Universiteit Utrecht

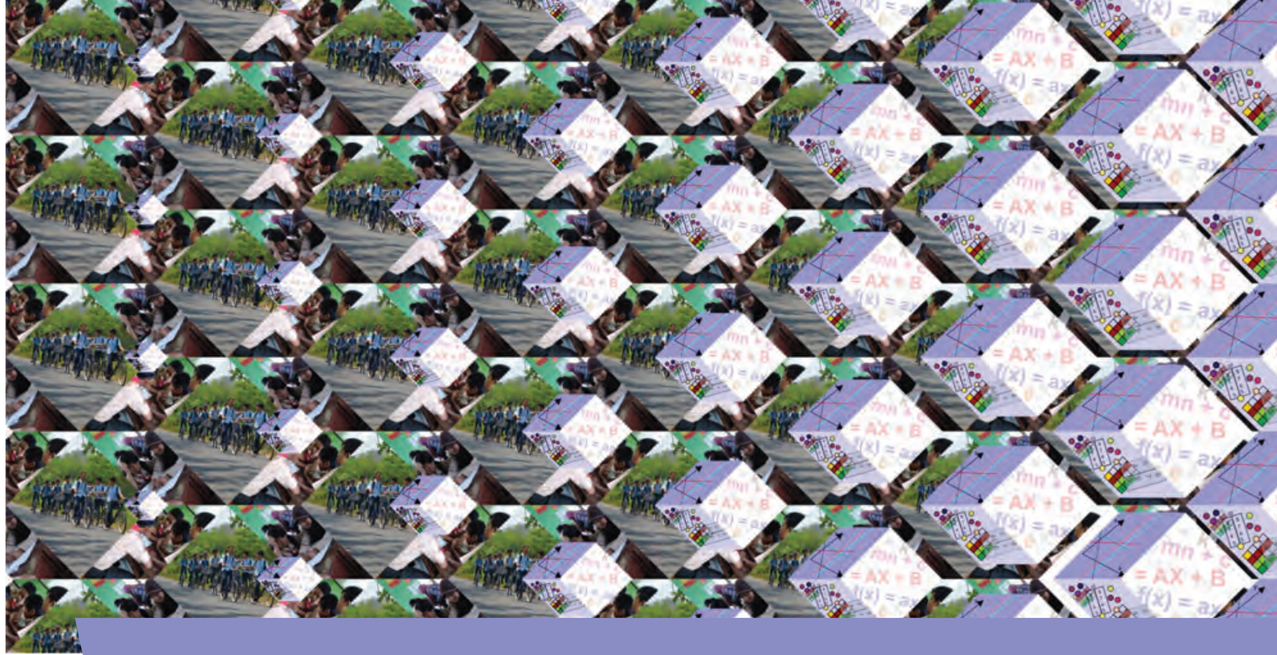
The Indonesian national curriculum mandates that mathematics education must be relevant to the needs of life and should offer students opportunities to develop the ability to apply their knowledge. Nevertheless, the low results of Indonesian students on the PISA studies (Programme for International Student Assessment) indicate the students' poor performance in applying mathematics to solve problems situated in real-world contexts. This situation induced the start of the CoMTI project ('Context-based Mathematics Tasks Indonesia'). The aim of the project was to investigate the reason of this poor performance and to explore ways for improvement. The results of the CoMTI project showed that Indonesian students mainly have difficulties in comprehending context-based tasks and in transforming these into a mathematical problem. With respect to possible reasons for these difficulties, a relation was found between students' difficulties and the *opportunity-to-learn* to solve context-based tasks that they got from textbooks and teachers' teaching practice.

FIsme Scientific Library

Ariyadi Wijaya

Context-based mathematics tasks in Indonesia: Toward better practice and achievement

88



Ariyadi Wijaya

Context-based mathematics tasks in Indonesia: Toward better practice and achievement

Faculteit Bètawetenschappen **FIsme**

Context-based mathematics tasks in Indonesia: Toward better practice and achievement

This research was funded by a grant from the Better Education through Reformed Management and Universal Teacher Upgrading (BERMUTU) project of the Indonesian Ministry of Education and Culture (project number: IDA CREDIT NO.4349-IND, LOAN NO.7476-IND).

This research was carried out in the context of the Dutch Interuniversity Centre for Educational Research.

ico

Wijaya, Ariyadi

Context-based mathematics tasks in Indonesia: Toward better practice and achievement / A. Wijaya – Utrecht: Freudenthal Institute for Science and Mathematics Education, Faculty of Science, Utrecht University / FIsme Scientific Library (formerly published as CD-β Scientific Library), no. 88, 2015.

Dissertation Utrecht University. With references. Met een samenvatting in het Nederlands.

ISBN: 978-90-70786-26-7
Key words: mathematics education / context-based tasks / students' difficulties / opportunity-to-learn / Indonesia

Cover design: Vormgeving Faculteit Bètawetenschappen
Printed by: Drukkerij Wilco BV

CONTEXT-BASED MATHEMATICS TASKS IN INDONESIA: TOWARD BETTER PRACTICE AND ACHIEVEMENT

CONTEXTOPGAVEN IN WISKUNDE: NAAR EEN BETERE ONDERWIJSPRAKTIJK EN BETERE PRESTATIES IN INDONESIË

(met een samenvatting in het Nederlands)

Proefschrift

ter verkrijging van de graad van doctor aan de Universiteit Utrecht op gezag van de rector magnificus, prof. dr. G. J. van der Zwaan, ingevolge het besluit van het college voor promoties in het openbaar te verdedigen op woensdag 21 januari 2015 des ochtends te 10.30 uur

door
Ariyadi Wijaya

geboren op 16 juli 1982
te Banjarnegara, Indonesië

Promotor: Prof. dr. M. H. A. M. van den Heuvel-Panhuizen

Copromotor: Dr. L. M. Doorman

Table of contents

For Elmi and Arka

Chapter 1

Introduction

Introduction

1 Background of the study

1.1 Ability to apply mathematics: An essential goal of mathematics education

A number of studies have reported that modern society requires more than content knowledge (see e.g., Ananiadou & Claro, 2009; Organisation for Economic Co-operation and Development [OECD], 2013; Partnership for 21st Century Skills [P21], 2002). To cope with the demands of modern life people need to be able to apply their knowledge. This situation has led to an emphasis on connecting education to students' lives (Graumann, 2011; P21, 2002), which implies that education should have as its goal to close the gap between how students learn in school and how they deal with everyday life. Educational practices, therefore, should provide students with not only knowledge, but also skills they need for life and community (Griffin, Care, & McGraw, 2012; Tomlinson, 2004).

