van Zanten, M., & Van den Heuvel-Panhuizen, M. (2018, April). *Het curriculum rekenen-wiskunde in het PO. Kansen en knelpunten* [The mathematics curriculum in primary education. Opportunities and bottlenecks]. Presentation at the "Staat van Het Onderwijs" [State of the Education], Educational Inspectorate, Utrecht, the Netherlands.

Van Zanten, M., & Van den Heuvel-Panhuizen, M. (2018, September). *What is in the books? Textbook analysis to reveal learning opportunities.* Presentation at the 6th International Realistic Mathematics Education Conference (RME-6), Grand Cayman, Cayman Islands.

Van Zanten, M., & Van den Heuvel-Panhuizen, M. (2018, October). *Rijke voorbeelden uit de geschiedenis van realistisch reken-wiskundeonderwijs* [Rich examples from the history of Realistic Mathematics Education]. Presentation at the Symposium XXIV from the Working Group History of the Dutch Association of Teachers of Mathematics, Utrecht, the Netherlands.

Van Zanten, M., Van den Heuvel-Panhuizen, M., & Veldhuis, M. (2018, November). *Realistic Mathematics Education in the Netherlands: Textbooks as carriers and barriers for reform.* Paper presented at the ICMI Study 24, School Mathematics Curriculum Reforms: Challenges, Changes and Opportunities, Tsukuba, Japan.

Van Zanten, M. (2019, January). *Het gereedschap van de vakman / vakvrouw. Kwaliteit (en) methode reken-wiskundeonderwijs* [The tool of the professional. Quality and (of) the mathematics textbook]. Invited talk at the Volgens Bartjens Rekenmiddag, Utrecht, the Netherlands.

Van Zanten, M., & Van den Heuvel-Panhuizen, M. (2019, January). *Probleemoplossen in reken-wiskundemethodes* [Problem solving in textbooks]. Presentation at the 37th Panama Conference, Veldhoven, the Netherlands.

Van Zanten, M., & Van den Heuvel-Panhuizen, M. (2019, September). *50 years of Realistic Mathematics Education ideas and their implementation in textbooks. The case of addition and subtraction.* Paper presented at the Third International Conference on Mathematics Textbook Research and development (ICMT-3), Paderborn, Germany.

Van Zanten, M., & Van den Heuvel-Panhuizen, M. (2020, January). *Leren optellen en aftrekken van 1970 tot nu* [Learning addition and subtraction from 1970 to present]. Presentation at the 38th Panama Conference, Zeist, the Netherlands.

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85. Lijnse, P. L. (2014). *Omzien in verwondering. Een persoonlijke terugblik op 40 jaar werken in de natuurkundedidactiek*.
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75. Smit, J. (2013). *Scaffolding language in multilingual mathematics classrooms.*
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68. Bokhove, Chr. (2011). *Use of ICT for acquiring, practicing and assessing algebraic expertise.*
67. Boerwinkel, D. J. & Waarlo, A. J. (2011). *Genomics education for decisionmaking. Proceedings of the second invitational workshop on genomics education, 2-3 December 2010.*
66. Kolovou, A. (2011). *Mathematical problem solving in primary school*.
65. Meijer, M. R. (2011). *Macro-meso-micro thinking with structure-property relations for chemistry. An explorative design-based study*.

