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Design Research in Mathematics Education:

Indonesian Traditional Games as Means to Support Second Graders' Learning of Linear Measurement

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Preface

"The Milky Way is nothing else but a mass of innumerable stars planted together in clusters."

- Galileo Galilei –

This thesis is nothing else but an idea that was supported by a mass of innumerable assist and help from many people surrounding me. First of all, I am very grateful to the teacher and all the students – in *Sekolah Dasar Negeri Percobaan 2 Yogyakarta, Indonesia* - involved in this research. A huge thanks is dedicated to Bu Budiyati – the classroom teacher – for her participation in this research and also for her spontaneous great ideas during the teaching experiment.

I am so glad to be supervised by Michiel Doorman and Ronald Keijzer because they did not only supervise and assist me in writing this thesis, but they also supported me to deeply excavate my idea. What they did were in line with the opinion of Galileo Galilei: ***"We cannot teach people anything; we can only help them discover it within themselves".*** Therefore, I would like to express my gratitude to them. Bapak Sutarto Hadi and Ibu Rosnawati are also two important persons behind this thesis. Bapak Sutarto Hadi helped me a lot in interpreting the videos of the activities and Ibu Rosnawati was an important "bridge" between me and the school.

The idea of this research itself came to my mind after having a discussion with Ibu Siti Aminah and Bu Rahma, therefore I would also like to thanks both of them for the inspiration. Another inspiration came from Ibu Annie Makkink when accompanying me in the first day of the sequence of teaching experiments, thank you very much Bu Annie. I would not have many data sources without a help from Anggit Prabowo, my student who was also interested in realistic mathematics education. The last but not the least, I would like to thank the following people: my fellow students for many discussions and also Mark Uwland for correcting my written English.

This thesis is my special dedication to my family – especially my father and my mother - for their never ending support.

Table of Contents

List of Figures

6. Conclusions and discussion

List of Appendices

Design Research in Mathematics Education:
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Abstract

Many prior researches revealed that most of young children tended to perform a measurement as an instrumental procedure, without a complete conceptual basis. This tendency may be due to the way in which linear measurement has been directly taught to young children as an isolated concept, separated from children's daily experiences. For this reason, a set of experience-based activities was designed to connect teaching and learning of linear measurement to children's daily life experiences.

This research aimed to investigate how Indonesian traditional games could be used to build upon students' reasoning and reach the mathematical goals of linear measurement. Consequently, design research was chosen as an appropriate means to achieve this research goal. In a design research approach, a sequence of instructional activities is designed and developed based on the investigation of students' learning processes. Forty-five students and a teacher of grade 2 in elementary school in Indonesia (*i.e. SD Percobaan 2 Yogyakarta*) were involved in this research.

The result of the teaching experiments showed that *fairness conflicts* in the game playing could stimulate students to acquire the idea of a standard measuring unit. Furthermore, the strategies and tools used by students in the game playing could gradually be developed, through *emergent modeling*, into a ruler as a standard measuring instrument. In the experience-based activities for learning linear measurement, emergent modeling played an important role in the shift of students' reasoning from concrete experiences in the situational level towards formal mathematical concepts of linear measurement.

Keywords: linear measurement; experience-based activities, Indonesian traditional games, design research, fairness conflict, emergent modeling

1. Introduction

Measurement has been a part of human life since centuries ago when some old civilizations used their body parts to measure the length of objects. The historical studies of ancient mathematics revealed the possibility that geometry and arithmetic were invented for counting and measurement purposes (Henshaw, 2006). Another example of the importance of measurement is how Nichomacus, a Greek mathematician, attempted to prove musical propositions by measuring the lengths of strings (Hodgkin, 2005).

Considering the importance of measurement in daily life, measurement has been taught since at elementary school in many countries. However, it is common that measurement is directly taught at the formal level of young children as an isolated concept (Castle & Needham, 2007; Kamii & Clark, 1997 and van de Walle & Folk, 2005). Teaching and learning of linear measurement mostly focuses on the use of a ruler as an instrumental procedure and then, rapidly, followed by conversion of unit measurements. The students' progress in acquiring the basic concepts of linear measurement when performing a measurement is not well-considered. Regarding this fact, there were two important issues that were well-considered as a reason to design and develop new instructional activities in this research.

The first issue is the finding of Van de Walle and Folk (2005) that young children have difficulty in understanding the basic concepts of linear measurement in the formal level. Although they can experience measurement using ruler or other measuring instruments, it cannot be guaranteed that they really understand the basic concepts of linear measurement. When children in grade 1 learn to measure the length of objects using non standard units, most of them know that they have to lay paper, pencil or other measuring units from end to end of the measured objects. Nevertheless, sometimes there is overlapping between the units and also empty spaces between the units. What students understand is they have to make an array of units. In the higher grades, most students in grade 2 until grade 4 could not give the correct measure of an object that was not aligned with the first stripe of the ruler (Kamii & Clark, 1997; Kenney & Kouba in Van de Walle, 2005 and Lehrer et al, 2003). These students merely focus on the number that matches to the edge of the measured object. These findings show that students tend to perform a measurement as an instrumental procedure, without a complete conceptual basis. Consequently, the teaching and learning of linear measurement need to focus on both how to use a measuring instrument and understand how this instrument works.

The need to focus on both how to use a measuring instrument and understanding how this instrument works directs to the emergence of the second issue, namely experience-based activities. The foundation of measurement education in kindergarten and elementary school needs to be laid on doing meaningful measuring experiences, through which a connection is made between informal measurement knowledge and the use of conventional and standard measuring instrument (Buys & de Moor, 2005 and Castle & Needham, 2007). Consequently, it is important to give young children experience-based activities that embody some basic concepts of linear measurement. Experience-based activities are relevant with Freudenthal's idea that stresses mathematics as a human activity, instead of subject matter that has to be transmitted (Freudenthal, 1991). Freudenthal (*ibid*) proposed the need to connect mathematics to reality through problem situation because experience-based activities could contribute to the emerging of mathematical practices. For young children, game playing could be a problem situation, which is experientially real for them and, therefore, can be used as a starting point for their learning process. In Indonesia, there are some traditional games that, without any consideration, are related to measurement activity. Some of those games, such as "*gundu*" (playing marble) and "*benthik*" embody some linear measurement concepts including comparing, estimating and measuring distances.

Considering the two aforementioned issues in the teaching and learning of linear measurement, namely students' tendency to do a measurement as an instrumental procedure and the need to connect mathematics to reality, we conjectured that game playing as a daily life experience could be used as a starting point for learning the basic concepts of linear measurement. The game playing can form a natural part of the experience-based and development-focused activities for the teaching and learning of linear measurement. Consequently, the central issue of this research is the use of Indonesian traditional games as experience-based activities for teaching and learning of linear measurement in grade 2 of elementary school. It is conjectured and expected that students' understanding of the basic concepts of linear measurement can be built upon students' natural experiences in their daily life, and that therefore students correctly and flexibly use a ruler.

Research questions

The main objective of this research was to investigate how Indonesian traditional games could be used to build upon students' reasoning and reach the mathematical goals of linear measurement. This research objective was split into two focuses to investigate the whole process of students' learning of linear measurement from experience-based activities to formal linear measurement. The first focus aimed at investigating the role of Indonesian traditional games to support students in promoting and eliciting the basic concepts of linear measurement. How Indonesian traditional games, as the contextual situation problem in learning measurement, could contribute to students' acquisition of basic concepts of linear measurement. The research question that was formulated to achieve this aim was:

> *How can students' game playing be used to elicit the issues and the basic concepts of linear measurement?*

The second focus arose when the instructional activities moved to the more formal mathematics, namely measuring using standard measuring instrument. The concrete mathematics that was elicited by Indonesian traditional games needed to be conveyed to the correct and meaningful use of a ruler as the formal mathematics of linear measurement. Hence, the second focus of this research was how to develop students' concrete knowledge of linear measurement to formal knowledge of linear measurement. The following question of this research was formulated as a guide in focusing on students' learning process in linear measurement.

> *How can students progress from a game playing to the more formal activities in learning linear measurement so that the mathematical concepts are connected to daily life reasoning?*

2. Theoretical framework

This chapter provides the theoretical framework that was addressed to construct groundwork of this research. Literature about linear measurement was studied to identify the basic concepts that are required to do a correct linear measurement. Furthermore, this literature was useful in designing instructional activities in which each of the basic concepts of linear measurement could be taught in the proper level of young children and also how linear measurement could be connected to daily life reasoning.

In this research, Indonesian traditional games were exploited as experience-based activities and contextual situation to build upon students' reasoning and reach the mathematical goals of linear measurement. Consequently, literature about realistic mathematics education was needed in explaining and investigating how mathematical reasoning in the experience-based activities as the contextual situations could be shifted towards the more formal mathematics.

This chapter also provides a short overview about linear measurement for elementary school in Indonesian curriculum in which this research was conducted.

2.1. Linear measurement

Van De Walle and Folk (2005) defined a measure as the number that indicates a comparison between the attribute of the object being measured and the same attribute of a given unit measurement. There are some stages that precede linear measurement, namely comparing length, estimating length, and measuring length. The sequences of a linear measurement procedure are described as follows:

a. Comparing length

   Comparison as the simplest measurement can be done by "filling", "covering" or "matching" the unit with the attribute of the measured objects. The simple way to express the relation of attributes between the compared objects is given by words, such as "longer-shorter".

   There are two kinds of comparison, namely:

   – Direct comparison

     This comparison is used if the compared objects can be placed next to another; therefore a direct comparison does not require a "third object".

- Indirect comparison

  When the compared objects cannot be placed next to another then we need to do indirect comparison. In an indirect comparison, a “third object” is required as a reference point that is gradually developed into a measuring unit for measurement.

b. Estimating length

   Estimating length of an object is more like a mental comparison because it tries to relate the length of the object with the benchmarks in mind.

   Benchmarks are needed as the points of reference in estimating the length of an object. Furthermore, according to Joram (2003), benchmarks can enhance the meaningfulness of standard units of measure and, therefore, benchmarks can be used as an important component of instruction on measurement and measurement estimation.

c. Measuring length

   The need of measurement is initiated in indirect comparison when the objects cannot be directly compared by placing them next to each other. Each object is compared to a “third object” and the relation between those two objects is derived from the relations between each object to the “third object”. In this process the “third object” becomes a unit for measuring.

Those measurement procedures are built upon a set of basic concepts of measurement. Barret in Stephen and Clement (2003) mentioned two basic concepts of linear measurement, namely *unitization* and *unit iteration*. *Unitization* occurs when we bring in a shorter object or mentally create a shorter object and compare its attribute to the attribute of other objects. In the next stage, this shorter object becomes a unit of measurement. By establishing a unit of measurement, we anticipate the second basic concept of linear measurement, which is *unit iteration*. *Unit iteration* is the process of finding how many units would match the attribute of the measured object. When a unit is not enough to cover up the attribute of the measured object, then the unit iteration is needed.

In addition to the idea of Stephen and Clement (2003) about linear measurement, Lehrer et al (2003) separated important ideas of linear measurement into two conceptual accomplishments, namely the conceptions of unit and the conceptions of scale. The basic concepts included in these two accomplishments are described in the following table.

	Basic concepts	**Description**
Conceptions of unit	• Iteration • Identical unit • Tiling • Partition • Additivity	A subdivision of a length is translated to obtain a measure Each subdivision is identical Units fill the space Units can be partitioned Measures are additive, so that a measure of 10 units can be thought of as a composition of 8 and 2
Conceptions of ruler	• Zero – point • Precision	Any point can serve as the origin or zero point on the scale The choice of units in relation to the object determines the relative precision of a measure. All measurement is inherently approximate.

Table 2.1. The basic concepts of linear measurement that are formulated by Lehrer

The combination between the procedure and basic concepts of measurement directs to a formulation of instructional activities for linear measurement. Van de Walle and Folk (2005) formulate a set of general instructional activities for linear measurement that are described as follows:

Conceptual knowledge to be developed	**Type of activity to use**
1. Understand the attribute being measured	1. Make comparisons based on the attribute
2. Understand how filling, covering, matching, or making other comparisons of an attribute with units produces what is known as a measure	2. Use physical models of measurement units (such as hand spans, foot, etc) to fill, cover, match, or make the desired comparison of the attribute with the unit. At the next stage, measuring instruments signifying physical models of unit (e.g. hand spans and foot).
3. Understand the way measuring instruments work	3. Combining the measuring instruments (ruler) and the actual unit models (such as string of beads) to compare how each works.

Table 2.2. The set of general activities for linear measurement generated by Van de Walle

2.2. Realistic Mathematics Education

According to Freudenthal, mathematics should be connected to reality through problem situations. The term "*reality*" means that the problem situation must be experientially real for students. In this research, Indonesian traditional games were set as the contextual problem situation for young children to learn linear measurement. Some Indonesian games embody measurement activities including *fairness conflict* as an important issue while comparing the distances in the game. Consequently, Indonesian traditional games served as the base of experience-based activities for linear measurement.

For the next question of how to proceed from situational activities to formal mathematics, the tenets of Realistic Mathematics Education (RME) offer clues and design heuristics.

2.2.1. Five tenets of realistic mathematics education

The process of designing a sequence of instructional activities that starts with experience-based activities in this research was inspired by five tenets for realistic mathematics education defined by Treffers (1987) that are described in the following ways:

1. *Phenomenological exploration*

   As the first instructional activity, a concrete context is used as the base of mathematical activity. The mathematical activity is not started from a formal level but from a situation that is experientially real for students. Consequently, this research employed Indonesian traditional games as the contextual situation.

2. *Using models and symbols for progressive mathematization*

   The second tenet of RME is bridging from a concrete level to a more formal level by using models and symbols. Students' informal knowledge as the result of experience-based activities needs to be developed into formal knowledge of linear measurement. Consequently, the "*making our own ruler*" activity in this research was drawn on to bridge from measuring activities in the games as the concrete level to using a ruler in measurement as the formal level of measurement.

3. *Using students' own construction*

   The freedom for students to use their own strategies could direct to the emergence of various solutions that can be used to develop the next learning process. The students' strategies in each activity were discussed in the following class discussion to support students' acquisition of the basic concepts of linear measurement. The student-made measuring instrument served as the bases of the emergence of a blank ruler as the preliminary of a normal ruler.

4. *Interactivity*

   The learning process of students is not merely an individual process, but it is also a social process. The learning process of students can be shortened when students communicate their works and thoughts in the social interaction emerging in the classroom. Game playing forms a natural situation for social interaction such as students' agreement in deciding a strategy for the fairness of their games.

5. *Intertwinement*

   Intertwinement suggests integrating various mathematics topics in one activity. The Indonesian traditional games used in this research did not merely support learning for linear measurement, moreover they also supported the development of students' number sense.

2.2.2. Emergent modeling

The implementation of the second tenet of RME produced a sequence of models that supported students' acquisition of the basic concepts of linear measurement. The process from using hand spans to using a measuring instrument in which the focus of activity changes and mathematical concepts of measurement develop can be characterized as emergent modeling.

Emergent modeling is one of the heuristics for realistic mathematics education in which Gravemeijer (1994) describes how *models-of* a certain situation can become *models-for* more formal reasoning. The levels of emergent modeling from situational to formal reasoning are shown in the following figure:

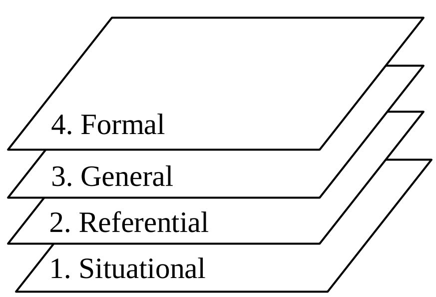


Figure 2.1. Levels of emergent modeling from situational to formal reasoning

The implementation of the four levels of emergent modeling in this research is described as follows:

1. Situational level

   Situational level is the basic level of emergent modeling where domain-specific, situational knowledge and strategies are used within the context of the situation. Game playing provides informal knowledge of linear measurement to students when students have to determine the closest distance in the games. There are some linear measurement concepts that are elicited by Indonesian traditional games, such as indirect comparison and measuring. In this level, students still use their body parts such as hand spans and steps as the main comparing and measuring tools.

2. Referential level

   The use of models and strategies in this level refers to the situation described in the problem or, in other words, referential level is the level of *models-of*.

   A class discussion encourages students to shift from situational level to referential level when students need to make representations (drawings) as the *models-of* their strategies and measuring tools in the game playing.

   As an addition, the "*making our own ruler*" activity also served as referential activity in which students produced their own ruler to represent their way in measuring distances. In this activity, student-made rulers became *model-of* the situation that signifies the iteration of hand spans and marbles.

3. General level

   In general level, *models-for* emerge in which the mathematical focus on strategies dominates over the reference to the contextual problem.

   Student-made rulers produced in "*making our own ruler*" became *models-for* measurement when they turned to be "blank rulers" as means for measuring. In this level, the blank rulers were independent from the students' strategies in the game playing.

4. Formal level

   In formal level, reasoning with conventional symbolizations is no longer dependent on the support of *models-for* mathematics activity. The focus of the discussion moves to more specific characteristics of models related to the *concepts of units, fairness* and *zero point* of measurement.

2.3. Linear measurement in the Indonesian curriculum for elementary school:

Linear measurement in Indonesia has been taught since the first grade in which students learn about comparison of length as the base of linear measurement. In second grade, students begin to learn how to use measuring instruments both non-standard and standard instruments. Table 3 described linear measurement for grade 1 and grade 2 in Indonesian curriculum.

Standard Competence	Basic Competence
The First Semester of Grade 1	
Geometry and Measurement 2. Measuring time and length	2.1. Determining time (morning, noon, evening), day and hours 2.2. Determining duration of time 2.3. Recognizing the terms long and short and also comparing length 2.4. Solving problems related to time and length
Standard Competence	**Basic Competence**
The First Semester of Grade 2	
Geometry and Measurement 2. Using measurement of time, length and weight in solving problems	2.1. Using time measuring instruments with hour as its unit measurement 2.2. Using non standard and standard measuring instruments for length 2.3. Using weight measuring instruments 2.4. Solving problems related to weight

Table 2.3. Linear measurement for elementary school in the Indonesian curriculum

2.4. Conclusion

A sequence of instructional activities for linear measurement with experience-based activities as its preliminary was designed based on three main components mentioned in the theoretical framework. These three components were the basic concepts of linear measurement, the sequence of measurement procedure and the emergent modeling. These three components also served as the base in designing the tools used in the instructional activities.

The overview of the proposed role of tools in the instructional sequence is summarized in the following table:

Tool	Imagery	Practice	Concept
Hand span, feet, marble		Indirect comparison	*Conservation of length*
Hand span, feet, stick	Signifies the "third object" in comparison become the measuring unit in measurement	Measuring	*Identical unit* and *unit iteration*
Strings of beads	Signifies the iteration of measuring unit, such as hand span, feet and marbles	Measuring and reasoning about activity of iterating a measuring unit	Standard measuring unit for the fairness and *precision* of measurement
Student-made measuring instrument	Signifies the need of a standard measuring instrument derived from the strings of beads	Measuring and reasoning about the need of a standard measuring unit	*Identical unit* and measuring as *covering spaces*
Blank ruler	Signifies the need of standard measuring instrument derived from the strings of beads	Reasoning about the need of standard measuring instrument and measuring as covering space	*Identical unit* and measuring as *covering spaces*
Normal ruler	Signifies the need of numbers on a blank ruler to make measuring easier and more efficient	Measuring long objects to stimulate students to consider the appearance and use of numbers on a ruler	Measuring as *covering spaces* and realizing that a number on a ruler could represent a measure
Broken ruler	Signifies the possibility to use a random starting point of measurement	Measuring the length of an object that was not aligned with the first stripe on the ruler	Any number can serve as *zero point* of measurement

Table 2.4. The overview of the proposed role of tools in the instructional sequence

The activities and the conjectures of students' strategies in using the tools to elicit and to promote the basic concepts of linear measurement in the experience-based activities are described in chapter 4, namely the instructional design.

3. Methodology

The basic research methodology and key elements of this research are described in this chapter. The issues that will be discussed in this chapter are: (a) research methodology, (b) research subjects, (c) hypothetical learning trajectory and local instruction theory, (d) data collection, and (e) data analysis including reliability and validity.

3.1. Research methodology

As described in chapter 1, the main objective of this research was to investigate how Indonesian traditional games could be used to build upon students' reasoning and reach the mathematical goals of linear measurement. For this purpose, design research was chosen as an appropriate means for answering the research questions and achieving the research goals. Wang & Hannafin (in Simonson; 2006) defined a design research as a systematic but flexible methodology aimed to improve educational practices through iterative analysis, (re)design, and implementation, based on collaboration among researchers and practitioners in daily life settings, and leading to contextually-sensitive design principles and theories. In this research, a set of experience-based activities was designed as a flexible approach to understand and improve educational practices in linear measurement for grade 2 of elementary school.

The phases in this design research are summarized below:

1. Preliminary design

   In the preliminary design, initial ideas were implemented, which were inspired by studying literature before designing the instructional activities.

   a. Studying literature

      This research was commenced by studying literature about linear measurement, realistic mathematics education, and design research as the bases for formulating initial conjectures in learning linear measurement.

   b. Designing the hypothetical learning trajectory (HLT)

      In this phase, a sequence of instructional activities containing conjectures of students' strategies and students' thinking was developed. The conjectured hypothetical learning trajectory was dynamic and could be adjusted to students' actual learning during the teaching experiments.

2. Pilot experiment

   The pilot experiment was a bridge between the preliminary design and the teaching experiment phase. This pilot experiment activity was conducted at the end of the academic year that was in May. The purposes of the pilot experiment activities were:

   - Investigating pre-knowledge of students

     The first tryout was implemented in grade 1 to investigate pre-knowledge of the students that would be the research subjects in the upcoming teaching experiment period. Charting this pre-knowledge of the students was important for the starting point of the instructional activities and adjusting the initial HLT.

   - Adjusting the initial HLT

     The main objective of the pilot experiment was collecting data to support the adjustment of the initial HLT. The initial HLT was tried out and the observed actual learning process of students was employed to make adjustments of the HLT. The tryout in grade 1 was aimed to make adjustments to HLT in non-standard measurement activities and the tryout in grade 2 was aimed to make adjustment of HLT in measurement activities using a ruler.

3. Teaching experiment

   The teaching experiment aimed at collecting data for answering the research questions. The ongoing process of the teaching experiments emphasizes that ideas and conjectures could be modified while interpreting students' reasoning and learning in the classroom. The teaching experiments were conducted in eight lessons in which the duration was 70 minutes for each lesson. Before doing a teaching experiment, teacher and researcher discussed the upcoming activity.

4. Retrospective analysis

   HLT was used in the retrospective analysis as guidelines and points of references in answering research questions. The extensive description of the data analysis was explained in subchapter 3.5, namely data analysis, reliability and validity.

3.2. Research subjects and timeline

Forty-five students and a teacher of grade 2 in an Indonesian elementary school in Yogyakarta - Indonesia, that was *SD Negeri Percobaan 2* Yogyakarta, were involved in this research. The students were about 7 to 8 years old and they had learnt about comparison of length in grade 1. *SD Negeri Percobaan 2* Yogyakarta has been involved in the *Pendidikan Matematika Realistik Indonesia* or Indonesian realistic mathematics education project since 2000.

The organization of this research is summarized in the following timeline:

	Date	**Description**
Preliminary design		
Studying literature and designing initial HLT	1 February – 30 April 2008	
Discussion with teacher	5 – 7 May 2008	Communicating the designed HLT
Pilot experiment		
Observation in grade 1	26 – 27 May 2008	Investigating students' pre-knowledge and social interaction among students
Tryout in grade 1	28 May 2008	Investigating students' pre-knowledge
Tryout in grade 2	30 May 2008	Trying out the initial HLT about measuring using blank, normal and broken ruler.
Teaching experiment		
Playing g*undu* activity	1 August 2008	Focusing on *conservation of length, identical unit* and *unit iteration*
Class discussion	2 August 2008	Focusing on *conservation of length, identical unit* and *unit iteration*
Playing *benthik*	4 August 2008	Focusing on *identical unit, unit iteration* and *covering space*
Class discussion and measuring activity	6 August 2008	Focusing on *identical unit, unit iteration* and *covering space*
Making our own ruler	8 August 2008	Focusing on *unit iteration* and *covering space*
Measuring using blank ruler	9 August 2008	Focusing on *covering space*
Measuring using normal ruler	11 August 2008	Focusing on *covering space* and the use of numbers on ruler
Measuring using broken ruler and final assessment	13 August 2008	Focusing on *covering space* and *zero point* of measurement

Table 2.5. The timeline of the research

3.3. Hypothetical learning trajectory and local instruction theory

A set of instructional activities was designed to investigate how Indonesian traditional games could be used to build upon students' reasoning and reach the mathematical goals of linear measurement.

The process of designing instructional activities in the classroom practices concerned on two important points that will be described in this chapter, namely *hypothetical learning trajectory* and *local instruction theory*.

3.3.1. Hypothetical learning trajectory

In designing an instructional activity, a teacher should hypothesize and consider students' reaction to each stage of the learning trajectories toward the learning goals. This hypothesize is elaborated in a day-to-day basis of a planning for instructional activities that is called as hypothetical learning trajectory (Gravemeijer, 2004). A hypothetical learning trajectory consists of learning goals for students, planned instructional activities, and a hypothesized learning process in which the teacher anticipates the collective mathematical development of the classroom community and how students' understanding might evolve as they participate in the learning activities of the classroom community (Simon, 1995).