In mathematics education a great deal of attention has been attached to developing students' ability to apply mathematics. The National Council of Teachers of Mathematics (NCTM), for example, states that the mathematics curriculum should focus on "mathematics that will prepare students for continued study and for solving problems in a variety of school, home and work settings" (NCTM, 2000, p. 14–15). Ability to apply mathematics in real-world contexts is also considered as a key area of mathematical competence in many European countries (Eurydice, 2011; Tomlinson, 2004). At a global level, the utilitarian purpose of mathematics in everyday life is also taken into consideration by the OECD through its Programme for International Student Assessment (PISA).

PISA is a large-scale assessment that examines students' ability to apply mathematics in a variety of situations. PISA introduces the term 'mathematical literacy' that is defined as "an individual's capacity to identify and understand the role that mathematics plays in the world, to make well-founded judgments and to use and engage with mathematics in ways that meet the needs of that individual's life as constructive, concerned and reflective citizens" (OECD, 2003, p. 24).

A focus on the application of mathematics implies the use of mathematics problems situated in real-world contexts in mathematics teaching and learning. De

Lange (2003) argued that students need to experience solving mathematics problems in different situations and contexts to develop their ability to transfer their knowledge from one area of application to another. Similarly, NCTM (2000) also pointed out the importance of providing students with opportunities to work on problems arising in contexts outside mathematics, which could be other subject disciplines or students' daily experiences. In the PISA studies, great value is also attached to "tasks that could be encountered in a variety of real-world situations, and that have a context in which the use of mathematics to solve the problem would be authentic" (OECD, 2003, p. 34). Such tasks are used in the PISA studies to assess mathematical literacy.

1.2 A discrepancy between the Indonesian mathematics curriculum and student achievement

Students' ability to apply mathematics is also considered as an important goal of mathematics education in Indonesia. In 2004, the Indonesian Ministry of National Education[1] started implementing a competence-based curriculum (*Kurikulum Berbasis Kompetensi* – KBK), which was revised into a school-based curriculum (*Kurikulum Tingkat Satuan Pendidikan*[2] – KTSP) in 2006. In contrast to previous curricula that were oriented on the content of subject matter and emphasized students' acquisition of knowledge, both KBK and KTSP placed a premium on developing students' ability to apply their knowledge. These two curricula considered that the subject of mathematics should target developing students' ability to: (1) understand the concepts of mathematics, explain the relevance of concepts, and apply the concepts or algorithms in a flexible way in problem-solving; (2) solve problems that require the ability to understand a problem, design and complete a mathematical model to solve it, and interpret the solution; and (3) appreciate the purpose of mathematics in life (Pusat Kurikulum [The National Curriculum Center], 2003; the Regulation of the Minister of National Education, No. 22, Year 2006, about Standards of Content). This educational goal is also considered in the newly implemented Curriculum 2013 in which the Indonesian government clearly mandates that education must be relevant to the needs of life and offers students opportunities to apply their knowledge in society (Kementerian

[1] The Ministry of National Education was renamed the Ministry of Education and Culture in 2011.

[2] As a consequence of the decentralization of education, the Indonesian Ministry of National Education revised KBK into KTSP by adding the principle of school-based management. In KTSP, the government determined the minimum standards (i.e. competences, content, and targeted performance) and every school had the freedom to develop or adjust the minimum standards based on the school's resources, students' needs, and district (local) educational goals.

Pendidikan dan Kebudayaan [the Ministry of Education and Culture], 2012). In relation to the objectives of Indonesian mathematics education, KBK and KTSP recommended teachers using mathematics problems situated in real-world contexts in their classroom practices. In line with these curricula, there have been educational movements in Indonesia that emphasize the application of mathematics and promote the use of real-world problems in learning mathematics: i.e. *Pendidikan Matematika Realistik Indonesia* (Indonesian Realistic Mathematics Education) (see e.g., Sembiring, Hadi, & Dolk, 2008; Sembiring, Hoogland, & Dolk, 2010) and *Pembelajaran Kontekstual* (Contextual Teaching and Learning) (Supinah, 2008).

The attention for students' ability to apply mathematics is also reflected in Indonesia's regular participation in PISA since the first survey in 2000. Despite strong attention for and interest in the application of mathematics in Indonesian curricula, however, the PISA results show Indonesian students' low performance on mathematics problems situated in real-world contexts. In the latest PISA 2012 study, for example, less than 1% of Indonesian students could solve mathematics problems that are situated in complex situation and require mathematical modeling and well-developed reasoning skills (OECD, 2013). Three quarters of Indonesian students could only solve mathematics problems that use familiar contexts, have clear questions, and present all relevant information. These students could identify relevant information and carry out routine mathematical procedures if explicit instructions were given.

The low performance of Indonesian students on mathematics problems situated in real-world contexts, which in this thesis are called 'context-based tasks', prompted us to carry out this PhD research in the Context-based Mathematics Tasks Indonesia (CoMTI) project. The main purpose of this PhD research was to gain a better insight into Indonesian students´ low performance on context-based tasks and to generate recommendations for improving student performance.

