Design Research as a Means for Building a Knowledge Base for Teachers and Teaching in Mathematics Education

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Abstract

Building on the claim that teachers need to know how innovative instructional approaches work to be able to adapt them to their own classrooms, design research is presented as a research method that aims to offer exactly that kind of information. We elaborate design research that aims to develop a local instruction theory—a theory about the process by which students learn a given topic in mathematics and theories about the means of support for that learning process. We will illustrate this with the example of design research on a local instruction theory on addition and subtraction to 100. We further discuss design research that combines the 2 goals of teacher learning and student learning in 1 project as a special case of research on teacher learning. In closing, we briefly look into the relation between design research and teacher research.

Educational research may have various goals and various audiences. School administrators, for instance, will be interested in objective knowledge that will enable them to make educational policy decisions, such as whether or not to adopt innovative curricula. In response to this need, educational researchers have developed various methods to determine which interventions are the most effective on average. Though such information may be valuable for administrators, teachers need a different kind of information that is less general. As the Research Advisory Committee of the National Council of Teachers of Mathematics (Cobb, 1996) argues, comparative research offers teachers little basis for knowing how to adapt curricula—which are proven to be effective—to their own situation. Teachers are better served with well-substantiated

theories about *how* these curricula (or other innovative recommendations) work. They may take those theories as conjectures, which they can test and modify in their own classroom. In this manner, the role of the teacher changes. Instead of being passive consumers of knowledge produced by others, teachers can make their own contributions to further developing such theories.

In this article, we discuss a type of research that is meant to provide theories about how an innovation works: design research (Gravemeijer & Cobb, 2006). We focus on the type of design research that is tailored to developing subject- and topic-specific instruction theories in mathematics education. The central aim of this kind of research project is to develop a theory about the learning process by which students come to grips with a given mathematical topic, complemented with theories about how to support such a learning process. Those means of support primarily concern instructional activities that may foster the mental activities, and shifts in student thinking, that constitute the learning process. The research, however, may also concern more general characteristics of the learning situation that are presumed essential or the conditions presumed to foster certain learning processes.

We note that this type of design research carries many similarities with what teachers do when they plan, enact, and evaluate their lessons. Design research may therefore also be taken as a point of reference for teacher research. We will have to take into account, however, that teachers have less time and resources than professional researchers. We therefore espouse a combination of researcher research on local instruction theories, and teacher research on the elaboration and adaptation of such theories in various instructional settings. Ideally, there will be a close collaboration in which an exchange of ideas and findings makes the research more practical and the teaching more scientific.

We close this general introduction with the observation that the use of this design research method is not limited to student learning. It may, of course, also be applied to research on teacher learning (of which we will present an example later) and teacher education (see also Dolk, den Hertog, & Gravemeijer, 2002; Simon, 2000).

This article is organized as follows: We start by elaborating design research as a method for developing local instruction theories in an iterative process of designing and improving instruction, while elucidating the need for understanding as a central element of this type of research. We complement this with an example of a local instruction theory. Next we address the fact that both the teacher and the students are learning in a design experiment. We then present an example of dual design research that has both teacher learning and student learning as its goal. We finish by noting that teachers can engage in this work by carrying out similar activities as design researchers. In fact, we start by elaborating the latter in order to situate design research.

Hypothetical Learning Trajectories and Local Instruction Theories

According to current views on mathematics education, the task of the teacher is to help students build on their own thinking while constructing more sophisticated mathematics. In this respect, one speaks of a transition from *instruction* to *construction*. What makes the latter difficult is that the teacher can only indirectly influence what the students construct. So, the question is how to reconcile the constructivist stance that students construct their own knowledge with the obligation of formal education to strive for pre-given educational goals. Simon's (1995) solution to this problem is that teachers are to choose instructional activities on the basis of how they expect students to act and reason. Teachers must try to anticipate what mental activities their students will

engage in when they participate in the envisioned instructional activities, while at the same time considering how those mental activities relate to the intended end goals. Simon (1995, p. 135) calls this a "hypothetical learning trajectory." The envisioned learning trajectory is hypothetical in that the actual learning trajectory may differ. The teacher, therefore, has to investigate whether the students' actual learning corresponds with what was conjectured. This will lead to new insights by the teacher regarding the students' thinking. These new insights will form the basis for a modified hypothetical learning trajectory for subsequent instructional activities. The alternation of anticipation, enactment, evaluation, and revision creates a cyclic, iterative process that Simon (1995) calls a mathematical teaching cycle. This cyclic, iterative process shows a strong resemblance to the core activity of design research, in which teaching experiments are not seen as tests of preconceived designs, but function as learning situations for researchers.