During the preliminary and teaching experiment phases, HLT was used as a guideline for conducting teaching practices in which instructional activities are supposed to support students' learning processes. Furthermore, HLT was also used in the retrospective analysis as guidelines and points of references in answering the research questions. As mentioned by Bakker (2004), an HLT is the link between an instruction theory and a concrete teaching experiment, therefore the HLT supports this design research in generating empirical grounded theories in linear measurement.

The following is an example of HLT used in this design research:

- Activity : Playing *gundu*
- Goals : Stimulate students considering the need of a "third object" in indirect comparison that afterward becomes a measuring unit in the next activity
- Description:

  The winner of the game is the player who can throw a marble in the closest distance to a given circle.

- Conjectures of student strategies:
  One possible strategy of students is to use different pencils to measure the distance of each marble and then to make a mark on each pencil.

Figure 3.1. An example of students' strategy in comparing length

To compare the distances, students just simply compare the position of the mark on the pencils. This strategy does not provide any unit iteration, but this strategy shows a strong transitivity. Students use an object that is longer than the distance to represent this distance. And students finally do direct comparison, namely comparing the first pencil (as the representation of the first distance) to the second pencil (as the representation of the second distance). This strategy matches to Piaget's idea (Castle & Needham, 2007); transitivity develops before unit iteration.

3.3.2. Local instruction theory

Local instruction theory is defined as a theory that provides a description of the envisioned learning route for a specific topic, a set of instructional activities and means to support it (Gravemeijer, 2004 and Cobb et al, 2003; Gravemeijer, 1994 and Gravemeijer in Doorman, 2005). In educational practices, a local instruction theory offers teachers a framework of reference for designing and engaging students in a sequence of instructional activities for a specific topic.

The relation between a hypothetical learning trajectory and a local instruction theory can be deduced from their definition. A local instruction theory provides a complete plan for a specific topic. From the local instruction theory, a teacher could design a hypothetical learning trajectory for a lesson by choosing instructional activities and adjusting them to the conjectured learning process of the students.

The core elements of the local instructional theory in this research were learning goals, instructional activities, the role of the tools and imagery (Gravemeijer, 2004), that also can be found in table 2.4.

3.4. Data collection

Various data sources were collected from videotaping and written data to get an extensive visualization of students' acquisition of the basic concepts of linear measurement.

The data collection of this research is described as follows:

1. Video

   The strategies used by students when measuring in the game playing were more as practical data, instead of written data, therefore students' measuring strategies were more observable from video. Short discussion with students during the game playing and the class discussion were also conducted and recorded as means to investigate students' reasoning for their idea.

   The videotaping during the teaching experiments was recorded by two cameras; one camera as a static camera to record the whole class activities and the other camera as a dynamic camera to record the activities in some groups of students.

2. Written data

   As an addition to the video data, the written data provided more information about students' achievement in solving the measurement problems. However, most of these data merely provided the final answers of students without detailed steps in finding those answers. These data were used for investigating students' achievement because students' learning processes were observed through videotaping and participating observatory.

   The written data included students' work during the teaching experiment, observation sheets, the results of assessments including the final assessment and some notes gathered during the teaching experiment.

3.5. Data analysis, reliability and validity

As mentioned in subchapter 3.1, the data were analyzed retrospectively with the HLT as the guideline. The data analysis was accomplished by the researcher with cooperation and review from supervisors to improve the reliability and validity of this research.

3.5.1. Data analysis

Doorman (2005) mentioned that the result of a design research is not a design that works but the underlying principles explaining how and why this design works. Hence, in the retrospective analysis the HLT was compared with students' actual learning to investigate and to explain how students acquire of the basic concepts of linear measurement that were elicited by Indonesian traditional games.

The main data that were needed to answer the first research question were the videotaping of the traditional games activity and the class discussion following the game playing. The videos of the Indonesian traditional games activities were transcribed to figure out *how* students perform the measurement during the game playing. The reasoning of *why* students use a particular strategy in the games was investigated from students' argument in the class discussion.

Student-made measuring instruments as the students' own construction (*the third tenet of RME*) were needed as the additional data to answer the second research question because the student-made measuring instrument served as a bridge to formal measurement using a ruler. Hence, analyzing the student-made measuring instrument aimed to explain students' progress from game playing to the more formal measurement using a ruler. This analysis was supported by the analysis of students' reasoning in the class discussion.

3.5.2. Reliability

Despite the use of assessments during the teaching experiment, the reliability of this design research was not accomplished in a quantitative way. Instead, qualitative reliability was used to preserve the consistency of data analysis.

The qualitative reliability was conducted in two following ways:

a. Data triangulation

   The data triangulation engages different data sources, namely the videotaping of the activities, the students' works and some notes from either teacher or observer.

   All activities were video recorded and the students' works were collected. The combination of the videotaping and students' works were chosen to check the reliability of interpretations based upon one video clip or one field note.

b. Cross interpretation

   The parts of the data of this research (especially the video data) were also cross interpreted with colleagues (i.e. the supervisors). This was conducted to reduce the subjectivity of the researcher's point of view.

3.5.3. Validity

To keep the methodology of this research as valid as possible and to answer the research questions in the right direction, the following methods of validity were used in the data analysis:

a. HLT as means to support validity

   As mentioned in the retrospective analysis in subchapter 3.1, the HLT was used in this retrospective analysis as a guideline and a point of reference in answering research questions. This aimed to connect and evaluate the initial conjectures to the gathered data and prevented systematic bias.

b. Trackability of the conclusions

   The educational process is documented by video recordings, field notes and by collecting written answers of the students. With this extensive data, we were able to describe the situation and the findings in detail to give sufficient information for our reasoning. This information enables the reader to reconstruct the reasoning and to trace the arguments that underpin the conclusions

4. The instructional design

Analyzing students' learning line or learning trajectory for a particular domain is a crucial part in designing instructional activities for students. Every stage of instructional activities should be adjusted to the level of students. Consequently, the hypothesized students' learning line for linear measurement was analyzed before designing a sequence of instructional activities for learning measurement. The following is a general overview of student's learning line for linear measurement in grade 2:

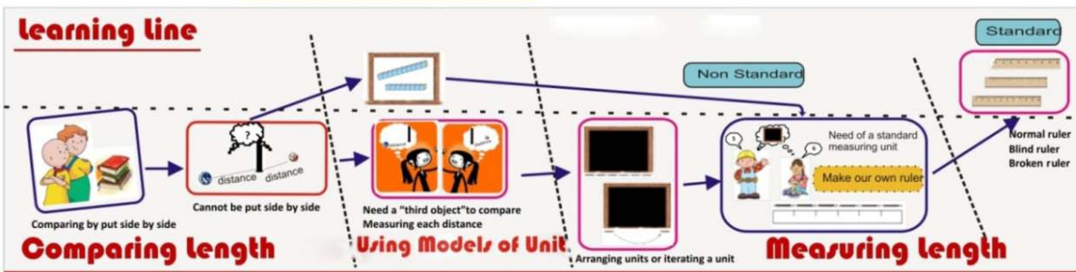


Figure 4.1. The learning line of students in learning linear measurement

The students' learning line for linear measurement is partitioned in three main stages; namely comparing length, using models of unit and measuring length in sequence.

a. Comparing length

   The concept of conservation of length is the main core of comparison. When students already perceive the idea of conservation of length, they will be able to do comparison of length (Kamii & Clark, 1997). Comparison itself serves as the base of measurement; therefore comparing activities embodied in Indonesian traditional games were used as preliminary for teaching and learning of linear measurement for grade 2.

   The need of "third objects" in indirect comparison supports the emergence of a unit measurement. An Indonesian traditional game, called g*undu*, was used to encourage students in learning the concept of indirect comparison and the emergence of a non-standard measuring unit.

b. Using models of unit

   At the beginning of measurement process, people are used to use non-standard measuring units. Therefore, the use of non-standard units at the beginning of measurement activities is crucial and beneficial at all grade levels. The first benefit is that non-standard units help students to focus directly on the attribute being measured. As the second benefit, the use of non-standard units at the beginning of measurement activities provides a good rationale for work with standard units. Using models of unit emerges when a “third object” is acquired to compare the length of objects which cannot be directly compared.

A discussion of the need for a standard unit will be more meaningful to students after they have measured objects using their own non-standard units. The different non-standard measuring units used by students in the game playing activities could be employed as a conflict to stimulate and support the emergence of standard measuring unit. The need to have a “fair” game was also expected to stimulate student to “standardize” the measuring units that were used in the game. Consequently, the emergence of a standard measuring unit was expected to be acquired in the class discussion. The agreement-based standard measuring unit as the result of standardization became the starting point of the emergence of a standard measuring unit in the formal measurement.

c. Measuring length

Measuring length requires the second basic concepts of linear measurement proposed by Barret in Stephen and Clement (2003), namely *unit iteration*. There are two kinds of unit iteration, namely:

– Arranging a number of similar units to cover the attribute of the measured objects.
– Iterating a unit from one to another end of the measured object.

Measuring length is also built up by the concept of *covering space* and any number as *zero point* of measurement. A problem that frequently occurs when young children measure the length of objects using paper strips is counting the number of stripes, instead of the number of spaces between two stripes. This fact shows that many young children do not fully perceive the idea of measuring as *covering space*. Consequently, the concept of *covering space* became the focus in measuring activity using strings of beads, making our own ruler activity and measuring using blank ruler activity in this research.

Many prior researches revealed that young children also have difficulty to give the correct measure of an object that is not aligned with number zero on the ruler (Kamii & Clark, 1997 and Kenney & Kouba in Van de Walle, 2005 and Lehrer et al, 2003). It indicates that many young children do not seemed to know that any number can serve as *zero point* of measurement. Hence, the use of broken ruler aimed to help students in understanding the concept that any number can serve as *zero point* of measurement.

A set of instructional activities for linear measurement was designed based on this hypothesized students' learning line and thinking process. This set of instructional activities was divided into seven different activities that were accomplished in eight days. Each day activity was aimed to achieve students' understanding in one or more basic concepts of linear measurement. Similarly, some of basic concepts of linear measurement were achieved from different activities. The relation among students' learning line, instructional activities and the basic concepts of linear measurement that need to be acquired is shown in the following diagram.

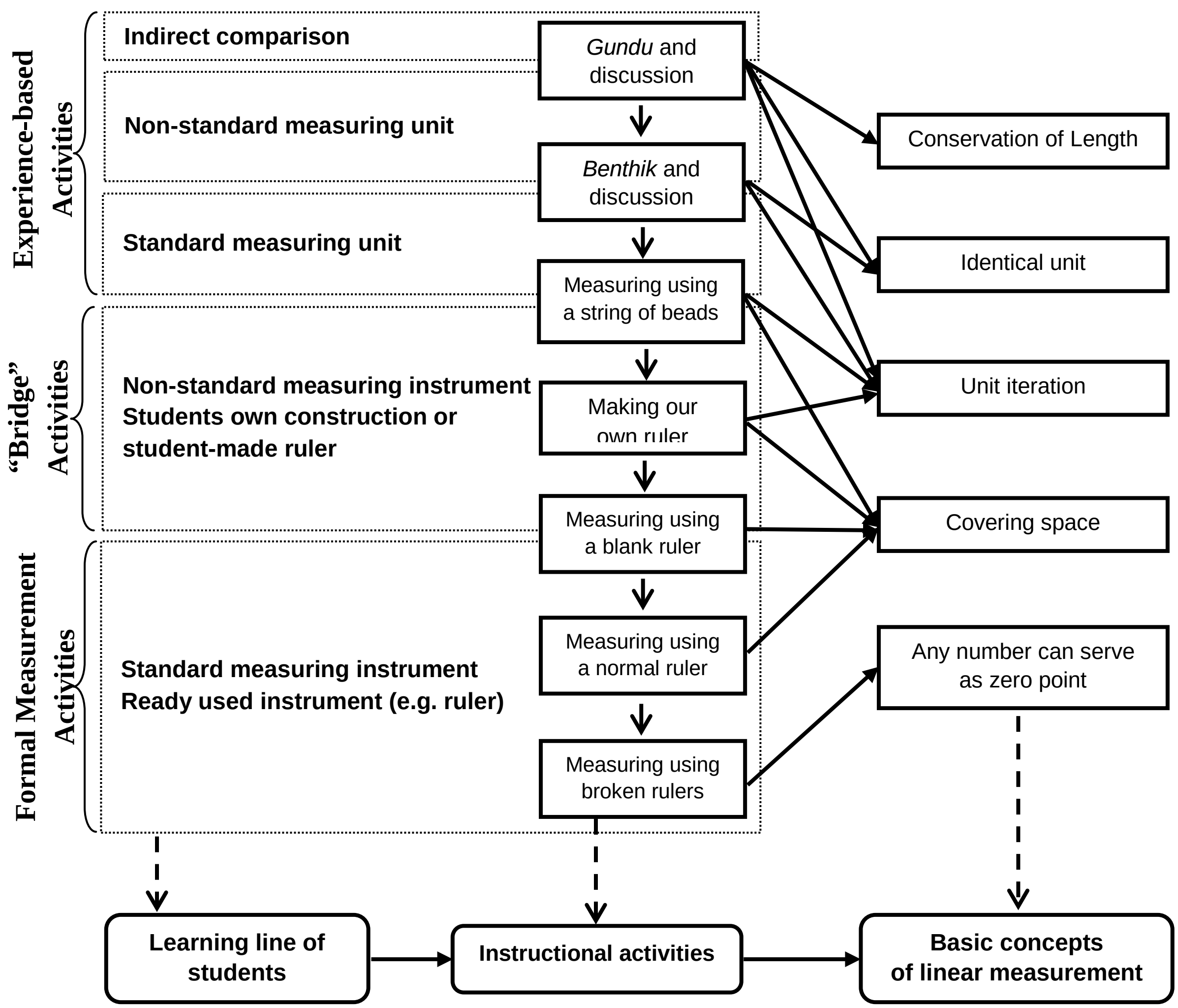


Figure 4.2. The main framework of experienced-based activities for learning linear measurement

The instructional activities for learning linear measurement that were embedded in the hypothetical learning trajectory are described as follows:

4.1. Playing *gundu* (Playing marble)

This activity aimed to stimulate students considering the need of “third object” in indirect comparison that afterward becomes a measuring unit in the next activity.

Rules (*adopted and adjusted from Siti M. Amin, 2006)*:

1. Each player in the group has to throw a marble to a circle on the ground (the distance between the circle and start point is approximately 2 – 3 meters). Player who can throw his/her marble into the circle will obtain 5 points.
2. The distance of the marbles to the circle on the ground are compared and the player whose marble has the shortest distance to the circle can play in first turn.
3. Next step is each player has to try to throw his/her marble into the circle. If the player cannot throw the marble into the circle then the next player throws his/her marble into the circle and so on until all player throw the marble.
4. If the player in step (3) are able to throw in his/her marble into the circle then he/she gets 1 extra point. This player also has a “rights” to hit the marbles of the other players to obtain more point.
5. The game is finished when all marbles are already in the circle or the remaining marbles are already hit by some player.
6. The winner is the player who obtains the bigger points.

Conjecture of students’ strategies:

Direct and indirect comparison activities are important because they do not require dealing with numbers and units and, therefore, they direct students to focus on understanding the length as the measurable attribute and the basic processes of measuring (Grant & Kline, 2003). Furthermore, indirect comparison is very close to the idea of transitivity, therefore children who already understood the basic concept of transitivity will have benefit in using a third object as a benchmark for the comparison. The third object that is used by children when comparing length could be a physical object or a mental benchmark (if children just imagine when comparing).

Conjectured strategies that are used by students:

- When the difference of the distances between each marble to the pole is large, students will decide the winner by simple estimation.

This strategy is an example of comparison using a mental benchmark. Students use the shorter distance as a mental benchmark to compare with the longer distances.

- Each group uses different pencils to measure the distance of each marble and then giving mark on each pencil.

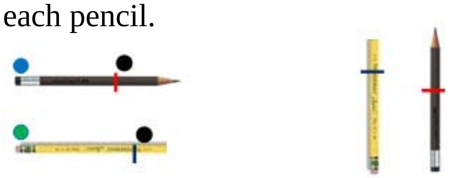


Figure 4.3. The strategy of students in comparing length

To compare the distances, students just simply compare the position of the mark on the pencils. This strategy does not provide any *unit iteration*, but this strategy shows a strong transitivity. This strategy matches to the finding of Castle and Needham (2007), namely that at both the beginning and end of the school year, more students demonstrated *transitivity* than *unit iteration*.

- Students decide the winner by measuring the distance using their body parts.
  To compare the distance, students compare the number of spans they need for each distance. This strategy provides *unit iteration* as one basic concept of measurement. However, according to Castle & Needham (2007), young children do not consider the different size of their body parts. This fact can cause a *fairness conflict* if in the game each player uses his/her own body part to determine the distance of his/her own marble.
- Students measure the distance using particular objects that can be iterated, such as marbles.
  Similar to using body part, the following strategies also use the idea of *unit iteration*.
- Students arrange marbles (as measuring instruments) to measure the distance
  Students who use this strategy seemed to think that all spaces that are being measured must be covered by physical unit measurements.
- Students only use one marble and then they iterate the marble to measure the distance.
  Students who use this strategy seemed to perceive that measuring does not have to physically cover all spaces.

Mathematical ideas that are embodied in this game are:

- Measurement

  Students do comparison (as a part of measurement) when they compare the distance of the marbles to decide the order of the player.

- Addition

  Students do addition when the sum up all points they have obtained in the game.

4.2. Class discussion

Teacher reminds students to the g*undu* game in the previous day's activity. Teacher can pose some question about comparison (as a part of measurement) related to the game, for instance:

1. "*How did you compare the distances of the marbles?*"

   The question is given to the students to investigate the strategies used by students to do indirect comparison (i.e. comparing the distances of the marbles).

   In direct comparison we can directly compare the length of the objects by arrange the objects in parallel way, but in indirect comparison we need a third object as a benchmark. The emergence of the third object is the main idea of measurement.

   There are some strategies that may be will be used by students, namely:

   - Using span
   - Using objects which its length is longer than the distances, for instance using pencil as shown in figure 4.2.

     Students who use this strategy do not seem to acquire the idea of unit iteration and they still think about simple transitivity.

   - Using objects that can be iterated, such as marble, that can be done in two different ways as follows:
   - Arranging an array of objects to cover the measured distances

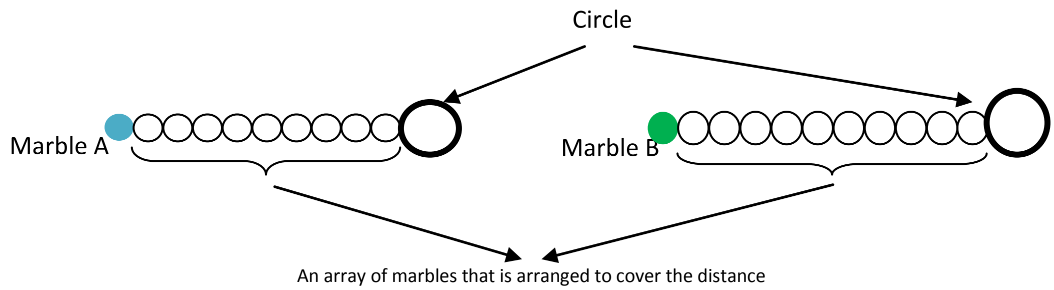


Figure 4.4. Arranging an array of objects to measure a distance

– Iterating an object to cover the measured distance.

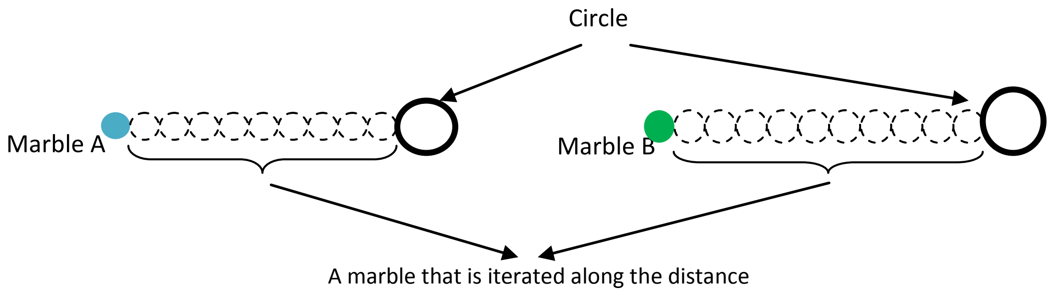


Figure 4.5. Iterating an object to measure a distance

2. *"Why did you use that strategy? Can you think other simpler strategies than your strategy?"*

   These questions aim to investigate students' argument about their strategy. It is possible that some students use a strategy because they are familiar with it, for instance they see adults do this kind of strategy.

Teacher also can give some problems to students, for instance:

1. *"The distance of Andi's marble to circle is shorter than that of Shafa's marble and the distance of Shafa's marble to the circle is shorter than the distance of Elok's marble to the circle. Whose marble is nearest to the circle?"*

   This question is posed to investigate students' understanding about the concept of transitivity.
2.

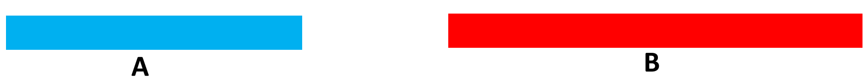


Figure 4.6. An example of comparison problem

Stick A is … than stick B.

Draw a new stick that is longer than stick A, but shorter than stick B.

4.3. Playing *benthik*

Materials : two wooden stick (long and short)

Player : the game is played by 2 groups and there are 5 students in each group.

Rules of the game (*adopted and adjusted from Siti M. Amin (2006))*:

1. The game is played by 2 groups; one group as batter team and the other group as guard. Rule number 2 is used to decide which group will be the batter.
2. A member of each group throws the short stick, the group whose member can throw the short stick in further distance will be in the first turn (i.e. as batter).

Figure 4.7. A group of students are playing *Benthik*

3. A member of the batter group throws the short stick and a member of the guard.
   - If one of the members of the guard team can catch the short stick then the guard team gets 10 points and the game is continued to rule /step number 4.
   - If the member of the guard team cannot catch the short stick then the game is continued to rule /step number 4.
4. The distance of the fallen short stick to the hit point is measured and the obtained distance becomes the point for the batter group.
5. Step 1 to step 4 is repeated until all members of the batter group already throw the stick. If all members of the batter group already throw the sticks then the role of the group is turned.
6. The winner is the group that obtains bigger points.

Mathematical ideas that are elicited by *benthik* game are:

- Non standard measurement

  This activity is done when students measure the distance of the stick.
- Addition

  Students do addition when they sum up the points they get in the game.

Conjecture:

It is expected that the long distances that are being measured will stimulate students to use big unit (such as using steps instead of spans) and then iterate this unit to measure the distance. Most students will use their paces because maybe they are familiar with this strategy.

Usually, in Indonesian traditional game there will be more than one "judge" to determine the result of a game. Hence, based on this culture, it is expected that there will be more than one student who measure a distance.

It is expected that there will be a conflict triggered by the different sizes of student's paces and, therefore, they will obtain different result for the same distance. This conflict can be used for introducing the use of same unit to obtain same result for same distance.

4.4. Class discussion and measuring using a string of beads

Goals:

This activity aimed to introduce a standard unit measurement

Activities:

In this class discussion, the different strategies used by students when playing *benthik* are discussed. Furthermore, this discussion aims to encourage students to reinvent some basic concepts of linear measurement.

At the beginning of the activity, teacher tells a story about *benthik* game in Indonesia.

Example of story:

"*I have a friend in Kalimantan. Last night she called me and told me that she and her students also play benthik game in Kalimantan. Last week they played benthik game at school and the winner could throw the stick quite far that was 25 sticks in length.* (The teacher shows a figure of stick). *Yesterday, our best team obtained 26 sticks in length for the distance of the stick* (the teacher shows another figure of stick).

Now, can you decide which team will be the winner; our team or their team?"

Conjecture of students' thinking:

Most students may spontaneously answer that they are the winner because they have a bigger number that is 26. These students do not realize the different lengths of the sticks. However, it is expected that there are some students who answer that the Kalimantan team is the winner because they use a longer stick to measure. These students already realize that the size of measurement unit is important for measurement.

If all students answer that they are the winner, that is because they neglect the different size of the sticks, teacher can give stimulating question.

"Look at the sticks, what do you think about the size of them. Do those sticks have same length?"

Young children do not indicate a need to account for differences among the body parts of different children (Castle & Needham 2007). Similarly, young children also do not indicate a need to account for differences among unit measurement.

According to Piaget's (Castle & Needham, 2007), transitivity develops before unit iteration. Hence, if students are asked to focus on comparison between the sticks they will begin consider the difference between the sticks. Students begin realize that the length of sticks that were used in the game is different because they use the tiles as the third object to do indirect comparison. Students compare each stick to the tiles as the reference point of comparison (note: the size of tiles in Indonesian school is almost always in same size, namely about 30 cm × 30 cm).

Figure 4.8. Different measuring units

Transitivity and conservation have a relationship with the inverse relation between the size of the unit and the number of those units (Clements, 1999). The bigger the size of a unit, the less number of the unit is needed to cover up a space or distance. Hence, it is expected that students will rethink about the winner of the game after they realize.

If students still do not realize the inverse relation between the size of the unit and the number of those units, teacher can give some stimulating question such as:

"Measure your table using your book and also your pencil. What do you think about the result of those measurements?"