2 What does a context-based task mean in this PhD thesis?

A variety of terms has been used to refer to problems that are situated in real-world contexts; for example 'word problems' (see, e.g. Bernardo, 1999; Verschaffel, Greer, & De Corte, 2000; Verschaffel, Van Dooren, Greer, & Mukhopadhyay, 2010), 'authentic tasks' (see, e.g. Palm, 2008; Kramarski, Mevarech, & Arami, 2002), 'context problems' (see, e.g. Van den Heuvel-Panhuizen, 1996; 2005), or 'modeling tasks' (see, e.g. Blum, 2011; Maass, 2010; Tasova & Delice, 2012). According to Maass (2010), 'word problems' are often seen as "very simple, often artificial tasks with text" (p. 290). Daily experience can often be ignored when solving a word problem because the contexts are artificial. Such a conception leads to an impression that the solution of a word problem can be obtained by operating all numbers given in the text with procedures that are provided explicitly (Maass, 2010). However, there is a more open definition of word problems that refers to mathematical modeling (see, e.g. Verschaffel et al., 2000; 2010). According to these studies, word problems should be used to support the development of modeling competences and cannot be solved by simply taking and combining all numbers provided in the text. The latter definition of a word problem is close to a so-called 'authentic task' that also refers to mathematical modeling. An authentic task is a mathematics task that represents a real life problem or situation (Palm, 2008) and does not have a ready-made algorithm (Kramarski et al., 2002). Furthermore, an authentic task often employs realistic data that are incomplete or inconsistent and asks solvers to use different representations in their solutions. The authenticity of contexts is also highlighted by Van den Heuvel-Panhuizen (1996; 2005) who argued that a 'context problem' should be authentic for students and should require them to think within the context of the problem. "By imagining themselves in the situation to which the problem refers, the students can solve the problem in a way that was inspired, as it were, by the situation" (Van den Heuvel-Panhuizen, 2005, p. 6). It means that context problems can suggest strategies to find a solution, such as using drawings, tables, or graphs. Another term that is used to refer to problems situated in real-world contexts is 'modeling tasks' (see, e.g. Blum, 2011; Maass, 2010; Tasova & Delice, 2012). Modeling tasks emphasize modeling or translation between real-world contexts and mathematics.

Considering the abovementioned descriptions of problems that use real-world contexts, in this thesis we define a context-based mathematics task as a mathematics task that is situated in real-world settings and provides elements or information that need to be organized and modeled mathematically. We do not use the term word problems or story problems because contexts can also be depicted in non-verbal descriptions such as pictures. A consequence of our definition is that we attach a high value to the following characteristics of context-based tasks: the tasks have more or less information than what is needed to find the solution and should not explicitly provide the required mathematical procedures.

3 Context-based tasks and modeling

Solving a problem situated in a real-world context requires interplay between the real world and mathematics (Blum et al., 2002; Blum, 2011), which is often referred to as modeling (Greer, 1997). According to Maass (2010), modeling is not simply the process of making a mathematical representation of a real-world problem, but it is a whole process that includes "understanding a realistic problem, setting up a model of the problem and finding a solution by working on the model mathematically" (p. 287). This process is in general similar to what in PISA is called 'mathematization', which is considered as the fundamental process to solve real-life problems (OECD, 2003).

Modeling has been considered in many studies on problems situated in real-world contexts. Maass (2010) used the modeling process as a key reference to make a classification of tasks that have real-world contexts. She distinguished seven categories of tasks based on the focus of the tasks on the stages of the modeling process; i.e. tasks that focus on: (1) the whole process of modeling, (2) understanding the real-world situation; (3) setting up a real model; (4) mathematizing; (5) working within the mathematical model; (6) interpreting; and (7) validating. With respect to the teaching of modelling, Blum (2011) and Blum and Ferri (2009) considered the modeling process as a specific strategic tool that needs to be acquired and used by teachers to support their students' learning. For this a four-step schema was developed, which comprised understanding the task, establishing a model, using mathematics, and explaining the result, to be used as "an aid for difficulties that might occur in the course of the solution process" (Blum & Ferri, 2009, p. 55)

Considering the relevance of the modeling process to problems situated in real-world contexts, in this PhD research we use the modeling process as one of the main references to get a better insight into Indonesian students' process of solving context-based tasks.

4 Structure of the thesis

The PhD research that forms the basis of this PhD thesis comprised four studies that were set up to investigate Indonesian students´ low performance on context-based tasks and to provide recommendations to improve student performance. The four studies are presented as the chapters of this thesis. Table 1 illustrates the structure of the thesis and the topic addressed in each chapter.

Table 1
Structure of thesis

	Topic that is addressed in the chapter
Chapter 1	Introduction
Chapter 2	What difficulties do Indonesian students have when solving context-based tasks?
Chapter 3	What are possible reasons for Indonesian students' difficulties when solving context-based tasks?
Chapter 4	
Chapter 5	How can Indonesian student performance on context-based tasks be improved?
Chapter 6	Conclusion

Chapter 2 reports a study in which we investigated Indonesian students' difficulties in solving context-based tasks. To identify these difficulties the types of errors made by students were examined. The study addressed the following research questions:

- *What errors do Indonesian students make when solving context-based mathematics tasks?*
- *What is the relation between the types of errors, the types of context-based tasks, and the student performance level?*

After examining the difficulties experienced by Indonesian students, two studies were conducted to investigate possible reasons for the difficulties. In these studies, which are reported in *Chapter 3* and *Chapter 4*, we used the concept of opportunity-to-learn.

Chapter 3 describes a study in which the opportunity-to-learn to solve context-based tasks offered in Indonesian mathematics textbooks was examined. The investigation was done by analyzing the characteristics of tasks in the textbooks. This study addressed the following research questions:

- *What are the amount of exposure to and the purpose of context-based tasks in Indonesian mathematics textbooks?*
- *To what extent are different types of information provided in tasks in Indonesian mathematics textbooks?*
- *What are the cognitive demands of tasks in Indonesian mathematics textbooks?*

Another issue dealt with in this study was the relation between opportunity-to-learn to solve context-based tasks provided in the textbooks and students' difficulties in solving such tasks. For this, we addressed the following research question:

- *What is the connection between students' errors when solving context-based tasks and the characteristics of tasks in Indonesian mathematics textbooks?*

Textbooks might be not the only factor influencing student performance, because how a textbook is used in the classroom is determined by teachers. Therefore, to get a better insight into possible explanations for students' difficulties we also took the teaching practices of Indonesian mathematics teachers into consideration.