Design research is especially relevant when the focus is on long-term learning processes that aim at more encompassing educational goals. This is a complicated task that surpasses what may be expected of teachers. We argue, therefore, that teachers should be offered some form of support that enables them to design hypothetical learning trajectories on a daily basis. Such support can be offered by a local instruction theory—a theory about a possible learning process for a given topic along with the means of supporting that process. We call such a theory local to distinguish it from domain-specific and more general instruction theories. A domain-specific instruction theory limits itself to a certain domain, such as mathematics. An example of such a domain-specific theory is the theory for realistic mathematics education (RME; Gravemeijer, 1994, 2004), which plays a significant role in the examples of design research that we will discuss here. RME harks back to the ideas of the mathematician and mathematics educator Hans Freudenthal. Freudenthal (1973) argued that students should engage in "mathematics as a human activity" instead of being taught mathematics as a "ready-made product." According to Freudenthal, students should be given the opportunity to reinvent mathematics with the help of teachers and well-chosen tasks. This point of departure formed the basis for some decades of design research in the Netherlands and elsewhere, which resulted in what is now called RME theory.

In contrast to a domain-specific instruction theory, a local instruction theory is tailored to a specific topic, such as addition of fractions, multiplication of decimals, or data analysis. The idea is that a local instruction theory offers a framework of reference for a teacher who is construing a hypothetical learning trajectory for his or her classroom at a given moment in time. To support teachers further, a set of exemplary instructional activities and materials may be produced—as a spin-off of the design research project—that teachers may adopt and adapt. It is important to note that the objective is not to produce textbook-like, ready-made scripts. Instead, the idea is to offer teachers additional resources from which they can choose and to which they can make their own adaptations. Thus, although the teacher may be informed by theories and tasks developed by others, there is still room and a need for the teacher to construe his or her own hypothetical learning trajectories. After all, tasks will have to be adapted to the teacher's goals and teaching style, and to his or her classroom at a given moment in time. To make such adaptations, the teacher must construe a hypothetical learning trajectory, but, when doing so, a local instruction theory can be used as a framework of reference. As a caveat, we note that although local instruction theories tend to focus on learning processes, activities, and tasks, they do assume that the teacher will play a proactive role and establish the appropriate classroom social norms—to use Cobb and Yackel's (1996) terms for describ-

ing the respective expectations and obligations of teachers and students in a classroom.

Design Research

The general goal of design research is to investigate the possibilities for educational improvement by bringing about and studying new forms of learning. The main topic of this article will be design research that aims at developing a local instruction theory. However, design research may also have other goals. One of the objectives of design research may, for instance, be to investigate specific elements of instructional settings that may be essential for bringing about the intended learning. This may concern, for instance, the classroom culture, the teacher's role, the role of symbols, or the role of language in mathematics education. In this article, however, the main focus will be on local instruction theories.

When looking at design research as a research method, we may discern two salient characteristics: (1) the specific role of design and (2) the specific role of the experiment. As design research aims at educational improvement and new forms of learning, a well-considered instructional design is in order. Great care must be taken to ensure that the design experiment is based on prior research and well-founded design principles, or design theories. In this sense, design experiments have two faces. On the one hand, theories offer guidance for both the design and the research. On the other hand, those same theories are put to the test. The testing especially concerns the local instruction theory that underpins the series of instructional activities that constitute the teaching experiment in the classroom. That is why the instruction theory with which a design experiment starts out is called a conjectured local instruction theory. This theory is potentially revisable. In other words, the expectation is that the local instruction theory will be revised and improved in the course of the teaching experiment.

The term *teaching experiment* sometimes causes confusion because the word experiment is often associated with quasi-experimental research, which presupposes a control group along with an experimental group. In design research, the role of the experiment is different. The goal is to explore, probe, and investigate in an experimental educational setting, not to compare a pre-given form of experimental education with conventional education. Experimenting in design research refers to exploring, adapting, and refining the local instruction theory. Researchers use these activities, along with retrospective analysis, to learn about the innovative practice. The yield of design research is, in essence, what the researchers have learned in terms of empirically grounded theories about how the intervention works.

We may discern three phases in each design experiment: (1) the preparation for the experiment, (2) the teaching experiment, and (3) the retrospective analysis. In the first phase, some preliminary design will be carried out, but the actual instructional activities are worked out, revised, or redesigned in the course of the classroom teaching experiment. In relation to the role of the preliminary design, we may borrow Simon's (1995) metaphor of a "travel plan." The preliminary design may be seen as the travel plan, and the teaching experiment as the actual journey, which in practice will differ from the original plan.

The methodological point of departure is that the instructional activities that researchers develop embody conjectures about how to shape the intended innovative instruction. In preparing the instructional activities, the researchers conduct anticipatory thought experiments by envisioning how students might participate in the proposed activities when they are realized in the classroom. These thought experiments reflect the researchers' conjectured local instruction theory that is

tried, elaborated, and improved upon in the teaching experiment. Analyses of the students' participation and inferred learning will inform the revision of the activities and the design of follow-up activities. Because the goal of design research is to deepen, refine, adjust, and improve instruction theories, analyses of what is going on in the classroom and how this relates to initial conjectures and revision decisions will be instrumental in furthering those theories. Therefore, it is essential to carefully document both what is going on in the classroom, and the rationales for the conjectures and revisions. The teaching experiment is followed by a retrospective analysis, which involves a careful review of the data and a reflection on the process of the teaching experiment in order to develop an explanatory model of what induced the changes observed in the learning ecologies.

