School Algebra struggle, what about algebra computer games?

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abstract

For many students, traditional (school) algebra is formal and abstract. This algebra deals with symbols and letters, and at some point students who want to move on in mathematics and science have to learn how to move around and manipulate with these symbols. The symbolic part of school algebra is little sense making and it is not always immediately clear why you have to learn algebra.

Given the situation that (some) students have to learn (some) traditional algebraic skills, we have explored in what ways little computer programs can help students to develop algebraic understanding (concepts) and algebraic skills. Especially little programs (applets) that provide feedback with scoring facilities have potential for learning algebra.

Focus of the paper: The problem of algebraic skills

A person cannot read a newspaper, shop, engage in most games or sports, or work without dealing with numbers. So we never have to ask why all students should study arithmetic. But (formal) equations, functions, and expressions with variables are noticeably lacking from these everyday activities. Furthermore, many of the so-called "real-world" algebra problems we give students can be solved using only arithmetic. So we are obliged to examine whether it is wise to require all students to learn a significant amount of algebra. Is it only "mathematical patriotism" that causes us to support teaching all students algebra, or are there significant reasons to learn the subject? Does the existence of computer algebra systems help or hurt the case for teaching algebra to all students? (Usiskin 2004).

These questions raised by Zal Usiskin wonderfully summarize the choices to be made in school algebra. These are the major issues of school algebra that we are facing when we are reforming algebra instruction. In the research that we report on in this paper we focus on only a few aspects of the major problem of the reform of algebra instruction. We have made choices and restricted ourselves:

- We deal with the “traditional” school algebra – that is the algebra that deals with symbols and letters, and the skills to manipulate with these symbols; expressions and formulas, graphing and solving equations.
- Although we deal with traditional school algebra topics and skills, we have developed and investigated strategies and environments in which students can learn the algebra in sense-making and “fun” ways.

The purpose of this research and development project - given the situation that some students have to learn (some) traditional algebraic skills - is to explore methods to help students to develop algebraic understanding (insights and concepts) and algebraic skills in sense-making ways. The methods and strategies that we have used in the project to accomplish this goal are technological tools and environments. Especially little programs (applets) that provide feedback and have scoring facilities seem to have potential for learning "traditional" algebra.

Theoretical Framework: Realistic computer algebra environments

The theoretical framework underlying our research and development work is that of Realistic Mathematics Education (e.g. Gravemeijer 1994, Freudenthal 1991). One of the features of this theory on learning and teaching mathematics is that learning should be meaningful and sense making to students. This is often interpreted as that in Realistic Mathematics Education (RME) one should always use realistic contexts and problem situations. Realistic is often interpreted as originating from "real life". However, sense making and meaningful problem situations can also be embedded in contexts that are artificial or mathematical. The key is that the problems are real and meaningful to the students; that the problems are sense making and worth while solving.

This implies that a computer game, a computer algebra environment, or an abstract equation with no "real life" context can be real to students when they are embedded in a sense making environment. The task of a curriculum developer is thus to create an environment that challenges and motivates students and that promotes learning.

Computer games can offer such an environment in which students can learn in sense making and meaningful ways. The context of a computer game does not need to be real to have the player enjoy the game. However, not any computer game environment is good to use, because there are good and bad computer games. So when one wants to integrate computer games in education, you need to take account of theories on computer games. Overmars (2003), following Rolling (2003) defines a computer game as interactive, it presents a virtual world (an environment) and has players. During the game the player develops skills and knowledge in order to reach one or more goals. In a good computer game the player gets involved[1] and a good computer game is in balance. Balance in many dimensions: some luck is OK but not too much; a player should improve during the game; the computer should do the work; there should be some kind of reward; one should play with the game not against. A very important aspect of good computer games is the flow; the balance between skills and challenges should be such that it keeps motivating the player to continue playing. On the popular website www.gamemaker.com, Mark Overmars has provided various examples of good games and bad games and an elaborated overview of criteria for good computer games.

Another aspect of RME is that students "reinvent" the mathematics. In contrast to radical constructivist theories, RME learning is a "guided" reinvention process (Gravemeijer 1994), in which the guidance is provided by the learning environment (the instructional materials) and the teacher.

[1] Too much involvement can lead too addiction.