For instance, students obtain that the length of the table is 6 books length (when measure using books) or nine pencils length (when measure using pencil).
If students do not perceive the distance yet, teacher can give another question:
"You said that the length of your table is six books length or nine pencils length. Are those six books length and nine pencils length different in length?"
It is expected that students realize that the length of an object can be represented in different number if they use different size of unit measurements. Furthermore, students are expected to perceive the idea of inverse relation between the size of the unit and the number of those units.

Next activity:
The next activity is making strings of beads that will be used to measure the length of objects in the class. This activity aims to introduce a standard measuring instrument.
Beads are used in this activity because beads are, in shape, close to marbles that have been used as unit in the previous activity. In other word, beads are the duplication of the marbles. Students are given a rope and 50 beads and then they are asked to make their own measuring instruments to measure the length of objects in the class.

4.5. Making our own ruler

Goals:
This activity aims as an introduction to standard measurement instruments.
After students have understood the need of standard **unit** measurement, they are introduced to standard measuring **instruments** as the follow up of the standard **unit** measurement.

Activity:
This activity needs to use students' production in the previous day, namely strings of beads. Students are given their strings of to measure various objects surrounding them (either indoor/classroom or outdoor) from short objects (e.g. books) to long objects (e.g. the wide of the classroom).
Then teacher tells students that one of his/her friend is going to borrow the strings of beads, but students still have to measure some other objects. Therefore, teacher asks

students how to measure the rest of the objects if they still want to use the same size of unit of measurement, namely the bead.

Students are given the string of beads and a sheet of paper to make their own ruler. It is not an obligatory for students to put numbers on their ruler.

Conjecture of students' thinking:

Students are able to make a "blank" ruler by copying their strings of beads on a paper.

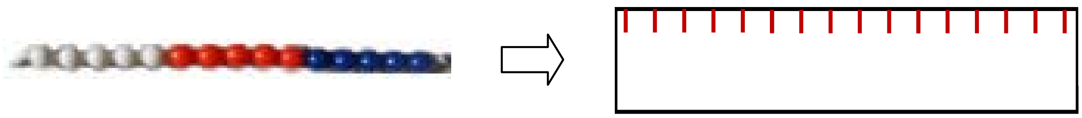


Figure 4.9. Conjectured students' strategy in making their own ruler

4.6. Measuring using a blank ruler

Goals:

In the pilot experiment, most of students in grade 2 still counted the number of the stripes instead of the number of spaces between two stripes. It means that those students do not seem to perceive the concept of *covering space* yet. Consequently, this activity aims to bring students to the understanding about the concept of *covering space* in measurement.

Part 1: Measuring

Students are given a "blank" ruler and then asked to measure the length of the figures on the worksheet.

Possible strategies that are used by students are:

1. Students put the edge of the measured object at the edge of the ruler.

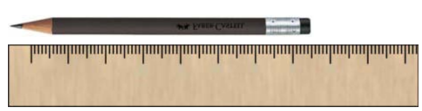


Figure 4.10. First conjectured strategy of students in measuring using blank ruler

- Students measure the length of objects by counting the number of stripes.
  Students who use this strategy do not seem to perceive the concept of *covering space* in measurement because they do not count the spaces that cover the length of the object.
- Students measure the length of object by counting the number of spaces
  These students already perceived the idea of *covering space* in measurement.

2. Students put the edge of the measured object at the first stripes of the ruler

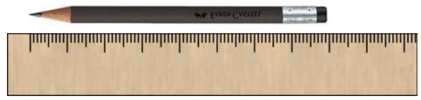


Figure 4.11. Second conjectured strategy of students in measuring using blank ruler

- Students measure the length of objects by counting the number of stripes
  Students who use this strategy do not perceive the concept of *covering space* in measurement because they do not count the spaces that cover the length of the object.
- Students measure the length of object by counting the number of spaces
  These students already perceived the idea of *covering space* in measurement.

Part 2: Class Discussion

After finishing the measuring activities, students are directed to a class discussion in which all students have to present the result of their measuring activity.

If there are students who do not perceive the idea of *covering space* in measurement of length, then the following discussion can be conducted:

1. Teacher can ask students student to measure other objects that the length is getting shorter. Teacher could draw the following figures on the blackboard:
   Note:
   The figures are drawn one by one from left figure (long stick) to right (to shorter stick).

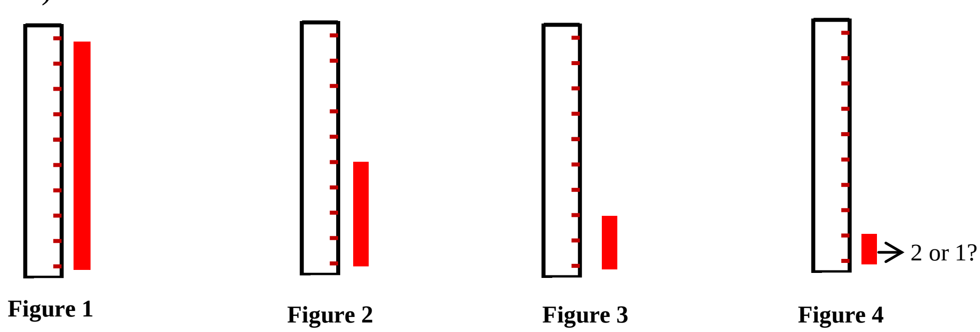


Figure 4.12. Drawings to stimulate students in acquiring the concept of covering space

Every time teacher finishes drawing a figure, students are asked about the length of the figure. The answers of students are not discussed further before the last figure is drawn.

It is possible that students still count the number of stripes until the third figure, but it is expected that the last figure can give conflict to students. It is expected that from the last figure students start to realize that they must count the number of spaces, instead of the number of stripes.

If students still count the number of stripes until the last figure then students are asked to draw a stick which its length is 1 cm.

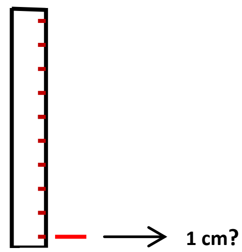


Figure 4.13. A drawing to stimulate students in acquiring the concept of covering space

It is expected that this last task will bring students to the understanding of the concept *covering of space*.

2. Teacher can use string of beads

   Teacher also can use beads to guide students in mastering the concept of *covering space*.

   At first, students are given a “blank” ruler and asked to measure the length of an object.

   For instance:

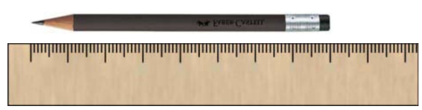


Figure 4.14. Measuring the length of an object using a blank ruler

Students who measure by counting the number of stripes will obtain nine as the length of the stick. Therefore, these students are given a string of beads and asked to measure the stick using the string of beads.

Note:

The diameter of a bead is equal to the length of one “space”, namely exactly 1 cm.

Figure 4.15. Measuring the length of object using a string of beads

Discussion is held after students finish measure the length of objects using string of beads. Students are asked to compare the way they measure using string of beads and the way they measure using "blank" ruler. The discussion is held to lead students into the understanding of the concept of "covering space".

4.7. Measuring using a normal ruler

Goals:

- Introduction to a standard measuring instrument
- To investigate how do students measure the length of objects, whether children just simply read the number related to the edge of the object or they consider the number of spaces between two successive numbers.

Part 1: Measuring the length of objects using "blank" ruler

This activity is a repetition of the previous day's activity. However, it is expected that from this activity students commence to realize the need to write numbers on their measuring instruments. Therefore, teacher should guide and stimulate students to the emergence of numbers on ruler.

Teacher can pose some question to do it, such as:

- Can you help me to measure the length of objects in a quicker way?
- What can we do to our ruler to measure in a quick way?

  It is expected that students come to idea to put numbers on the "blank" ruler.

Part 2: Writing down numbers on the ruler

It is expected that students can figure out how to number the ruler. Students are asked to put numbers on their "blank" ruler.

There are various ways that might be used by students, such as:

1. Students start to put numbers on the **stripes** and start from number "1"

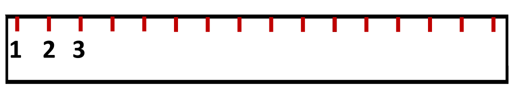


Figure 4.16. Numbering is started from "1" and written at the stripes

2. Students start to put numbers on the **spaces** and start from number "1"

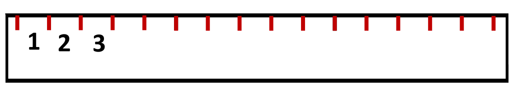


Figure 4.17. Numbering is started from "1" and written at the spaces

3. Students start to put numbers on the **stripes** and start from 0.

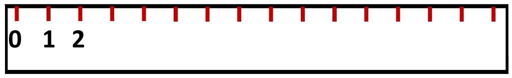


Figure 4.18. Numbering is started from “0” and written at the spaces

The next activity is discussing the proper way to put number on the ruler.

- The first ruler is created by students who measure by counting the number of stripes, or in other words they do not understand yet about the concept of *covering space*.
- The second ruler reflects that students already understand the concept of *covering space* because they count the number of spaces. However, the ways they write the number make it difficult to read the result of measurement.
- There are some possible conjectures derived from the third ruler, namely:
  - Students create this ruler because they are already familiar with the appearance of ruler in their daily life.
  - Students already understand the concept of *covering space* and they, furthermore, also understand that the second stripe reflects the first space; the third stripe reflects the second space; and so on.

Part 3: Measuring the length of objects using normal ruler

When students already understand the need and the advantage of numbers on a ruler then they are asked to measure the length of some objects using a normal ruler.

Goals:

This activity aims to investigate how students measure the length of objects, whether children just simply read the number related to the edge of the object or they have considered the number of spaces covering the measured objects.

Conjecture:

- Children put the edge of the pencil at the edge of the ruler, instead of at the first stripe.

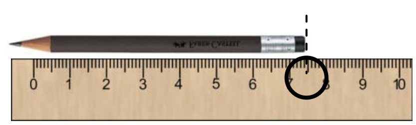


Figure 4.19. Matching the edge of the ruler to the edge of the object and reading the number

Children simply read the number related to the edge of the pencil. In this case children do not seem to really measure.

– Children put the edge of the pencil at the edge of the pencil at the first stripe (i.e. at “0”).

Figure 4.20. Matching the “0” to the edge of the object and reading the number

It is expected that there will be a conflict when children count the number of stripes because they will obtain the length of the pencil is nine stripes but when they read the number on the ruler they will obtain eight. This conflict can be used to emphasize that measurement using ruler is not counting the number of stripes, but counting the number of spaces between two stripes. However, from this conflict can emerge a new conflict that is number “0”.

Children count the number of spaces between two stripes and, then, they will obtain 8. This result is synchronic to the number on the ruler.

4.8. Measuring using broken rulers

Goals:

– Students are able to use standard measuring instruments
– Students are able to understand the concept of *zero point* of measurement.

Activity:

In the previous activity students measured the length of object by ruler that was started from “0”, but in this activity the ruler is started from various numbers. After students perceiving the use of normal ruler, they are directed to the next activity to learn about the concept of *zero point* of measuring.

Lehrer et al (2003) said that any point can serve as zero point or starting point of linear measurement. Thus, in this activity we use a broken ruler, namely the ruler that is not started from “0”.

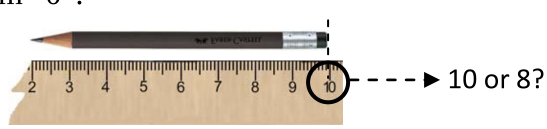


Figure 4.21. Measuring the length of an object using a broken ruler

Conjecture:

- Some students who understand *conservation of length* will directly answer that the length of the pencil is eight because they use the same strategy as they used in the previous activity.
- Some children still simply read the number on the ruler that matches to the edge of the pencil.

  For students who use this strategy, teacher could give a new extreme broken ruler (e.g. started from 15) and a short object (e.g. 1 cm) then ask them to measure the object using that broken ruler. If students still use their previous strategy (directly look at the number), they will obtain that the length of the object is 16 cm (because the edge of the object matches to number 16 on the ruler).

  The expectation from this task is that the “extreme” situation could make students start to realize that measuring with ruler is not simply done by reading the number that matches to the end of the measured object, but they have to consider the starting point or the zero point of the measurement.
- Students ignore the numbers on the ruler and still count the number of stripes.

  For students who use this strategy, teacher can ask them to give an object whose length is 1 cm. Students will realize that it is impossible for them to have 1 cm length object if they use their strategy, because 1 cm length in their strategy is merely a stripe.
- Students ignore the numbers on the ruler and count the number of spaces between two stripes.

  For students who use this strategy, teacher can give a long object and then ask students to give the length of the object as soon as possible. If students find that counting the spaces on the ruler takes time, it is expected that they will try to find easier strategy to determine the length of the object. Thus, it is expected that they will consider the zero or starting point and also the end point and finally they obtain the length of the object is the number at the end point subtracted by the number at starting point.

Class Discussion:

The discussion is conducted to guide students in understanding the concepts of *covering space* and *zero point* of measurement. The following activities can be done during the discussion:

1. Students are asked to measure the same objects using “blank” ruler and normal ruler. Then, they are asked to compare and discuss the results. Class discusses the right result of the measurement.
2. Using strings of beads to measure the length of objects.

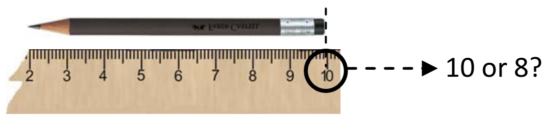


Figure 4.22. Measuring the length of an object using a broken ruler

Students who answer that the length of the pencil is 10 cm then are asked to measure the pencil using strings of beads. It is expected that students will answer that the length of the pencil is eight beads. It is expected that using strings of beads can stimulate students to understand the concept of *zero point* of measurement. Furthermore, students are expected to understand that the result of the measurement is obtained from subtracting the number matches to the end of the object by the number matches to the beginning of the object.

5. Retrospective analysis

In this chapter, the retrospective analysis of data collected from both the pilot experiment and the teaching experiment activities are described. As mentioned in chapter 3, the result of this research is not a design that works but the underlying principles explaining how and why this design works. Consequently, the hypothetical learning trajectory served as a guideline in the retrospective analysis to investigate and explain students' acquisition of the basic concepts of linear measurement that were elicited by Indonesian traditional games as experience-based activities.

5.1. Pilot experiment for investigating students' pre-knowledge

The designed hypothetical learning trajectory that was tested in grade 1 consisted of two activities, namely comparing length and non standard measurement activities. Meanwhile, standard measurement activities using blank ruler, normal ruler and broken ruler were tested in grade 2.

5.1.1. Pilot experiment in grade 1

Ten students in grade one were involved in this pilot experiment. They were given tasks about comparison and measurement of length.

Comparing activity:

The tasks in comparing activity were given both in written and oral tasks. The following was one of problems in written task.

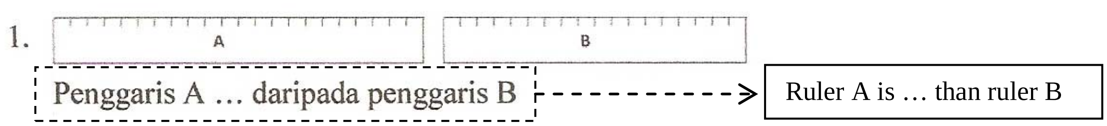


Figure 5.1. An example of written comparison problem

Two out of ten students answered that ruler A was longer than ruler B because ruler B was shorter than ruler A. Two other students did not give any reasoning although they had the correct answer. The last six students answered that ruler A was longer than ruler B because there were more stripes on ruler B. Furthermore, a student, Govi, redrew the rulers and written numbers on both rulers as an addition to his answer. However, these numbers were not synchronic with his answer because he numbered from 0 to 6 on both rulers although he answered that there were more stripes. This shows that numbers on rulers seemed to be meaningless to Govi because he did not really consider the numbers when comparing the rulers.

After doing the written task, students were involved in oral task in which they were asked to communicate their idea in comparing two lines drawn on the floor. Seven students compared the length of lines by their span and three students used strings of beads to compare the length of the lines. Four out of seven students who used their span compared the lines not in constant span (i.e. they bent their span to adjust to the edge of each line, namely to have an integer number of iteration). There are two conjectures are drawn out from this finding. The first conjecture is that students did not completely perceive the concept of *identical unit*. The second conjecture is that students do not seem to pay attention to the preciseness of the measure or comparison.

In general, the students involved in this tryout seemed to already understand the need of a “*third object*”, such as hand span, steps and other objects, as the most principle requirement to do indirect comparison. These findings match to the conjectures formulated before this activity, namely the use of body parts as tools in comparing.

Measuring activity:

In this activity students were asked to measure the length of a table. Seven students used their span to measure the table and three students used strings of beads.

When measuring the length of the table, Govi counted a half span as a complete span. Actually he got 6,5 of his spans as the length of table but he counted them as 7 spans, although he did not bend his hand span. Govi knew that he had to iterate his span from end to end of the table but he did not realize that he had to keep his span constant in length. It seems that Govi only perceived the concept of *unit iteration* but not for the concept of *identical unit* in measurement.

Another interesting finding was the reasoning of Putri. She used strings of beads to measure and arranged the strings on the table with some overlapping between the strings.

Figure 5.2. Overlapping in measuring length

As well as Govi, Putri already perceived the concept of unit iteration, but she did seem to fully understand the concept of covering spaces in measurement.

The general conclusion from this activity is that students have understood the need to have a “third object” as measuring unit and, furthermore, to *iterate unit* in measurement. The *unit iteration* and overlapping in measurement observed during the pilot experiment match to the conjecture about how students perform a measurement. However, one finding was out of the conjecture namely when a student bent his hand span. The conjectures derived from this occurrence are either the student did not completely perceive the idea of *identical unit* or he did not yet consider the *precision* in the measurement.

5.1.2. Pilot experiment in grade 2

Twenty students were involved in this activity, but only five students who involved in the discussion (the other students were picked up by their parents). The focus on this activity was investigating students’ acquisition of the concept of zero point in measurement. For this purpose, students were given a worksheet and a set of broken rulers. They were requested to measure the length of figures in the worksheet.

The different strategies used by students when measure using blank ruler were observed in this try out, namely:

1) Directly looked at the number that matched to the edge of the measured objects.
2) Counted the number of stripes and start counting from 1

   Students who use this strategy count from the first stripe and recite the numbers from “1”. They say “one” while touching the first stripe, say “two” while touching the second stripe and so on until the last stripe.
3) Counted the number of stripes but started reciting number from “0” instead of from “1”. (Note: this strategy emerged in the discussion after students finished working with their worksheet).

Researcher : *How long is this pencil* (point to the figure of a sharp pencil) *if we measure it using ruler number 4* (i.e. a ruler that is started from number 2)?

Ofar measures the length of the sharp pencil using ruler number 4

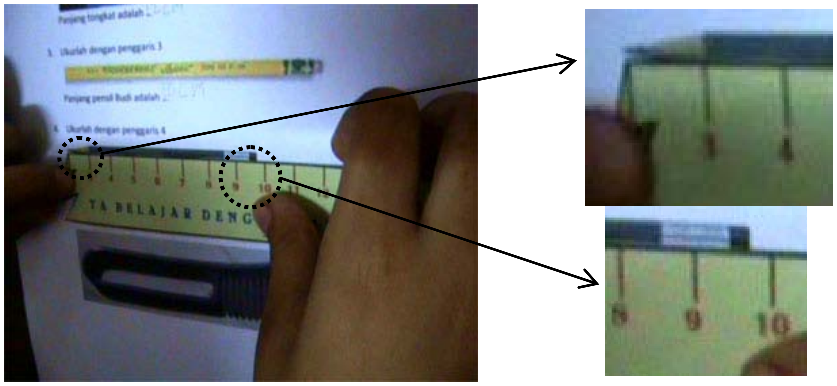


Figure 5.3. Ofar directly looks at the number that matches to the last edge of the pencil

Ofar : *The length of the pencil is close to 10.*

Researcher : *Now, measure this pencil again but using ruler number 3* (i.e. a ruler that is started from number 1)

Ofar measures the length of the sharp pencil using ruler number 3

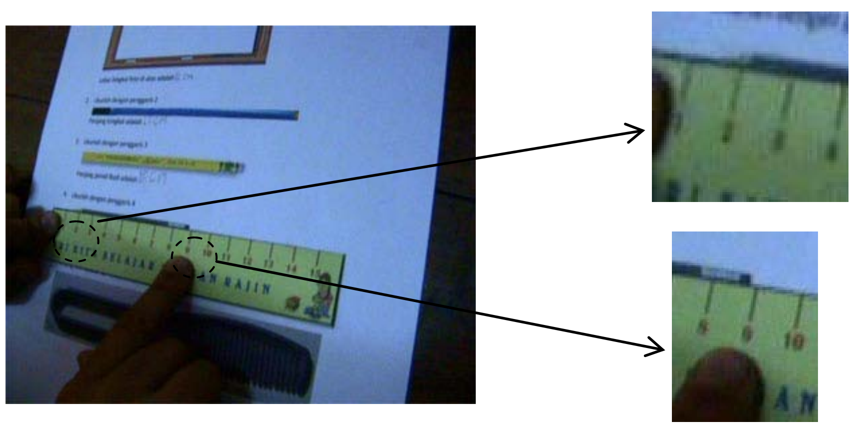


Figure 5.4. Ofar directly look at the number that matches to the last edge of the pencil

Ofar : *The length of this pencil is almost nine*

Researcher : *Now, measure again using ruler number 2* (i.e. a normal ruler)

Dandi measures the length of the pencil using a normal ruler

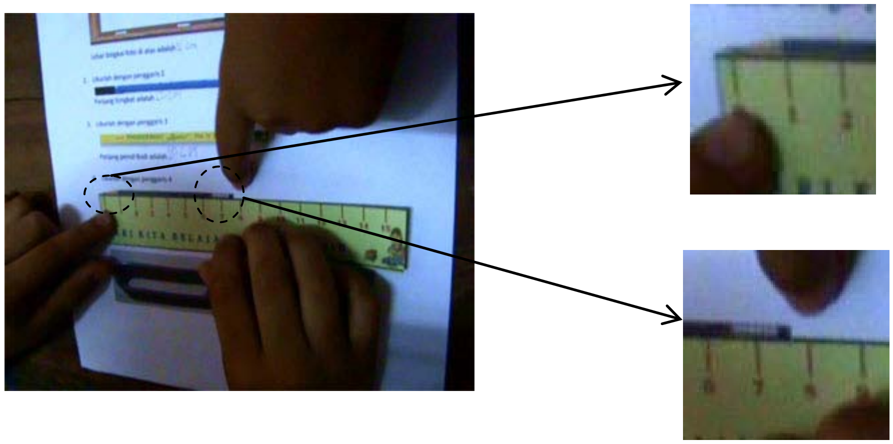


Figure 5.5. Dandi and Ofar directly look at the number that matches to the last edge of the pencil

Ofar : *It's almost eight*
Researcher : *How is it possible that we obtain different measure for the same object?*
Dandi : *Because the length of the rulers was different*
Researcher : *Which one is the correct measure?*
Dandi : *All measures are correct; depend on the ruler we use to measure.*

From figure 5.3 to figure 5.5, it can be concluded that the students did not seem to consider the starting point of their measurement because they directly looked at the number matched to the edge of the measured object. It means that these students did not seem to perceive the concept of *zero point* of measurement. However, these findings match to the conjecture that was formulated based on the literature as discussed in chapter 2, namely students have difficulty to measure the length of an object that is not aligned at the first stripe of a ruler.

5.1.3. General conclusion of the pilot experiment activities

The tryout in grade 1 showed that the prospective research subjects (i.e. grade 1 students) already perceived the idea of transitivity, the need of a third object for indirect comparison and the unit *iteration* in measurement. However, to develop students' understanding about these concepts it was decided to keep indirect comparison activities in the instructional activities for the teaching experiment. Other findings from the tryout in grade 1 were that students did not completely perceive the idea of *identical unit* and how to *iterate unit* when performing a measurement. Moreover, students did not seem to consider the need of *precision* in measurement. These findings satisfied our predictions, because we would address these topics with our activities for grade 2.

Students in grade 2 showed that they understood that a number on the ruler represented a length or a measure, but they showed difficulty in measuring the length of objects that were not aligned to the first stripe of the ruler. However, the discussion in the try out in grade 2 showed that the use of various broken rulers to measure each single object could encourage students' to perceive the concept of *zero point* in measurement.

In general, the main purpose of the pilot experiment was improving the designed HLT. However, because the limited activities in the pilot experiment did not provide enough arguments to make adjustments to the HLT so real changes of the HLT were not necessary.

5.2. Teaching experiment

In general, the retrospective analysis in the teaching experiment was worked out in a similar categorization shown on figure 4.2 in chapter 4. The analysis was elaborated in each stage of students' learning line, instead of in each activity. This aimed at explaining how students were acquiring the basic concepts of measurement that afterwards could be generalized for instructional design.

5.2.1. Indonesian traditional games as the experience-based activities

The first tenet of RME, *phenomenological exploration*, focuses on using a concrete context as the base of mathematical activity. For this reason, the Indonesian traditional games were used as the experience-based activities. Considering their rich measurement context, playing g*undu* and playing *benthik* were chosen in this research. The aim of these games was providing a contextual problem situation to build a base for the sense of measurement to students. Consequently, each of these Indonesian traditional games was followed by a class discussion.

Measurement activity in this phase was still a non formal measurement. In general, the expectation from these activities was students would demonstrate the *unit iteration* and use various measuring units. For more specific purpose, a fairness conflict could become an issue that could stimulate student to feel the need for *precision* and to come up with a standard measuring unit.

