Modelling motion for the learning of calculus and kinematics

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Introduction

In this paper we present results of a design research project on modelling and symbolizing in mathematics education. With symbolizing we refer to theories on how we create, use and adapt symbols and how this relates to concept development. Here we will focus on the role of graphical models in the process of teaching and learning the principles of calculus and kinematics.
Instructional materials are thought to be transparent for the students, in the sense that students can see and understand the scientific concepts through the materials. The way graphs are used in calculus and kinematics education suggests a similar transparency. For instance, they are used as models for describing change of velocity or distance travelled during a time interval. Students are expected to give meaning to the relation between distance travelled and velocity through characteristics of these graphs such as area and slope.
It is argued that the use of such instructional materials is based on a representational view (Cobb et all 1992), which assumes that scientific knowledge can be represented and made accessible by instructional models. Cobb et all criticize this idea and argue that you must take into account the limitations of the knowledge base of the students into which the scientific concepts have to be integrated. Often it shows that students lack the knowledge needed to see or understand the scientific concepts that are represented by the materials. Elements of this representational view can be found in ‘discovery learning’ approaches. Approaches in which students’ are expected to discover concepts while exploring formal representations in computer simulations. A considerable amount of research shows that students have difficulties with interpreting and using graphs in calculus and kinematics in non-standard situations.
As an alternative for the above-mentioned approaches we will demonstrate how models of motion can co-evolve in a learning process. Theories on symbolizing give rise to heuristics for designing a learning route within which the mathematical and scientific knowledge emerges from the activity of the students (Gravemeijer et all 2000). In this route, the creation, use and adaptation of various graphical models are interwoven with learners’ activities in a series of science-practices. These practices vary from modelling discrete measurements to reasoning with continuous models of motion. We wanted to investigate the students’ contributions during these practices and the way they reason. Consequently, for understanding their activities we used a design research approach of planning and testing the envisioned trajectory in classroom situations. A research approach that aims at *how* a trajectory works, instead of trying to decide *whether* it works.

Guided re-invention by emergent modelling

Continuous velocity-time and distance-time graphs are – often theoretical - descriptions of a situation for mathematical reasoning. Their appearance and conventions (e.g. time-axis horizontal) are the result of a long period of scientific research on the calculus of change. During this period – prior to the continuous graphs - other models were developed and used for different purposes. After a period of almost 2000 years the graphs that we use nowadays were created (e.g. Clagett 1959). These continuous time graphs are models to be used for reasonings about motion with graphical properties like area and slope. Our claim is that many problems of students with calculus and kinematics are due to the fact that they don’t really understand why they can use these graphs for their reasonings.

We investigated an alternative approach that aims at a process in which the mathematics stays related to their understanding of the physical properties of motion, and emerges from the modelling activities of the students. This is also an objective of realistic mathematics education, where instructional design is aimed at creating optimal opportunities for the emergence of formal mathematical knowledge. During this process students can preserve the connection between the mathematical concepts and what is described by these concepts. The students' final understanding of the formal mathematics should stay connected with, or as Freudenthal would say, should be "rooted in", their understanding of these experientially real, everyday-life phenomena (Freudenthal, 1991). The core mathematical activity is "mathematizing", which stands for organizing from a mathematical perspective. Freudenthal sees this activity of the students as a way to reinvent mathematics. However, the students are not expected to reinvent everything by themselves. In relation to this, Freudenthal (1991) speaks of *guided reinvention*; the emphasis is on the character of the learning process rather than on the invention as such. The idea is to allow learners to come to regard the knowledge they acquire as their own private knowledge, knowledge for which they themselves are responsible.

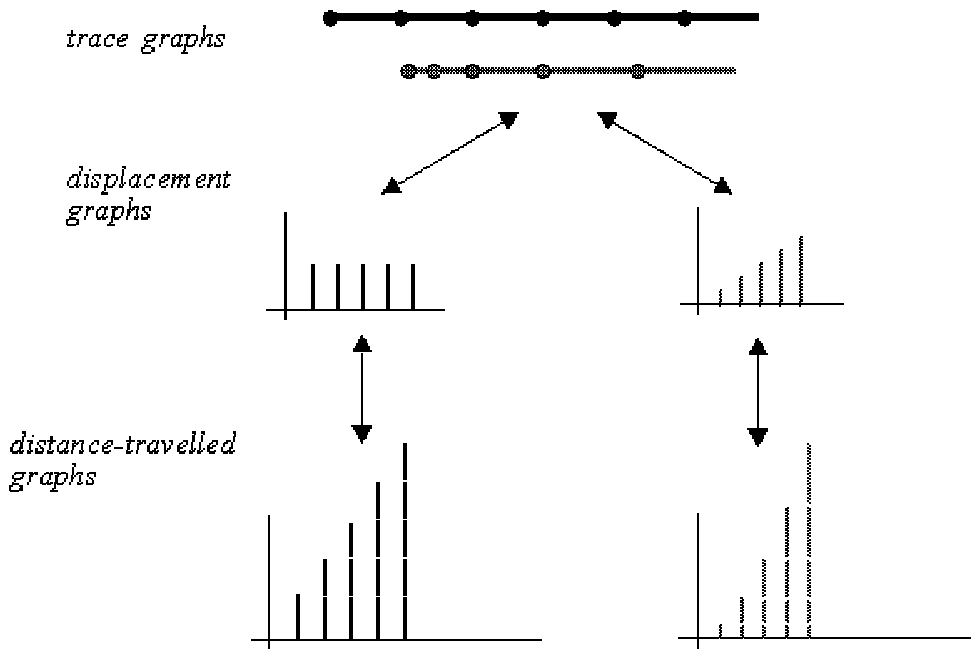
In a reinvention approach, the problem situations for the students play a key role. Well-chosen context problems offer opportunities for the students to develop informal, highly context-specific models and solving strategies. These informal solving procedures then may function as foothold inventions for formalization and generalization, in other words: for progressive mathematization. The instructional designer tries to construe a set of problems that can lead to a series of processes that together result in the reinvention of the intended mathematics. Basically, the guiding question for the designers is: How could I have invented this?

