THE STANDARDS APPLIED: TEACHING DATA VISUALIZATION

Martin van Reeuwijk
Freudenthal Institute, Utrecht University, The Netherlands

As part of the project Design, Development and Assessment in Mathematics Education, I spent four weeks at Whitnall High School in Milwaukee, Wisconsin, testing a booklet on descriptive statistics called *Data Visualization* (Freudenthal Institute 1989). This textbook was developed in 1989 by the Freudenthal Institute's Research Group on Mathematics Education of the University of Utrecht in The Netherlands in cooperation with Gail Burrill of Whitnall High School. Especially written for this project, the booklet is based on the NCTM's *Curriculum and Evaluation Standards* (1989) and on the philosophy of 'realistic mathematics education' developed by the research group of Utrecht University.

According to this philosophy mathematics originates from daily life and should be a useful tool in solving problems in real-life situations. Mathematics is seen as an integrated subject in which such parts as geometry, algebra, arithmetic, calculus, and statistics are very much related. Some of the ingredients of 'realistic mathematics education' are developing a critical attitude, understanding the underlying concepts, and using mathematics in problem-solving situations. These ingredients are also reflected in the Data Visualization program.

Goals and materials

One goal of the project at Whitnall High School was to identify the problems that students and teachers encountered in using a textbook that presented mathematics according to the philosophy of realistic mathematics education, a new departure for the students and teachers at Whitnall. Another goal of the project was to develop what Thomas Romberg calls an 'existence proof' for the curriculum and evaluation standards, that is, to demonstrate the possibility of teaching mathematics as described in the *Standards* document by using realistic contexts and problems. This method of instruction does not isolate mathematics but embeds it in real-life situations, presenting the opportunity to teach not only lower-level mathematics content but such equally important higher-order skills such as problem solving, modeling, and critical attitude. The American standards are in many ways comparable to the Dutch philosophy of realistic mathematics education. To reflect the change in teaching mathematics, a new method of teaching was also devised.

Besides using the book *Data Visualization* (Freudenthal Institute 1989), a set of tests was constructed and tried out during the four-week project.

Some first impressions

In September 1989, the teachers of the mathematics department at Whitnall High School began algebra classes using *Data Visualization* instead of the regular algebra textbook. The book was used in seven algebra classes: two low-level classes (11th- and 12th-grade students), four regular classes (grades 9 and 10), and one honors class (9th-grade students). Six different teachers were involved.
The number of students and their ages are given in table 1. No significant differences were found in gender distribution; every class had as many boys as girls.

For three and a half weeks, I attended most of the classes, either sitting in the back of the classroom or walking around while the students were working in groups or by themselves. I made notes of what was happening and talked with the students and their teachers. The findings were both exciting and discouraging.

Martin van Reeuwijk teaches at the Freudenthal Institute, Utrecht University, The Netherlands. He has been in-

volved with reform in upper secondary mathematics education in The Netherlands and coordinates the Middle School Project as carried out by the University of Wisconsin-Madison and the Freudenthal Institute.

The author is a researcher working at the Research Group on Mathematics Education and the Mathematical Computer Centre (OW&OC) of the University of Utrecht in The Netherlands. Professor Jan de Lange is the present director of this research group. The project Design, Development and Assessment in Mathematics Education was carried out by the Freudenthal Institute and NCRSME (Madison, Wisconsin) at Whitnall High School, Greenfield, Wisconsin. The *Data Visualization* project in 1989 had a follow-up in 1990 with the booklet *Matrices* (NCTM 1988).

TABLE 1

Classes and Students Involved in the *Data Visualization* Project

Class	Level	Number of Students	Grade	Age
1	Honors	23	9	13–14
2	Regular	28	9	13–14
3	Regular	28	9, 10	13–15
4	Regular	30	9, 10	13–15
5	Regular	27	9	13–14
6	Low level	29	11, 12	16–18
7	Low level	26	11, 12	16–18

During the first week, we had to concentrate on reviewing such basic skills as percents and fractions, since the students seemed to have lost some of the basic mathematics and arithmetic skills over the summer. After refreshing these basic skills, the students remembered the mathematical rules and tricks, but other difficulties developed. The students did not know how to use the mathematics in new situations. Although they were able to answer a question that looked familiar or to solve a problem they had seen before, in new situations they did not know what to do or where to start in solving a problem. The transfer of mathematical knowledge was quite low.