64. Kortland, J., & Klaassen, C. J. W. M. (2010). *Designing theory-based teaching-learning sequences for science. Proceedings of the symposium in honour of Piet Lijnse at the time of his retirement as professor of Physics Didactics at Utrecht University*.
63. Prins, G. T. (2010). *Teaching and learning of modelling in chemistry education. Authentic practices as contexts for learning.*
62. Boerwinkel, D. J., & Waarlo, A. J. (2010). *Rethinking science curricula in the genomics era. Proceedings of an invitational workshop*.
61. Ormel, B. J. B. (2010). *Het natuurwetenschappelijk modelleren van dynamische systemen. Naar een didactiek voor het voortgezet onderwijs*.
60. Hammann, M., Waarlo, A. J., & Boersma, K. Th. (Eds.). (2010). *The nature of research in biological education: Old and new perspectives on theoretical and methodological issues – A selection of papers presented at the VIIth Conference of European Researchers in Didactics of Biology*.
59. Van Nes, F. (2009). *Young children's spatial structuring ability and emerging number sense*.
58. Engelbarts, M. (2009*). Op weg naar een didactiek voor natuurkunde-experimenten op afstand. Ontwerp en evaluatie van een via internet uitvoerbaar experiment voor leerlingen uit het voortgezet onderwijs*.
57. Buijs, K. (2008). *Leren vermenigvuldigen met meercijferige getallen.*
56. Westra, R. H. V. (2008). *Learning and teaching ecosystem behaviour in secondary education: Systems thinking and modelling in authentic practices*.
55. Hovinga, D. (2007). *Ont-dekken en toe-dekken: Leren over de veelvormige relatie van mensen met natuur in NME-leertrajecten duurzame ontwikkeling.*
54. Westra, A. S. (2006). *A new approach to teaching and learning mechanics*.
53. Van Berkel, B. (2005). *The structure of school chemistry: A quest for conditions for escape.*
52. Westbroek, H. B. (2005). *Characteristics of meaningful chemistry education: The case of water quality*.
51. Doorman, L. M. (2005). *Modelling motion: from trace graphs to instantaneous change.*
50. Bakker, A. (2004). *Design research in statistics education: on symbolizing and computer tools.*
49. Verhoeff, R. P. (2003). *Towards systems thinking in cell biology education*.
48. Drijvers, P. (2003). *Learning algebra in a computer algebra environment. Design research on the understanding of the concept of parameter.*
47. Van den Boer, C. (2003). *Een zoektocht naar verklaringen voor achterblijvende prestaties van allochtone leerlingen in het wiskundeonderwijs.*
46. Boerwinkel, D. J. (2003). *Het vormfunctieperspectief als leerdoel van natuuronderwijs. Leren kijken door de ontwerpersbril*.

45. Keijzer, R. (2003). *Teaching formal mathematics in primary education. Fraction learning as mathematising process.*
44. Smits, Th. J. M. (2003). *Werken aan kwaliteitsverbetering van leerlingonderzoek: Een studie naar de ontwikkeling en het resultaat van een scholing voor docenten.*
43. Knippels, M. C. P. J. (2002). *Coping with the abstract and complex nature of genetics in biology education – The yo-yo learning and teaching strategy.*
42. Dressler, M. (2002). *Education in Israel on collaborative management of shared water resources.*
41. Van Amerom, B.A. (2002). *Reinvention of early algebra: Developmental research on the transition from arithmetic to algebra.*
40. Van Groenestijn, M. (2002). *A gateway to numeracy. A study of numeracy in adult basic education.*
39. Menne, J. J. M. (2001). *Met sprongen vooruit: een productief oefenprogramma voor zwakke rekenaars in het getallengebied tot 100 – een onderwijsexperiment.*
38. De Jong, O., Savelsbergh, E.R., & Alblas, A. (2001). *Teaching for scientific literacy: context, competency, and curriculum.*
37. Kortland, J. (2001). *A problem-posing approach to teaching decision making about the waste issue.*
36. Lijmbach, S., Broens, M., & Hovinga, D. (2000). *Duurzaamheid als leergebied; conceptuele analyse en educatieve uitwerking.*
35. Margadant-van Arcken, M., & Van den Berg, C. (2000). *Natuur in pluralistisch perspectief – Theoretisch kader en voorbeeldlesmateriaal voor het omgaan met een veelheid aan natuurbeelden.*
34. *Janssen, F. J. J. M. (1999). Ontwerpend leren in het biologieonderwijs. Uitgewerkt en beproefd voor immunologie in het voortgezet onderwijs.*
33. De Moor, E. W. A. (1999). *Van vormleer naar realistische meetkunde – Een historisch-didactisch onderzoek van het meetkundeonderwijs aan kinderen van vier tot veertien jaar in Nederland gedurende de negentiende en twintigste eeuw.*
32. Van den Heuvel-Panhuizen, M., & Vermeer, H. J. (1999). *Verschillen tussen meisjes en jongens bij het vak rekenen-wiskunde op de basisschool – Eindrapport MOOJ-onderzoek.*
31. Beeftink, C. (2000). *Met het oog op integratie – Een studie over integratie van leerstof uit de natuurwetenschappelijke vakken in de tweede fase van het voortgezet onderwijs.*
30. Vollebregt, M. J. (1998). *A problem posing approach to teaching an initial particle model.*
29. Klein, A. S. (1998). *Flexibilization of mental arithmeticsstrategies on a different knowledge base – The empty number line in a realistic versus gradual program design.*