Methodology: Algebra with applets

Using RME and Overmars' and Rolling's theory on computer gaming as theoretical frameworks, we developed a series of small computer programs to facilitate learning algebra for students of 12 to 15 years old. In our work we have restricted ourselves to students in the honors tracks; students at secondary education who will move on to College or University after graduating from secondary school[2]. For these students, (formal or "traditional") algebra is part of the existing curriculum. In the first three years of secondary school the algebra topics include: formulas and expressions (constructing, using, combining, also in relation to graphs and tables), expressions and equations (creating equations and solving equations using various strategies), algebraic manipulation skills needed to rewrite or solve equations and expressions (working with parentheses, manipulating linear and quadratic equations).

The computer programs we have developed are java applets that run over the internet. The applets have been published on the WisWeb-site (www.wisweb.nl) a website that has been especially designed for use by students (and teachers) at secondary schools. The advantage of using applets over the internet is that students can use them anywhere with an internet connection and any time.

Since 2000 we have developed more than 20 applets for algebra instruction, and many of these applets have different versions that fit certain specific algebra topics or skills. For grades 8 and 9 we have developed instructional materials in which many of these applets have been integrated. These instructional materials cover all algebra for these grade levels and can be used to replace the existing textbooks. The materials consist of a mix of paper worksheets and computer lab activities. Because of practical reasons (not enough computers available at the schools), not all lessons can be given in a computer lab and the instructional materials contain computer activities and paper-and-pencil activities. The pedagogical setting is also a mix of lessons in regular classrooms without computer (occasionally the teacher may have a demonstration computer with projection facilities available) and lessons in computer labs in which students work with the applets.

The applets and instructional materials were developed by a team of researchers and curriculum developers at the Freudenthal Institute in collaboration with teachers from four secondary schools. All was field tested extensively at these schools, and the lessons were observed by the researchers. Based on these experiences applets and instructional materials were revised and field tested again. Several articles in Dutch mathematics education journals have been published (see the list with references at the end of this paper).

[2] These are students in the Havo and VWO tracks at secondary schools in The Netherlands

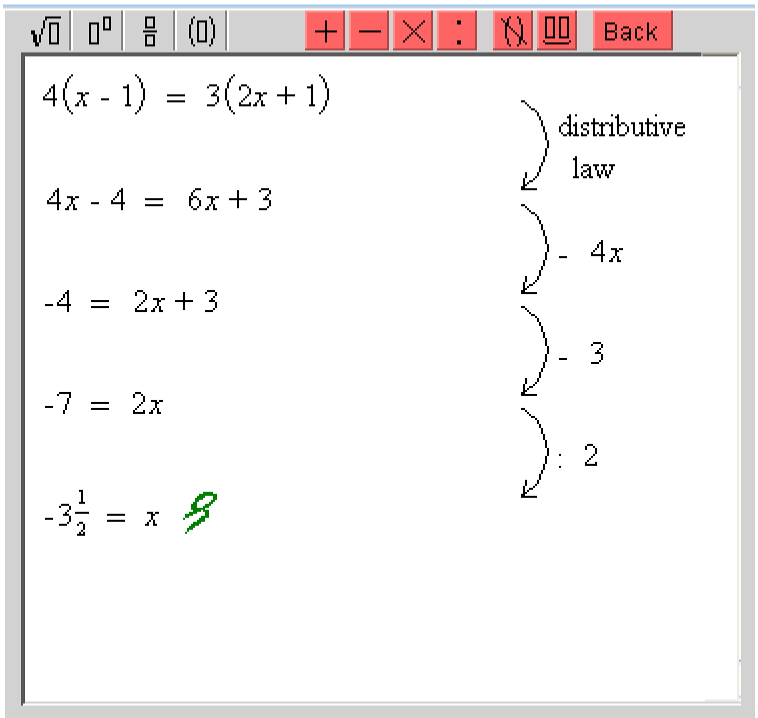

Figure 1, “Balance”, solving equations with the balance strategy, a WisWeb-applet

Experiment: Abstract algebra made virtually real

To illustrate the potential of these environments to learn algebra, we will use a few algebra applets that have aspects of computer games. We start with the applet “Balance” (see figure 1). This applet is about “solving equations” and uses the balance strategy.