For example, bar graphs were drawn on the map of the world, representing the urban and rural population in several parts of the world from 1950 to 2025 (predicted numbers). See figure1.

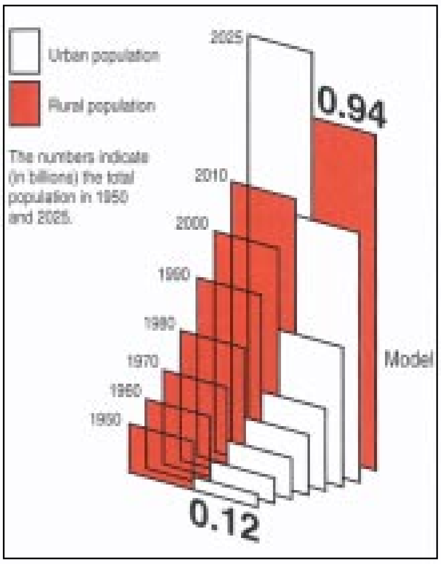

Figure 1: Growth of urban population compared to that of rural population (1950-2025)

Many students did not know what to do when asked to compute the growth of the urban population compared with the growth of the rural population. The calculation was not that difficult, but the problem for them was where to start. The first challenge was to read the graphs; then they had to translate the information into mathematical form (mathematization); and finally they had to do the calculation.

For example, to find the urban population in 1950 one has to estimate the height of the white bar compared with the height of the black one. The black bar is three times as high as the white bar; so one-fourth of the total population in 1950 (0.12 billion) was living in cities.The difficulty was the integration of measuring, estimation, and simple arithmetic to find out that the urban population in 1950 was 30 000 000 people. The problem was not the mathematics but rather how to use it in a real-life situation.

The idea that a question can have more than one right answer or that a discussion about a problem, even in mathematics, is possible were also new experiences to most of the students. Expecting the teacher to know the one and only answer, the students had no experience in deriving an answer through discussion, that is, thinking about possible answers and giving reasons why they chose a particular answer to a question.

A good example of a problem that elicits discussion is the problem about the number of police officers killed by hour of day in 1966-1975. The data were presented in a bar chart having a bar for every hour, or twenty-four bars. But instead of on a horizontal axis representing hours, the data were presented in the format of a twenty-four-hour clock (see fig. 2).

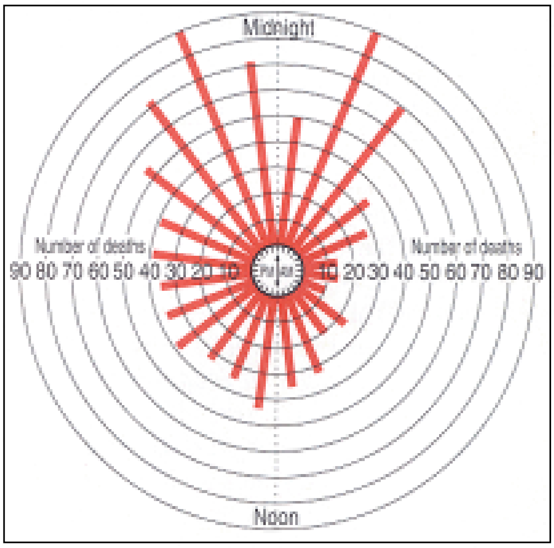

Figure 2: Law-enforcement officers killed by hour of day: 1966-1975

Questions like ‘Compute the number of officers killed per hour’ or ‘Draw a bar chart with a horizontal axis’ caused no problems for the students. But the last question on this problem, ‘For which group of persons is this graph interesting and in what way could it be used,’ seemed to be very ‘difficult’. The students did not know how to answer. So the teacher tried to stimulate them by giving a possible answer, ‘For politicians who want to make the neighborhood safer.’

