How to deal with Algebraic Skills in Realistic Mathematics Education?

Monica M. Wijers
Freudenthal Institute, The Netherlands
<m.wijers@fi.uu.nl>

In the Netherlands, the mathematics curriculum for lower secondary has been modernised since 1992 and reflects a realistic approach to mathematics. For algebra this means that the focus is more on using algebra than on studying algebra as an abstract subject. Some years after the implementation of the new curriculum a survey among teachers and researchers revealed difficulties with the teaching and learning of algebra Tension was felt between: skills and understanding; tools and structure etc. Difficulties surfaced with the transition from lower to upper secondary. The Freudenthal Institute is investigating these difficulties and is experimenting with possible solutions.

Realising a sound algebra strand in an integrated mathematics curriculum based on the principles of Realistic Mathematics Education (RME) is not an easy task. In the Netherlands this has been an ongoing struggle for at least the past 20 years.

In 1992 in the Netherlands an educational change was implemented: the so called Basisvorming (basic education) was introduced. This change involved the introduction of a core curriculum with15 subjects for all students aged 12-14. For each subject attainment targets were formulated, describing a minimum program consisting of 'what everybody should know and master of a certain subject'. At the same time a new mathematics curriculum for students aged 12-16 years was developed (team W12-16, 1992). This curriculum for grades 7 through 10 was to be implemented at the same time as the 'basic education' program. Figure 1 shows the Dutch school system.

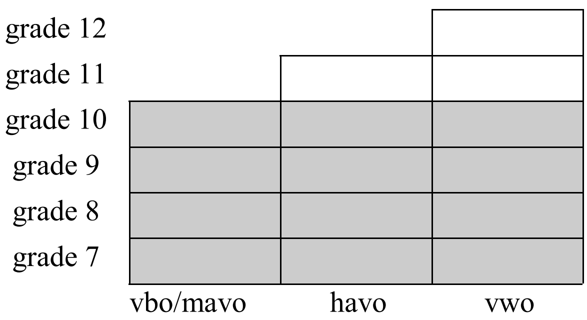

Figure 1. The Dutch school system

The algebra in this new curriculum was intended to be sense making for all students, also for those that would not need any mathematics after the basic education. The old curriculum didn't fulfil this requirement. It can be said that in the old situation the algebra strand (as well as the whole mathematics curriculum) for the lower ability students (vbo/mavo) consisted of a poor rendering of the more abstract algebra program for college preparatory (havo/vwo) students. In the past this had led to many difficulties especially in the lower ability stream (vbo/mavo). Students had difficulties mastering the abstract algebraic skills and teachers had a hard time trying to give sense to the abstract algebra. A new more sense making algebra program based on the principles of RME was believed to overcome these difficulties. One program with one set of attainment targets was to be developed for all students aged 12-16. However, more in line with the structure of the Dutch school system (figure 1), for the actual realisation of the targets in student materials a distinction was made between the two higher level streams (vwo and havo) and the two lower level streams (mavo and vbo).

Characteristics of the W12-16 algebra strand

The approach to algebra in the W12-16 curriculum can be characterised by the following four aspects.

Firstly, emphasis is more on students *using* algebra than on students developing algebra. Of course some co-developing of algebra is included. Learning develops by ways of gaining and building insight. After all, a good user understands his methods and tools.

The second characteristic is that *interpreting* algebraic representations and language is more important than using algebraic techniques to operate on algebraic objects themselves. This implies that algebraic techniques follow from and after interpretational activities. Operating on abstract formulas gets some attention, but mainly for the students in higher level streams.

Third a choice is made to use algebraic techniques that are *widely applicable*. For instance for solving equations, techniques are chosen which can be used on a broad range of types of equations and which require not too much algebraic manipulations on formulas. Techniques that depend more on the type of formula – such as specific techniques for solving quadratic equations - go deeper into the formal algebra, but are less widely applicable.

The fourth aspect has to do with the *alignment* of the algebra strand. Chosen is to gradually develop the algebra broadly, instead of stacking things up. This means that topics are revisited regularly during the school years. First at a more informal level and gradually they develop towards a more formal level. In the 'stacking' model, one topic is developed completely and on top of it the next topic is build.

All four aspects, that are based on antitheses, have a slightly different accent for the different streams. As can be expected, for mavo/vbo the more informal side is stressed. In havo/vwo a little more of the formal algebra is included, students not only learn how to use a variety of algebraic tools; reflecting on this use must lead to an understanding of the structure of algebra. These students must also learn to reason about formulas and their structure separated from the underlying situation.

A very important reason for including more formal algebra in grades 7 and 8 of havo/vwo is that these students need to be able to choose at the end of grade 9 between the more informal mathematics A curriculum and the more formal mathematics B curriculum.