It is important to note that although we discern three phases, the boundaries between them are not fixed. How far instructional activities are designed in advance, during the preparation phase, or during the teaching experiment may vary; although a golden rule holds that the closer the design of instructional activities is tied to what is learned in the classroom, the better. As far as the boundary between the teaching experiment and the retrospective analysis is concerned, we may note that the data analysis starts during the teaching experiment. The retrospective analysis, however, creates an opportunity for a more thorough and systematic analysis of the same data—if the teaching experiment phase is well documented. In addition, retrospective analysis may also involve a whole new look at the data when new topics of interest emerge.

Preparing for an Experiment

Like any other research, design research is carried out within a certain theoretical framework. Such a framework will encompass broad background theories, such as cognitivism, socioconstructivism, or sociocultural theory, but also domain-specific instruction theories such as RME. This theoretical stance will have consequences for how potential instructional goals are framed, and even for what instructional goals are set. One of the first preparatory activities will therefore be to think through the potential instructional goals.[1] Another aspect of the preparation for the design experiment will be to specify assumptions about the cognitive starting points, which, in turn, will call for investigation into how the students who will participate in the teaching experiment actually think and reason. In addition to the cognitive starting points, the researchers will also need to specify assumptions about the kind of classroom social norms that may be required to facilitate the envisioned form of learning.

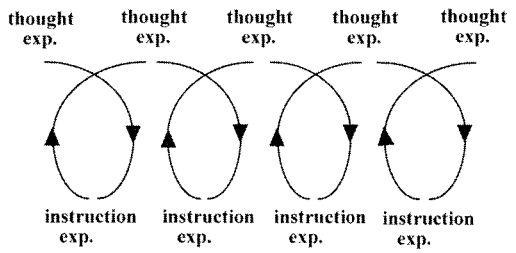

FIG. 1.—An iterative process of micro design cycles

Conducting a Teaching Experiment

During the teaching experiment, a series of instructional activities is designed, tested, and revised. The activities embody testable conjectures about both the students' mental activities and the specific means of supporting those mental activities. As conjectures are generated and perhaps refuted, new conjectures are developed and tested. The result is an iterative design process of daily micro design cycles that consist of anticipatory thought experiments, enactment of the instructional activities, observations and analyses of what is going on in the classroom, and evaluation of the conjectures that are embodied in the instructional activities (see Fig. 1).

We may compare those micro design

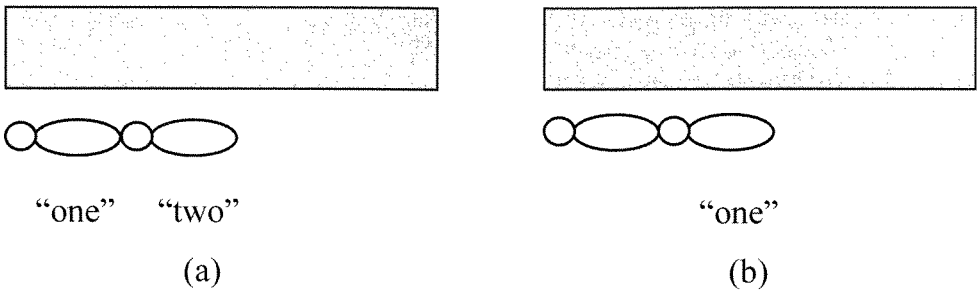

FIG. 2.—Two methods of counting as students paced the length of a carpet

cycles with the idea of testing and revising in the classical empirical cycle, although there are significant differences. While each empirical cycle starts afresh, subsequent micro design cycles take place in the same setting. The cumulative character of the micro design cycles complicates the analysis because what happens in a given cycle will be influenced by what happened in the preceding cycles, whether a given design cycle concerns a revised activity following an unsatisfactory activity, or a new activity building on what occurred in earlier activities. A second difference is that the analysis and evaluation of the instructional activities concern the mental activities of the students, not (just) observable behavior. The goal of the analysis in the micro design cycles is to understand how the students' inferred mental activities relate to their participation in the instructional activities.