After a while the students began to understand the problem, and then they produced several answers. One of them made the critical observation, ‘There is not enough information given; this graph doesn’t tell me for what city and in what situation the officers were killed. Maybe we have to split the graph in different graphs.’ The teacher had not thought about this aspect of the problem and so felt insecure about having lost control in class. The teacher stopped the discussion and went on to the next problem, just when the students had begun to show initiative. Another difficulty arose when the class finally agreed on an answer: the students had to formulate their answers in correct English sentences. They were used to giving an answer in the ‘answer box’ on the sheet, and answers to mathematics questions were mostly numbers or yes and no answers. Here they had to explain their answers, and they had problems with that approach. Writing and communication within mathematics is certainly not a trivial goal.

Group work and cooperation

To facilitate the discussion and thinking, we formed groups of four or five students. For many, working in a group and cooperating with each other were new experiences. Some found the concept of group work difficult because it entails the cooperation of individuals in a group. Some aims of the group work were to help students develop a feeling of responsibility to the other members of the group, to organize the group work, to help each other if necessary, and to divide the work among the members of the group. During the first or second lesson, the groups were formed. Every group consisted of a facilitator, a reporter, a leader, and someone who did the computations.

The booklet *Data Visualization* started with an urbanization map of the world. The problems in the first chapter were all associated with this map, and the students were asked to do these problems in their groups. In the first week while they sat in their groups, they did the problems independently. Everyone wrote down the answers: no discussion or cooperation took place. But group work improved during the course of the project.

For example, a problem from chapter 1 on which the students worked individually within their groups is shown in figure 3. The two circle diagrams represent the urbanization of China in 1949 and 1980. ‘What factor, the total growth of the population or the trend of moving to cities, plays the most important role in China?’

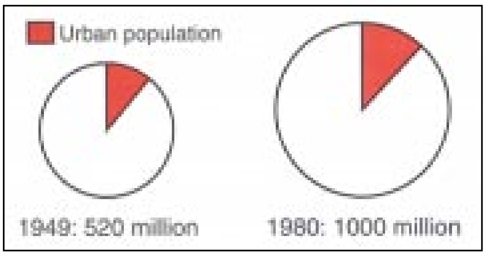

Figure 3: Urbanization in China for the years 1949 and 1980

To answer this question one has to look very carefully at the two circle diagrams and do some computations. In many groups the students tended to think individually about this problem. When I approached a group of students and asked them how they answered the question, they individually told me their answers and wanted to continue with the next problem. When I asked about their reaction to the answers of their fellow students, only then did they start to think about and discuss the different answers. As yet no group work had occurred spontaneously: I was the facilitator.

After some weeks the situation improved. For example, in chapter 6 a list with the top thirty television programs was presented. The list was based on the Nielsen ratings, and for every program the sponsoring network was also mentioned. A portion of the list is given in table 2. The Nielsen ratings reflect the percents of the viewing audience that watched the show.

In class the students were asked to make three separate tables for the three networks NBC, CBS and ABC, and then to make three box plots and compare them. The result of one student’s work is given in figure 4. Finally the students were asked, ‘Is it right to compare the three box plots, and whose interest do the box plots serve?’ The students in the groups divided the work; one member of each group made a table for each network. One student read the numbers from the text, and another wrote them down. Then they compared the three different tables, and a discussion followed to decide on the best way to draw the three different box plots: ‘When we have to compare them, we have to draw them on the same scale.’ In response to the question ‘Is it right to compare the three box plots?’ the group came up with different answers: ‘Yes, because ...’ and ‘No, because ...’ They could defend every answer, and the student with the clearest handwriting wrote down the answers of the group on a single sheet of paper. The group work improved, and seeing the change was exciting.

TABLE 2			
Part of the Nielsen Ratings for Television Programs from September 1985 to April 1986			
Nielsen Rating	Program	Network	Percent of Viewing Audience
1	The Cosby Show	NBC	34.0
2	Family Ties	NBC	30.5
3	Murder, She Wrote	CBS	25.3
4	60 Minutes	CBS	23.9
5	Cheers	NBC	23.7
6	Dallas	CBS	21.9
7	The Golden Girls	NBC	21.8
8	Dynasty	ABC	21.8
9	Miami Vice	NBC	21.3
10	Who's the Boss?	ABC	21.1
11	Night Court	NBC	20.9
12	Highway to Heaven	NBC	20.3
13	Kate & Allie	CBS	20.0
14	Monday Night Football	ABC	19.8

After the first week of the project, problems with the new mathematics decreased drastically. Students were interested in the class and commented that they liked mathematics now more than before, that it was not so boring, and that they had discovered that mathematics can be used in real-life situations. When questions arose about homework, they came after school to discuss them. Even the low-level and least motivated students got involved in the data-visualization unit and liked it.