Research on the implementation of the new 12-16 curriculum

Curriculum reform for mathematics in the Netherlands is mostly funded by the government and carried out by project teams based at the Freudenthal institute. These project teams design the new curricula by means of developmental research. This is a cyclic process of developing exemplary student materials, conducting classroom experiments, reflection and revision. In large scale reforms, such as the design of the new 12-16 curriculum, the curriculum as a whole is described as concrete as possible and exemplary materials are developed to illustrate how this can be elaborated. Commercial publishers use these products to design their textbooks. Due to a variety of reasons, among which time pressure on the publishers is a main one, certain aspects of the algebra especially the meaningful incorporation of algebraic skills (like manipulating formulas and expressions), has not really succeeded in the intended way.

In 1992 the first textbooks reflecting the reformed mathematics curriculum were implemented at the schools. The whole implementation process took 4 years. In 1996 teachers were able to have an overall view of the way in which the new algebra took form in the textbooks. They got worried about the transition of havo/vwo students from lower secondary (basisvorming) to upper secondary.

To get better insight in the difficulties teachers and their students have with respect to the algebra, in the period from 1996-1999, surveys were held, interviews were taken and lessons were observed (Wijers, 2000, Wijers and Kemme 2000). Summarised the results show:

- the reformed math program including the approach to algebra is seen as an improvement for vbo/mavo students (lower level streams)
- the reformed approach toward mathematics including algebra is a valuable one
- the emphasis on understanding and meaning is important
- mathematics in grade 7 (age 12) is too easy for havo/vwo students
- there is not enough formal algebra in the lower grades: it starts too late
- students do not have enough training in algebraic (instrumental) skills and do not master them
- upper secondary teachers consider operating on algebraic expressions as the core of mathematics

The above results show some discrepancies, such as: 'reformed algebra is good versus formal algebra is needed'; ' emphasis on understanding and meaning is important versus more training of algebraic skills is necessary'. In the next section we will try to refine the results. We will use examples to clarify what difficulties students have and why.

Students' difficulties with formal algebraic skills

The reformed approach toward mathematics in lower secondary indeed did make algebra more sense-making, but the question remains how the more formal algebra can be incorporated. Of course there is also the underlying problem of what formal algebra is needed for what students at what age?

The results presented above make clear that formal algebra is typically seen as important for the higher level streams and for upper secondary and not for the lower level streams. For students in the lower level streams (vbo/mavo) the learning of algebra (its structure and symbols) is not a goal in itself. Algebra is a tool to solve problems. These students therefore do not need to master formal algebraic skills outside the contextual problems. Actually the reformed curriculum - including the algebra - is not seen as problematic for students in vbo/mavo.

So we can conclude that the difficulties with algebra are limited to the higher level streams (havo/vwo). The textbooks indeed do reveal that in grade 7 and in he first half of grade 8 not much time is spent on formal - out of context - algebraic skills. The chapters on algebra show a diversity of realistic problems in context. Emphasis is on interpreting algebraic representations such as graphs, tables and (word) formulas. Problems can be solved within and with help of the context. The context provides students with clues about appropriate solution methods and strategies. Most teachers report that most students do not have difficulties with this part of the algebra. Classroom observations support this.

Students' difficulties with the algebra start at the end of grade 8 and continue in grade 9. The chapters on algebra suddenly include more formal looking problems. Also the emphasis seems to shift more to bare, out of context, formal problems in which algebraic instrumental skills are taught and trained. The chapters that focus on ‘bare’ algebraic skills do contain problems in context, but the majority of the problems in these chapters look kind of old-fashioned, they have a lot of variables and symbols, and have as the only brief text ‘factorise’ or ‘solve’ etc. Teachers report that students initially have a lot of difficulties with these chapters. These difficulties however are not as much on the level of ‘doing the problems’, the ‘how’, as on understanding ‘what this is all about’ and ‘what is the use of this’, the ‘why’. It seems there is a sudden transition to a more formal approach to algebra. All the formal instrumental algebraic skills seem to be concentrated in a few chapters, mainly in grade 9. These chapters are crowded with all kinds of different skills, and algorithms.

For instance one chapter on solving equations deals with all of the following methods:

- simple equations like $3(n+2)=90$ can be solved by using common sense strategies
- same type of equations solved by using the so called ‘covering’ method (figure 2)

uit: MW 1b hv p.90/91

Figure 2. Solving an equation with the ‘covering’-method

- finding the intersection of two graphs by using a table
- the balance method (same operation on each side of the equation) is used in combination with graphs and tables
- the balance method without graphs and tables
- solving inequalities by means of solving the equation and using substitution

Students often loose sight on what they are doing and why. They can solve equations often by using more informal methods like trial and improve or by graphical/numerical methods using graphs and tables. But now they need to use the more formal methods and algorithms they just learned. Of course this leads to confusion and students trying to apply at random algorithms as can be seen in the examples of student work in figure 3.