5.2.1.1. Playing *gundu* and its contribution in supporting students' acquisition of the concepts of *identical unit* and *unit iteration*

When being asked to compare the distances of the marbles, there was no student that stated that the distances were incomparable due to their impossibility to be put next to each other. Instead, all students directly compared the distances. This fact showed that students already understood that a comparison does not always require putting the measured object next to each other. Moreover, the use of a "third object" as a benchmark in comparison showed that the use of a measuring unit started to emerge in this activity.

The strategies that were used by students to compare the distances at the beginning of the game were using pencil and hand spans, as shown in the following figures:

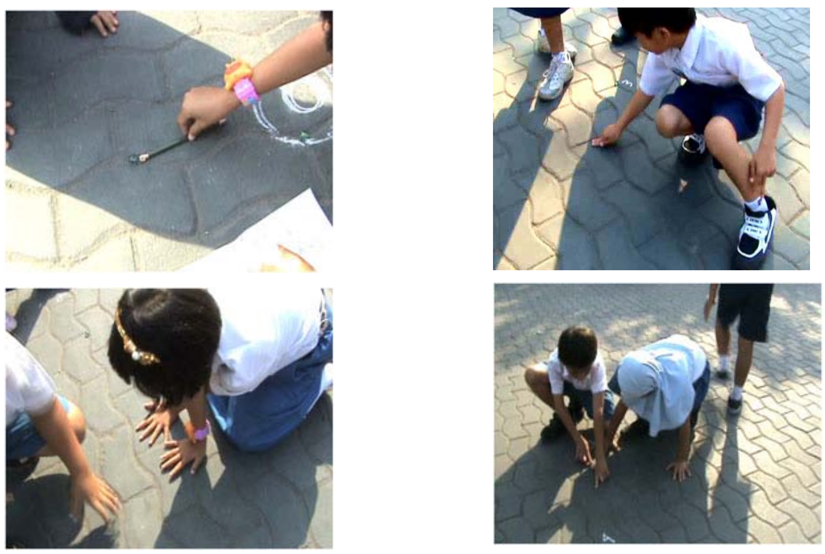
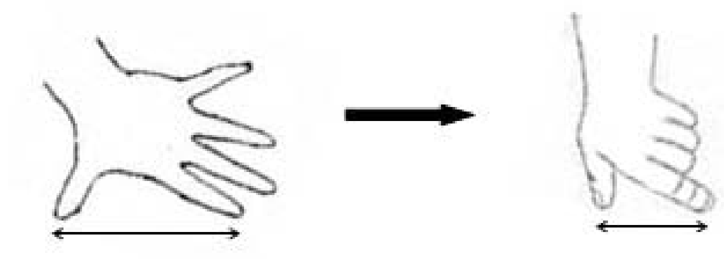


Figure 5.6. Various students' strategies to compare the distances

At the beginning of this activity, students were still measuring the distance of their own marble. Each marble was measured by the student who threw the marble or, in other words, there was no single person who measured all distances. However, at this moment there was neither conflict nor discussion among students.

In the last 15 minutes a *fairness conflict* occurred that, afterwards, inspired students to discuss this problem. The first conflict emerged when there were two marbles that seemed to have a same distance to the circle, namely 3 spans in length. In fact, the distances of these marbles to the circle were different (i.e. about $2\frac{1}{2}$ and $2\frac{1}{3}$

spans). However, both students bent their span to have a complete or integer span for the last span and, therefore, they obtained similar measures for the two marbles.



A “normal”span A bent span but counted as a “normal” span

Figure 5.7. Students bent their span to adjust the measure

There were two conjectures drawn out from this bent hand span. The first possibility was that the students did not completely consider the *identical unit* in measurement. The second possibility was that the students did not consider the need of *precision* in measurement.

When the students were asked whether they had an idea to solve this problem, they came to an idea to use a chalk to compare the distances from each marble to the circle. Finally, they were able to determine the nearest marble because the chalk exceeded the circle for the second measure.

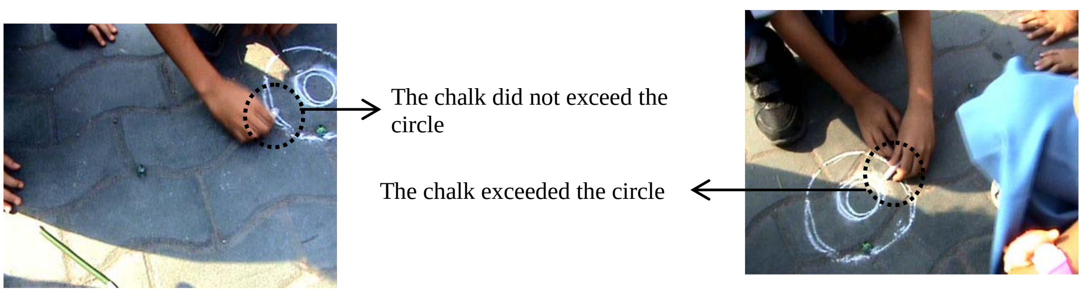


Figure 5.8. A chalk to compare the distances

Although the bent spans of the students showed that the students still used a non identical unit of measurement. Students’ decision to change the unit measurement, i.e. from hand span to a chalk, showed that they commenced to perceive the concept of *precision* as one of the conceptual accomplishments of rule in measurement (see table 2.1). The students seemed to realize that the choice of the unit size determines the precision of the measure. This occurrence exceeded the conjectures in the HLT, because this activity was not designed to accomplish the concept of *precision* in measurement.

The second conflict emerged when the distances of the marbles to the circle were getting closer in which it was getting difficult to determine the nearest marble. At first, students used a chalk to compare the distances.

Figure 5.9. Which marble is the nearest to the circle?

After measuring using a chalk, students got similar measure for both distances. In fact, the distances of these marbles were not exactly equal because the chalk exceeded the circle a little bit more for one of the distances.

Figure 5.10. The distances seem to be equal when measured using a chalk

However, students did not realize the differences between these measures and, therefore, they discussed to come up with another new strategy.

Dea : *We can use a marble to compare the distances*

Researcher : *Why do you suggest us to use marble for replacing the chalk?*

Dea : *Because a marble is shorter than this chalk*

The phrase "*because a marble is shorter than this chalk*" showed that Dea seemed to realize that the size of a unit determine the preciseness of the measure. In this situation, Dea seemed to perceive the idea of *precision* because she knew that a shorter measuring unit, in this case a marble, could give a more precise measure than a longer measuring unit.

Figure 5.11. A marble was used to give more precise measures

At last, students were able to determine the nearest marble. Nonetheless, in conceptual perspective students did not seem to understand the concept of *covering space* because they left (identical) empty spaces between the iteration of marble. Students use their finger to mark the iteration and their finger left identical empty spaces between the iteration of marbles.

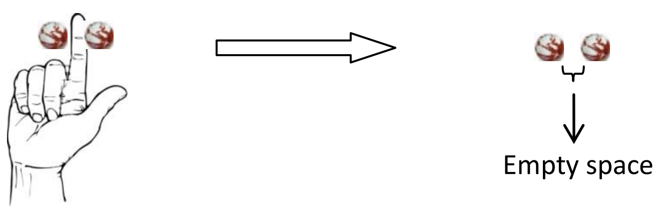


Figure 5.12. Empty spaces between iteration

From playing *gundu*, it can be concluded that students commenced to use "third objects" as benchmarks in comparing distances which this was a start of a measurement. Students also commenced to consider the *unit iteration*, *precisions* and *adjusting the unit* to the size of distance that is being measured. However, in this activity students did not yet completely perceive the concept of *identical unit.*

5.2.1.2. Class discussion and a *conflict* situation as stimuli and supports for students' acquisition of the basic concepts of linear measurement

A class discussion was always conducted after each game playing to support and to develop students' acquisition of the basic concepts of linear measurement elicited by the experience-based activity. The class discussion also aimed to develop students' *interactivity*, as the fourth tenet of RME, in learning linear measurement. Approximately only 50% of students were active in the class discussion, the other half of students were passive both in answering teacher's question and initiating their own idea. During the class discussion, the teacher created some conflicts among students by comparing and discussing their different strategies to stimulate and develop their acquisition of the basic concepts of linear measurement linear measurement.

From the following vignette, it is confirmed that students commenced to shift from comparison to measurement.

Teacher : When playing gundu, how did you determine the nearest marble?

Almost all students communicate their strategy in previous day activity

Students : I just simply look at the marbles [mental comparison] … I used a pencil to compare the distance … I ***measured*** *the distances by my* ***hand span*** *… I* ***walked*** *to* ***measure*** *the distance*

Teacher : Why did you use those strategies?

Gilang : Because ***the smallest measure is the nearest marble***

The word "*measure*" as the answer of "*how did you determine the nearest marble?*" showed that students commence to use measurement as means to compare the distance (i.e. "*determine the nearest*"). The phrase "*smallest measure*" also indicated a measurement and, on the other hand, the phrase "*nearest marble*" indicated a comparison. Hence, Gilang showed that he used a measurement as a means to compare when he said that "*the smallest measure is the nearest distance*. The pencil, hand span and pace that were used by students in comparing distances showed that students, in general, have commenced to use a non-standard measuring unit.

In this class discussion, the teacher also created a situation as a means to support students' acquisition of concept *identical unit*. The crucial guides by the teacher are shown in the following excerpt.

Teacher : Now the problem is how to determine the nearest marble?

Uam : I used my span

Teacher : OK, Uam come here. Anybody else who used span? [Elok rises up her hand] Elok, come here. How about Fahmi, what did you use? Did you use your span? If so, come here.

Teacher : *Now, all three of you show your span to your friends. Elok , Fahmi and Uam, show your span please.*

Figure 5.13. Will these spans give a fair measure?

Teacher : *If we practice to measure the distance of Rakka's marble using these three different spans, will we obtain same measures?*

Gilang : *No, they are not. Because the sizes of their spans are different so we will obtain different result of measurement.* [The other students agree with Gilang's opinion]

What the teacher did by showing different hand spans of Uam, Elok and Fahmi was an example of creating a situation in which students realize that their strategies are not sufficient for more sophisticated instruments. This situation encouraged students to focus on the effect of the size of measuring unit to the result of the measurement [*as shown in Gilang's opinion*].

Another stimulus created by the teacher in the class discussion was by using the word "*fair*". The *fairness* was a natural principle in this situation, when playing games. Hence, the word "*fair*" was used by the teacher to support the emergence of a standard measuring unit.

Teacher : *Is it **fair** for our game if we measure the distances using different hand span?*

Gilang : *No, it is not*

Teacher : *Is it **fair** if we measure the distances using only Fahmi's span?*

Students : *Yes, it is …*

Teacher : *Can you derive a conclusion from this fact?*

Students are discussing with their partner for about 5 minutes until Haya proposes her opinion.

Teacher : Let's we listen to Haya's opinion

Haya : The game is not ***fair*** *if there are many students measure the distances because the different size of steps will give different result (of measurement)*

Teacher : So … ***can we use*** *different steps to measure the distances in our game?*

Students : No it is not because it is not ***fair***

Teacher : What should we do?

Haya : In a game, we will have a ***fair*** *game if there is only one person who measures the distances because the different size of steps will give different result (of measurement)*

Haya came up with a standardization of measuring unit by merely using one person's body part as measuring unit. This showed that the need of standard measuring unit has commenced to emerge, although students still emphasized on body parts as measuring units. In this situation the measurement still depended on the presence of a single person. For this reason, the teacher triggered a new conflict to guide the students to come up with an *independent standard measuring unit*. Independent standard measuring unit means that the standard measuring unit is independent from a single person as the "operator".

In the next excerpt, a phrase "*no-one is willing to measure*" was posed by teacher to encourage students to eliminate body parts as measuring unit.

Teacher : If in a game there is ***no-one is willing to measure*** *the distances using his/her steps. What should we do (in measuring the distances) to obtain fair results?*

Students : We can use a bamboo stick

Teacher : Why should we use a bamboo stick?

Gilang : Because the length of stick is constant (unchanged)

Students : We also can use ruler, pencil, eraser, marker …

Teacher : Yes, you all are right. Can anyone of you give a conclusion?

Haya : We need to measure distances using an object (measuring unit) to have fair measurement of various distances.

The opinion of Haya and Gilang showed that "*no-one is willing to measure*" could stimulate students to make a shift from using a person's body part as a standard measuring unit to using independent object that could be used by every person. By saying "*because the length of stick is constant*", Gilang used the concept of *conservation of length* to support his acquisition of concept *identical unit* and the emergence of standard measuring unit in measurement. The final conclusion of Haya became a base for students in perceiving the concept of *identical unit* and, moreover, the need of a standard measuring unit for fair measurement.

From the playing *gundu* and the class discussion, students commenced to acquire some basic concepts of measurement, namely *unit iteration*, *identical unit* and *precisions*. Students' learning line in these activities also shifted from *comparison* to *measurement* that engaged *a non-standard measuring unit*.

5.2.1.3. Playing *benthik*: The shift from a non-standard measuring unit towards a standard measuring unit

The acquisition of the concept *unit iteration* and the use of *a non-standard measuring unit* prolonged in playing *benthik* activity and also in the class discussion after the game. In playing *benthik*, the *unit iteration* and *non-standard measuring units* were utilized by students when measuring the distances.

Figure 5.14. *Unit iteration* and *non-standard measuring unit* in playing *benthik* activity

The initial acquisition of *identical unit* shown in playing *gundu* was developed as a base for a standard measuring unit in playing *benthik.* The following vignette shows the emergence of a standard measuring unit in the playing *benthik.*

Researcher : Will it be a problem for our game if Deva's team uses hand span to measure and Rakka's team uses their steps?

D'Chia : ***It is not a problem because (the length of) a step and a hand span are same.***

D'Chia is showing and comparing the length of his hand span and the length of Fahmi's foot [Note: D'Chia is a member of Rakka's team and Fahmi is a member of Deva's team]

Despite the different measuring unit proposed by D'Chia, his attempt to emphasize on the similar length of the two different units showed that it was a start of the emergence of a standard measuring unit. This process developed when a *conflict* occurred when two different measures for a single distance were obtained by Deva and Ivan. Both Deva and Ivan measured the distance using their paces, but Deva got 38 feet and, on the other hands, Ivan got 46 feet.

Researcher : Why did we get different results for the same distance? ***How can this happen?***

Suddenly (without any discussion) students answer the question

Students : Because that foot (point to Deva's foot) is big and this one (point to Ivan's foot) is small.

Figure 5.15. Different length of units result different measures

Students' reasoning of the different measures showed that they already understood the relation of the size of measuring unit and the result of measurement. Hence, the discussion was targeted to the emergence of a standard measuring unit.

Researcher : ***Which result should we use?***

Students do not answer this question

Researcher : Is it ***fair*** *for our game?*

Students : No, it is not

Researcher : So, what we need to do to have a fair game?

Deva : We can use this (point to the long stick) to measure the distance

The short vignette above reflected that a natural conflict, for instance prompted by the word "*fair*", was very important in stimulating students to come up with a solution. Students did not give any reaction when they were asked by a non conflict question, "*which result should we use?*". The question "*is it fair for our game?*" seemed to be more natural and stimulating in this situation, because students were able to come up with a solution for the problem when being asked by this question.

Deva proposed to use the long stick to measure the distances.
Suddenly Deva shows the long stick and suggests using it to measure the distance

Figure 5.16. A stick to measure the distances

Researcher : Why do we need that stick?
*Deva : Because its **length is constant***

The concept of *conservation length*, shown by the phrase "*the length is constant*", underlay Deva's acquisition of *identical unit* concept. Moreover, Deva had arrived at the level of *standard measuring unit*. Deva's achievement in *identical unit* and standard measurement was shared and developed in the next class discussion.

At the end of the previous class discussion (i.e. after playing *gundu*), students seemed to realize the need of a standard measuring unit to obtain a fair measure. However, at the beginning of playing *benthik* students did not use this idea to measure the distances. Students started again from various measuring units and they shifted to a standard measuring unit after the occurrence of a conflict when measuring using different foot sizes. In short, students showed an inconsistency in acquiring the need of a standard measuring unit.

5.2.1.4. Class discussion: Communicating and developing ideas

The class discussion was conducted to facilitate and develop students' acquisition of the basics concepts of linear measurement. The benefit gained from the class discussion was not merely communicating a student's idea, but also stimulating other students to develop various strategies. The inconsistency shown by students in perceiving the need of a standard measuring unit was expected to be solved in the class discussion.

Considering the importance and the use of conflicts to stimulate students perceiving the basic concepts of linear measurement, the teacher started the discussion by providing a conflict based on the playing *benthik* activity.

Teacher : Deva obtained 24 long sticks and I still remember my brother told me that Agung, his best student, in Kalimantan got 50 short sticks in length.

While telling the story, the teacher draws the representation of Deva's stick and Agung's stick on the board.

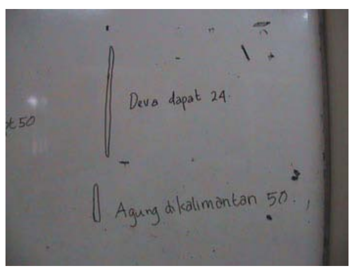


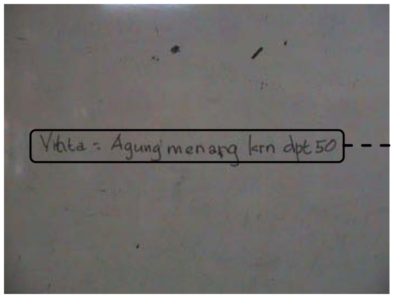
Figure 5.17. Who is the winner?

Presenting different strategies on the board is a way to emphasize the communication of the problem because students occasionally ignored an oral problem. The combination of oral and written problems will engage students in more active thinking and discussion.

Teacher : Who is the winner between Deva and Agung? Who did throw stick in the further distance?

Vinta : Agung is the winner because Agung obtained a bigger number, namely 50.

The teacher writes Vinta's opinion on the board to share and communicate it to the whole class.



Agung is the winner because he got 50

Figure 5.18. Communicating a students' idea to the whole class

The teacher tried to engage all students in the discussion by writing Viola's opinion on the board and asking for other opinions. This strategy was successful because not all students did pay attention to the discussion.

Gilang : "Deva's stick is longer than Agung's stick, so the length (of those sticks) is different.
Teacher : Yes … and what is the next?
Gilang : The number (the measure) is also different
Teacher : Who is the winner?
Gilang : Agung is the winner.

Despite his attention to the different stick [shown in "*Deva's stick is longer … so the length is different*"], Gilang still seemed to focus on the number or measure when determining the winner. It was reflected from the combination between "*the number (the measure) is also different*" and "*Agung is the winner*" that were posed by Gilang. Gilang showed his inconsistency in perceiving the need of a standard measuring unit, because in playing *gundu* Gilang seemed to realize the need of a standard measuring unit.

Teacher : Any other opinion? Is there any winner?
Some students : No
Teacher : Who was saying "no"?
Shafa rises up her hand
Teacher : Shafa, ***why did you answer "no"****?*
Shafa : Deva's stick is longer than Agung's stick so it is impossible for Deva to be the winner

The attempt to connect the different length of measuring unit to the impossibility to determine the winner showed Shafa's consistency in understanding of the need of a standard measuring unit. Both in playing *gundu* and playing *benthik*, Shafa was consistent in noticing the need of a standard measuring unit.

Teacher : Shafa said that Deva was not the winner because his stick was longer than Agung's stick (teacher shares Shafa's opinion to the class)

Teacher : So, who is the real winner?

Gilang : Agung is the winner

Teacher : We still have different solutions for his problem so now all of you think and discuss again in pairs with your partner (about this problem). Discuss about who the winner is.

The teacher attempted to engage all students in active discussion by asking them to discuss the problem in pairs.

Aira : We cannot determine the winner

Teacher : ***Why did you say that we could not determine the winner?***

Aira : Because one stick is longer than the other

Teacher : Who is the winner between Deva and Agung?

Aira is shaking her head to indicate that in her opinion there is no winner between Deva and Agung

Teacher : Any other idea?

Teacher : OK D'Chia, share your opinion to your friends

D'Chia : The length of the sticks is different so if we measure the long stick with the short stick we will know how many short sticks will match the long stick. So 20 is added to 20 is 40 and 4 is added by 4 is equal to 8 and the sum of those is 48. Then the winner is Agung (because Agung got 50).

Teacher : D'Chia said that Agung is the winner

Uam : I agree that Agung is the winner because he has 50

As well as Shafa and Aira, D'Chia tried to connect the short stick to the long stick. The sentence "*if we measure the long stick with the short stick we will know how many short sticks will match the long stick*" showed that D'Chia tried to standardize the measuring unit in the term of short stick. Furthermore, D'Chia compared the length of Deva's stick to that of Agung's stick but he did not really compare the short stick to the long stick. Therefore, Deva only came to an assumption that the length of the long stick is as twice as the long of the short stick.

Shafa, Aira and D'Chia seemed to perceive the need of a standard measuring unit. In contrast, the opinion of Vinta, Gilang and Uam showed that they did not notice the difference between the measuring units. Instead, they still focused on the number of measures, instead of on the different sticks. Therefore, the teacher attempted to direct students to come to the need of a standard measuring unit by demonstrating the measurement of Agung and Deva. Students seemed to be more enthusiastic when they did the demonstration. After demonstrating the measurement, students found that Deva is the actual winner. This demonstration stimulated students to notice that they need to consider the length of measuring units, as shown in the following excerpt.

Teacher : What can we do to determine the winner?
Gilang : We can use marbles because marbles are always in the same size
Teacher : Yes it is possible to use marbles but the distances in benthik game is too far to be compared using marble. Is there any idea?
Gilang : We cut the sticks and make them similar in length
Shafa : Yes, we have to use measuring units that have similar length

Gilang showed his achievement in acquiring the need of a standard measuring unit when he proposed to use marble and make the sticks in similar length by cutting one of them.

5.2.1.5. Summary of the experience-based activities

As shown in figure 4.2, the objective of the experience-based activities was building and developing students' understanding of the concepts of *conservation of length*, *identical unit*, *unit iteration* and *covering space*. From subchapter 5.2.1.1 to 5.2.1.4, it was found that from the game playing students commenced to perceive the idea of the concepts of *conservation of length*, *identical unit*, *unit iteration* and *covering space*. Moreover, students also started to perceive the concept of *precisions* and the relation between the size of the unit and the result of measurement. The initial knowledge of the basic concepts of linear measurement that was gained from the game playing was developed in the class discussion.

However, most of these concepts were still perceived by students as informal knowledge. Consequently, the next important step in the instructional sequence was providing "bridge" activities to develop students' informal knowledge of linear measurement into the more formal knowledge of linear measurement.

5.2.2. "Making our own ruler" as a bridge from a situational knowledge to the formal measurement

This activity referred to the second and the third tenet of RME, namely *using models and symbols for progressive mathematization* and *using students' own construction*.

Making our own ruler activities aimed to bridge students' informal knowledge of linear measurement to a formal measurement. Formal measurement in this term was defined as the correct and meaningful use of ruler in measurement.

Making ruler activities were conducted in a series of three activities as follows:

- Measuring using strings of beads
- Making our own measuring instrument
- Shifting from a student-made measuring instrument into a blank ruler as a start of a normal ruler

In the first activity, measuring using strings of beads, students were directed to get acquainted with a non standard measuring **instrument**. The string of beads was chosen as the measuring instrument because it was the imitation of an array of marble that was used to measure the distance [*we can see figure 5.11 and Gilang's idea shown at the last vignette in subchapter 5.2.1.4 in which students started to think and use marble as measuring unit*]. Consequently, using measuring unit to measure (as the focus of previous activity) was turned to using measuring instrument to measure (as the focus of this activity).


Figure 5.19. Strings of beads to measure length

In general, this activity was successful because almost all students were able to correctly measure the length of various objects using strings of beads. However, there was an interesting finding when Elok's group was measuring the height of a chair. This group obtained one and a half of strings of beads, but these students said that the height of the chair was **50 and a half**. The 50 referred to 50 beads in the string, while the half referred to a half of the string's length.

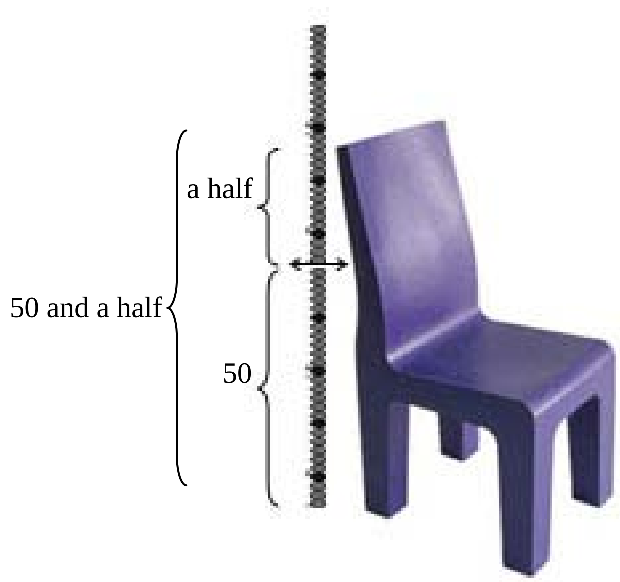


Figure 5.20. Illustration of Elok's measure

The conjecture derived from this finding is that these students were confused about the difference between the beads as measuring units and the string of beads as the measuring instrument.