Chapter 4 describes our study on examining a possible explanation for students' difficulties from the perspective of the opportunity-to-learn (OTL) to solve context-based tasks offered by teachers through their teaching practices. In this study the teachers' teaching practices were seen from two perspectives, i.e. the teachers' reported practices and observed practices. The former focused on the characteristics of context-based tasks presented by teachers in their lessons, whereas the latter involved the teaching approaches used by teachers to help their students learn to solve context-based tasks. Considering the potential influence of beliefs on teachers' teaching practices, we also investigated teachers' beliefs about mathematics, the learning and teaching of mathematics, and context-based tasks. The following research questions were addressed:

- *What beliefs do Indonesian teachers have regarding mathematics, teaching and learning of mathematics, and context-based tasks?*
- *What OTL to solve context-based tasks do teachers offer students in their classroom practice?*
  - *What kinds of context-based tasks do Indonesian teachers offer their students?*
  - *What teaching approach do Indonesian teachers use to teach context-based tasks?*
- *Is there a relationship between the OTL to solve context-based tasks offered by teachers and students' errors when solving such tasks?*

Chapter 5 describes a study that investigated whether Indonesian students' performance in solving context-based tasks can be improved by offering students opportunity-to-learn to solve such tasks. This opportunity-to-learn, which comprised a set of context-based tasks and a consultative teaching approach, was developed based on our findings about what was lacking in Indonesian mathematics textbooks (reported in *Chapter 3*), and in Indonesian teachers' teaching practices (reported in *Chapter 4*). In this final study we focused on answering the following question:

- *Does providing students with opportunity-to-learn to solve context-based tasks contribute to students' performance when solving these tasks and, more specifically, is there any effect on the correctness of the answers and on the types of errors?*

Chapter 6 summarizes the findings from the four studies and discusses practical implications of the findings on the teaching and learning of context-based tasks in Indonesia. In this chapter we also provide suggestions for further research on context-based tasks.

References

Ananiadou, K., & Claro, M. (2009). 21st Century Skills and competencies for new millennium in OECD countries. *OECD Education Working Papers, No. 41*. OECD Publishing. httpx://dx.doi.org/10.1787/218525261154

Bernardo, A. B. I. (1999). Overcoming obstacles to understanding and solving word problems in mathematics. *Educational Psychology, 19*(2), 149–163.

Blum, W. et al. (2002). ICMI Study 14: Application and modelling in mathematics education – Discussion Document. *Educational Studies in Mathematics, 51*(1), 149–171.

Blum, W., & Ferri, R. B. (2009). Mathematical modelling: Can it be taught and learnt? *Journal of Mathematical Modelling and Application, 1*(1), 45–58.

Blum, W. (2011). Can modelling be taught and learnt? Some answers from empirical research. In G. Kaiser, W. Blum, R. B. Ferri & G. Stillman (Eds.), *Trends in teaching and learning of mathematical modelling* (pp. 15–30). New York: Springer.

De Lange, J. (2003). Mathematics for literacy. In B. L. Madison & L. A. Steen (Eds.), *Quantitative Literacy: Why numeracy matters for schools and colleges* (pp. 75–89). Princeton, NJ: National Council on Education and Disciplines.

Eurydice. (2011). *Mathematics education in Europe: Common challenges and national policies*. Brussels: Education, Audiovisual and Culture Executive Agency.

Graumann, G. (2011). Mathematics for problem in the everyday world. In J. Maasz & J. O'Donoghue (Eds.), *Real-world problems for secondary school mathematics students: Case studies* (pp. 113–122). Rotterdam: Sense Publishers.

Greer, B. (1997). Modelling reality in mathematics classrooms: The case of word problems. *Learning and Instruction, 7*(4), 293–307.

Griffin, P., Care, E., & McGaw, B. (2012). The changing role of education and schools. In P. Griffin, B. McGraw & E. Care (Eds.), *Assessment and Teaching of 21st Century Skills* (pp. 1–16). New York: Springer.

Kementerian Pendidikan dan Kebudayaan. (2012). *Dokumen Kurikulum 2013* (The document of the Curriculum 2013). Jakarta: Author.

Kramarski, B., Mevarech, Z. R., & Arami, M. (2002). The effects of metacognitive instruction on solving mathematical authentic tasks. *Educational Studies in Mathematics, 49*(2), 225–250.

Maass, K. (2010). Classification scheme for modelling tasks. *Journal für Mathematik-Didaktik, 31*(2), 285–311.

National Council of Teachers of Mathematics. (2000). *Principles and standard for school mathematics*. Reston: Author.

OECD. (2003). *The PISA 2003 assessment framework - Mathematics, reading, science, and problem solving knowledge and skills*. Paris: Author.

OECD. (2013). *PISA 2012 Results: What students know and can do. Student performance in mathematics, reading and science*. Paris: Author.

Palm, T. (2008). Impact of authenticity on sense making in word problem solving. *Educational Studies in Mathematics, 67*(1), 37–58.

Partnership for 21st Century Skills. (2002). *Learning for the 21st century. A report and mile guide for 21st century skills*. Tucson, AZ: Author

Pusat Kurikulum. (2003). Kurikulum 2004. *Standar kompetensi mata pelajaran matematika Sekolah Menengah Pertama dan Madrasah Tsanawiyah* (The curriculum 2004: The competence standards for mathematics in Junior High School and Islamic Junior High School). Jakarta: Departemen Pendidikan Nasional.

Sembiring, R., Hoogland, K., & Dolk, M. (2010). *A decade of PMRI in Indonesia*. Utrecht: APS.

Sembiring, R. K., Hadi, S., & Dolk, M. (2008). Reforming mathematics learning in Indonesian classrooms through RME. *ZDM The International Journal on Mathematics Education, 40*(6), 927–939.

Supinah. (2008). *Pembelajaran matematika SD dengan pendekatan kontekstual dalam melaksanakan KTSP* (Contextual approach in teaching elementary mathematics as a means to implement KTSP). Yogyakarta: PPPPTK Matematika.

Tasova, H. I., & Delice, A. (2012). An analysis of pre-service mathematics teachers' performance in modelling tasks in terms of spatial visualisation ability. *Research in Mathematics Education, 14*(3), 297–298.

The Regulation of the Minister of National Education, No. 22, Year 2006, about Standards of Content.

Tomlinson, M. (2004). *14-19 Curriculum and qualifications reform. Final report of the Working Group on 14-19 Reform*. London: Department for Education and Skills.

Van den Heuvel-Panhuizen, M. (1996). *Assessment and Realistic Mathematics Education*. Utrecht: CD-β Press, Utrecht University.

Van den Heuvel-Panhuizen, M. (2005). The role of context in assessment problems in mathematics. *For the Learning of Mathematics, 25*(2), 2–9 + 23.

Verschaffel, L., Greer, B., & De Corte, E. (2000). *Making sense of word problems*. Lisse: Swets & Zeitlinger.

Verschaffel, L., Van Dooren, W., Greer, B., & Mukhopadhyay, S. (2010). Reconceptualising word problems as exercises in mathematical modelling. *Journal für Mathematik-Didaktik, 31*(1), 9–29.