Figure 3. Using algorithms and rules at random, not seeing the purpose of the problem

In fact the goal of the chapter outlined above is *not* to have students find solutions of equations, but to have students learn how to use and notate some specific algebraic methods and strategies. The main difficulty may be that this is never made explicit to the students.

The same analysis holds for chapters on operating on expressions and formulas out of context. This involves skills such as: taking together like terms; eliminating or inserting parentheses; knowing the order of operations; substituting, etc. For students these are many different aspects to be learned and used at the same time. It is not so that these students can not learn to master these skills, but they do view the chapter as the odd one out. They feel insecure. They see no connection to the algebra they did before and still do in some of the other algebra chapters on relationships and functions. They even have a hard time connecting the problems in context with the bare problems on the same page.

All in all research so far shows a disconnection between learning instrumental algebraic skills and solving algebraic problems. In the textbooks no proper balance between stressing meaning and understanding and training algebraic skills has been realised. Algebraic (instrumental) skills are presented 'in a hurry' and there is no well thought-out approach towards the training of these skills. There seem to be two different not well connected algebras in lower secondary. This has consequences on how students deal with the algebra (and calculus) in upper secondary. In order to be able to solve complex algebraic problems, both instrumental skills and general higher order skills (problem solving, interpreting, etc) are needed. Both types of skills therefore need explicit attention in connection with each other in the learning process: they both need to be pointed out and they need to be trained in connection to one another.

Possible solutions

The above conclusions already indicate possible solutions: design a more coherent and balanced algebra strand; starting in earlier grades with both aspects of algebra; making more explicit what is the purpose and the use of specific skills, etc. Some steps to realise these solutions have already been taken or are being taken right now.

The revised editions of the textbooks for lower secondary (implemented from 98/'99 on starting in grade 7) show a more gradual build up of algebraic skills. Formal algebra starts earlier and is more spread out over the years. Unfortunately ongoing observations seem to indicate that the approach towards algebraic skills has not really changed: instrumental skills and general skills are still being taught in a rather disconnected way.

At the Freudenthal Institute research is done into the possibilities of using different technological means, to make the algebra program more balanced. For instance classroom experiments are carried out with the use of computer algebra systems in grade 9 of one of the higher level streams (vwo). Results so far show that connecting the development of mathematical (algebraic) conceptions to machine-procedures and technical skills is a necessary condition for having students develop real understanding of variables and parameters (Drijvers, 2000).

Another way to use technology in order to realise a more balanced algebra program is the use of applets. These small computer applications form a learning environment in which the use of both instrumental and general skills can be integrated. The 'how' and 'why' of symbolic manipulations (instrumental algebraic skills)can be connected for instance by using 'geometrical algebra' to deal with equivalent expressions among other things, and jigsaw-puzzle algebra' to investigate algebraic patterns (Kindt, 1999). Both these ideas and others have been concreted in applets, that can be viewed at http://www.fi.uu.nl/wisweb. Classroom research on the use of algebra applets so far has revealed that working with the applets students develop and use algebraic instrumental skills on their own level. Students own constructions lead to a deeper insight in the 'why' and also in the 'how' of certain instrumental algebraic skills. The different 'levels' built into these applets (like in computer games) correspond to levels of 'formality' of the instrumental algebraic skills involved (van Reeuwijk, 2001).

It is too early to tell if these solutions will eventually lead to a more coherent and balanced algebra program in which skills and understanding are well connected. The results so far seem promising, but more research needs to be done. The Freudenthal Institute together with teachers and other researchers will continue the struggle to design such an 'ideal' algebra program.

References

Team W12-16. (1992). *Achtergronden van het nieuwe leerplan wiskunde 12-16 (Translation: Background to the new mathematics curriculum 12-16)* . Utrecht, Freudenthal Institute/SLO

Drijvers, P., van Herwaarden, O. (2000). Instrumentatie van ICT-gereedschap: algebra met computeralgebra. [Translation: Instrumentation of ICT-tools: algebra with computeralgebra] *Nieuwe Wiskrant , (20)1*, 38-44.

Reeuwijk, M. van (2001). Bollen schieten op eigen niveau [Translation: Shoot balls on one's own level] *Nieuwe Wiskrant , (20)3*.

Wijers, M. (2000). Algebra, een praktijkprobleem?. [Translation: Algebra, a practical problem?] *Nieuwe Wiskrant , (19)3*, 36-42

Wijers, M., Kemme, S. (2000). Welke Algebra heb je nodig in 4 vwo?. [Translation: What algebra do you need in grade 11 of the college preparatory stream(vwo) ?] *Nieuwe Wiskrant , (20)1*, 22-26.