The applet presents an equation that is to be solved step by step. With the red buttons on top, the student can select an operation that is then performed on both sides of the equation. The goal is to simplify the equation and eventually end with $x =$ solution. With this applet, a student has to think and decide what operation to perform and the applet actually performs the calculations. When the end of the solving process is reached, the applet gives feedback with a green mark, and a student can go on with the next equation from the list on the bottom of the applet window (figure 2). This applet serves the objective “understanding and using the balance strategy for solving equations.”

Of this applet we developed a version with gaming features (see figure 2). In this version, the student also needs to type in the next line. The applet does not perform the operation, but only gives feedback on the correctness of the next line in the solving process. Students can earn points for entering the correct next line. When they solve the problem themselves without asking the applet for help, they can get the maximum number of points: 10. When a student asks for help, the applet performs the operation to get the next line in the solving process (as in figure 1), but that costs points.

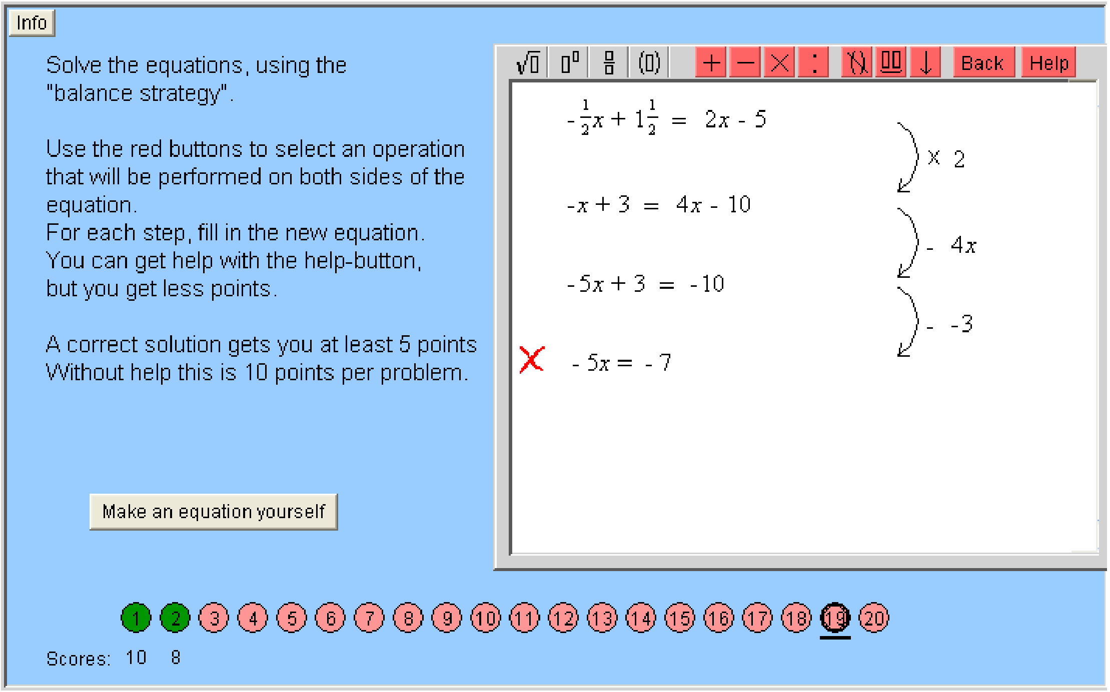

Figure 2, game version of the Balance applet.
Students have to solve the equation and have to type in the next line themselves.
With the help button, the applet performs the operation, but that costs points.
A correct solution gives 5 points; when no help is used that is 10 points.

Students get feedback with a green mark (correct) or a red cross (wrong). There is one correct solution, but there are many ways leading to this solution. Students have a lot of freedom, and they get immediate feedback for each line in the solving process they enter.

The balance applets are two examples of the kind of applets we developed for students to learn and practice solving equations. The “Balance game” applet illustrates how we have implemented features of computer games. In this same philosophy we developed more applets with two versions, an open environment and a gaming environment where students get feedback and can earn points.