In relation to this, it may be helpful to distinguish between two complementary ways of identifying causal relations, the regularity conception of causality that is connected to observed regularities, and a process-oriented conception of causal explanation "that sees causality as fundamentally referring to the actual causal mechanisms and processes that are involved in particular events and situations" (Maxwell, 2004, p. 4). Within the latter conception of causality, "causal explanation" refers to "the mechanisms through which and the conditions under which that causal relationship holds" (Shadish, Cook, & Campbell, 2002, cited in Maxwell, 2004, p. 4). Coming to grips with the mechanisms that explain how a given instructional activity works is exactly what design researchers aim to accomplish. In this sense, the micro cycles of thought and instruction experiments correspond to a process-oriented conception of causal explanation, while the classical empirical cycle corresponds with a regularity conception of causality. It is important to note that the process-oriented casual explanations we aim for here must encompass both the learning of individual students and the learning process of the classroom as a whole.

We can elucidate the two forms of causality with the help of a classroom episode in which students who measured the length of a carpet by pacing heel to toe appeared to measure the carpet in different ways (taken from Stephan, Bowers, & Cobb, with Gravemeijer, 2003). Some students counted "one" when they placed their first foot down in alignment with the beginning of the carpet, and "two" with the placement of their next foot (see Fig. 2a). Other students did not start counting when they put their first foot down, but started with "one" when they placed their other foot (see Fig. 2b).

It was assumed that for those students who counted paces in the second manner (Fig. 2b), the goal of pacing was to count the number of steps it took them to reach the end of the carpet rather than to establish a length. The teacher therefore orchestrated a whole-class discussion in which these two ways of counting paces were con-

trasted. At the end of a lengthy discussion, the first way of measuring was treated as legitimate by the classroom community, together with forms of argumentation that reflected the notion of measuring as covering space.

After the whole-class discussion, all students used the second way of counting. The latter observation could be taken as an indicator that a regularity-oriented causal relation exists between the whole-class discussion and the subsequent change in behavior of the students (although establishing a regularity-oriented causal relation would require a more rigorous proof of the presumed regularity). This regularity-oriented causal relation does not involve explanations of how this change is induced. We may contrast this with a process-oriented causal explanation, which would explicate the mechanism that causes the change by building on the above exposition and clarifying what makes students start thinking of measuring as covering space. Such an explanation would describe (1) how the nonconventional way of counting originates in a conception of measuring as counting steps, and (2) how a whole-class discussion of the two ways of counting stimulates students to start thinking of measuring as covering space.

In concluding this section on the teaching experiment, we return to the dual goal of understanding and improvement. The researchers' overriding concern is what they can do to effectuate the instructional intent. Moreover, adaptations are not only asked for when things do not turn out as expected. Adaptations are also in order when actual forms of reasoning that emerge in the classroom open up unforeseen potential learning routes.

The mini cycles of thought and instruction experiments serve the improvement of the local instruction theory. In fact, there is a reflexive relation between the thought and instruction experiments and the local instruction theory that is being developed. On the one hand, the conjectured local instruction theory guides the thought and instruction experiments, and, on the other hand, the micro cycles of design and analysis shape the local instruction theory (see Fig. 3). Thus, the instruction experiments are connected to the conjectured local instruction theory via the thought experiments, and to the revised local instruction theory via the revision decisions. Since the local instruction theory and its empirical foundation is the main object of the research, it is of crucial importance to document what thought experiments, what empirical data, and which theoretical considerations led to the design or revision of instructional activities. To enable a thorough retrospective analysis, these data have to be complemented with video recordings of all classroom lessons, observer logs, video-recorded individual interviews with students before and after the experiment, and copies of the students' written work.

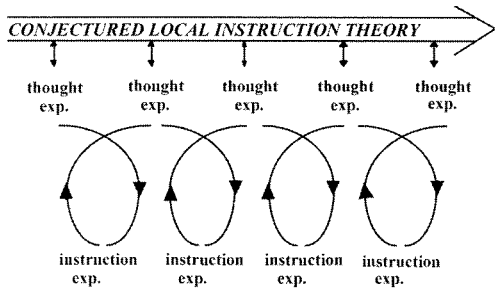

FIG. 3.—Reflexive relation between theory and experiments.

Retrospective Analysis

Since the goal of design research projects may vary, the content of the retrospective analysis may vary. Constructing a local instruction theory on some topic will almost always be one of the goals. The retrospective analysis then aims at the reconstruction of an improved local instruction theory on the basis of what is learned from the series of micro design cycles in the teaching experiment. In a sense, the data analysis actually starts during the teaching experi-

ment—although with a slightly different focus—and will be complemented with the retrospective analysis of the entire data set collected during the experiment.

When a specific aspect of the learning ecologies is investigated, interventions during the teaching experiment—and corresponding micro design cycles—may be less salient or even absent. In the latter case, the data analysis will consist solely of a retrospective analysis, for which an approach such as Glaser and Strauss's (1967) constant comparative method may be used. Central to this approach is an iterative process of looking for patterns in the data, framing assumed patterns as conjectures about the data, testing those conjectures on the complete data set, and using the findings as data for a subsequent round of analysis. Final claims and assertions can then be justified by backtracking through the various levels of the analysis. Finally, we should note that it is not uncommon to repeat a design experiment, either to explore insights gained during the experiment or because of external developments. In this respect, we may discern macro design cycles, which span entire experiments, and thus encompass a series of micro design cycles. From such macro design cycles a more robust local instructional theory may emerge.