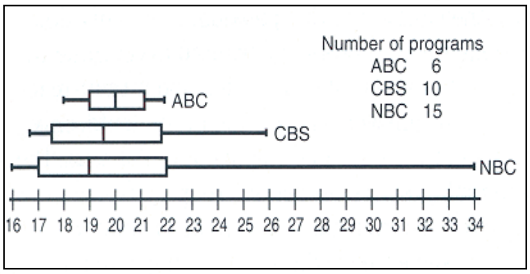

Figure 4: One student's work showing box plots for major networks; Nielsen Ratings are on the horizontal axis. NSC has the best-rated TV program.

For example, one Malaysian student whose spoken English was hard to understand had no contact with his fellow students. He sat in the back of the classroom, and the teacher told me that he had great difficulty with learning. During the four weeks he started to enjoy mathematics, he asked questions in class - very rudimentary ones, but exhibiting his new involvement. He tried to understand the discussion, and even after school he came to the teacher to get answers to his questions. At the end of four weeks, when I was leaving, he told me that he now had discovered the use of mathematics and that he liked it. He showed me his solution to the problem 'Compute the average size of a family.' The problem is shown in figure 5. This problem was from the second chapter of the textbook, and when it had first been assigned, the boy hadn't understood how to solve it. But after four weeks, he told me that he finally understood the question and he was satisfied that he had learned something. Rather than talking about cars, girlfriends or boyfriends, or sports, I observed that the students were now concentrating on mathematics for the duration of the fiftyminute class periods.

Some Data on
the Size of Families in the Netherlands

No. of Children per Family	No. of Families (in Thousands)
0	1176
1	810
2	1016
3	417
4	149
5	59
6	23
7 or more	16

Is it possible to compute
the average size of a family?

Figure 5: A problem from 'Data Visualization' (Freudenthal Institute 1989) on family size. The question is to compute the average size of a family.

To teach or not to teach

The teachers, however, appeared to have more difficulty with the new approach to mathematics than did the students. Traditionally, teachers tell students about a new theory or explain a new mathematical rule, do some examples on the chalkboard or overhead projector, and assign problems in the textbook. They found it difficult to accept the notion that 'the best way of teaching is not to teach,' as the director of the Freudenthal Institute, Jan de Lange, has said. For a teacher to change teaching methods, developed over years of experience and practice, and to allow students to discover solutions by themselves instead of explaining them requires a considerable amount of rethinking and concentration on a new philosophy. Good preparation of the teachers of a project like this is very important. Developers cannot tell a teacher, 'You have to change, throw away your teaching method, and let the students discover the mathematics hy themselves.' A teacher, like a student, has to find out by himself or herself what it is to do mathematics trough a new approach.

In this project, however, the teacher preparation was poor. The project started in September 1989. The first classes were conducted on a Wednesday, the books had arrived on the previous Monday, and I, as the person who could give some support, had arrived on the day in between. Too little time was allowed to preview the project with the six teachers involved. This poor preparation might have been the cause of some of the difficulties the teachers experienced during the first weeks.

During the course of the project we had meetings every week to talk about the problems that they had encountered. The teachers expected that I would tell them what to do and how to teach; they felt insure because a lot of the methods of teaching they had developed were not appropriate for the content of this project. They found it difficult to be confronted with students who gave answers that they hadn't thought about and to facilitate a discussion of which they could lose control. Teacher and students are more nearly equal in this new situation, and teachers need time to become accustomed to this idea.