Results: The power of feedback and scoring points

From the field tests of the applets we learned that students develop and practice algebraic skills when using these kinds of applets. Students performed quite well on the – paper and pencil – tests administered after a chapter working with the instructional materials in which the applets were embedded. Students scored better than students in previous years who only worked with the textbooks. From the observations and interviews with students we learned that students appreciated the feedback given by the applet. A “wrong” (red cross) by the applet is interpreted as “not right yet” and stimulates to go on and improve. Students often interpret a “wrong” by the teacher as negative; as “end of the story, I can’t do it”. Students were challenged to experiment with the various options provided by the applet. They played around and investigated what happened when you perform “strange” operations on the equations, like e.g. multiplying by a very large number. Students also wanted to explore and find out when the applet would crash. The learning environment (the applet), although virtual and not connected to a real life problem situation, provoked real problem solving within the context of formal equations. Apparently the applets encouraged students to develop and execute algebraic thinking and algebraic problem solving.

The game versions of the applets had more impact on students' motivation to solve problems than we had anticipated. Students wanted to score the maximum number of points, and did not want to ask for "help". In "Balance game" applet students did not bother about all the work of typing in the next line. They very soon became quite proficient in typing in complex algebraic expressions and when working in pairs they helped each other and tried to convince their partners to type in the "correct" next line. There was a lot of interaction going on between pairs of students working together on one computer.

Students had fun with the algebraic activities. They enjoyed themselves. A nice unanticipated side-effect of working with the balance applets was the improved notation schemes of the students they used when solving equations on paper-and-pencil. In the applet the next line of the solving process is nicely placed under the previous line, and the operation that is performed is put at the right with an arrow downwanrs. Students incorporated this notation in their own way of writing down the solving process when they had to solve an equation with paper and pencil.

Conclusion: Is a computer game the same as educational software

After four years of working with applets integrated in mathematics class, we can carefully conclude that the use of applets has add-on value: They are fun and motivate students; they allow students to work at their own level of thinking and thus better address individual differences between students; the visual, interactive and dynamic features of applets makes the mathematics more easy to understand; thanks to the calculation power of the applets one can focus on the mathematical concepts and models; students are more creative and get more self esteem; the applets form a model students can fall back on; the practice and feedback features are much more powerful than paper-and-pencil exercises[3]. However, to take fully advantage of these opportunities of applets, applets should be integrated in the daily mathematics class routine. Using the computer should not be a voluntary thing that is extra beyond regular math class.

With respect to the game features of educational software, we need to be aware of the differences between games and play yards. In a play yard environment (like the regular "Balance" applet), students' creativity is important and the end result is often a nice product. A play yard is something else than a game in which students have goals to accomplish (e.g. getting scores). For some purposes a play yard environment is better suited than a game environment. The pressure of scoring may prevent students from experimenting and exploring.

Another issue to be aware of is that playing is not (always) the same as learning. One should not force a playing situation in a learning environment when it is not sense making or meaningful. When playing, you often learn, but in educational games too often students learn other things than the mathematics that is intended; e.g. students learn how to fool the program and how to find shortcuts to get points, instead of learning how to solve equations.

[3] On the WisWe-site (www.wisweb.nl) we published various notes and comments about the experiences that led to this list of add-on value of applets.

From the theory on Computer Games in combination with the experiences from the field tests, we learned of more features to keep in mind in the future development of applets. The problems in the applets need to match the educational setting. That is the flow is to be respected, thus the kind of equations presented should make sense to the students. The reward needs to match the students; e.g. a green mark will do for secondary students, but a funny cartoon that appears when the correct solution is found, may be a better reward. Another challenge is to explore the possibilities of adaptive applets; those are applets that adapt the level of abstraction or difficulty to the level of the player. The interface needs to be simple and self explanatory to the player (that is the student). In our existing applets, winning is not really associated with good playing; the choice of strategy and efficiency should pay off in some way. A final question that we have not systematically explored so far is how “realistic” the applets have to be. How can we make the applets more real in look and feel? And is this important?

In our research and development work on using applets in mathematics education we use various kinds of applets (play yard environments, games, practice software), and each kind has its pros and cons. We have learned that we need to be clear in the choices that we make and the purposes and objectives of the applets we use. Some applets serve practice objectives and goals, while other applets serve concept development goals. The variety is powerful, as long as we not strive for that universal best kind of computer program for learning and teaching mathematics. That piece of software does not exist!

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