Design Research on a Local Instruction Theory on Addition and Subtraction up to 100

We will illustrate the method of design research with the research on addition and subtraction up to 100 that was carried out at Vanderbilt University (Cobb, Gravemeijer, Yackel, McClain, & Whitenack, 1997; Gravemeijer, 2004; Stephan et al., 2003). We will first briefly sketch the local instruction theory that emerged.

The general idea is that measuring by iterating measurement units of 10 and one can give rise to the construction of a ruler, which can subsequently support arithmetical reasoning about incrementing, decrementing, and comparing measures. The instruction essentially evolves as follows: After a series of preparatory activities, the students start measuring with stacks of 10 Unifix cubes (small plastic blocks that one can click together). They first iterate units of 10, and then adjust by adding or subtracting ones. As part of this process of measuring with 10s and ones, students appropriate decuple-related number relations, such as $48 = 50 - 2$ and $48 + 10 = 58$. Next, a paper strip of 10 cubes that can be used for both iterating units of 10 and reading of individual units of one is introduced as a measurement device. With that, a basis is laid for the construction of a measurement strip that comprises 10 units of 10, each subdivided into 10 units of one cube.

The conjecture is that because of the students' learning history, measuring with the measurement strip is grounded in the imagery of measuring with units of 10 and one. Thus, for the students, measuring with the strip signifies iterating a unit of 10 cubes and a unit of one cube. Next, a shift is made from actually measuring items to reasoning about given lengths when solving tasks around incrementing, decrementing, and comparing lengths of objects that are not physically present. These tasks offer opportunities for developing solution methods based on curtailed counting—using the decimal structure as a framework of reference. The difference in length, for instance, between two planks that are 48 and 75 Unifix cubes long, respectively, may be found by using 50, 60, and 70 as benchmarks, while reasoning $48 + 2 = 50$, $50 + 10 + 10 = 70$, $70 + 5 = 75$, thus the difference is $2 + 20 + 5 = 27$. To record and discuss such solution methods, the empty number line is introduced as a means for symbolizing and scaffolding measurement-strip-specific arithmetical solution methods that are grounded in reasoning with 10s and ones (see Fig. 4). A jump on the number line describes a shift on the measurement strip that in turn can be seen as referring to iterating individual

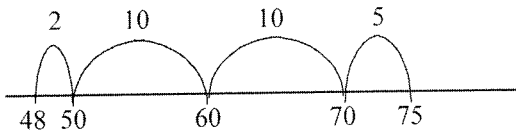

FIG. 4.—Jumps on the empty number line

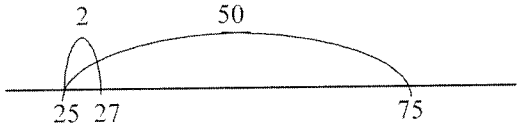

FIG. 5.—Illustrating 75 − 48 = 75 − 50 + 2

Unifix cubes or units of 10 cubes. It is important, however, that the students realize that their objective should not be to make a schematic drawing of a measuring device, but to make a drawing that shows their arithmetical reasoning. When the students have become more proficient with number relations, they may start to base their reasoning on those relations. For example, they may reason that 75 − 48 = 75 − 50 + 2 and use jumps on the number line to elucidate this (see Fig. 5).

Before we launch into a description of the actual research, we will first briefly identify the two components of a local instruction theory that we mentioned earlier—the theory on the learning process of the students and theories on means of support for that learning process. The learning process on flexible arithmetic up to 100 may be described broadly as developing a network of number relations up to 100 based on the decimal number structure and learning to use this network of number relations as a basis for derived-fact strategies for addition and subtraction. We will limit ourselves to this global description, as the theories on the means of support will add more detail.

Measuring with units of a different rank may be seen as one of the means of support because it creates a situation in which students may start to coordinate iterating units of 10 and one and, in the process, develop decuples-based number relations. Measuring may also be seen as a means of support in that cardinal and ordinal number aspects get integrated in the context of measuring; a measurement number such as 48 may both be seen as a quantity of 48 units (cardinal) and as a position on a measurement strip (ordinal). The benefit of this connection is that it enables one to solve problems on combining and splitting quantities by counting on or counting back in the number row. So, in addition to developing a network of number relations, students also become familiar with skip counting as a means for solving addition and subtraction problems.

The number line is then introduced as a means for scaffolding the process of skip counting by multiples of 10 and one. In this manner, the use of the empty number line can be seen as a means of support for fostering counting by jumps and for discussing and curtailing such arithmetical strategies. The number line also supports a shift from thinking about measures and measuring to thinking in terms of number relations. This, in turn, is expected to facilitate a generalization from magnitudes to numerical quantities in general.