The strings of beads were still used in the next activity, namely when students were asked to make their own measuring instruments based on their string of beads. As a note, the word "ruler" was not yet used in this activity.

There were two different strategies used by students to make their own measuring instrument. The first strategy was imitating the shape and size of the string of beads on a paper. The second strategy was directly drawing two straight lines and then put some stripes on the straight line by imitating the size of the string of beads.

Figure 5.21. Two different strategies in making our own measuring instrument

The new measuring instrument (not yet a ruler) constructed by students showed the process of emergent modeling how a model emerged from a situational level to formal level.

In playing *benthik* a student (i.e. Gilang) came up with an idea to use marbles to measure the distances in the game (*look at the last vignette at subchapter 5.2.1.4*). Gilang's idea represented one of *situational level* in the experience-based activities in which Gilang explained how his interpretation and solution of the problem developed based on how to act in the setting of marbles as measuring tools (see subchapter 2.2.2).

In the *referential level* Gilang's idea was followed up by the use of strings of beads as representation of iterated marbles. Moreover, the strings of beads became the base of the emergence of student-made measuring instrument as the *model of* the situation that signifies the iteration of marbles.

The numbers written on students' new measuring instrument (as shown in figure 5.23) showed how students commenced to consider that a number represented a measure. In this phase students started to use their instrument as *model for* measuring the length of objects. The use of student-made measuring instrument as the *model for* measurement showed that *general level* of modeling has been attained by students.

The last level of emergent modeling, *the formal level*, started to be accomplished when some students draw a ruler as their new measuring instrument. This kind of instrument became the preliminary of the use of ruler to accomplish the concept of *zero point* in measurement. In the formal level students' reasoning with conventional symbolizations started to be independent from the support of models for mathematics activity. In this level, the focus of discussion move to more specific characteristics of models related to the concepts of units, fairness and zero point of measurement.

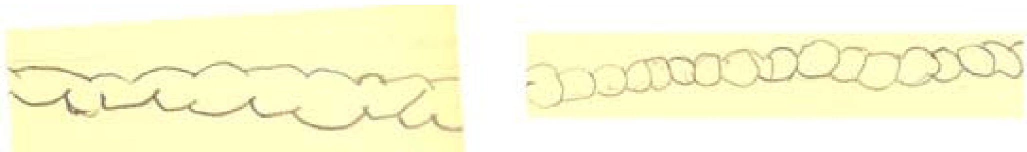


Figure 5.22. New measuring instruments as the *models-of* situation that signifies the iteration of marbles

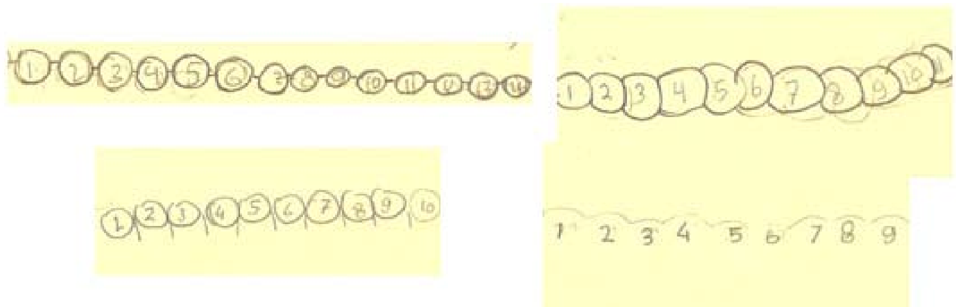


Figure 5.23. The students' new instruments as the *models-for* measurement

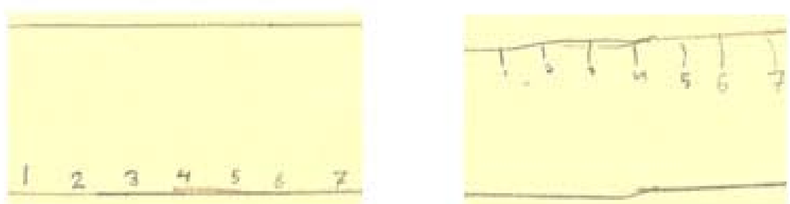


Figure 5.24. Student-made ruler as the starting point of the formal level of emergent modeling

As the conclusion of "*making our own ruler*" activity, students showed their progress to shift from experience-based activity to formal measurement. The student-made measuring instrument reflected that students started to measure in a more formal way. The student-made measuring instruments were subsequently developed into blank rulers as a start of normal ruler as ready-used measuring instrument.

5.2.3. A blank ruler: The student-made ruler as the beginning of a standard measuring instrument

From the result of the pilot experiment in grade 1, it was conjectured that most students did not perceive the concept of *covering space* and, moreover, they did not realize that there were spaces within a ruler. Students in the pilot experiment seemed to focus on the stripes, instead of on the spaces. Consequently, the concept of *covering space* became the key issue in the blank ruler activity. The student-made measuring instruments were developed into a broken ruler that subsequently was used as means to emphasize concept of *covering space* and, furthermore, to introduce a ruler as a standard measuring instrument.

The activity was started by working with worksheets that preceding the class discussion. The worksheet contained five problems and had been solved by 43 students that worked in pair. From the students' answers of the worksheet, it is obtained that the "level of accomplishment" for concept *covering space* is 43,81% [*this result can be found at appendix E on page 113-114*]. The level of accomplishment was defined as the ratio of the number of correct answers to the number of the total answer.

However, it was difficult to conclude whether those correct solutions reflected a correct way of measuring using blank ruler because the students' worksheet merely provided the final answer of solutions without any record about students' strategies. For this reason, the following analysis of students' reasoning based on video recording aimed to investigate students' learning process and level in acquiring the concept of *covering space*.

In general, there were three strategies used by students when measuring using a blank ruler. These strategies are described as follows:

Fist strategy:

The first strategy of students was placing the edge of the ruler on the edge of the measured object and then counting the number of stripes. The following excerpt is an example of a student who used this strategy.

Researcher : Laras, how did you measure the length of this figure?

Laras : I put this ruler here (match the edge of the ruler to the edge of the measured figure) and then counted this (point to the stripes) starting from here (point to the first stripe) to left side. Therefore the length of this figure is 10

Researcher : What is "10"?

Laras : "10" is the number of these (point to the stripes)

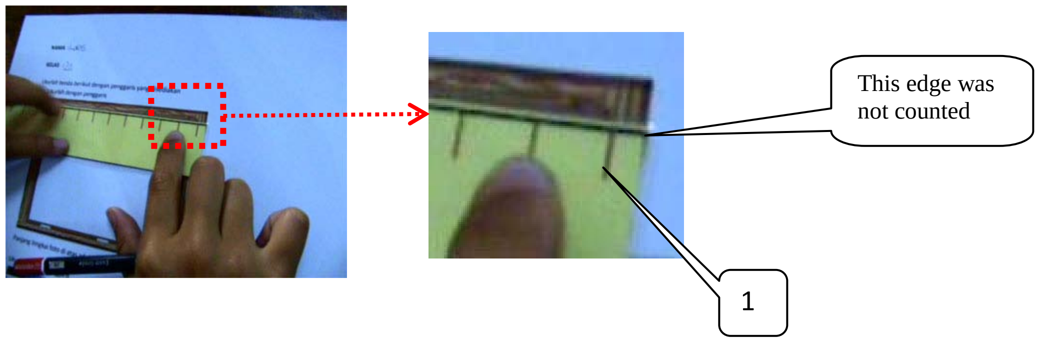


Figure 5.25. The object is aligned to the edge of the ruler, instead of to the first stripe

Some conjectures are derived from this strategy, namely:

1. Concept of *zero point* of measurement
   The conjecture of this situation is that the students who used this strategy assumed that the edge of measuring instrument was the zero point of measurement, similar to how they operated stick as measuring instrument.
2. Concept of measuring as *covering space*
   These students still counted the number of stripes, instead of the number of spaces between two stripes. These students showed that they *cover the spaces* when measuring using strings of beads, but they did not realize the spaces when these spaces were transformed from beads into spaces between two stripes on ruler. The conjecture of this occurrence is that these students did not perceive the concept of *covering space* in more formal measurement that is when using ruler.

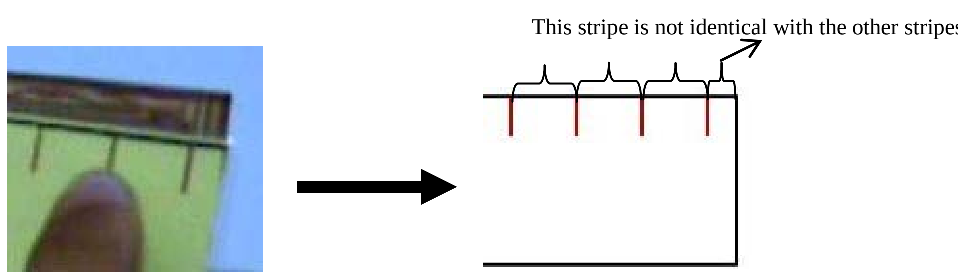


Figure 5.26. A **non** *identical unit* in measurement

In this situation, this strategy also could lead to the emergence of a non *identical unit* in measurement when students did not align the ruler to the object in proper way [see figure 5.26]. The length of the first space is different with the length of other stripes.

The second strategy:

The following excerpt shows another strategy used by students when measuring using a blank ruler.

Researcher : Rangga, how did you measure the length of this figure?

Rangga : Like this … (Rangga matches the last stripe of the ruler to the edge of the objects and then he counted the number of the stripes from right side)

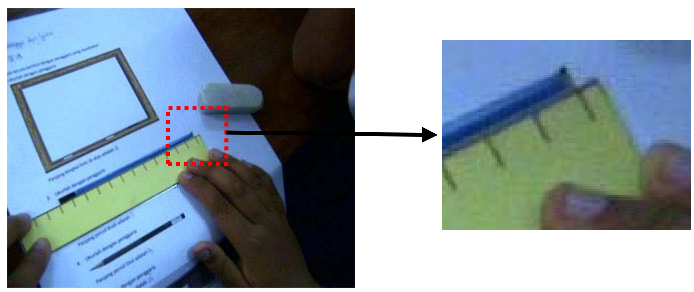


Figure 5.27. Align the edge of the object to the first or last stripe of the ruler

To measure the length of the given figures, Rangga was placing the first stripe of the ruler on the edge of the measured object and then counting the number of stripes. As well as Laras, Rangga showed that he did not perceive the concept of measuring as *covering spaces*. Compare to Laras, Rangga showed a better understanding about the concept of *zero point* of measurement.

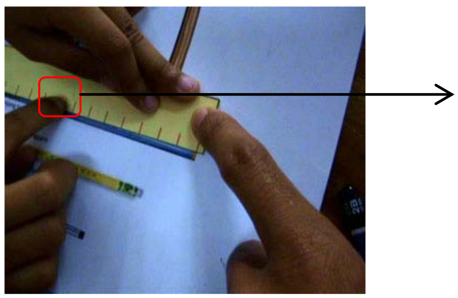
The third strategy:

The third strategy shows the highest level of students in perceiving the basic concepts of linear measurement. This achievement was shown by how students placed the first stripe of the ruler on the edge of the measured object and then counted the number of spaces between two stripes.

The following two excerpts show how student perceive the concept of *covering spaces* in linear measurement.

The first excerpt is the discussion between researcher and Vira, after Vira measuring the length of the figure of stick.

Researcher : Vira, how long is this stick?
Vira : The length of this stick is 12
Researcher : Can you show me how you measured this stick?
Vira : Just counting
Researcher : What was being counted?
Vira : This (point to the "space" between stripes)
Vira matches the last stripe of the ruler to the edge of the objects and then he counted the number of the "spaces" from right side



Count the number of spaces

Figure 5.28. Counting the number of spaces to measure the length

The second excerpt shows that there was a discussion among some students. Shafa, Abi, Hikari and Alvin were discussing about what that should be counted when they measure the length of objects using a blank ruler.

Hikari : Sir, what that should be counted? The stripes or the "holes" (students' term for "spaces")
Researcher : What do you think?
Hikari : I do not know
Researcher : Who said that the holes that should be counted?
Hikari : Shafa
Researcher : Shafa, Why did you say to Hikari that we had to count the number of "holes"?
Shafa : I counted the number of the "holes" because if I counted the stripes, ***there is no bead preceding the first stripe and therefore it was not a complete measure*** *yet.*

The phrase "*there is no bead preceding the firs stripe*" show that Shafa connected her experience when measuring using strings of beads to measuring using a blank ruler. This phrase also shows that the "*holes*" (Shafa's term for spaces) represented the beads, therefore she counted the number of holes in measuring as well as she counted the number of beads when using strings of beads to measure. Shafa's reasoning shows that she seemed to perceive the concept measuring as *covering spaces*.

Class discussion:

In the class discussion after measuring activity, teacher used both the strings of beads and student-made ruler to stimulate students perceiving the concept of *covering space*.

Teacher : ***Do you remember when we measured using strings of beads?*** *Where did we put our finger when we count "1"? Is it at this point (point to the beginning of a bead); this point (point on a bead) or this point (point to the "end" of a bead)?*

Teacher : *When we touch this point (point to the beginning of a bead), has it already been "one"? Is there any bead that precedes the first bead?*

Haya : *No, there is no bead before the first stripe*

Teacher : *Haya, can you show to us the first distance?*

Haya draws a horizontal line below the figure of the first bead to show the first distance

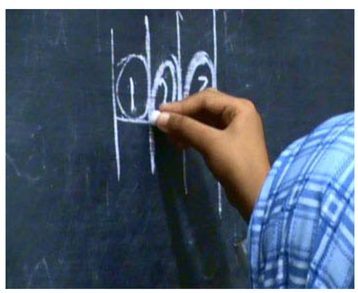


Figure 5.29. A horizontal line to indicate a distance

Teacher : *Why do not we start counting from the first stripe?*

Deva : *Because there is no bead precedes that stripe.*

Deva's reasoning in the class discussion is similar to Shafa's reasoning when she discussed with Hikari, Abi and Alvin.

From the reasoning of Haya and Deva that arose after teacher's stimulation using strings of beads, it seems that the strings of beads served as important bridge to help students move to more formal measurement (i.e. using ruler) and perceive the concept of *covering space*. The horizontal line drawn by Haya showed a distance on a ruler as a result of transformation from a bead into a space between stripes on ruler. Consequently, the spaces between two stripes become the unit within a ruler that should be counted in the measurement.

The following are the steps used by teacher to stimulate students to perceive the idea measuring as covering spaces. Teacher emphasizes on the "distance of a bead" to lead students focus on the spaces between two stripes.

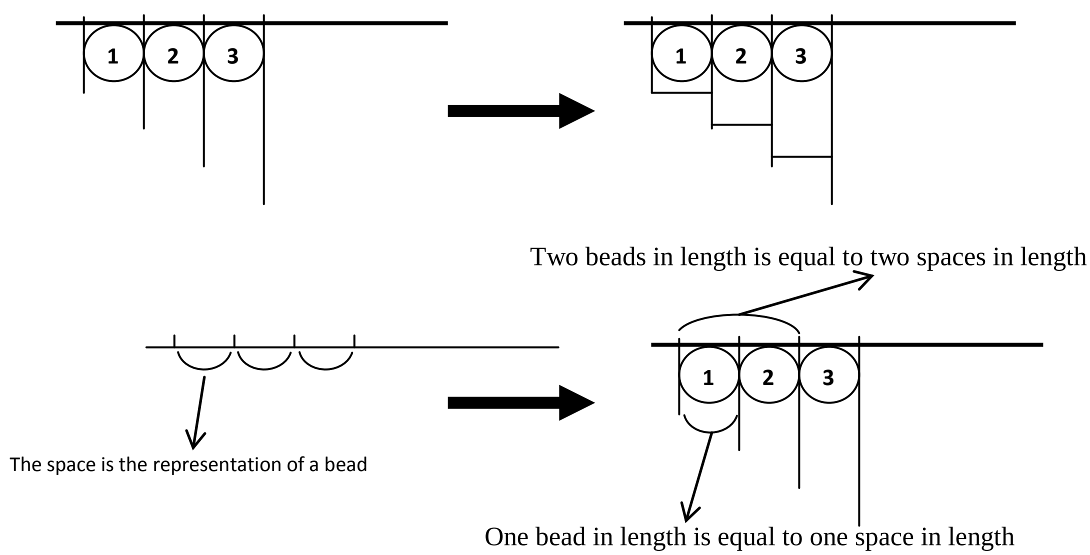


Figure 5.30. The sequence of guidance to support student in perceiving concept *covering space*

After students perceiving the concept of *covering space*, teacher orchestrated the next discussion to introduce students to a normal ruler. The emergence of numbers on a ruler was the main issues in this discussion.

Teacher : How do we write numbers on this blank ruler?
Deva : We start numbering from "1" on the first stripe
Teacher : Why do you do that?
Deva does not answer the question, but he starts reciting number from 1
Teacher : If we put this string of beads on a blank ruler, where is the first bead? Remember that when we measure using string of

beads, we count the number of beads. So where should we put number "1" on the ruler? Is it at the first stripe or at the second stripe?

Deva : The first stripe.

D'Chia : I measured by counting the number of stripes but starting from the second stripe because ***the second stripe is the end of the first bead****.*

D'Chia had moved to the next step of learning when he said "*...because the second stripe is the end of the first bead*". This will make the process of numbering became easier because he write numbers on the stripes, instead of on the spaces. Furthermore, it was expected that Deva would write number "1" on the second stripe (*because the second stripe is the end of the first bead*).

Teacher : So, where should we write down number "1"?

Gilang : At the second stripe

Teacher : ***What is your reason, Gilang?***

Gilang : Because there is nothing before the first stripe so we cannot put number "1" on the first ruler.

Teacher : Is there any other opinion?

Aira : We write "0" on the first stripe because we did not count the first stripe

Teacher : Where should we give mark to indicate the first bead?

Students : Write number "1" on the second stripe

Teacher : How to indicate the third bead?

Students : We should write number "3" on the fourth stripe

The reasoning of Gilang and Aira support D'Chia's opinion that a stripe on a ruler indicated the space preceding it and therefore the numbers should be written on the stripes.

General conclusion of the measuring using blank ruler activity:

Based on students' answers in the worksheet and students' reasoning during the measuring activity and the class discussion, it is conjectured that most students still had difficulty in perceiving the concept of *covering space*.

The progress of students' reasoning in the class discussion showed that the use of strings of beads played an important role in encouraging students to consider the concept of *covering space*. At the end of the class discussion students started to perceive the concept of *covering spaces* and focused on the stripes as indicator of the spaces preceding them. Furthermore, students commenced to realize the need to write numbers on their ruler.

5.2.4. A normal ruler: What do numbers on a ruler aim at?

This activity was preceded by measuring using the blank ruler to investigate students' progress in acquiring the concept of *covering space* after the class discussion at the end of the previous activity (i.e. *measuring using blank ruler*). The improvement of the "*level of accomplishment*" in acquiring the concept of *covering spaces* when operating a blank ruler was not significance because there was merely 3,29% improvement from 43,81% [*achieved from the previous day's activity; see appendix E on page 113-114*] to 47,1% [*see appendix F on page 115-117*)]. It is conjectured that the class discussion did not successfully support the development from an individual's accomplishment to social's accomplishment of the concept of *covering space*.

The measuring using normal ruler activity was conducted as the follow up of the emergence of numbers on a ruler. In the previous activity students commenced to realize the use of number on the ruler and also how they should write the numbers on the ruler. Therefore, the main focus in this activity was how to correctly and meaningfully measure using a normal ruler.

The worksheet revealed that for the same problems in the worksheet, the "*level of accomplishment*" for measuring using normal ruler was merely 3,63% higher than that of measuring using blank ruler, namely 50,73% compare to 47,1% (*see appendix F on page 115-117*). It is conjectured that in this situation the appearance of numbers on the ruler did not seem to give a significant effect to achieve a correct measure. Most students seemed to neglect the numbers on the ruler when measuring the objects. Therefore, to investigate this finding, the analysis was focused on students' strategies when operating the normal ruler to measure.

The strategies used by students when operating normal ruler were relatively similar to the strategies used when operating the blank ruler. Students' strategies in operating the normal ruler to measure length are described as follows:

a. Matching the "0" to the edge of the measured object and counting the number of stripes. The counting is started from the second stripe but the reciting (of number) is started from 1.

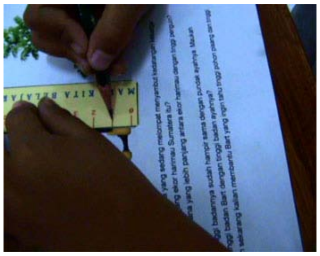


Figure 5.31. Measuring by counting the number of spaces

Fahmi starts counting from the second stripe (he does not count/tag the first stripe), but he start reciting numbers from 1. He says "one" while touching the second stripe, "two" for the third stripe and so on.

The fact that Fahmi still counted the number of stripes excluding the first stripe (as indication that Fahmi actually counted the number of spaces) indicated that he did not give attention to the numbers on ruler.

b. Matching the "1" to the edge of measured object and counting the number of stripes.

Students who used this strategy seemed to have difficulty to perceive the concept of *covering spaces* in measurement because they did not completely achieve the concept in the previous day discussion. They also have difficulty to determine the zero point of measurement because they did not consider the "0" on the ruler. The conjecture of this occurrence is that students still assume that measuring was merely a counting; therefore they used "1" as the starting point as well as "1" as starting point of counting a set of objects.

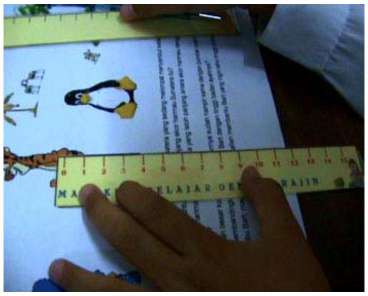


Figure 5.32. Measuring by counting the number of stripes

Both Salma and Azka, although they were not partners, measure the length of Tiger's tail by matching the edge of the tail to number "1", instead of number "0". To determine the length of the tail, she directly looks at the number that corresponds to the last edge of the tail (i.e. 4) without counting anything (neither the number of stripes nor the number of spaces).

Researcher : Why don't you match the "0" to the edge of the tail?
Salma : Because "0" is nothing so we do not use "0"

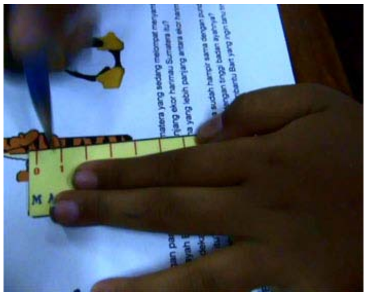
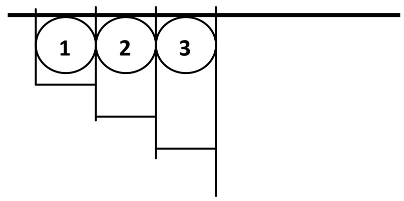


Figure 5.33. Zero is nothing, therefore we do not need it

When Salma did not use the "0" and said that "*zero is nothing*", it seemed that she still thought measuring as counting a set of object (cardinality).
To solve this problem, the teacher conducted a class discussion and reminded students to the discussion in the previous day when they discussed the way to write numbers on a ruler.

Teacher : Where should we put the edge of the measured objects on the ruler? Is it on "0" or "1"?
Students : We should put the edge of the measured objects on the "0"
Dea : No. We should start from number "1"
Teacher : Why do we start from number "1"?
*Dea : **Because zero is nothing***

The teacher used the similar figure as figure 5.28 to encourage students in perceiving the "normal" *zero point* of measurement. At the beginning of linear measurement, students are introduced "0" as the "normal" starting point of measurement. Therefore, this activity was also used to help students perceive this idea. Other *zero points* of measurement would be introduced to students at the end of instructional activities for learning linear measurement, namely measuring using broken ruler.



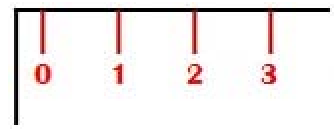


Figure 5.34. Where should we start to measure?

Teacher : ***How did we put the strings of beads when we measure the length of objects?***

Students : *We matched the first bead to the edge of measured objects*

Teacher : *Now, do you remember how we wrote numbers on our ruler yesterday*

Students : *We started numbering from "0"*

Teacher : *Compare the figure on the black board to your ruler and then discuss with your friend how we put the ruler to measure.*

After around a 5 minutes discussion, some students communicated their idea.

Shafa : ***We should put the "0" at the edge of the object because "0" is the beginning of the first bead.***

The teacher attempted to connect this activity to measuring using strings of beads by proposing a question, "*how did we put the strings of beads when we measure the length of objects?*. This kind of guidance from the teacher and students' reaction to this guidance showed that strings of beads played an important role as a bridge from experience-based activities to formal linear measurement.

When there was no student who directly looked at the number on the ruler to determine the length of the object, the teacher asked students to measure longer objects. It was expected that students came up with a new strategy that was more efficient than counting the number of spaces when measuring long objects.

There were six students who rise up their hand when the teacher asks how many students directly look at the numbers on the ruler without counting. It showed that only 13% of the students started considering the appearance of numbers on a ruler.

Caca : *We do not need to count the spaces; I just look at this (point to the last number on her ruler). Then I sum up the number (she sum up all measure in each iteration)*

Teacher : *So how long is the width of your table?*

Caca : *15+15+12*

Teacher : Fahmi, how did you count when measuring?

Fahmi counted the number of stripes but he started reciting number from zero. He said "0" while tagging the first stripe and so on until the last stripe.