Chapter 2

Difficulties in solving context-based PISA mathematics tasks: An analysis of students' errors

Wijaya, A., Van den Heuvel-Panhuizen, M., Doorman, M., & Robitzsch, A. (2014). Difficulties in solving context-based PISA mathematics tasks: An analysis of students' errors. *The Mathematics Enthusiast, 11*(3), 555-584.

Difficulties in solving context-based PISA mathematics tasks: An analysis of students' errors

1 Introduction

Employer dissatisfaction with school graduates' inability to apply mathematics stimulated a movement favoring using mathematics in everyday situations (Boaler, 1993a) and a practice-orientated mathematics education (Graumann, 2011). The main objective of this movement is to develop students' ability to apply mathematics in everyday life (Graumann, 2011) which is seen as a core goal of mathematics education (Biembengut, 2007; Greer, Verschaffel, & Mukhopadhyay, 2007). In addition, this innovation was also motivated by theoretical developments in educational psychology such as situated cognition theory (Henning, 2004; Nunez, Edwards, & Matos, 1999) and socio-cultural theory (Henning, 2004). Finally, this context-connected approach to mathematics education emerged from studies of mathematics in out-of-school settings such as supermarkets (Lave, 1988) and street markets (Nunes, Schliemann, & Carraher, 1993).

In line with this emphasis on application in mathematics education, the utilitarian purpose of mathematics in everyday life has also become a concern of the Programme for International Student Assessment (PISA) which is organized by the Organisation for Economic Co-operation and Development (OECD). PISA is a large-scale assessment which aims to determine whether students can apply mathematics in a variety of situations. For this purpose, PISA uses real-world problems that require quantitative reasoning, spatial reasoning or problem solving (OECD, 2003b). An analysis of PISA results showed that the competencies measured in PISA surveys are better predictors for 15 year-old students' later success than the qualifications reflected in school marks (Schleicher, 2007). Therefore, the PISA survey has become an influential factor in reforming educational practices (Liang, 2010) and making decisions about educational policy (Grek, 2009; Yore, Anderson, & Hung Chiu, 2010).

Despite the importance of contexts for learning mathematics, several studies (Cummins, Kintsch, Reusser, & Weimer, 1988; Palm, 2008; Verschaffel, Greer, & De Corte, 2000) indicate that contexts can also be problematic for students when they are used in mathematics tasks. Students often miscomprehend the meaning of context-based tasks (Cummins et al., 1988) and give solutions that are not relevant to the situation described in the tasks (Palm, 2008). Considering these findings, the intention of this study was to clarify students' obstacles or difficulties when solving context-based tasks.

As a focus we chose the difficulties students in Indonesia have with context-based problems. The PISA 2009 study (OECD, 2010) showed that only one third of the Indonesian students could answer the types of mathematics tasks involving familiar contexts including all information necessary to solve the tasks and having clearly defined questions. Furthermore, less than one percent of the Indonesian students could work with tasks situated in complex situations which require mathematical modeling, and well-developed thinking and reasoning skills. These poor results ask for further research because the characteristics of PISA tasks are relevant to the mathematics learning goals mandated in the Indonesian Curriculum 2004. For example, one of the goals is that students are able to solve problems that require students to understand the problem, and design and complete a mathematical model of it, and interpret the solution (Pusat Kurikulum, 2003).

Although the Indonesian Curriculum 2004 takes the application aspect of mathematical concepts in daily life into account (Pusat Kurikulum, 2003), the PISA results clearly showed that this curriculum did not yet raise Indonesian students' achievement in solving context-based mathematics tasks. This finding was the main reason to set up the 'Context-based Mathematics Tasks Indonesia' project, or in short the CoMTI project. The general goal of this project is to contribute to the improvement of the Indonesian students' performance on context-based mathematics tasks. The present study is the first part of this project and aimed at clarifying Indonesian students' difficulties or obstacles when solving context-based mathematics tasks. Having insight in where students get stuck will provide us with a key to improve their achievement. Moreover, this insight can contribute to the theoretical knowledge about the teaching and learning of mathematics in context.

2 Theoretical background

2.1 Learning mathematics in context

Contexts are recognized as important levers for mathematics learning because they offer various opportunities for students to learn mathematics. The use of contexts reduces students' perception of mathematics as a remote body of knowledge (Boaler, 1993b), and by means of contexts students can develop a better insight about the usefulness of mathematics for solving daily-life problems (De Lange, 1987). Another benefit of contexts is that they provide students with strategies to solve mathematical problems (Van den Heuvel-Panhuizen, 1996). When solving a context-based problem, students might connect the situation of the problem to their experiences. As a result, students might use not only formal mathematical procedures, but also informal strategies, such as using repeated subtraction instead of a formal digit-based long division. In the teaching and learning process, students' daily experiences and informal strategies can also be used as a starting point to introduce mathematics concepts. For example, covering a floor with squared tiles can be used as the starting point to discuss the formula for the area of a rectangle. In this way, contexts support the development of students' mathematical understanding (De Lange, 1987; Gravemeijer & Doorman, 1999; Van den Heuvel-Panhuizen, 1996).

In mathematics education, the use of contexts can imply different types of contexts. According to Van den Heuvel-Panhuizen (2005) contexts may refer to real-world settings, fantasy situations or even to the formal world of mathematics. This is a wide interpretation of context in which contexts are not restricted to real-world settings. What is important is that contexts create situations for students that are experienced as real and related to their common-sense understanding. In addition, a crucial characteristic of a context for learning mathematics is that there are possibilities for mathematization. A context should provide information that can be organized mathematically and should offer opportunities for students to work within the context by using their pre-existing knowledge and experiences (Van den Heuvel-Panhuizen, 2005).

The PISA study also uses a broad interpretation of context, defining it as a specific setting within a 'situation' which includes all detailed elements used to formulate the problem (OECD, 2003b, p. 32). In this definition, 'situation' refers to the part of the students' world in which the tasks are placed. This includes personal, educational/occupational, public, and scientific situation types. As well as Van den Heuvel-Panhuizen (2005), the PISA researchers also see that a formal mathematics setting can be seen as a context. Such context is called an 'intra-mathematical context' (OECD, 2003b, p. 33) and refers only to mathematical objects, symbols, or structures without any reference to the real world. However, PISA only uses a limited number of such contexts in its surveys and places most value on real-world contexts, which are called 'extra-mathematical contexts' (OECD, 2003b, p. 33). To solve tasks which use extra-mathematical contexts, students need to translate the contexts into a mathematical form through the process of mathematization (OECD, 2003b).