The local instruction theory described above was developed in two design research projects (Cobb et al., 1997; Stephan et al., 2003) that built on the idea of using the empty number line as a means of support in learning how to add and subtract flexibly up to 100 (Treffers, 1991; Whitney, 1988). The original instructional activities incorporated the use of a string of 100 red and white beads, with alternating colors every 10 beads. In this setup, counting and skip counting beads on the bead string functioned as the foundational imagery for reasoning with the empty number line. A weakness, however, was the artificial character of the activity of counting beads on a bead string. However, when the bead string was skipped in the first teaching experiment, the importance of this kind of imagery showed (Cobb et al., 1997). Without the underlying imagery, the students developed different interpretations of the

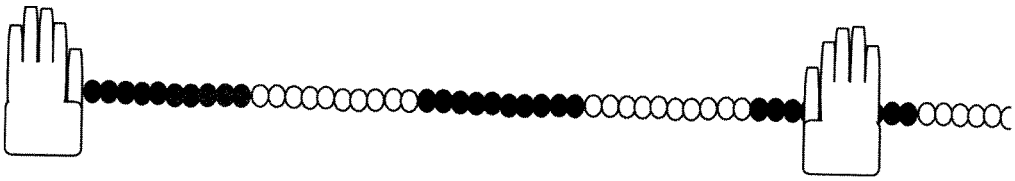

FIG. 6.—Showing a quantity of 43 beads

number line. For instance, some students interpreted a hash mark on the number line labeled "43" as indicating the forty-third instance of counting, whereas others interpreted it as the result of having counted 43 times. In the case of the bead string, this would be equivalent to deciding whether you are talking about the forty-third bead or about a quantity of 43 beads (see Fig. 6).

Without the imagery of counting on the bead string, the students could not resolve such differences. So, an alternative for the bead string had to be sought that did not share its lack of real-life motivation. Such an alternative was found in linear measurement. This formed the basis for a follow-up design experiment (see Stephan et al., 2003), which we will discuss below, starting with the *preparation for the number-line experiment.*

According to the preliminary plan for the second teaching experiment, measuring with Unifix cubes would be preceded by measuring by pacing heel to toe. This would create the need for a standardized measurement unit, which would be dealt with by introducing a scenario in which the king's foot functioned as the measurement unit. Measuring with Unifix cubes was to be introduced with a story about smurfs (little blue dwarfs) who measured with their food cans that happened to have the size of Unifix cubes. The idea of measuring with the smurf bar—consisting of 10 Unifix cubes—would be introduced by having the students discuss how to measure more efficiently. Measuring by 10s and ones would lay the basis for the introduction of a measurement strip that was made of a paper strip on which 10 units of 10 were iterated, each subdivided into 10 units of one cube. Measuring with the measurement strip would be followed with a shift toward using the measurement strip to reason about comparing, incrementing, and decrementing lengths of objects. To further foster a shift away from measuring to the use of arithmetical solution strategies, another tool, the measurement stick (which showed similarity with the bead string, see Fig. 7), would be introduced along with a scenario about varying heights of the water in a canal. Next, the empty number line would be introduced, and, as a final step, the students would be encouraged to reason with the empty number line when solving problems that involved contexts other than measuring.

During the *number-line teaching experiment,* more detailed instructional activities were worked out, tried, and revised. Changes concerned, for instance, an extension of the king's-foot scenario because the shift to the tasks in the smurf scenario proved too demanding. The students had to interpret a stack of 41 cubes, for instance, as the *results* of measuring, while they only had experience with *creating* the measure of an object in the activity of pacing. Therefore, activities were inserted that would

FIG. 7.—Measurement stick (in the context of water heights the stick is positioned vertically)

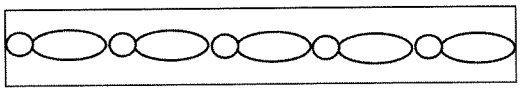

FIG. 8.—A foot strip of five feet

foster a process of reification, on the basis of which students would start to treat the results of measuring as object-like entities. These activities consisted of iterating collections of paces symbolized on paper (see Fig. 8).

Another change to the instructional activities concerned the use of a paper 10 strip. Not envisioned in advance of the teaching experiment was the fact that when the students were asked to design a paper measurement strip on the basis of their experience with measuring with the smurf bar, they only marked multiples of 10 on the strip. When asked to design a 10 strip that could replace the smurf bar, they drew individual cubes as well. While making the 10 strip they apparently realized that they used the smurf bar both for iterating 10s and for counting individual cubes. By pasting 10 strips together, a measurement strip was created that signified the earlier activity of iterating 10s *and* ones.

A third change concerned the shift from the measurement strip to the measurement stick (Fig. 7), which, in essence, proved dispensable. Working with the measurement strip, the students already developed a form of arithmetical reasoning that lent itself perfectly to be described with jumps on an empty number line.

