Celia Hoyles on Freudenthal - an interview

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Last year, Celia Hoyles received the first Hans Freudenthal Award for a cumulative programme of research in mathematics education. Celia Hoyles studied mathematics at the University of Manchester. She began her career as a secondary teacher, and then became a lecturer at the Polytechnic of North London (now London Metropolitan University). She entered the field of mathematics education research in the late 1970s and became professor of Mathematics Education at the Institute of Education, University of London, in 1984. In December 2004, she was appointed Chief Adviser for Mathematics at the Department for Education and Skills of the UK Government.

Celia Hoyles (*CH*) is interviewed by Arthur Bakker (*AB*), who worked at the Freudenthal Institute but now works as a research officer in the 'Techno-mathematical Literacies in the Workplace' project, which is directed by C. Hoyles and R. Noss.

1 The interview

AB: *How do you remember Freudenthal?*

CH: I have this vivid image of Freudenthal with his bow tie at a plenary session at the Psychology of Mathematics Education conference in 1983. I remember it so well as if it were yesterday: there was this incredible discussion between him and Fischbein – these two brilliant guys who spoke from a different point of view. Fischbein spoke from a psychological viewpoint and Freudenthal from his particular take on mathematics and didactical phenomenology. They got so heated! They were having a real go.
I think it was about common sense, I think with Freudenthal seeing mathematics as an extension of common sense, but Fischbein did not agree. That was the first time I had come across the ideas of Freudenthal.

Freudenthal was a mathematician who was obviously held in huge esteem in mathematics, and then he dedicated himself to mathematics education. Actually, there were only a few mathematicians, wise mathematicians, whose ideas about education worked out well and who understood that mathematics education is much broader than educating future mathematicians. He also thought that mathematics teaching needed to be motivated by solving 'real' problems so that learners from the start would appreciate the power of mathematics. This resonated with many educators who wished that their subject to be more grounded, so students would be able to apply their knowledge and at the same time would not be bored.

In my view, mathematics education should more generally be about investigating structures and symbolising them. And I think that for school students problems need not always be realistic.

AB: *Do you think him being a mathematician made the approach of Realistic Mathematics Education (RME) successful?*

CH: I have not quite got to the bottom of why his approach of RME turned out to be so appropriate for the culture and tradition in the Netherlands, but I suspect his stature made a huge impact, along of course with the power and resonance of his ideas. I actually think it would not work here in England in the same way, but I might well be wrong. We just don't have the same culture; for instance, teachers teach in different ways and we have different traditions in which the curriculum and assessment procedures are shaped.

What was it that built up a culture in the Netherlands, where these ideas could germinate and grow? I think it must also have been Freudenthal's presence, bringing the mathematics and mathematics education communities together. If you can only achieve that synergy between mathematicians and educators around a few key concepts, then you can form a culture in schools and beyond.
The success of RME was due to the community of teachers, teacher educators, curriculum developers and researchers - all these people around the country working with a similar agenda. Of course, they all have their own take, but they don't have to negotiate the starting points. That is why RME imported here wouldn't work: you need this common culture first.

AB: *What were the ideas of Freudenthal that inspired you most?*

CH: Freudenthal gave kids a voice. Mathematics had to be real and meaningful for the learners, and by starting with realistic contexts, you can draw more people in. In those days, I was doing research on the affective side of learning mathematics, which is why I liked his ideas. In particular, in his 'Didactical Phenomenology' (1983), Freudenthal wrote very insightful things about geometry, which was an area I worked in. All educators acknowledged that pupils should explore shape and space, but then there was always a huge step to formal geometry. In England, geometry education at that time either focussed on the space explorations or the theorems and proofs, and Freudenthal's notion of reinvention was important to make the connection between the exploration of geometrical ideas and the more formal geometry. I thought that was really impressive.

AB: *I assume Freudenthal was one of many thinkers who influenced you?*

CH: Yes, there were others - Piaget and, for me centrally, Papert. Freudenthal's ideas on geometry were important to me, as I felt very strongly about the importance of geometry and the need to link all the beauty and intuition of the subject with the deductive side - which for me had been a separate domain. All these thinkers link in for me with many other influences such as constructivism and so on. RME is very much in parallel - great minds think alike. One issue that disappointed me in RME was the way in which computers were discussed in relation to mathematics education. Freudenthal did not seem to appreciate the importance of computers, how they could be used as mediating tools as part of the problem solving process, whereas for me computers - Logo at the time - were very important (see for example Noss & Hoyles, 1996). Perhaps he just didn't put his great mind to this.