The extra-mathematical contexts defined by PISA are similar to Roth's (1996) definition of contexts, which also focuses on the modeling perspective. Roth (1996, p. 491) defined a context as "a real-world phenomenon that can be modeled by mathematical form." In comparison to Van den Heuvel-Panhuizen and the PISA researchers, Roth takes a narrower perspective on contexts, because he restricts contexts only to real-world phenomena. However, despite this restriction, Roth's focus on the mathematical modeling of the context is close to the idea of mathematization as used in PISA.

Based on the aforementioned definitions of context, in the present study we restricted contexts to situations which provide opportunities for mathematization and are connected to daily life. This aligns with the aim of PISA to assess students' abilities to apply mathematics in everyday life. In conclusion, we defined context-based mathematics tasks as tasks situated in real-world settings which provide elements or information that need to be organized and modeled mathematically.

2.2 Solving context-based mathematics tasks

Solving context-based mathematics tasks requires an interplay between the real world and mathematics (Schwarzkopf, 2007), which is often described as a modeling process (Maass, 2010) or mathematization (OECD, 2003b). The process of modeling begins with a real-world problem, ends with a real-world solution (Maass, 2010) and is considered to be carried out in seven steps (Blum & Leiss, as cited in Maass, 2010). As the first step, a solver needs to establish a 'situation model' to understand the real-world problem. The situation model is then developed into a 'real model' through the process of simplifying and structuring. In the next step, the solver needs to construct a 'mathematical model' by mathematizing the real model. After the mathematical model is established, the solver can carry out mathematical procedures to get a mathematical solution of the problem. Then, the mathematical solution has to be interpreted and validated in terms of the real-world problem. As the final step, the real-world solution has to be presented in terms of the real-world situation of the problem.

In PISA, the process required to solve a real-world problem is called 'mathematization' (OECD, 2003b). This involves: understanding the problem situated in reality; organizing the real-world problem according to mathematical concepts and identifying the relevant mathematics; transforming the real-world problem into a mathematical problem which represents the situation; solving the mathematical problem; and interpreting the mathematical solution in terms of the real situation (OECD, 2003b). In general, the stages of PISA's mathematization are similar to those of the modeling process. To successfully mathematize, a student needs to possess mathematical competences which are related to the cognitive demands of context-based tasks (OECD, 2003b). Concerning the cognitive demands of a context-based task, PISA defines three types of tasks:

1. *Reproduction tasks*: These tasks require recalling mathematical objects and properties, performing routine procedures, applying standard algorithms, and applying technical skills.
2. *Connection tasks*: These tasks require the integration and connection from different mathematical curriculum strands, or the linking of different representations of a problem. The tasks are non-routine and ask for transformation between the context and the mathematical world.
3. *Reflection tasks*: These tasks include complex problem situations in which it is not obvious in advance which mathematical procedures have to be carried out.

Regarding students' performance on context-based tasks, PISA (OECD, 2009a) found that cognitive demands are crucial aspects of context-based tasks because they are – among other task characteristics, such as the length of text, the item format, the mathematical content, and the contexts – the most important factors influencing item difficulty.

2.3 Analyzing students' errors in solving context-based mathematics tasks

To analyze students' difficulties when solving mathematical word problems, Newman (1977, 1983) developed a model which is known as Newman Error Analysis (see also Clarkson, 1991; Clements, 1980). Newman proposed five categories of errors based on the process of solving mathematical word problems, namely errors of reading, comprehension, transformation, process skills, and encoding. To figure out whether Newman's error categories are also suitable for analyzing students' errors in solving context-based tasks which provide information that needs to be organized and modeled mathematically, we compared Newman's error categories with the stages of Blum and Leiss' modeling process (as cited in Maass, 2010) and the PISA's mathematization stages (OECD, 2003b).

Table 1 shows that of Newman's five error categories, only the first category that refers to the technical aspect of reading cannot be matched to a modeling or mathematization stage of the solution process. The category comprehension errors, which focuses on students' inability to understand a problem, corresponds to the first stage of the modeling process ("understanding problem by establishing situational model") and to the first phase of the mathematization process ("understanding problem situated in reality"). The transformation errors refer to errors in constructing a mathematical problem or mathematical model of a real-world problem, which is also a stage in the modeling process and in mathematization. Newman's category of errors in mathematical procedures relates to the modeling stage of working mathematically and the mathematization stage of solving mathematical problems. Lastly, Newman's encoding errors correspond to the final stage of the modeling process and mathematization at which the mathematical solution is interpreted in terms of the real-world problem situation. Considering these similarities, it might be concluded that Newman's error categories can be used to analyze students' errors in solving context-based mathematics tasks.

2.4 Research questions

The CoMTI project aims at improving Indonesian students' performance in solving context-based mathematics tasks. To find indications of how to improve this performance, the first CoMTI study looked for explanations for the low scores in the PISA surveys by investigating, on the basis of Newman's error categories, the difficulties students have when solving context-based mathematics tasks such as used in the PISA surveys.

Generally expressed our first research question was:

1. *What errors do Indonesian students make when solving context-based mathematics tasks?*

A further goal of this study was to investigate the students' errors in connection with the cognitive demands of the tasks and the student performance level. Therefore, our second research question was:

2. *What is the relation between the types of errors, the types of context-based tasks (in the sense of cognitive demands), and the student performance level?*

Table 1
Newman's error categories and stages in solving context-based mathematics tasks

Newman's error categories[a]	Stages in solving context-based mathematics tasks	
	Stages in Blum and Leiss' Modeling[b]	Stages in PISA's Mathematization[c]
Reading: Error in simple recognition of words and symbols	--	--
Comprehension: Error in understanding the meaning of a problem	Understanding problem by establishing situational model	Understanding problem situated in reality
--	Establishing real model by simplifying situational model	--
--	--	Organizing real-world problems according to mathematical concepts and identifying relevant mathematics
Transformation: Error in transforming a word problem into an appropriate mathematical problem	Constructing mathematical model by mathematizing real model	Transforming real-world problem into mathematical problem which represents the problem situation
Process skills: Error in performing mathematical procedures	Working mathematically to get mathematical solution	Solving mathematical problems
Encoding: Error in representing the mathematical solution into acceptable written form	Interpreting mathematical solution in relation to original problem situation Validating interpreted mathematical solution by checking whether this is appropriate and reasonable for its purpose	Interpreting mathematical solution in terms of real situation
--	Communicating the real-world solution	--