28. Genseberger, R. (1997). *Interessegeoriënteerd natuur- en scheikundeonderwijs – Een studie naar onderwijsontwikkeling op de Open Schoolgemeenschap Bijlmer.*
27. Kaper, W. H. (1997). *Thermodynamica leren onderwijzen*.
26. Gravemeijer, K. (1997). *The role of context and models in the development of mathematical strategies and procedures*.
25. Acampo, J. J. C. (1997). *Teaching electrochemical cells – A study on teachers' conceptions and teaching problems in secondary education*.
24. Reygel, P. C. F. (1997). *Het thema 'reproductie' in het schoolvak biologie*.
23. Roebertsen, H. (1996). *Integratie en toepassing van biologische kennis – Ontwikkeling en onderzoek van een curriculum rond het thema 'Lichaamsprocessen en Vergift'*.
22. Lijnse, P. L., & Wubbels, T. (1996). *Over natuurkundedidactiek, curriculumontwikkeling en lerarenopleiding*.
21. Buddingh', J. (1997). *Regulatie en homeostase als onderwijsthema: een biologie-didactisch onderzoek*.
20. Van Hoeve-Brouwer G. M. (1996). *Teaching structures in chemistry – An educational structure for chemical bonding*.
19. Van den Heuvel-Panhuizen, M. (1996). *Assessment and realistic mathematics education.*
18. Klaassen, C. W. J. M. (1995). *A problem-posing approach to teaching the topic of radioactivity.*
17. De Jong, O., Van Roon, P. H., & De Vos, W. (1995). *Perspectives on research in chemical education.*
16. Van Keulen, H. (1995). *Making sense – Simulation-of-research in organic chemistry education.*
15. Doorman, L. M., Drijvers, P. & Kindt, M. (1994). *De grafische rekenmachine in het wiskundeonderwijs.*
14. Gravemeijer, K. (1994). *Realistic mathematics education.*
13. Lijnse, P. L. (Ed.) (1993). *European research in science education.*
12. Zuidema, J., & Van der Gaag, L. (1993). *De volgende opgave van de computer.*
11. Gravemeijer, K., Van den Heuvel-Panhuizen, M., Van Donselaar, G., Ruesink, N., Streefland, L., Vermeulen, W., Te Woerd, E., & Van der Ploeg, D. (1993). *Methoden in het reken-wiskundeonderwijs, een rijke context voor vergelijkend onderzoek.*
10. Van der Valk, A. E. (1992). *Ontwikkeling in Energieonderwijs*.
9. Streefland, L. (Ed.) (1991). *Realistic mathematics education in primary schools.*
8. Van Galen, F., Dolk, M., Feijs, E., & Jonker, V. (1991). *Interactieve video in de nascholing reken-wiskunde.*
7. Elzenga, H. E. (1991). *Kwaliteit van kwantiteit.*

6. Lijnse, P. L., Licht, P., De Vos, W., & Waarlo, A. J. (Eds.). (1990). *Relating macroscopic phenomena to microscopic particles: a central problem in secondary science education.*
5. Van Driel, J. H. (1990). *Betrokken bij evenwicht.*
4. Vogelezang, M. J. (1990). *Een onverdeelbare eenheid.*
3. Wierstra, R. F. A. (1990). *Natuurkunde-onderwijs tussen leefwereld en vakstructuur.*
2. Eijkelhof, H. M. C. (1990). *Radiation and risk in physics education.*
1. Lijnse, P. L., & De Vos, W. (Eds.). (1990). *Didactiek in perspectief.*

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In mathematics education, textbooks are a main resource for daily teaching practice. Therefore, textbooks have a major impact on the learning opportunities offered to students. The aim of this PhD study was to get a better understanding of Dutch primary school mathematics textbooks regarding their contribution to the opportunity to learn. To this end, textbooks based on different didactical approaches and published in different time periods were studied on three features: the included learning content, the articulated performance expectations, and the incorporated learning facilitators.

The research revealed that textbooks differ greatly in the learning opportunities they provide to students. These differences were found to be related to the textbook's didactical approach. The textbooks appeared not only to vary in the learning facilitators they provide – which could be expected when the didactical approach differs – but also in the content and performance expectations. Furthermore, it was found that textbooks differ in their alignment with the formally intended curriculum in the Netherlands.

An important consequence of the difference in learning opportunities is that not all students are exposed to the same content and performance expectations. This is even the case when students are taught with the same textbook, because due to the organizational structure of textbooks, not all students are presented all parts of them.