Let us consider another mathematical question that can be answered in different ways. None of the answers is 'the best'; the answer depends on the argumentation. Take a look at the information in figure 6. By adding the results of all students in one class and dividing the sum by the number of students in that class, solvers can compute the mean average. That's not difficult. The mean average for class A is higher than the mean average for class B. So from the teachers perspective class A performed better on the test, and when one of the students shared her opinion, the question was settled and the lesson went on. However, one of the groups had a discussion about another possible correct answer. They concluded that class B performed better because in that class the chance to get a higher score is higher than the chance in class A. They didn't look at the average but at the number of results lower than 50. The teacher didn't notice the discussion in this group, and a good opportunity for interaction was missed. In situations like these, teachers felt uncomfortable because their answer was not exclusively the right answer to the question.

A problem from *Data Visualization* (Freudenthal Institute 1989) on family size. The question is to compute the average size of a family.

Class A		Class B
2	1	23
9	2	
8	3	
9	4	
87	5	
87	6	
98	7	00122346
76532210	8	012448
91	9	0139

Does this table suffice to judge which class performed better?

Figure 6: Results of two classes of a mathematics test presented in a stem-leaf display. The numbers in the middle column represent the tens, for example, two students in class A have a score of 82 and only one student in class B has a score of 82.

However, the teachers were willing to change, and after four weeks they had become more comfortable with the approach, explaining less and allowing the students to explore on their own.

Some impressions on assessment

The first test that we tried out in the project was a traditional, restricted-time written test that had to be completed in fifty minutes and was given after two weeks. The problems presented in the test asked for thinking, not merely reproducing what was learned during the first two weeks. Although the students had difficulty with this new approach, they showed some nice, creative solutions.

The second test was an open test; the students were asked to design a test for their fellow students that could be used at the end of the project. In pairs and in groups the students designed very original and creative tests. They collected information from magazines and newspapers to design problems that were very related to real life.

The third test was also a restricted-time written test, and the fourth was a take-home test: Students were asked to rewrite an article on the problem of overpopulation in Indonesia using the statistical techniques and graphical-representation methods they had learned during the four weeks. Especially the second and fourth assessment tasks were part of the educational process, and students got the opportunity to show what they did know instead of what they didn't know. The four tests showed us that the differences between the low-level classes and the honors class were not big. Most of the students could solve the problems, and the solutions differed primarily in the level of formalization and abstraction.

Summary

The four weeks of this project were interesting and demonstrated that it is possible to change mathematics education in the United States. The difficulties that students and teachers had in reaction to a new approach to mathematics were the same as those experienced in The Netherlands when the mathematics curriculum was changed. It is interesting to see that American students don't differ compared with the students in The Netherlands. After some weeks, they see the value of this new way of learning mathematics, and they get involved and like it. When one can apply mathematics in real-life situations, it becomes much more motivating to study mathematics.

For teachers, on the contrary, the change is much harder. The teacher is no longer the presenter of new theories and explainer of new ideas but more a facilitator. We learned that good preparation and support of teachers is very important. The best way to ensure that teachers understand and embrace the changes is to have them first play the role of students and go through the learning process themselves. Creating opportunities for teachers to share experiences and to learn from each other is also important to the success of the endeavor.

Implementing the curriculum and evaluation standards with the current teaching materials appears to be hard. Theses materials don't have open questions, real-life problems, and applications in the way we tried in *Data Visualization*.
Although this unit's main focus is on descriptive statistics, connections are made with such other mathematical areas as geometry and algebra. Real-life situations are used not only to apply the mathematics but also to promote the mathematics to start with; mathematics is 'reinvented' by the students in this method. The new approach gives students the opportunity to solve problems on different levels of abstraction and formalization. Students get involved and, it is hoped, increase their self-confidence when they discover that they can solve mathematical problems. Furthermore, if assessment is part of the educational process and tests and alternative assessment tasks are related to what is done in class, students will be increasingly motivated to learn mathematics.

Although changing the mathematics curriculum is not unduly difficult, it appears that a change in teaching methods will be a much more difficult challenge. But the fact that students become excited about working on a more equal basis with teachers and doing useful things in mathematics class will certainly help to make the change easier. To illustrate the feelings of students and teachers at the end of the project, I conclude with two quotations. One student inquired, 'Do we really have to go back to number math and stop doing thinking math?' And one teacher summarized, 'It was a struggle but worthwhile because I know that all my students have learned some useful mathematics.'

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