[a] (Newman, 1977, 1983; Clarkson, 1991; Clements, 1980); [b] (as cited in Maass, 2010); [c] (OECD, 2003b)

3 Method

3.1 Mathematics tasks

A so-called 'CoMTI test' was administered to collect data about students' errors when solving context-based mathematics tasks. The test items were selected from the PISA's (OECD, 2009b) released 'mathematics units'[1] and included only those units which were situated in extra-mathematical context. Furthermore, to get a broad view of the kinds of difficulties Indonesian students encounter, units were selected in which Indonesian students in the PISA 2003 survey (OECD, 2009b) had either a notably low or high percentage of correct answer. In total we arrived at 19 mathematics units consisting of 34 questions. Hereafter, we will call these questions 'tasks', because they are not just additional questions to a main problem but complete problems on their own, which can be solved independently of each other. Based on the PISA qualification of the tasks we included 15 reproduction, 15 connection and 4 reflection tasks. The tasks were equally distributed over four different booklets according to their difficulty level, as reflected in the percentage correct answers found in the PISA 2003 survey (OECD, 2009b). Six of the tasks were used as anchor tasks and were included in all booklets. Every student took one booklet consisting of 12 to 14 tasks.

The CoMTI test was administered in the period from 16 May to 2 July, 2011, which is in agreement with the testing period of PISA (which is generally between March 1 and August 31) (OECD, 2005). In the CoMTI test the students were asked to show how they solved each task, while in the PISA survey this was only asked for the open constructed-response tasks. Consequently, the time allocated for solving the tasks in the CoMTI test was longer (50 minutes for 12 to 14 tasks) than in the PISA surveys (35 minutes for 16 tasks) (OECD, 2005).

3.2 Participants

The total sample in this CoMTI study included 362 students who were recruited from eleven schools located in rural and urban areas in the province of Yogyakarta, Indonesia. Although this selection might have as a consequence that the students

[1] In PISA (see OECD, 2003b, p. 52) a 'mathematics unit' consists of one or more questions which can be answered independently. These questions are based on the same context which is generally presented by a written description and a graphic or another representation.

in our sample were at a higher academic level than the national average[2], we chose this province for carrying out the present study for reasons of convenience (the first author originates from this province).

To have the sample of this study close to the age range of 15 years and 3 months to 16 years and 2 months, which is taken in the PISA surveys as the operationalization of fifteen-year-olds (OECD, 2005), and which also applies to the Indonesian sample in the PISA surveys, we did this study with Grade 9 and Grade 10 students who generally are of this age. However, it turned out that in our sample the students were in the age range from 14 years and 2 months to 18 years and 6 months (see Table 2), which means that our sample had younger and older students than in the PISA sample.

Table 2

Composition of the sample

Grade	Boys	Girls	Total	Min. age	Max. age	Mean age (SD)
Grade 9	85	148	233	14 y; 2 m	16 y; 7 m	15 y; 3m (5 m)[a]
Grade 10	59	70	129	14 y; 10 m	18 y; 6 m	16 y; 4 m (7 m)
Total	144	218	362			

[a] y= year; m = month

Before analyzing students' errors, we also checked whether the ability level of the students in our sample was comparable to the level of the Indonesian students who participated in the PISA surveys. For this purpose, we compared the percentages of correct answers of 17 tasks included in the PISA 2003 survey (OECD, 2009b), with the scores we found in our sample.

To obtain the percentages of correct answers in our sample, we scored the students' responses according to the PISA marking scheme, which uses three categories: full credit, partial credit, and no credit (OECD, 2009b). The interrater reliability of this scoring was checked by conducting a second scoring by an external judge for approximately 15% of students' responses to the open constructed-response tasks. The multiple-choice and closed constructed-response tasks were not included in the check of the interrater reliability, because the scoring

[2] For example, of the 33 provinces in Indonesia the province of Yogyakarta occupied the 6th place in the national examination in the academic year of 2007/2008 (Mardapi & Kartowagiran, 2009).

for these tasks was straightforward. We obtained a Cohen's Kappa of .76, which indicates that the scoring was reliable (Landis & Koch, 1977).

The calculation of the Pearson correlation coefficient between the percentages correct answers of the 17 tasks in the PISA 2003 survey and in the CoMTI sample revealed a significant correlation, $r\,(15) = .83$, $p < .05$. This result indicates that the tasks which were difficult for Indonesian students in the PISA 2003 survey were also difficult for the students participating in the CoMTI study (see Figure 1). However, the students in the CoMTI study performed better than the Indonesian students in the PISA 2003 survey. The mean percentage of correct answers of the students in the present study was 61%, which is a remarkably higher result than the 29% correct answers of the Indonesian students in the PISA survey. We assume that this result was due to the higher academic performance of students in the province of Yogyakarta compared to the performance of Indonesian students in general (see Footnote 2).

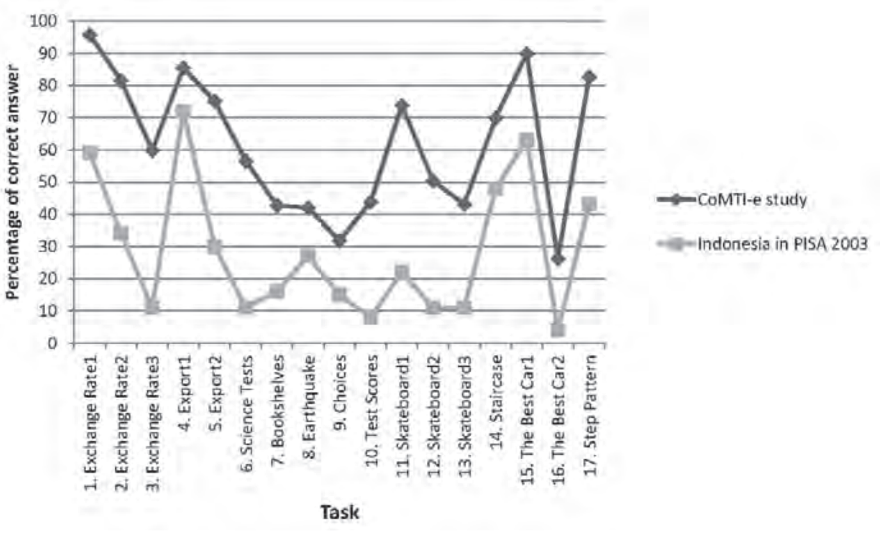


Figure 1. Percentage correct answers in the CoMTI sample and the Indonesia PISA 2003 sample

3.3 Procedure of coding the errors

To investigate the errors, only the students' incorrect responses, i.e., the responses with no credit or partial credit, were coded. Missing responses which were also categorized as no credit were not coded and were excluded from the analysis because students' errors cannot be identified from a blank response.