Although Fahmi tagged the stripes, his reciting strategy reflected that it seemed to be merely reading.

Teacher : Do we need to count the number of spaces if there are numbers on our ruler?

Students : No, we do not need to do that

Salma : We just need to sum up the numbers (from each iteration)

The argument of Caca and Salma became the bases of the conjecture that students begin to realize the use of numbers on a ruler. Students seemed to realize that a number on a ruler indicate a measure. However, at this stage it is still difficult to determine whether students have chosen the right number on the ruler to indicate the right measure of an object. This could be investigated by conducting a new activity that focuses on the *zero point* of measurement. Consequently, the following measuring using broken ruler activity would be used to give more correct and meaningful use of ruler.

General conclusions of the measuring using normal ruler activity:

The low improvement of students' "level of accomplishment" in acquiring the concept of *covering space* after the class discussion [i.e. 3,29%*; see appendix E on page 113-114* and *appendix F on page 115-117*] shows that the class discussion need to be developed. The class discussion can be developed by engaging more students to communicate their ideas. More measuring practices can also be done by students within the class discussion. The measuring practices aim to help students to get acquainted with operating a ruler. Hence, conducting practices within the class discussion offers a balance between the progress in acquiring the concepts of linear measurement and the practice of measuring.

From this activity, students seemed to consider that a number on a ruler represented a measure. Furthermore, some students commenced to consider and use the numbers when they measure. However, it was not yet obvious whether students merely read the numbers [*see Fahmi strategy on page 76*], or already correctly operated the normal ruler. Consequently, new activity was needed to investigate students' ability in operating a normal ruler, especially about the *zero point* of measurement. Measuring using broken ruler was, afterwards, chosen to investigate students' accomplishment of the concept of *zero point* of measurement.

5.2.5. A broken ruler: Where and how should we start measuring?

At the end of the normal ruler activity, students commenced to realize that a number on a ruler indicate a measure. Therefore, the broken ruler activity was conducted to develop this understanding to a correct and meaningful use of a ruler.

The teacher started the activity by giving students a task to measure the length of a figure on the board. The teacher asked some students to measure the length of the figure using rulers that had different starting points. The results of students' measure were summarized by the teacher in the following table:

Student	**Aira**	**Salma**	**Rangga**	**Rakka**	**Dea**
Start	0	1	2	3	3
End	15 and 4 (2 iterations)	20	21	22	22
Length	19 (Directly look at the last number)	20 (Directly look at the last number)	21 (Directly look at the last number)	20 (count the stripes)	37 (count both short and long stripes)

Table 5.4. Various measures from various rulers

Dea had a unique measure (very long compared to the others') because she counted both the centimeter stripes and the five millimeter stripes. As well as Rakka, the appearance of numbers on the ruler did not seem to be meaningful to Dea because she still counted the number of spaces. Moreover, Dea seemed to be confused by different kinds of stripes on the ruler, therefore she counted both kinds of stripes.

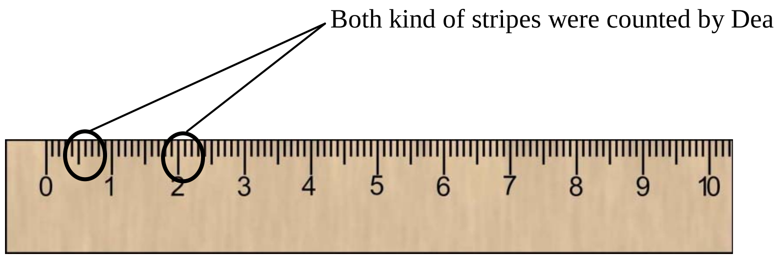


Figure 5.35. Different kinds of stripes on ruler that were counted by Dea

The various results of measurement for a single object became the main issue of the class discussion.

Teacher : ***How is it possible that we have different measures for a single object****? Which measure is the correct one?*

Elok : ***Rakka's measure is the right one because Rakka did counting***

Teacher : *Rakka, can you show to your friends how you measured by counting?*

Rakka : *I started counting from the stripe that numbered "3". I counted the numbered "3" stripe as one.*

Elok's argument - "*Rakka's measure is the right one because Rakka did counting*" - shows that for Elok a measurement was still a counting.

Teacher : *How about Rangga? You started measuring from the stripe that numbered "2" and ended at stripe that numbered "21", but how could you obtain 21 as your measure?*

Rangga : *I just looked at the last number (number that matched to the last edge of the stick)*

Rangga, Salma and Aira seemed to consider that a number on ruler indicate a measure, therefore they directly look at the numbers on ruler. However, they did not consider the starting point of their measurement and, therefore, they did not choose the correct number on ruler to indicate the correct ruler.

Teacher : *Let's we focus on the measures in the table. When we started from 0, we ended at 19. When we started from 1, we ended at 20. When we started from 2, we ended at 21. And when we* *started from 3, we ended at 22. But, how can these strategies give different measure?*

Teacher : Do you have any idea about those measures?

Students do not give any reaction to this question. They look puzzled

Teacher : Do you remember our activity measuring use normal ruler? ***Where did we start measuring?***

The teacher attempted to connect a broken ruler to a normal ruler to encourage students to realize that these rulers have different starting point. Furthermore, the question "*where did we start measuring*" was proposed by the teacher to encourage student to consider the starting point of their measurement.

Students : We started from "0"

Teacher : Now, look at Salma's measure. Salma started from "1" and ended at "20". Yesterday we started measuring from "0" so if we changed the "1" of Salma to "0", what will the "20" become?

As the next guidance, the teacher compared Salma's measuring process to the measuring process they did in previous day activity. By comparing those measurements, it was expected that students were encourage to realize that different starting point would give different "last number" on ruler. Consequently, the starting point of measurement played an important role in determining the measure.

D'Chia : The "20" will become "19" because we ***move backward*** *one step from "1" to "0" and, therefore, from "20" to "19"*

*Teacher : What do you mean with "***move backward***"?*

D'Chia : I subtracted "20" by "1".

Teacher : Now, how about Rangga's measure? If we change the "2" into "0", what will the "21" be?

Elok proposed the same idea as D'Chia's idea to solve the result of Rangga's measurement.

Teacher : The strategies of D'Chia and Elok are right. Is there any other idea?

Most students still looks puzzled; therefore teacher gives guide by drawing a ruler on the black board.

The teacher makes the representation of Salma's measure, namely by drawing a stick above the ruler. This stick is started from "1" and ended at "20". Then the teacher asks students to measure in similar strategy as what they used in measuring using blank ruler activity. Students begin count the number of spaces and teacher gives marks (i.e. arcs) above each space.

Figure 5.36. The representation of Salma's measure on the board

Students end counting at "19", then they say that the length of Salma's stick is "19". The next, teacher draws the representation of Aira's measure (figure of a stick that lays from "0" to "19").

The teacher makes drawings of all students' measure (except Dea's measure because at the end Dea's measure was similar to Raka's)

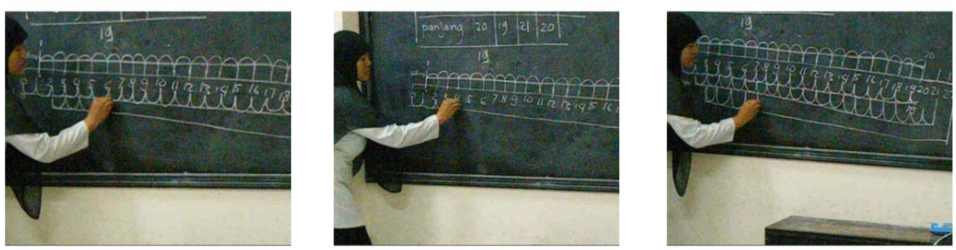


Figure 5.37. "Jumping" to determine the real length of stick

The new measurements give the similar new measure, namely 19.

Teacher : We have proven that the length of the stick is 19. It does not matter which ruler we used to measure. Does anyone of you have idea about it?

Students started thinking and some of them discussed with their partner. This took about 5 minutes until Haya proposed her idea.

*Haya : **The length of an object does not change** although the rulers that we use are different.*

Haya's statement, "*the length of an object does not change*", showed that she considered the concept of *conservation of length* to argue that they could measure the length of an object using any ruler.

Teacher : Yes, you are right. But, what should we do to measure using different rulers?

*Haya : **Subtracted by the number that we use as starting point.***

Teacher : What number that should be subtracted?

Haya : The result of measurement (the number that corresponds to the last edge of the measured object)

Teacher : Haya's opinion is correct. The measure of an object can be determined by subtracting the last number by the first number. Although we only have a broken ruler, we are still able to give the correct measure.

Haya consider the starting point of measurement determine the result of measurement. Furthermore, she seemed to understand that she needed to subtract the last number by the first number to get the correct measure.

General conclusion of the measuring using broken ruler activity:

At the beginning of this activity, most students did not consider the numbers on the ruler. Counting the spaces seemed to be more meaningful and reasonable for them [*see Elok's opinion at the beginning of the class discussion*].

Despite the acquisition of the concept of *zero point* of measurement shown by Haya and some students at the end of the class discussion, the result of the final assessment informed that there were merely about 52,38% of the students seemed to correctly measure the length of objects that were not aligned at number "0" on the ruler [*see appendix G on page 118-119 and appendix H on page 120*]. So, it is conjectured that the students still need to do more measuring practices using the broken ruler.

6. Conclusions and discussion

This chapter contains three main components, namely *conclusions* as answers to the research questions, a *discussion* to provide information about important issues in this research and *recommendations* for further educational research especially in linear measurement education. Those three components will be elaborated in the three following subchapters.

6.1. Conclusions

There are two research questions mentioned in the first chapter of this research, namely:

1. *How can students' game playing be used to elicit the issues and the basic concepts of linear measurement?*
2. *How can students progress from game playing to the more formal activities in learning linear measurement so that the mathematical concepts are connected to daily life reasoning?*

The first research question will be answered by summarizing the analysis of the first four activities of this research as described in subchapter 5.2.1 (from sub 5.2.1.1 to 5.2.1.4). The second research question will be answered by focusing on the "*making our own ruler*" activity.

The last part of the conclusions will be a local instruction theory for the teaching and learning of linear measurement in grade 2 of elementary school.

6.1.1. Answer to the first research question

The basic concepts of linear measurement that were expected to be elicited in game playing were the concept of *identical unit*, *unit iteration* and the need of *standard measuring unit*. Before going to the further discussion, it is important to differentiate between *identical unit* and *standard measuring unit*. An *identical unit* signifies that the unit being used **within a** measurement must be identical or constant in size. On the other hand, *a standard measuring unit* emphasizes on the need to use an identical unit, or a standard unit, for **parallel measurements** to give a **fair result**. In short, the *identical unit* focuses on a single measurement and the *standard measuring unit* aims at a fairness of many measurements.

Indonesian games that were used in this research (i.e. playing *gundu* and *benthik*) were rich with linear measurement activities including comparing length and measuring length. Consequently, it is interesting to answer the following question:

> *How can students' game playing be used to elicit the issues and the basic concepts of linear measurement*?

The summary of the result of students' game playing as elaborated in subchapter 5.2.1.1 to 5.2.1.4 will be used to answer this research question. The students' acquisition of basic concepts of linear measurement that were elicited in Indonesian traditional games was elaborated in four stages. These four stages are described as follows:

1. ***"A third object" as a benchmark for indirect comparison***

   In indirect comparison, "third objects" were used by students as benchmarks when comparing the separated distances in playing *gundu*. There was no student who stated that the distances were incomparable due to their impossibility to be put next to each other. At the beginning of the game, students used the most natural benchmark, namely their body parts including hand span and feet, to compare the distances.

   The flexibility of hand span led to the emergence of a preciseness conflict when there were students who bent their hand span when comparing the distances. Two students bent their hand spans when measuring the distance of their marbles to the circle, therefore they obtained 3 hand spans although in fact one distance was 2 and a half spans and the other distance was 2 and three quarters of spans.

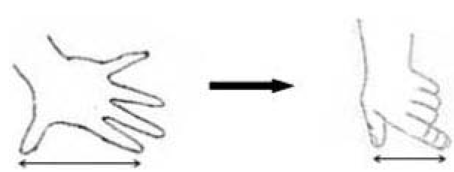


Figure 6.1. A non identical measuring unit

   The similar measures as a result of the bent hand spans directed students to come up with a more precise measure. Consequently, a chalk was used to substitute hand spans because the chalk was rigid and could not be bent. However, new conflict arose when the chalk gave similar measures of two distances.

Figure 6.2. The size of measuring unit determines the result of measurement

To solve this problem, students chose a marble as the new benchmark. It seemed that students commenced to understand the relation between the size of unit and the precision of measurement because they preferred to use a shorter object as the benchmark. Students also showed their understanding about the relation between the size of unit and the result of measurement when they played *benthik*.

As a conclusion, students perceived the idea that separated distances or objects were comparable. Moreover, students commenced to use third objects as benchmarks of indirect comparison. When Deva and Ivan obtained different measures for a single distance, students knew that this problem was caused by the different size of the feet of Deva and Ivan.

2. *A shift from indirect comparison to measurement*

Students frequently used the word "measure" when they explained their strategies to determine the shortest distances in the game [*look at the beginning of subchapter 5.2.1.2*]. This fact shows that students commenced to consider the measurement as a means for indirect comparison.

According to Barret in Stephen and Clement (2003), unitization and unit iteration are the most principle concepts of linear measurement. The idea of unitization obviously emerged in playing *benthik*. In playing *benthik*, the use of hand span and foot shifted from benchmarks for indirect comparison to units in measurement.

The long distances in playing *benthik*, as compared to the distances in playing *gundu*, directed students to iterate the measuring unit. Consequently, *unit iteration* as the second principle of linear measurement emerged in this activity.

3. ***Standard measuring unit***

Students' tendency to use their own measuring unit (either their body part or their tool) led to the emergence of a fairness conflict. Fairness is a natural conflict in young children's playing. For this reason, it was used to encourage students to elicit the issues and basic concepts of linear measurement.

The need to have a fair game in playing *benthik* encouraged students to standardize the measuring unit they use. During the class discussion, the teacher gave some guidance to stimulate students to come up with a standard measuring unit. The two stages of the process of unit standardization developed by students are described as follow:

a. A body part of a single person as the judge

An employment of a referee or judge is natural in game playing. Hence, the first stage of standardization emerged when students came up with an idea to hire a person as referee or judge to measure the distances in the game using his/her body part.

b. An independent measuring unit

As a result of guidance from teacher, students considered to reduce the dependency of measuring process on a single person. Consequently, they commenced to use an independent measuring unit that can be operated by any player. This independent measuring unit became a starting point for creating a measuring instrument.

Summary

Students' process in acquiring the basic concepts of linear measurements was started from *using a benchmark for indirect comparison* (either merely compared or iterated), which afterwards developed into *a unit for measuring*. The natural fairness conflict in the game playing encouraged students to came up with *a standardization of the measuring unit*.

The general scheme of students' process in eliciting the basic concepts of linear measurement from the game playing is shown in figure 6.3.

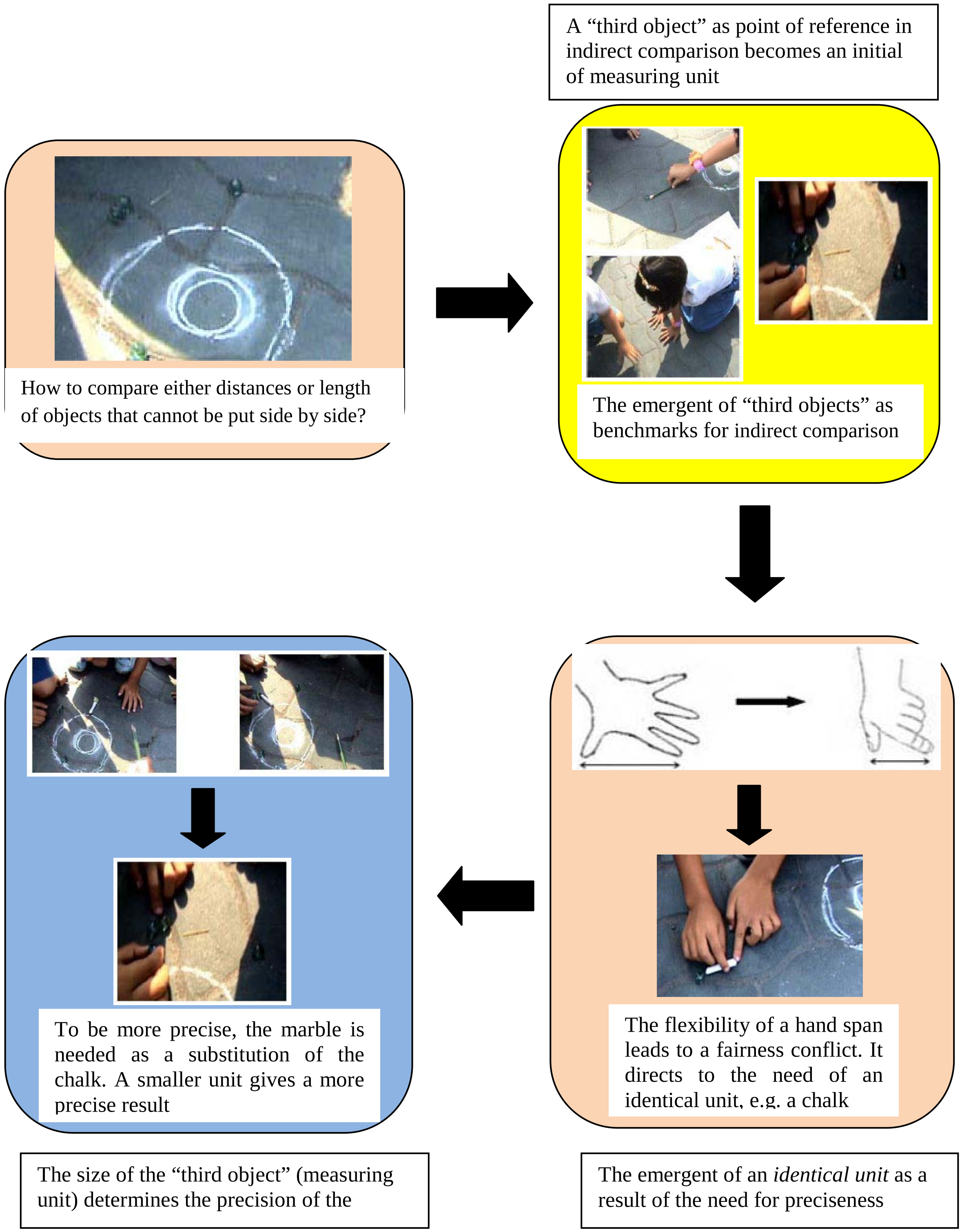


Figure 6.3. The scheme of students’ process in eliciting basic concepts of linear measurement in Indonesian traditional games

6.1.2. Answer to the second research question

The second research question focused on the shift of students' progress from game playing towards the more formal linear measurement using a ruler.

How can students progress from game playing to the more formal activities in learning linear measurement so that the mathematical concepts are connected to daily life reasoning?

The emergent modeling as described in subchapter 2.2.2 serves as the base for answering the second research question. The activities in this research that were aimed at the emergent modeling were "*measuring using strings of beads*", "*making our own ruler*" and "*measuring using ruler*" activities. Those activities and their role in supporting students' learning are described in the following way:

1. ***"Measuring using strings of beads": A shift from measuring unit to measuring instrument***

   The measuring units and their iteration that were used by students during the game playing were transformed into strings of beads as non-standard measuring instruments. This transformation aimed as an initial bridge to more formal linear measurement using a ruler.

   In this activity, students commenced to consider a measuring instrument as a set of iterated measuring units. Almost all students already used the beads in the strings as the unit, instead of the string itself. Only a few students showed an inconsistency in choosing the measuring unit as shown in figure 6.2.

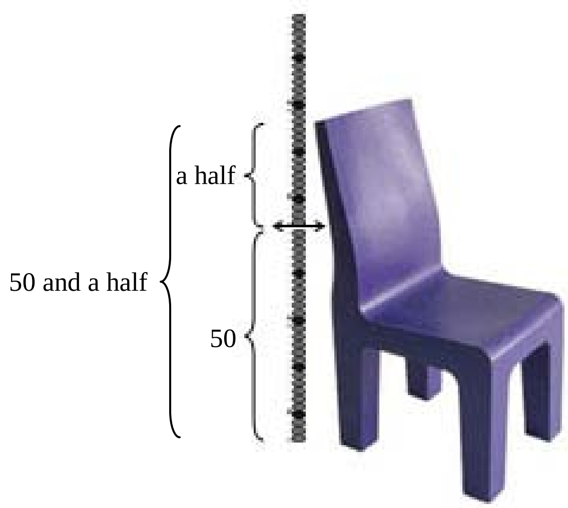


Figure 6.4. Is the string a measuring unit or measuring instrument?

Students who used a strategy as shown in figure 6.4 seemed to be confused about the difference between the string as a measuring unit and as a measuring instrument.

In general, students have accomplished the *situational level* of emergent modeling when they explained their interpretation and solution for determining the closest distance (in playing *gundu*) and the distances when playing *benthik*. Afterwards the accomplishment of the *referential level* was showed by the use of strings of beads as representations of iterated marbles. Moreover, the strings of beads became the base of the emergence of student-made measuring instruments as the *models-of* the situation that signifies the iteration of marbles. However, the units that were contained in the measuring instrument were still a "concrete" unit, namely a bead, which can be easily operated by students.

Students' progress from performing a measurement using a measuring unit to using a measuring instrument needs to be developed into more formal measurement using a ruler. Therefore, the next activity focused on the shift from using strings of beads towards using a ruler.

2. ***"Making our own ruler": An introduction to a standard measuring instrument***

The concrete characteristic (i.e. easy to be observed) of a bead as a unit of string of beads made it relatively easy to be considered and operated by students. When an object was covered by 10 beads, students were easily able to say that the length of the object was 10. However, the result of the pilot experiment shows that students have difficulty when performing a measurement with a ruler. It seemed easier to count the stripes on the ruler than to count the spaces, therefore most students considered the stripes as the units of a ruler. For this reason, the focus of the unit of a measuring instrument was transformed from "a bead" into "a space". This process was facilitated by "*making our own ruler*" activity. Furthermore, this activity also aimed to introduce a ruler as a standard measuring instrument to students.

The "*making our own ruler*" activity promoted the accomplishment of the next levels of emergent modeling. The numbers written on students' new measuring instruments showed how students commenced to consider that a number represented a measure. In this phase students started to use their instruments as *models- for* measuring the length of objects. The use of the student-made measuring instruments as the *models-for* measurement showed that *general level* of modeling has been attained by students.

The *formal level* started to be accomplished when some students drew a ruler as their new measuring instrument. This kind of instrument became the preliminary of the use of a ruler to accomplish the concept of *zero point* in measurement.

3. ***"Measuring using a ruler": How to correctly and meaningfully measure using a ruler***

   There were three kinds of rulers that were used in this activity, namely a blank ruler, a normal ruler and a broken ruler.

   Student-made measuring instruments were developed into a blank ruler that aimed to develop students' acquisition for concept *covering space*. The combination between strings of beads and the blank ruler encourage students to perceive that the beads in the strings were represented by the spaces on the ruler. Consequently, this eliminated students' tendency to count the number of stripes, instead of the number of spaces.

   The normal ruler was used to encourage students to perceive that a number on the ruler could represent a measure. At the beginning of this activity, many students still counted either the number of stripes or spaces. The appearance of numbers on the ruler seemed to be not meaningful to students for measurement. At the end of this activity, students started to consider the numbers on a ruler.

   Students' initial consideration of numbers on the ruler was developed to the understanding of the concept of the *zero point* in linear measurement that was facilitated by *measuring using a broken ruler* activity. Through this activity students were encouraged to perceive the concept that any number can serve as starting point for measurement.