Research on the design of primary-school RME sequences has shown, that the concept of *emergent models* can function as a powerful design heuristic (Gravemeijer, 1998). First, context problems that offer the students the opportunity to develop situation-specific methods are selected. Second, these methods are modelled during classroom discussions and subsequent activities. In this sense, the models emerge from the activity of the students. Even if the models are not actually invented by the students, great care is taken to approximate student invention as closely as possible by choosing models that link up with the learning history of the students. Another criterion is in the potential of the models to support mathematization. The idea is to look for models that can be generalized and formalized to develop into entities of their own, which as such can become models for mathematical reasoning (Gravemeijer & Doorman, 1999). The shift from informal *models of* realistic situations to *models for* mathematical reasoning concurs with a shift in the way the student thinks about the model, from models that derive their meaning from the modelled context situation, to thinking about mathematical relations. In this context the term 'model' must be understood in a broad sense. It is not just the physical representation, but everything that comes with it (e.g. activity and purpose) that constitutes the model (Cobb 1999). As a consequence, during the activities of the students the model and the situation being modelled co-evolve. Modelling in this view is a process of reorganizing both activities and the situation. The situation becomes to be structured in terms of mathematical concepts and relationships.

A modeling motion trajectory

We tried to develop an instructional sequence in which discrete graphs come to the fore as *models of* changing location of a hurricane, and develop into *models for* reasoning about the relation between displacements in time intervals and total distance travelled.

A situation in which it makes sense to describe modelling motion seems the weather forecast, especially the change of position of hurricanes: when will it reach land? This problem is posed as a leading question throughout the unit as a context for the need of grasping change. After being introduced to time series, students work with situations that are described with stroboscopic photographs. The idea is that students come up with measurements of displacements and that it makes sense to display them graphically (based upon Boyd et al. 1996). The key issue that should arise in the discussion is how to describe change (of position) in order to view patterns and to be able to do predictions. Two types of discrete graphs are discussed: graphs of displacements and graphs of total distance travelled. In following activities the students work with a computer tool to analyse various situations with these graphs. During the discussions and their activities, students should find relations between discrete velocity graphs and discrete graphs of distances travelled.
Note that a key element of the notion of reinvention is that the models first come to the fore as models of situations that are experientially real for the students. It is in line with this notion that discrete graphs are not introduced as an arbitrary symbol system, but as models of discrete approximations of a motion that link up with prior activities or students' experiences.



The starting point for the transition of displacements to modelling velocity, is in the medieval notion of instantaneous speed, which is introduced in the context of a narrative about Galileo's work. Instantaneous velocity will be defined in accordance with this medieval notion, in terms of the distance covered if the moving object maintains its instantaneous velocity for a given period of time. In this context, the problem is posed of how to verify Galileo's hypothesis on free fall: velocity increases constantly, and is proportional to time. Figuring this out demands of the students that they come to grips with the relationship between the motion, the representation, and the approximation. During this process, the way of modelling motion, and the conceptualisation of the motion that is being modelled, co-evolve.
A shift is made from problems cast in terms of everyday-life contexts to a focus on the mathematical and physical concepts and relations. In order to make such a shift possible for the students, they have to develop a mathematical framework of reference that enables them to look at these types of problems mathematically (see also Simon 1995). It is exactly the emergence of such a framework that this sequence tries to foster. It is this framework that

enables the students to trace the origin of the mathematical models and to anticipate on what is to come.

Teaching experiments

This learning route is tested out and revised during teaching experiments in four Dutch tenth-grade classes. We collected data by video and audio taping of whole class discussions and group work. The videotapes were used to analyse students' discourses and students' written materials according to a constant comparative method (Glaser and Strauss 1967).

We used the concept of a hypothetical learning trajectory (Simon 1995, Lijnse 1995) to describe and analyse the conjectured teaching and learning process. Reflection on what happened during the experiments results in recognising and optimising successful patterns in the teaching and learning processes. After several teaching experiments these explanatory patterns evolve in recommendations for a local instruction theory.

The qualitative analyses show that during the practices students re-invent and develop graphical symbolizations, as well as the scientific concepts aimed at. However, the analyses left us with some questions concerning the transition of working with discrete measurements to interpreting graphs of a continuously changing composite magnitude velocity. Additionally, we found that the teacher had a crucial role during the classroom experiments. The students' activities were especially productive when the teacher introduced situations and arranged classroom discussions of students' contributions in such a way, that the students themselves came up with the problems that had to be solved.

Conclusion

In this guided re-invention approach the construction and interpretation of graphical models and the scientific concepts are rooted in the activity of the students through a cascade of inscriptions (Roth e.a. 1997). This ensures that mathematical and physical models aimed at, are firmly rooted in the students' understanding of everyday-life phenomena. We conclude that in such teaching and learning processes students learn to model and to organize new situations from a mathematical perspective. We advise that design heuristics for realizing classroom discussions where students pose problems to be solved accompany the emergent modelling heuristic.
On the basis of our findings we like to discuss the above-sketched results and the implications of the use of theories on modelling and symbolizing in science and mathematics education design and research.

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