In addition to the above revisions that were part of the daily micro design cycles, the *retrospective analysis of the number-line experiment* helped in identifying more fundamental aspects of the instructional sequence, such as important shifts in the students' thinking, and the role of symbols and models. Those shifts concerned, for instance, the shift from measuring as counting the number of steps to measuring as covering space, which we described earlier in connection with the two distinct ways of counting paces that were observed. As an example of one of the additional objectives of the above design research—next to that of developing a local instruction theory—we may mention the research on the role of symbols and models. This was investigated in the context of the instructional design heuristic called emergent modeling (Gravemeijer, 2004). The objective of this design heuristic is to organize the students' modeling activity in such a manner that it supports the process of reinventing more formal mathematics, by fostering a dialectic process of modeling and meaning making.

Dual Design Research: The Role of Language in Mathematics Education as an Example

In the preceding sections, student learning was our main focus. However, teachers will also be learning in a teaching experiment. Instead of taking those learning processes for granted, we should acknowledge that in many cases the teacher will need to learn in order to conduct a successful teaching experiment. Therefore, we can set goals for the learning of the teacher as well as the students. We might call design research of this kind dual design research since there are two teaching experiments going on at the same time, one in which the students are the learners and one in which the teacher is the learner.

We can illustrate dual design research with a study on the role of language in mathematics education in the first grade of secondary education (students 12–14 years of age). This study built on earlier research by Boer (2003), who investigated how the limited language proficiency of Dutch ethnic minority students resulted in poor mathematics results. She showed how language-avoiding strategies of the students created barriers for learning that were not identified because of the lack of interaction between teacher and student. The problem remained hidden and was underestimated by both teachers and stu-

dents. The aim of the new design study was to tackle this problem by integrating mathematics and language learning for students in multiethnic classrooms.

During the *preparation phase,* a theoretical framework on teaching and learning mathematics and language was developed by integrating theories on second-language learning and RME theory (Eerde, Hajer, & Prenger, 2008). This resulted in three key characteristics that teachers would have to promote: (1) the interactive, joint construction of meaning of the concepts and of the contexts of the mathematical problems; (2) the opportunity for students to actively talk while working on the mathematical problems; and (3) the feedback on both the students' mathematics (content) and language (form).

An interview and a concept-mapping task showed that the teacher of the experimental classroom had a general awareness of the role of language in his mathematics lessons. He especially noted that students often do not understand their textbook assignment. However, he did not mention ways in which he might contribute to the language development of his students. Classroom observations showed that the students did not know the mathematical concepts they were working with, which was probably caused by the lack of interaction. Students worked individually on their textbook tasks. They did not get much opportunity to talk or write. The classroom culture seemed to discourage students from talking and writing in the mathematics classroom. The researchers decided to aim at designing mathematical problems that beg for interaction and create the need for students to talk and write about mathematical issues. The idea was that this would force a change in social norms and that students' language problems would become salient in their talking and writing.

During the first lessons of *the teaching experiment,* the teacher gave the students opportunities for group- and whole-classroom discussions on the basis of the mathematical problems and the researchers' didactical suggestions. In a subsequent video-stimulated interview, the teacher reported that by listening to the students, he became aware that they lacked language or had insufficient language to talk about mathematical issues. However, it was the results of a writing task that really alarmed him and made him aware of how the students' insufficient language proficiency impeded their mathematical learning processes. The students had been asked to write sentences using concepts such as "gradual," which had been used in the context of describing graphs in which the curve was gradually rising or descending. The students wrote:

- If two lines are at the same height you call them gradual.
- The temperature in the Netherlands is gradual.
- If a graph does not rise nor descend but stays at the same height we say it is gradual.

The answers showed that the students confused the word gradual with other words. The first student confused the word gradual with the word parallel. The second student probably confused the Dutch word for gradual, *geleidelijk,* with the word *gelijk,* which means "the same." The third answer showed that the student did not know the mathematical concept *constant.* These misunderstandings had remained unnoticed until the students wrote about them.

The researchers further observed that the teacher had a very dominant role in the whole-class discussions. This led the researchers to suggest that the teacher plan for student presentations and change his role when the students were presenting their work. This change could indeed be observed in the classroom. During a debriefing, the teacher reported that he was surprised how well students were able to play a leading role during presentations. In addition, these presentations offered the students the opportunity to show their

identities by using examples from their own lives.

Another observation during the teaching experiment was that the students were using daily language to talk about mathematics—language directly related to the here and now. When talking about graphs, students were saying things such as, "Here he is going this way" and "He goes up and then straight." They used informal daily language that could only be shared with someone that was present and could hear what was said and see what was done. The development of mathematical concepts, however, requires a common mathematical language that is independent of gestures and pointing. The students needed to learn to use concepts such as curve, rising, constant, horizontal and vertical axis, and origin.