Summary

The strings of beads and student-made measuring instruments played an important role in bridging the experience-based activities to formal linear measurement. The students' progress from eliciting the basic concepts of linear measurement in traditional games to acquiring more basic concepts in formal linear measurement using a ruler is summarized in the following scheme:

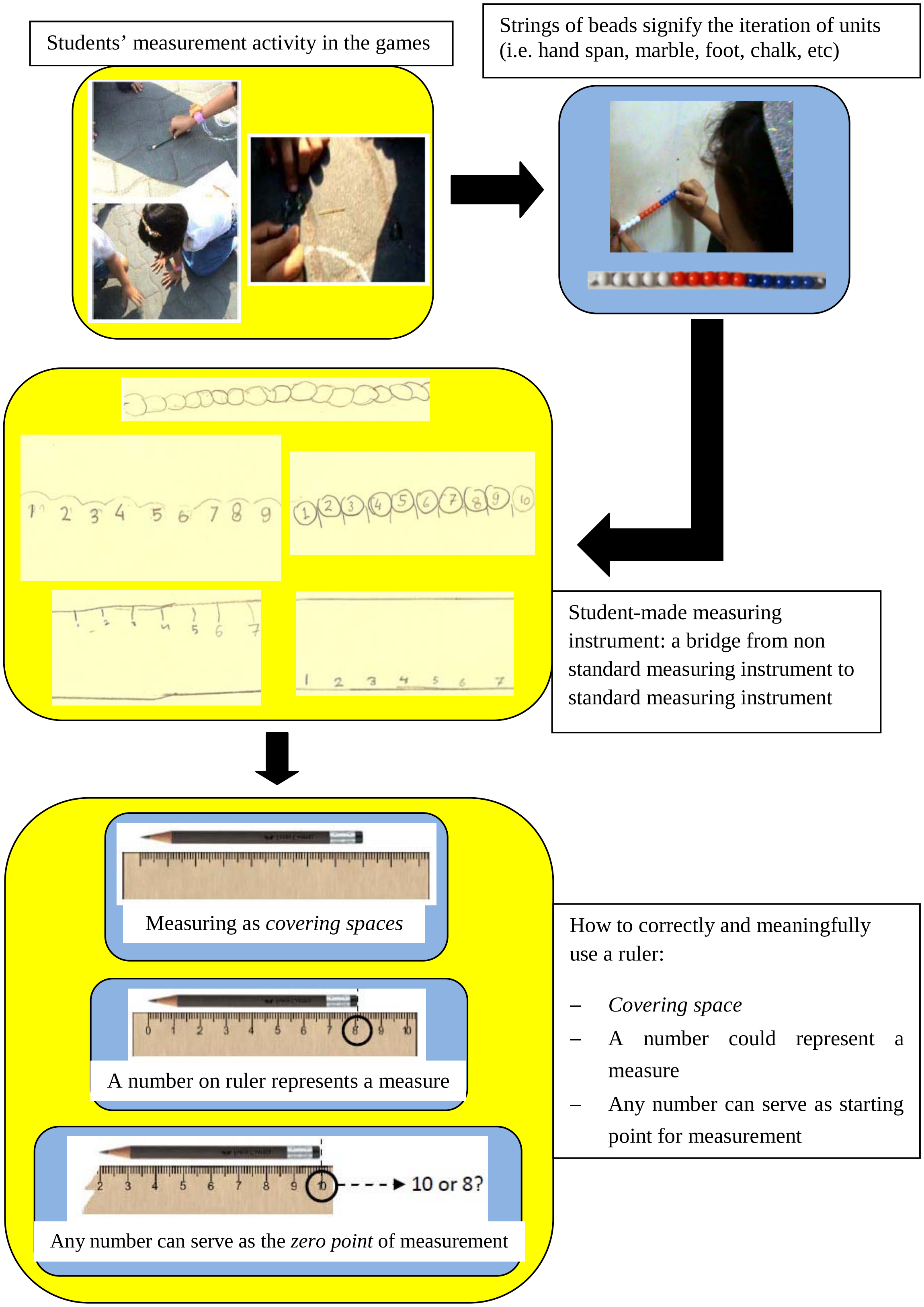


Figure 6.5. The scheme of students' progress in acquiring the basic concepts of linear measurement from experience-based activities to formal measurement

6.1.3. Local instruction theory for teaching and learning of linear measurement in grade 2 of elementary school

In chapter 3, it was mentioned that a design research is a systematic and flexible methodology aimed to design principles and theories for improving educational practice (Wang & Hannafin in Simonson; 2006). Consequently, this research aimed to contribute in formulating and developing a local instruction theory for teaching and learning of linear measurement in grade 2 of elementary school.

The local instruction theory with respect to the sequence of activities and the intended concept development for the teaching and learning of linear measurement was summarized in the table 6.1. This table shows the interaction between the development of the tools that were used and the acquisition of the mathematical concepts (Doorman, 2005 and Gravemeijer, 2003). The role of the teacher is essential in this interactive process; therefore it is thoroughly discussed in subchapter 6.2.2.

<table>
<tr><th>Activity</th><th>Tool</th><th>Imagery</th><th>Practice</th><th>Concept</th></tr>
<tr><td rowspan="2">Indonesian traditional games: playing gundu</td><td rowspan="2">Hand span, feet, marble</td><td rowspan="2"></td><td>Indirect comparison</td><td>Conservation of length
Emergence of a benchmark for indirect comparison</td></tr>
<tr><td colspan="2">The activity of iterating the benchmarks of comparison should become the focus to the introduction of measurement.</td></tr>
<tr><td rowspan="3">Indonesian traditional games: playing benthik</td><td rowspan="3">Hand span, feet, stick</td><td rowspan="3">Signifies that the “third object” in comparison becomes the measuring unit in measurement</td><td colspan="2">Playing benthik provides an opportunity to develop the use of “a third object” as benchmarks for indirect comparison, which becomes measuring unit</td></tr>
<tr><td>Measuring as the development of indirect comparison</td><td>Identical unit and unit iteration</td></tr>
<tr><td colspan="2">The fairness conflict in the game could lead to the need for a standard measuring unit</td></tr>
</table>

<table>
<tr><th>Activity</th><th>Tool</th><th>Imagery</th><th>Practice</th><th>Concept</th></tr>
<tr><td rowspan="2">Measuring using strings of beads</td><td rowspan="2">Strings of beads</td><td rowspan="2">Signifies the iteration of measuring unit, such as hand span, feet and marbles</td><td>Measuring and reasoning about activity of iterating and counting a measuring unit.
A beginning of using a standard measuring unit.</td><td>Standard measuring unit for the fairness and precision of measurement</td></tr>
<tr><td colspan="2">The use of strings of beads should shift the focus of learning process from measuring units to measuring instruments</td></tr>
<tr><td rowspan="3">Make our own ruler</td><td rowspan="3">Student-made measuring instrument</td><td rowspan="3">Signifies the need of standard measuring instrument derived from the strings of beads</td><td colspan="2">The use of a measuring instrument (as the successor of a measuring unit) should be started from students’ own construction (the third tenet of RME).</td></tr>
<tr><td>Measuring and reasoning about the need of standard measuring unit</td><td>Identical unit and measuring as covering and counting the spaces</td></tr>
<tr><td colspan="2">Student-made ruler should be used as the base for an introduction of ready-used ruler. The circles (as representations of beads) on the student-made ruler should be diminished and developed into “spaces” of a ruler</td></tr>
<tr><td rowspan="2">Measuring using blank ruler</td><td rowspan="2">Blank ruler</td><td rowspan="2">Signifies the need of a standard measuring instrument derived from the strings of beads</td><td>Reasoning about the need of a standard measuring instrument and measuring as covering space</td><td>Identical unit and measuring as covering and counting the spaces</td></tr>
<tr><td colspan="2">The need for a more efficient way to measure long distances should lead to the emergence of numbers on ruler</td></tr>
</table>

Activity	Tool	Imagery	Practice	Concept
Measuring using normal ruler	Normal ruler	Signifies the need of numbers on a blank ruler to make measuring easier and more efficient	Measuring long objects to stimulate students to consider the appearance and use of numbers on a ruler	Measuring as covering spaces and realizing that a number on a ruler represents a measure.
			The normal ruler has zero at the starting point for measurement. Therefore, to investigate students' understanding about the *zero point* of measurement, a new conflict needs to be given to students.	
Measuring using broken ruler	Broken ruler	Signifies the possibility to use a random starting point of normal ruler	Measuring the length of an object that was not aligned with the first stripe on the ruler	Any number can serve as *zero point* of measurement

Tabel 6.1. Local instruction theory for linear measurement in grade 2

6.2. Discussion

The implementation of RME in this design research reflects from how the tenets of RME underlay the activities in this research. This implementation will be elaborated on in the following chapters: *traditional games as experience-based activities for learning linear measurement*, *class discussion: teacher's role and students' social interaction and emergent modeling*.

6.2.1. Indonesian traditional games as experience-based activities for learning linear measurement

The first tenet of RME is *the phenomenological exploration* as the base and preliminary of the sequence of instructional activities. As the first instructional activity, a situation that is experientially real for student is used as the base for mathematical activity. Considering the experiential characteristics and the richness of linear measurement concept in some Indonesian traditional games, Indonesian traditional games were chosen as the contextual situation of the instructional activities in this research.

In education, games can be applied as a powerful support for traditional methods to reach the objectives of learning process. The use of games for teaching could give important implications for understanding how informal and formal learning can support and accelerate students' learning process and also increase students' motivation in learning (de Freites & Oliver, 2006 and Pietarinen, 2003). According to Pietarinen (2003), games provide an interpretational device and also could guide students towards a better understanding of the concepts and cognitive reasoning.

However, using games in mathematics education needs to be supported by a class discussion as a reflective session. In the reflective session, students' concrete experiences from game playing were shared and focused and transformed into initial concepts of linear measurement. Considering the importance of a class discussion as the reflective session, teachers should be able to organize the class discussion to reach the objectives of students' learning processes.

6.2.2. Class discussion: Teacher's role and students' social interaction

Interactivity as the fourth tenet of RME emphasizes on students' social interaction to support the individual's learning process. The learning process of students is not merely an individual process, but it is also a social process, and these both perform simultaneously (Cooke & Buchholz, 2005; Lave & Wenger in Lopez & Allal, 2007; Stephen & Cobb, 2003 and Zack & Graves, 2002). The learning process of students can be shortened when students communicate their works and thoughts in the social interaction both in the game playing and in the class discussion.

Game playing provides a natural situation for social interaction, such as students' agreement in deciding a strategy for the fairness of their games. However, game playing needs to be supported by a class discussion to develop students' concrete experiences into mathematical concepts. In the class discussion, the teacher plays an important role in orchestrating social interaction to reach the objectives both for individual and social learning (Cooke & Buchholz, 2005 and Doorman & Gravemeijer, in press).

The roles of teacher in the class discussion will be elaborated in the five manners described in this chapter:

1. **Providing students an opportunity to present their idea**

   According to the third tenet of RME, it is important to start the class discussion by using *students' own construction*, such as students' strategies. The teacher, as the orchestrator of the class discussion, should stimulate students to present their ideas as the starting point of the class discussion (Cooke & Buchholz, 2005 and Sherin, 2002).

   The following are examples of questions that were used by the teacher to stimulate students to express their ideas:

   – *"When playing gundu, how did you determine the nearest marble?"*

   – *"What did you use to measure the distances?"*

2. **Stimulating social interaction**

   According to Vigotsky in Zack & Graves (2001), social interaction is the core of learning process because learners first construct knowledge in their interactions with people and activity context. Therefore, a teacher should be a good orchestrator in provoking students' social interaction. The teacher could provoke social interaction by either making groups of students or asking some questions. Generating micro discussions in a macro discussion in the class discussion can be the first step to stimulate the students to share and discuss their strategies. It was observed from the class discussion when some students who were passive in the class discussion were able to communicate and discuss their ideas within their group.

   The second strategy for stimulating social interaction was posing appropriate questions (Cooke & Buchholz, 2005). During the class discussion, it was observed that the teacher occasionally posed the following questions to stimulate students' social interaction.

   – *"Any other idea?"*

     This kind of question could serve both as a way of providing opportunity for student's self expression and also as a way for stimulating social interaction among students.

   – *"Do you agree?"*

     It was natural in a class (that was also observed from this research) that not all participants are really involved in the discussion. Therefore, this kind of question can stimulate students to pay attention to the others' idea and argument.

– *"Can you show to your friend … ?"* and *"Can you draw your strategie?"* These questions aimed to encourage students to communicate their idea.

3. **Connecting activities**

   In supporting students' learning, it is important for the teacher to help children communicate and develop their ideas by elaborating upon what they already know. An example of this manner was when the teacher encouraged students to perceive the concept of *measuring as covering space*. The teacher connected the *blank ruler activity* to *measuring using string of beads* activity by posing the following questions:

   "Do you remember when we measured the distances using strings of beads? Where did we put our finger when we counted "1"?"

   By connecting the strings of beads to the blank ruler, the teacher tried to emphasize that a space on a blank ruler was a representation of a bead on a string of beads. Consequently, students should count the number of spaces, instead of the number of stripes, when they measure using a blank ruler.

4. **Eliciting the mathematical concepts**

   The most important objective of a class discussion is transforming students' concrete experiences into mathematical concepts as mentioned by Cooke & Buchholz (2005) and Kolb in de Freites & Oliver (2006).

   An example of transforming a concrete experience into a mathematical concept was observed in the class discussion when the teacher use the fairness conflict to stimulate students to come up with an idea of a standard measuring unit. The teacher frequently used the word "*fair*" to stimulate students to think about a standard measuring unit.

   "Is it fair for our game?"

   When this question merely resulted in a single person as a measurer, instead of using a tool [not a person] as a standard unit, the teacher posed the following question to give more guidance:

   "If in a game there is no-one who is willing to measure the distances using his/her own hand span [to be a referee or a judge], *what should we do to obtain a fair result?"*

5. **Asking for clarification**

Asking for students' clarification is important for the learning process because it can investigate students' reasoning about their idea or strategies that could reveal both students' difficulty and students' achievement in their learning process.

The following vignette is an example of a critical part in a student's learning process that was revealed through asking clarification.

Dea : No. We should start from number "1"
Teacher : Why do we start from number "1"?
Dea : Because zero is nothing

The teacher's question is a kind of question for asking clarification and reasoning based on Dea's idea. From Dea's response, it seemed that Dea was still confused between measuring and counting object (cardinality). From this invention, the teacher learned which part of the learning process should be developed.

Another advantage of asking clarification is when students' reasoning gives information about the strength of particular methods or strategies that could support students' learning process. The following vignette shows how the word "*fair*" becomes really important in supporting students' learning process.

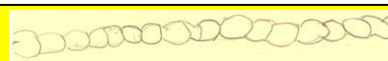
Teacher : So, can we use this strategy (i.e. using different steps) to measure the distances in our game?
Students : No it is not because it is not ***fair***
Teacher : What should we do?
Haya : In a game, we will have a ***fair*** *game if there is only one person who measures the distances …*

As a summary, by asking clarification we can know how a **weakness** of some students' progress could be diminished by providing a proper **guidance**. How the **strength** of particular methods or strategies offers an **opportunity** to develop students' learning process also can be found by asking clarification.

6.2.3. Emergent modeling

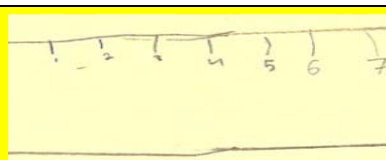
As mentioned in subchapter 2.2.2, the emergent modeling design heuristic could support students' progress from a concrete situation to a formal reasoning. Consequently, the second tenet of RME, *using models and symbols for progressive mathematization*, focuses on how a model and a symbol can be used as a bridge from the concrete level to the more formal level. The "*making our own ruler*" activity was drawn on to bridge from measuring activities in games - as the concrete level - to the more formal level of measurement, namely using a ruler in measurement.

Students' strategies in the games that were discussed in the class discussion showed *how students' own construction can be used* to support students' acquisition of the basic concepts of linear measurement. Furthermore, a new student-made measuring instrument as another students' own construction served as the bases of the emergence of the blank ruler as the preliminary of the normal ruler.



Students' measuring instrument as the representation of a string of beads became a *model of* the situation that signifies the iteration of marbles.

The number on students' measuring instrument signified the act of counting the beads. Therefore it became a *model for* measuring



Students' measuring instrument was developed into more formal ruler in which the beads were represented by spaces

Figure 6.7. The emergent modeling in the experience-based activities for linear measurement

6.3. Recommendations

The general recommendation in this chapter concerns with the local instruction theories for teaching and learning of linear measurement for grade 2 of elementary school. Therefore, this recommendation is addressed to both the practice of teaching and learning linear measurement (as the implementation of this research) and to further research in mathematics education for developing and improving mathematics education practices.

This recommendation is split into two focuses, namely classroom organization as the didactical component and intertwinement of mathematics topics as the mathematical component.

6.3.1. Classroom organization

The first recommendation in the didactical component is addressed to the practice of teaching and learning in linear measurement that is based on experience-based activities.

The class size should be well-considered when designing experience-based activities because it is difficult to be effective and efficient if one teacher organizes many students in the game playing as outdoor activity. There are two possible solutions for this problem. The first possible solution is hiring a teacher assistant to conduct the game playing. The second solution is conducting the game playing in two sessions. The first session is played by half of the students; meanwhile the other half is doing written tasks. On the second session, the role of students turns.

Another classroom organization that needs to be well-considered is the class discussion. The finding of this research, that only a few students were active in the class discussion, underlies the need to give tasks and practices in between the game playing and the class discussion. This task could be a micro discussion in small groups of students. The micro discussion is supposed to engage more students to actively discuss the concepts that are elicited by the games.

The teacher who was involved in this research (*her name is Budiyati*) is an experienced teacher. She has also been involved in *Pendidikan Matematika Realistik Indonesia* (Indonesian project for RME) since 2002. Therefore, she did an excellent job in conducting the experience-based activities, especially the way she conducted the class discussion. Related to this fact, the next question is *"what would this research be if the teacher were not an experienced teacher in RME?"*

Considering this point, it will be very important to do a research which also focuses on the teacher's role in students' learning process in an experience-based instructional activity. Another crucial point is how a teacher should give more attention to social interaction to develop both individual and social learning process.

6.3.2. Intertwinement of mathematics topics

In addition to the didactical component, the next recommendation focuses on the mathematics content. Considering the last tenet of RME, *intertwinement*, *s*ome activities used in this research could be developed to reach other mathematical concepts by intertwining with other mathematics topics.

Intertwine linear measurement with number operations

Another mathematics topic that is taught in grade 2 is addition and subtraction up to 500. Linear measurement is very close to addition, namely when students sum up all measures in the iteration of ruler (when measuring long distances). Therefore, measuring long distances using a ruler of 100 cm long can be used to intertwine linear measurement with addition up to 500.

Intertwine linear measurement with fractions

The bent hand span (see figure 6.1) shows that the result of measurement is not always an integer number. It is naturally and frequently encountered in the games that the measuring unit exceeded the measured object or distance. There were some students' reactions to this problem that were encountered in this research, namely:

- Students bent their hand span to match the measuring unit with the measured object and, therefore, they obtained an integer number as the measures.
- Students rounded the number either up or down. For instance, students said 6 as the measure when they obtained 5 and half of sticks.
- Students used the word "and a little" to indicate that the measuring unit exceed the measured lengths or distances
- Students already used "a half" and "a quarter" although their measures were not exactly "a half" or "a quarter" of the measuring unit.

Considering this finding, the suggestion for the next research is intertwining the linear measurement topic with the early fractions concept. This intertwinement can be done in the game playing when students measure the distances using a stick or hand span.

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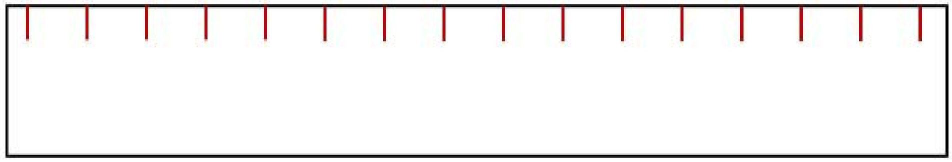
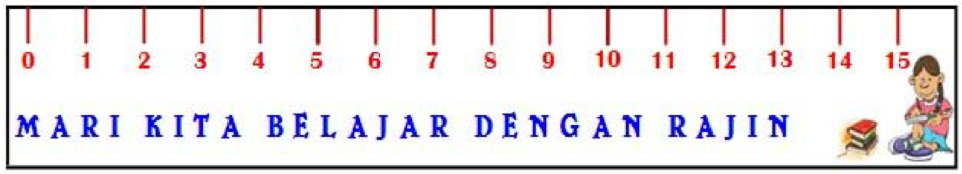
The following are the rulers that were used in this research (not in the real size):

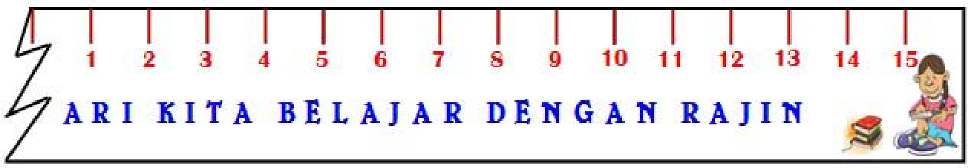
Ruler number 1:



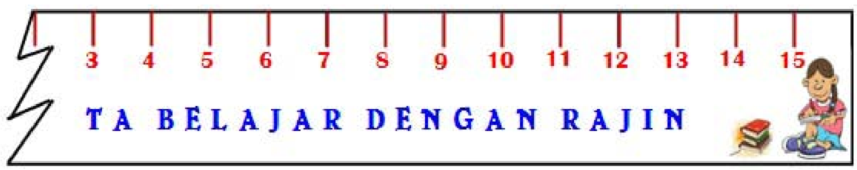
Ruler number 2:



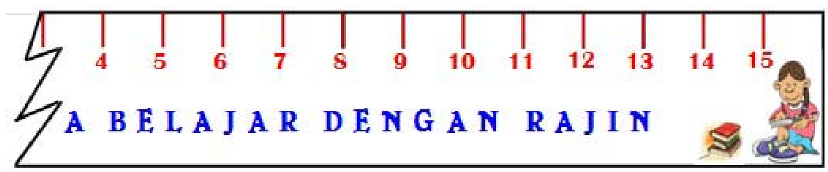
Ruler number 3:



Ruler number 4:

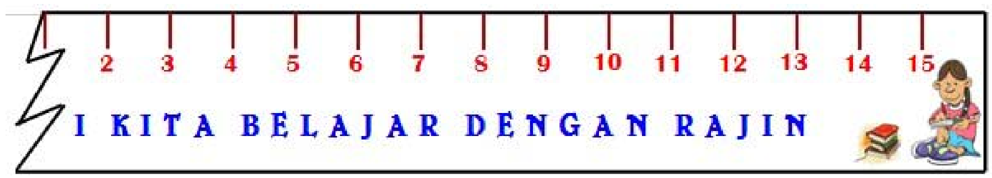


Ruler number 5:

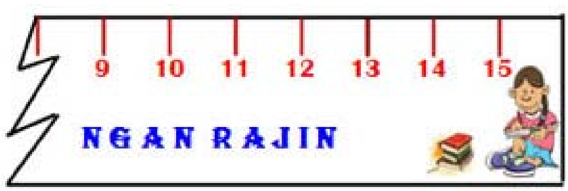


Ruler number 6:

Ruler number 7:



Ruler number 8:



Appendix B: Worksheet for measuring using blank ruler activity

NAME :

CLASS:

Measure the objects using the given ruler:

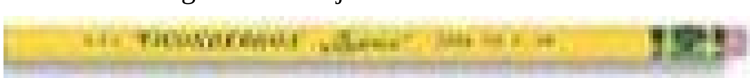
1. Measure the length of this object:

The length of the frame is ...

2. Measure the length of this object:

The length of the stick is ...

3. Measure the length of this object:



The length of Budi's pencil is ...

4. Measure the length of this object:

The length of Dini's pencil is ...

5. Measure the length of this object:
The length of this object is ...

Appendix C: Worksheet for measuring using normal ruler activity

Last Sunday Simpsons family went to the zoo in the city. There were many animals live at the zoo; such as Tiger from Sumatera - Indonesia, Penguin from the North pole, Giant bat from Africa, *Elang Brondol* (*Brondol* Eagle) from Bali – Indonesia, Squirrel from Thailand and even Deer that was born in a nursery in Wonosari – Indonesia.

The father of Bart Simpsons was astonished with the long tail of Sumatera Tiger that was jumping and welcoming Simpsons family. Now, can you help Bart’s father to measure the length of Sumatera Tiger’s tail?

Now, let’s look at the Penguin besides the Tiger. Can you decide which is longer between tiger’s tail and the height of the penguin? Can you also measure the height of the Penguin?

Ooopppss … I just realized that Bart Simpsons is quite tall right now. He is almost as high as his father’s shoulder. Isn’t he? Would you help Bart’s mother to compare the height of Bart and the height of his father?

After you are helping the father and mother of Bart, can you help Bart? He’s wondering how high the height of banana tree and coconut tree are. Can you help him?

Name :

Class :

Measure the length of these objects using ruler number 1:

1. The length of Sumatera Tiger's tail is .... (3 cm)
2. The height of the Penguin is ……… (4 cm)
3. The height of Bart Simpson's father is ……. (5 cm)
4. The height of Bart Simpson is ……. (3 cm)
5. The height of the banana tree is…… (3 cm)
6. The height of coconut tree is…. (7 cm)

Measure the length of these objects using ruler number 2:

7. The length of Sumatera Tiger's tail is ....
8. The height of the Penguin is ………
9. The height of Bart Simpson's father is …….
10. The height of Bart Simpson is …….
11. The height of the banana tree is……
12. The height of coconut tree is….

Appendix D: Worksheet for measuring using broken ruler activity

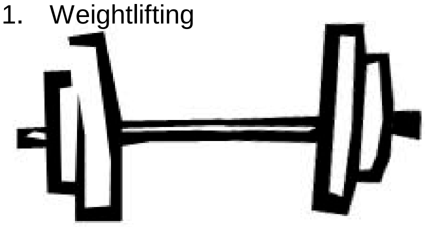
NAME :

Opening ceremony of Beijing Olympic Games was celebrated on 8 August 2008. Olympic Games is a world sport event party that is held once in 4 year. There are many branches of sport in the olympiad, such as:

- Weightlifting
- Basket ball
- Gymnastic
- Fencing
- Bicycle racing
- Running (sprint, middle distance, etc)

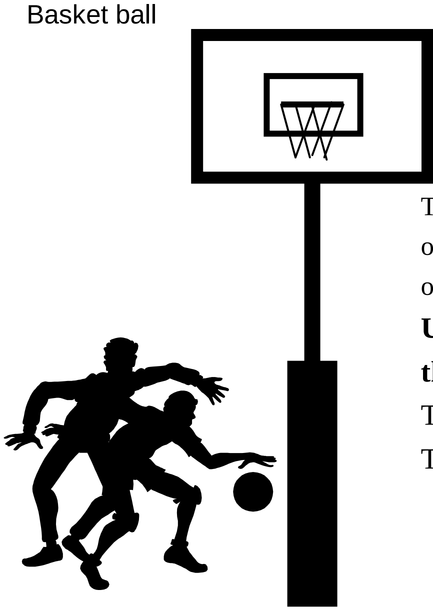
Let's look at the following figures related to Olympic games:

1. Weightlifting



The equipment that is used in weightlifting is called barbell.
Now look at the figure of barbell. Who knows the length of the "stick" of the barbell? Can you help coach to figure out how long it is?
Use the ruler number 4 to measure the length of barbell's "stick" .
The length of the barbell's stick is …..