Apart from his reinvention idea and engaging learners, there is something else I have to tell you. When I taught post-graduate students in mathematics education I used an article by Freudenthal, which was a critique of research in mathematics education. It was vicious! He was right, but he did 'nasty' things: he went out and looked up the references and showed that the references did not quite say what they were quoted as saying. You could see he was using a mathematical mind, which is slightly unfair in maths education research. When you quote a mathematical theorem you can use it wholesale, but research in maths education is not quite like that. You always reinterpret others' research. Yet having rigorous standards is good; I guess he also tried to make maths education a science (see Freudenthal, 1978).

In those days, research in mathematics education was mainly about doing pre and post-tests on errors and misconceptions, which seems easier, but Freudenthal, along with many others, moved away from that and based his ideas on student interviews. Nowadays this sounds rather ordinary of course, but at that time it was quite revolutionary. Interviews bring in the student's voice and that really appealed to me.

AB: *That approach also fits well with his idea that mathematics is a human activity - the idea of mathematising. We need a research methodology that acknowledges this mathematical activity.*

CH: Mathematising is absolutely crucial. When you make a model of a situation - and I do not mean modelling in a traditional applied mathematics sense (we call it 'situated modelling') - you get rid of all those aspects that are not so relevant and you can see something you haven't seen before: you see a structure. That is central to mathematics education, although it is often not made clear what the point of mathematising or modelling is. In an ideal world, the point of doing it has to be obvious: you suddenly see something you didn't see before and it is useful to you, surprises you, or enchants you.

What makes me sad sometimes is what has happened to the ideas of great visionary thinkers. The ideas sometimes become trivialised: let's get a context for this and then it is not real anymore. A good curriculum is a nice vehicle in the beginning, but it is not enough. RME is interpreted very widely and it sometimes loses its central meaning, although Freudenthal's goals are clear. I call this neutralisation and, unfortunately, there is no solution to it.

AB: *We have to breathe life into it over and over again. It is like composers and musicians: composers write the music, but musicians have to recreate the music in each performance.*

CH: Yes, but at least you have a community in the Netherlands that tries to do that.

2 Reflection

In this reflection on the interview, I would like to discuss a few interesting points that Hoyles raised.
First of all, why did Realistic Mathematics Education (RME) work in the Netherlands? I think Hoyles is right in that Freudenthal as a famous mathematician putting his energies into education helped in bringing together the mathematics and maths education communities. In addition, however, we also need to look at RME historically. In the 1970s students and their teachers suffered from New Maths in the Bourbaki style, which focussed on set theory and formalisation. It was very dry and there was a lack of reality. Freudenthal with his group got the chance to work on alternatives because the Ministry of Education supported improvement of mathematics education through the development of instructional materials and improvement of mathematics education.
The second point is the role of computers in RME. I never met Freudenthal in person, but from what I have heard from colleagues, he indeed was not very interested in their role for mediating the learning of mathematics.

Perhaps computers could not really do very useful things in education in his days, but I assume that using computer tools was then considered too restrictive for the RME approach.
The third point is the difference in culture. I think Hoyles is right about the importance of having a culture that agrees upon the importance of making mathematics more realistic to kids. In a broader sense I would like to speculate about a cultural difference between the Netherlands and Britain that has made it easier in the Netherlands to develop 'mathematics for all'. What I hear in England is that there is an elitist attitude towards education, which I will try to illustrate with a few examples. First of all, there are large differences between schools in England, and the same holds for universities.

In the Netherlands we do not have such big differences between schools: for example, we hardly have any private schools, which are quite common in England, and many parents perceive these schools as the best (though few can afford them). Another example of the focus on the best is that the UK Government nowadays gives extra funding to schools that specialise in particular topics (science, technology, languages). One school that deliberately did not want to apply for this 'specialist status' received considerable attention in the press (the Guardian, 30-11-2004).

The attention towards 'the best' is also apparent from the huge number of awards and prizes in English education, and the fact for example that the Institute of Education's mission statement is 'to pursue excellence in education'. In the Netherlands it is almost unthinkable to have such a phrase displayed on the building and on every institutional document. In the Netherlands it is much more common to wish to be 'normal'. Yet in mathematics education I think we have paid too much attention to 'mathematics for all', because those students who might have the talent to study mathematics or science at university are not being challenged enough during their school years. Initiatives such as the Junior College Utrecht seem to be a productive countermovement.

References

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Vorig jaar ontving Celia Hoyles de eerste Hans Freudenthal-prijs voor onderzoek naar wiskundeonderwijs. Celia Hoyles studeerde wiskunde aan de Universiteit van Manchester, begon haar carrière als leraar in het voortgezet onderwijs en werd daarna universitair docent aan de London Metropolitan University. Eind jaren zeventig werd ze onderzoeker en in 1984 werd ze benoemd tot hoogleraar in wiskundeonderwijs aan het Institute of Education van de Universiteit van Londen. In december 2004 is zij daarnaast benoemd tot hoofdadviseur voor wiskunde aan het Engelse Ministerie van Onderwijs.