The conclusion, therefore, was that one had to include the necessary mathematical concepts and language in the specification of instructional objectives. In this manner, teachers would become aware that they had to scaffold the students' language to help them develop more formal mathematical language.

In the interview at the end of the experiment, the teacher went into much more detail about the role of language in mathematics classes. He expressed more awareness of various linguistic aspects of mathematics learning, both at the level of the amount of text and of the vocabulary in the textbooks. He stressed that, whereas he formerly thought "the students would surely understand this," he now paid more attention to his explanations and to the language he used during instruction.

We may conclude with the main results of the *retrospective analysis.* An important goal for student learning was language production as a means for the development of mathematical concepts and mathematics-related language. The conjecture—tasks that beg for interaction would foster language production—proved correct. Also, presentations by students proved to be a valuable means for stimulating language production. The analysis also showed, however, that the students' language stayed informal, and it was concluded that additional efforts in that respect are needed.

As far as teacher learning is concerned, one of the conjectures was that creating tasks that beg for interaction would create a situation in which teachers would become aware of the students' language problems. This conjecture proved correct, but it demonstrated that seeing the results on a writing task was much more convincing than listening to the students talk. Another conjecture was that having the students present would challenge the existing social norms that seemed to discourage student discussion. This intervention also seemed to work. However, the changes that could be observed in the experimental lessons did not spill over to the regular lessons. It further showed that teachers need clear information on the mathematical and the language objectives, on how to help students appropriate more formal mathematical language, and on how to give feedback on students' written language. Giving appropriate feedback will not only focus the teachers' actions, but also raise the students' awareness of language as a tool for learning mathematics.

Finally, the lack of changes in the regular lessons showed that a change in social norms is needed. Teachers will have to promote student discussion during the mathematics lessons, and the students will have to be willing to stammer and express a lack of understanding. The teacher's awareness of the huge and unexpected variation in the students' language proficiency and the students' awareness of the role of language for learning mathematics proved to be a strong incentive for an integrated approach of language and mathematics learning (Eerde & Hajer, 2005).

Conclusion

We started this article by arguing that teachers might be better served with re-

search that informs them about *how* a proposed innovation works rather than research that proves that this innovation works better than a so-called conventional approach. With the former manner of research, we may also overcome the problem that teachers experience scientific knowledge that is produced in research as too abstract and too general to directly inform their practice (Stigler & Hiebert, 1999). We presented design research as a research method that aims at understanding how innovative instruction works. We discussed design research on local instruction theories as an example. We added that teacher learning might be an integral part of research on student learning. We denoted this dual design research and gave an example of teacher learning and student learning in the context of design research on the role of language in mathematics education. We showed how the teacher became involved in the cyclic process of designing, experimenting, and analyzing. The rationale for this is in the need to acknowledge the teacher's autonomy. The expectation is that teachers may change their teaching, or experiment with alternative approaches, on the basis of the conclusions they themselves draw from data from their own classrooms. This places the teacher on equal footing with the researchers. Ideally, teachers and researchers experience the project as a collective effort in which they together analyze video footage, student work, and other data to decide on the next steps. Note that the key principles here are—just as with the students—ownership and understanding.

It seems self-evident to extend the conception of design research we describe here to research that the teachers themselves may carry out in their own classrooms. In relation to this, we may observe a striking similarity in the iterative character of design research, Simon's (1995) mathematical teaching cycle, and the iterative character of models for teacher learning, such as lesson studies (Stigler & Hiebert, 1999). Following Fullan (2006), we would argue that teacher learning will flourish only if teachers work together. In this respect we may refer to Fullan's observation that one of the obstacles for teachers to learn about their own practice is that they do not have the opportunity to observe classrooms of other teachers or to be observed by their colleagues in their own classrooms. The core idea of lesson studies is that teachers collectively design, try out, observe, analyze, and improve innovative lessons (Stigler & Hiebert, 1999). In this manner, lesson studies offer an alternative for the top-down approaches to innovation in education that prove to be problematic. In addition to the fact that teachers are the key agents in the former approach, and therefore develop a strong sense of ownership, it is the iterative character of the lesson studies that makes them potentially powerful.

Design research may be connected to the research of teachers in two ways. First, design research may function as a catalyst for teachers to get involved in research. As design research provides teachers with theories on how instruction works in paradigm cases, teachers may take those theories as conjectures for experimenting in their own classrooms. Second, the metaphor of design research as capitalizing on the learning process of the researchers may also inspire teachers to explicate and document their own learning processes when pursuing the two main goals of understanding and improving instruction.

Note

1. In line with what Stigler and Thompson propose in their contribution to this special issue, we may stress that these goals have to be shared by the teacher and the research team. However, there is a difference, in that design-research goals will not be worked out in measurable terms, but in more conceptual terms.

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