2. Basket ball

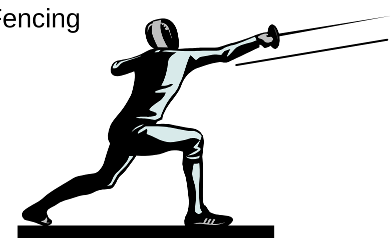


The object of the game is to outscore one's opponents by throwing the ball through the opponents' basket from above while preventing the opponents from doing so on their own. An attempt to score in this way is called a shot.
Use the ruler number 5 to measure the length of the basket board and also the height of the pole.
The length of the basket board is …..
The height of the pole …

3. Gymnastic

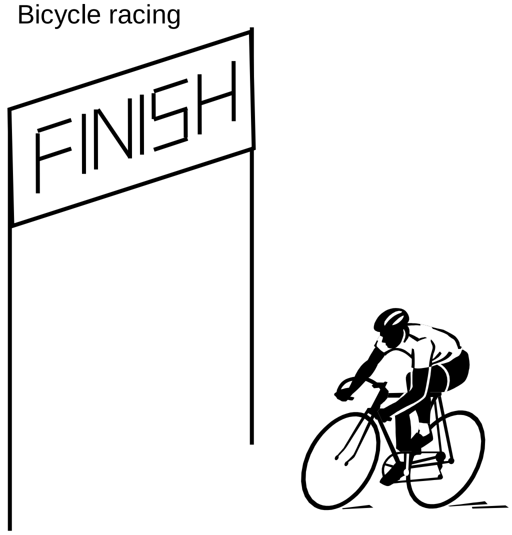
Can you measure the length of the right and left arm of that athlete?
Use the ruler number 6 to do it.
The length of the right arm is …..
The length of the left arm is …

4. Fencing



Let's we measure the length of the saber or sword on the figure.
How if we use the ruler number 7?
The length of the saber is …..

5. Bicycle racing

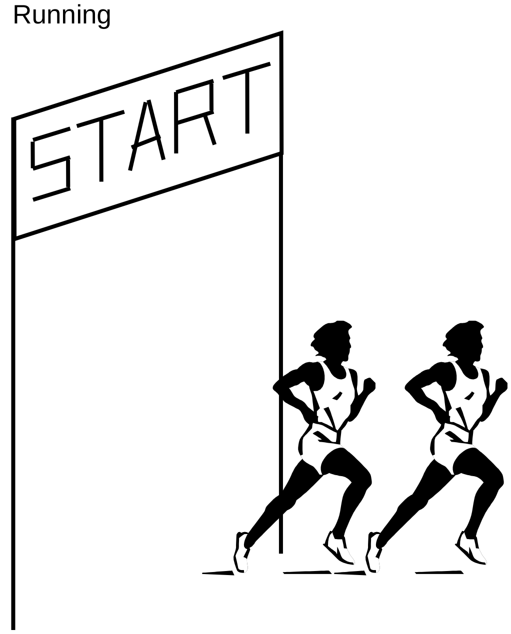


Look, that racer almost reaches the finish line.

Can you help me to figure out the height of the pole in the finish flag if we use the ruler number 8?

The height of the pole is …..

6. Running



Wow, the pole of start flag is quite tall. Hmmm … how long is it we measure it use the ruler number 6?

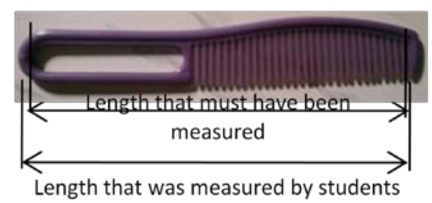
The height of the pole of start flag is …..

7. All the athletes that are participated in Olympic Games always take the official Olympic bus to go back and forth from their dormitory to the stadium.

   Now, let's measure the length of the bus if we use the ruler number 7.

   The length of the bus is ….

Appendix E: Analysis of students' answers for the worksheet in measuring using blank ruler activity (9 August 2008)

Number	Students	Questions					Percent age	Analysis
		1	2	3	4	5		
	ANSWER	**10**	**12**	**10**	**8**	**15**		
1	Icha and Rastra	11	12	10	8	14	60%	It seems that these students seemed to misunderstand the last questi on. 
2	Salma and Viana	10	13 cm	11 cm	9 cm	15 cm	40%	Question number 2,3 and 4 seemed to be solved by counting the number of stripes, but the first and the last question seemed to be solved by counting the number of spaces.
3	Obi and Shafa	10 cm	12 cm	10 cm	8 cm	14 cm	80%	It seems that these students seemed to misunderstand the last question. 
4	Alvin and Hikari	11	12	10	8	14 and a little	60%	Question number 2, 3 and 4 seemed to be solved by counting the spaces, the first question seemed to be solved by counting the stripes and “the little” for the last answer reflects that students did not put the ruler in a correct way.
5	Mutia and Vivi	11 rulers	13 rulers	11 rulers	9 rulers	13 rulers	0 %	These students seemed to count the number of stripes (except for the last question) and they call "rulers" as "units"
6	Hamzah and Jihan	11 and a half	13 and a half	11 and a half	9 and a half	15 and a half	0 %	The “half” in students’ answers shows that students seemed to put the edge of the measured objects at the edge of the ruler, instead of at the first stripe. These results also showed that the students measured the number of stripes.
7	Deva and Elok	11 stripes	13 stripes	11 stripes	9 stripes	15 stripes	20%	Students’ measure and also the word “stripes” that used by these students as the measuring unit reflect that they seemed to count the stripes. Only the last question that seemed to be answered by counting the spaces.

8	Fira and Hanan	10	13	10	8	15	80%	Fira and Hanan seemed to count the number of the spaces when measuring the objects, except for the second question.
9	Laras	10	13 and a half	11	9	15	40%	These results showed that students seemed to be inconsistent when measuring (between counting the number of stripes and the number of spaces) b question number 1 and 5 seemed to be solved by counting the stripes, but the other questions seemed to be solved by counting the stripes.
10	Aira and Puput	10	11	10	7	13	40%	As well as Laras; Aira and Puput seemed to be inconsistent when measuring (between counting the number of stripes and the number of spaces)
11	Rakka and Dea	11	13	11	9	15	20%	Rakka and Dea seemed to count the number of the stripes when measuring the objects, except for the last question.
12	Naima and D'Chia	10	13	10	$8\ ^3/_4$	15	60%	Only the second question that seemed to be solved by counting the stripes
13	Caesar and Vinta	10	12	10	8	15	100%	Caesar and Vinta seemed to count the spaces when measuring the objects
14	Nisa and Agil	36	12	10	8	15	80%	Nisa and Agil seemed to misunderstand the first question. They measured the perimeter of the figure of blackboard, instead of measured its length. However, actually they did the right measurement because they counted the number of spaces.
15	Tsamarah and Dina	10	13	11	8	15	60%	Question number 1, 4 and 5 seemed to be solved by counting the spaces, but the other questions seemed to be answered by counting the stripes.
16	Fahmi and Tifani	11 cm	13	11	9	15	20%	These students seemed to count the stripes (except for the last question
17	Yaya and Uam	10 and a half	12 and a half	10 and a half	8 and a half	14 and a half	0 %	The “half” in students’ answer shows that these students seemed to put the edge of the measured objects at the edge of the ruler, instead of at the first stripe. However, actually they did the right measurement because they counted the number of spaces, except for the last question.
18	Rama and Dinda	10	12 and a half	11	8 and a half	15	40%	These results showed that students seemed to be inconsistent when measuring (between counting the number of stripes and spaces)
19	Azka and Salya	10	13	11	9	15	40%	Question number 1and 5 seemed to be solved by counting the spaces, but the other questions seemed to be answered by counting the stripes.
20	Caca and Sekar	10	12	10	8 and a little	14	60%	Caca and Sekar seemed to count the spaces, except for the last question.
21	Rangga and Gina	11	13	11	9	15	20%	These students seem to count the number of stripes (except for the last question)
		52.38%	28.57%	42.86%	33.33%	61.9%	**43.81%**	**More than a half of students did not seem to correctly measure. Students' answers also showed students' inconsistency in measuring length. It seemed that for some problems they seemed to count the number of stripes, but for some other questions they seemed to count the number of spaces.**

Appendix F: Analysis of students' answers for the worksheet in measuring using normal ruler (11 August 2008)

Number	Students	Questions 1 / 7 (Answer 3 / 3)	2 / 8 (Answer 4 / 4)	3 / 9 (Answer 5 / 5)	4 / 10 (Answer 3 / 3)	5 / 11 (Answer 3 / 3)	6 / 12 (Answer 7 / 7)	Percentage	Analysis
1	Haya and Ivan	3 metres	4 metres	6 metres	4 metres	3 metres	6 metres	50%	Haya and Ivan seemed to count the stripes to answer question number 3 and 4. These students already used "meter" as their measuring unit.
		3 metres	3 and a third	5 metres	2 metres	3 metres	7 metres	67%	The numbers on the ruler did not seem to be so meaningful to Haya and Ivan because they merely increased one more correct answer.
2	Salma and Viana	4 cm	5 cm	6 cm	4 cm	4 cm	8 cm	0 %	The results show that Salma and Viana seemed to count the number of stripes. However, they already used "cm" as measuring unit.
		4 cm	5 cm	6 cm	4 cm	4 cm	8 cm	0 %	The results show that students seemed to count the number of stripes and it seems that numbers on the ruler were not meaningful for these students. However, they already used "cm" as measuring unit.. However, these results indicate these students' consistency when measuring used both blank ruler and normal ruler because they had same answers for both the blank ruler and the normal ruler.
3	Obi and Shafa	3 cm	4 cm	5 cm	3 cm	3 cm	7 cm	100%	Obi and Shafa seemed to count the number of spaces and they commenced to use "cm" as unit.
		3 cm	4 cm	5 cm	3 cm	3 cm	7 cm	100%	The same answers for both parts of the worksheet indicate students' consistency when measuring used both the blank ruler and the normal ruler. However, it is not obvious whether they already used the numbers on the ruler or not.
4	Alvin and Hikari	3	4	5	3	3	7	100%	Alvin and Hikari seemed to count the number of spaces
		3	4	5	3	3	6	83%	In general, these students seemed to be consistent when measuring used both blank ruler and normal ruler because they had same answers for both the blank ruler and the normal ruler. However, it is not obvious whether they already used the numbers on the ruler or not.
5	Mutia and	3	4	5	4	3	7	83%	In general, these students seemed to count the number of spaces

	Vivi	3	4	6	4	3	7	67%	In general, these students seemed to be consistent when measuring used both blank ruler and normal ruler because they had same answers for both the blank ruler and the normal ruler.
6	Hamzah and Jihan	3	5	6	4	4	7	33%	The results show that students counted the number of stripes.
		4	5	6	4	4	7	17%	The results show that students till counted the number of stripes. However, these results indicate students' consistency when measuring used both blank ruler and normal ruler. It seems that numbers on the ruler were not meaningful for students. However, they already used "cm" as measuring unit.
7	Deva and Elok	3	4	5	3	3	7	100%	These students seemed to count the number of spaces.
		3	4	5	3	3	7	100%	These students seemed to be consistent when measuring used both blank ruler and normal ruler because they had same answers for both the blank ruler and the normal ruler. However, it is not obvious whether they already used the numbers on the ruler or not.
8	Fira and Hanan	4	5	6	4	4	8	0 %	The results show that Fira and Hasan seemed to count the number of stripes because their answers "1" bigger than the correct answers
		3	4	5	3	4	8	67%	It is interesting because Fira and Hanan were not able to give any correct answer for the blank ruler activity, but they were able to give the correct answer for 4 out of 6 questions in the normal ruler activity. A conjecture that is derived from this fact is that these students seemed to read the numbers on the ruler.
9	Laras	4	5	6	4	4	7	17%	Laras seemed to count the number of stripes because her answers were "1" bigger than the correct answers (except for the last question).
		4	5	6	4	4	7	17 %	Laras seemed to count the stripes because her answers were "1" bigger than the correct answers (except for the last question). However, these results indicate Laras' consistency when measuring used both blank ruler and normal ruler because they had same answers for both the blank ruler and the normal ruler.
10	Aira and Puput	3	3	5	2	3	7	67%	For 4 out of 6 questions, Aira and Puput seemed to count the number of spaces.
		4	5	6	4	4	7	17 %	The numbers on the ruler did not seem to be meaningful for these students because their result in the normal ruler activity was worse than that of the blank ruler activity. The results show that students counted the number of stripes. Students were inconsistent in measuring use different measuring instruments (i.e. blank ruler and normal ruler).
11	Rakka and Dea	4	5	6	4	3	7	33%	The results show that students counted the number of stripes. For question number 4, Rakka and Dea seemed to count the number of stripes because their answer was "4" and the correct answer was "3".
		4	5	6	4	3	7	33%	The results show that students counted the number of stripes. However, these students were little bit consistent in measuring use different measuring instruments (i.e. blank ruler and normal ruler).
12	Naima and D'Chia	4	5	6	4	3	7	33%	For 5 out of 6 questions, Naima and D'Chia seemed to count the number of spaces because their answers were "1" bigger than the correct answers.
		3	4	5	3	4	6	67%	The increasing numbers of correct answers shows that Naima and D'Chia seemed to start considering the

									appearance of numbers on the ruler.
13	Caesar and Vinta	4	5	4	3	3	5	33%	Only the first question that seemed to be answered by counting the number of stripes. For the other questions, it is possible that Caesar and Vinta did not measure what that should be measured (note: the figures that should be measured were not "good" figures)
		4	5	5	3	3	5	50%	As well as Naima and D'Chia, Caesar and Vinta seemed to start considering the appearance of numbers on the ruler. It can be seen from the increasing number of the correct answers.
14	Nisa and Agil	4	5	6	4	3	7	33%	For 5 out of 6 questions, Nisa and Agil seemed to count the number of spaces because their answers were "1" bigger than the correct answers.
		4	5	6	4	4	7	17%	The numbers on the ruler did not seem to be meaningful for these students because their result in the normal ruler activity was worse than that of the blank ruler activity.
15	Tsamarah and Dina	4	5	6	3	4	7	33%	For 5 out of 6 questions, Tsamarah and Dina seemed to count the number of spaces because their answers were "1" bigger than the correct answers.
		4	5	5	4	4	8	17%	The numbers on the ruler did not seem to be meaningful for these students because their result in the normal ruler activity was worse than that of the blank ruler activity.
16	Fahmi and Tifani	3	4	5	3	3	6	83%	For 5 out of 6 questions, Fahmi and Tifani seemed to count the number of spaces
		3	4	5	3	3	6	83%	These results indicate these students' consistency when measuring used both blank ruler and normal ruler because they had same answers for both the blank ruler and the normal ruler.
17	Yaya and Uam	4	4	6	4	4	8	0 %	The results show that students seemed to count the number of stripes.
		3	4	6	3	3	7	83%	It is an interesting finding because Yaya and Uam were not able to give any correct answer for the blank ruler activity, but they were able to give the correct answer for 5 out of 6 questions in the normal ruler activity. A conjecture that is derived from this fact is that these students seemed to read the numbers on the ruler.
	Rama and Dinda	4	5	6	3	4	8	17%	Rama and Dinda seemed to count the number of stripes because their answers were "1" bigger than the correct answers (except for the last question).
		4	5	6	3	4	8	17%	Rama and Dinda still seemed to count the number of stripes on the ruler.
19	Azka and Salya	4	5	6	4	3	6	17%	Azka and Salya seemed to count the number of stripes because their answers were "1" bigger than the correct answers (except for the last question).
		4	5	6	4	3	7	33%	These results indicate these students' consistency when measuring used both blank ruler and normal ruler because they had same answers for both the blank ruler and the normal ruler.
20	Icha and Rastra	4	4	5	3	4	6	17%	Icha and Rastra seemed to count the number of stripes because their answers were "1" bigger than the correct answers (except for the last question).

		4	4	5	3	4	8	33%	In general, these students seemed to be consistent when measuring used both blank ruler and normal ruler because they had same answers for both the blank ruler and the normal ruler.
21	Gilang	3	4	6	4	4	8	33%	For 4 out of 6 questions, Gilang seemed to count the number of stripes because their answers were "1" bigger than the correct answers.
		4	5	6	4	4	7	17 %	The decreasing number of the correct answers reflected that the numbers on the ruler did not seem to be meaningful for Gilang
22	Caca and Sekar	4	4	5	4	4	7	50%	All the wrong answers of Caca and Sekar were "1" bigger than the correct answer, therefore it seemed that Caca and Sekar count the number of stripes for these questions.
		4	4	5	4	3	7	67%	In general, these students seemed to be consistent when measuring used both blank ruler and normal ruler because they had same answers for both the blank ruler and the normal ruler.
23	Rangga and Gina	3	4	5	3	3	7	100%	Rangga and Gilang seemed to count the number of spaces.
		3	4	5	3	3	7	100%	These results indicate students' consistency when measuring used both blank ruler and normal ruler. However, it is not obvious whether they already used the numbers on the ruler or not.
Blank Ruler		43.48 %	47.83 %	39.13 %	39.13 %	56.52 %	56.52 %	**47.1 %**	**The level of achievement in operating blank ruler compare to that of in operating normal ruler merely increase 3.63%. It means that the numbers on the ruler did not seem to be so meaningful to the students.**
Normal Ruler		43.48 %	47.83 %	52.17 %	47.83 %	52.17 %	60.87 %	**50.73 %**	**In general, a half of students seemed to be able to correctly measure. However, it is difficult from this table to investigate whether students still counted the number of stripes or started to realize/read the numbers on the ruler for the normal ruler activity. Consequently, discussions with students both within the activities and in the class discussion aimed to investigate students' strategy.**
General analysis for the set of questions in the worksheets									**The figures on the worksheet were not in a "good" shape, because the edge of the figures were not obvious (e.g. the height of Bart Simpson and the Penguin were difficult to be precisely measured). Therefore, some students seemed to have difficulty in measuring the length of the figures.**

Appendix G: Analysis of students' answers for the worksheet in measuring using broken ruler activity (on 13 August 2008)

Number	Students	Questions									Percentage	Analysis
		1	2a	2b	3a	3b	4	5	6	7		
ANSWER/LENGTH		**5**	**3**	**5**	**1**	**1**	**2**	**5**	**6**	**6**		
RULER No.		**4**	**5**	**5**	**6**	**6**	**7**	**8**	**6**	**7**		
START OF RULER		**2**	**3**	**3**	**4**	**4**	**1**	**8**	**4**	**1**		
1	Haya and Dinda	5	3	5	1	1	2	**5**	**6**	6	100%	These students seemed to correctly measure the length of the figures
2	Yaya and Vivi	7	6	8	5	5	7	14	10	11	0%	These students seemed to read the number on the ruler because their answers were the addition of each length and each start of ruler
3	Vinta and Elok	5	3	5	1	1	2	5	6	6	100%	These students seemed to correctly measure the length of the figures
4	Fira and Shafa	7	5	5	1	1	2	5	6	6	78%	For the first question, these students seemed to read the number on the ruler. However, for question number 3a to 7 these students seemed to correctly perform the measurement
5	Aira and Naima	5	6	8	5	5	7	13	9	11	11%	Except for question number 1,4 and 7, these students seemed to read the number on the ruler because their answers were the addition of each length and each start of ruler
6	Puput and Viana	5	5	8	6	5	7	14	10	11	11%	These students seemed to read the numbers on the ruler for question number 2b, 3b, 5 and 6.
7	Mutia and Jihan	3	3	8	1	1	2	5	6	5	67%	These students seemed to read the numbers on the ruler for question number 2b.
8	Salma and Gina	5	6	8	5	5	7	14	10	11	11%	These students seemed to read the numbers on the ruler for question number 2a, 2b, 3a, 3b, 5 and 6.
9	Tifani and Agil	3	3	6	1	1	3	4	4	6	44%	These students seemed to count the number of stripes for

												question number 2b and 4.
10	Tsamarah and Sekar	6	3	6	2	2	3	4	7	6	22%	These students seemed to count the number of stripes for question number 1, 2b, 3a, 3b, 4 and 6.
11	Ivan and Uam	3	3	6	2	2	2	2	6	7	22%	These students seemed to count the number of stripes for question number 2b, 3a and 3b.
12	Laras and Obi	5	3	6	2	2	2	4	5	5	22%	These students seemed to count the number of stripes for question number 2b, 3a and 3b.
13	Icha	5	3	5	1	1	2	5	6	6	100%	These students seemed to correctly measure the length of the figures
14	Dina and D'Chia	5	3	5	1	1	2	6	7	6	78%	These students seemed to count the number of stripes for question number 5 and 6.
15	Rangga and Alvin	5	3	5	1	1	2	5	6	6	100%	These students seemed to correctly measure the length of the figures
16	Gilang	5	3	5	1	1	2	5	6	6	100%	These students seemed to correctly measure the length of the figures
17	Hamzah and Rama	5	3	8	4	4	6	14	8	11	22%	These students seemed to read the numbers on the ruler for question number 2b and 5.
18	Rastra and Hanan	5	3	5	1	1	2	3	6	6	89%	It is difficult to figure out why these students answer "3" for the fifth question.
19	Deva and Rakka	5	3	5	1	1	2	6	7	6	78%	These students seemed to count the number of stripes for question number 5 and 6.
20	Caesar	3	12	6	2	2	3	6	7	7	0%	These students seemed to count the number of stripes for question number 3a, 3b, 5 and 7.
21	Salya and Fahmi	5	3	8	5	5	7	14	10	11	11%	These students seemed to read the numbers on the ruler for question number 2b, 3a, 3b, 5 and 6.
Percentage		**66.67%**	**71.43%**	**42.86%**	**52.83%**	**52.38%**	**57.14%**	**33.33%**	**42.86%**	**52.38%**	**52.38%**	**Only a half of students seemed to be able to measure the length of an object although the object was not aligned to "0"**

Appendix H: Analysis of students' answers for the worksheet in measuring using broken ruler activity

Question number	Analysis
1	There were 14 students who gave correct answer. Two students seemed to merely read the number on the ruler because they gave "7" as their answer. The broken ruler was started from "2" and the length of the figure was 5, therefore "7" was the number that matched to the edge of the figure. A student seemed to count the number of stripes because these students gave "6" as the answer. It is possible that four students who answer "3" did not correctly measure the object. They seemed to measure the stick between the weights, instead of the whole stick.
2a	There were 15 students who gave correct answer. Three students seemed to merely read the number on the ruler because they gave "6" as their answer. The broken ruler was started from "3" and the length of the figure was 3, therefore "6" was the number that matched to the edge of the figure. It is difficult to figure out why there some students who gave "5" and "12" as their answer.
2b	There were nine students who gave correct answer. Seven students seemed to merely read the number on the ruler because they gave "8" as their answer. The broken ruler was started from "3" and the length of the figure was 5, therefore "8" was the number that matched to the edge of the figure. Five students seemed to count the number of stripes because this students gave "6" as the answer
3a	There were 11 students who gave correct answer. Four students seemed to merely read the number on the ruler because they gave "4" as their answer. The broken ruler was started from "4" and the length of the figure was 1, therefore "5" was the number that matched to the edge of the figure. Four students seemed to count the number of stripes because these students gave "2" as the answer.
3b	There were 11 students who gave correct answer. Five students seemed to merely read the number on the ruler because they gave "4" as their answer. The broken ruler was started from "4" and the length of the figure was 1, therefore "5" was the number that matched to the edge of the figure. Four students seemed to count the number of stripes because this students gave "2" as the answer.
4	There were 12 students who gave correct answer. Three students seemed to either merely read the number on the ruler or counted the number of the stripes because they gave "3" as their answer. The broken ruler was started from "1" and the length of the figure was 2, therefore "3" was the number that matched to the edge of the figure. It is difficult figure out why some students answer "7" and "8" as the length of the figure.
5	There were seven students who gave correct answer. Six students seemed to merely read the number on the ruler because they gave either "13" or "14" as their answer. The broken ruler was started from "8" and the length of the figure was 5, therefore "13" was the number that matched to the edge of the figure. The students who answered "14" could also merely read the number but they put the ruler in wrong position. Four students seemed to count the number of stripes because they answer "6" these students answer "6". It is possible that students who answered either "3" or "4" measured the wrong figure, namely the width of the flag instead of the height of the pole.
6	There were nine students who gave correct answer. Four students seemed to merely read the number on the ruler because they gave "10" as their answer. The broken ruler was started from "4" therefore "10" was the number that matched to the edge of the figure. The students who answered "14" could also merely read the number but they put the ruler in wrong position. Four students seemed to count the number of stripes because they answer "7"
7	There were 11 students who gave correct answer. Two students seemed to either read the number on the ruler or counted the number of stripes because they gave "7" as their answer. The broken ruler was started from "1" and the length of the figure was 6, therefore "7" was the number that matched to the edge of the figure. It is difficult to figure out why some students gave answer "11"