The scheme that was used to code students' errors (see Table 3) was based on Newman's error categories and in agreement with Blum and Leiss' modeling process and PISA's mathematization stages. However, in this coding scheme we included only four of Newman's error categories, namely 'comprehension', 'transformation', 'mathematical processing', and 'encoding' errors. Instead of Newman's error category of 'process skills', we used the term 'mathematical processing', because in this way it is more clear that errors in process skills concern errors in processing mathematical procedures. The technical error type of 'reading' was not used because this type of error does not refer to understanding the meaning of a task. Moreover, the code 'unknown' was added in the coding scheme because in about 8% of the incorrect responses, the written responses did not provide enough information for coding the errors. These responses with the code 'unknown' were not included in the analysis.

To make the coding more fine-grained the four error types were specified into a number of sub-types (see Table 3), which was done on the basis of a first exploration of the data and a further literature review. For example, a study of Leinhardt, Zaslavsky, and Stein (1990) was used to establish sub-types related to the use of graphs, which resulted in three sub-types: 'treating a graph as a picture', 'point-interval confusion', and 'slope-height confusion'. The last two sub-types belonged clearly to the error type of mathematical processing. The sub-type 'treating a graph as a picture' was classified under the error type of transformation because it indicates that the students do not think about the mathematical properties of a graph. Because students can make more than one error when solving a task, a multiple coding was applied in which a response could be coded with more than one code.

The coding was carried out by the first author and afterwards the reliability of the coding was checked through an additional coding by an external coder. This extra coding was done on the basis of 22% of students' incorrect responses which were randomly selected from all mathematics units. In agreement with the multiple coding procedure, we calculated the interrater reliability for each error type and the code unknown, which resulted in Cohen's Kappa of .72 for comprehension errors, .73 for transformation errors, .79 for errors in mathematical processing, .89 for encoding errors, and .80 for unknown errors, which indicate that the coding was reliable (Landis & Koch, 1977).

3.4 Statistical analyses

To investigate the relationship between error types and task types, a chi-square test of independence was conducted on the basis of the students' responses. Because these responses are nested within students, a chi-square with a Rao-Scott adjustment for clustered data in the R survey package was used (Lumley, 2004; 2012).

For studying the relationship of student performance and error type, we applied a Rasch analysis to obtain scale scores of the students' performance. The reason for choosing this analysis is that it can take into account an incomplete test design (different students got different test booklets with a different set of tasks). A partial credit model was specified in ConQuest (Wu, Adams, Wilson, & Haldane, 2007). The scale scores were estimated within this item response model by weighted likelihood estimates (Warm, 1989) and were categorized into four almost equally distributed performance levels where Level 1 indicates the lowest performance and Level 4 the highest performance. To test whether the frequency of a specific error type differed between performance levels, we applied an analysis of variance based on linear mixed models (Bates, Maechler, & Bolker, 2011). This analysis was based on all responses where an error could be coded and treated the nesting of task responses within students by specifying a random effect for students.

4 Results

4.1 Overview of the observed types of errors

In total, there were 4707 possible responses (number of tasks done by all students in total) which included 2472 correct responses (53%), 1532 incorrect responses (33%), i.e., no credit or partial credit, and 703 missing responses (15%). The error analysis was carried out for the 1532 incorrect responses. The analysis of these responses, based on the multiple coding, revealed that 1718 errors were made by the students. Of these errors 38% were comprehension errors and 42% were transformation errors. Mathematical processing errors were less frequently made (17%) and encoding errors only occurred a few times (3%) (see Table 4).

Table 3

Coding Scheme for error types when solving context-based mathematics tasks

Error type	Sub-type	Explanation
Comprehension	Misunderstanding the instruction	Student incorrectly interprets what (s)he is asked to do.
	Misunderstanding a keyword	Student misunderstands a keyword, which is usually a mathematical term.
	Error in selecting information	Student is unable to distinguish between relevant and irrelevant information (e.g. using all information provided in a task or neglecting relevant information) or is unable to gather required information which is not provided in the task.
Transformation	Procedural tendency	Student tends to directly use a mathematical procedure such as formula or algorithm without analyzing whether or not it is needed.
	Taking too much account of the context	Student's answer only refers to the context or real-world situation without taking the perspective of the mathematics.
	Wrong mathematical operation/concept	Student uses mathematical procedures or concepts which are not relevant to the tasks.
	Treating a graph as a picture	Student treats a graph as a literal picture of a situation. (S)he interprets and focuses on the shape of the graph, instead of on the properties of the graph.
Mathematical Processing	Algebraic error	Error in solving algebraic expression or function.
	Arithmetical error	Error in calculation.

Error type	Sub-type	Explanation
Mathematical Processing	Error in mathematical interpretation of graph:	
	- Point-interval confusion	Student mistakenly focuses on a single point rather than on an interval.
	- Slope-height confusion	Student does not use the slope of the graph but only focuses on the vertical distance.
	Measurement error	Student cannot convert between standard units (e.g. from m/minute to km/h) or from a non-standard unit to a standard unit (e.g. from step/minute to m/minute).
	Improper use of scale	Student cannot select and use the scale of a map properly.
	Unfinished answer	Student uses a correct procedure, but (s)he does not finish it.
Encoding		Student is unable to correctly interpret and validate the mathematical solution in terms of the real-world problem. This error is reflected by an impossible or not realistic answer
Unknown		Type of error could not be identified due to limited information from student's work.

Table 4

Frequencies of error types

Type of error	N	%
Comprehension (C)	653	38
Transformation (T)	723	42
Mathematical processing (M)	291	17
Encoding (E)	51	3
Total of observed errors	1718[a]	100

[a] Because of multiple coding, the total of observed errors exceeds the number of incorrect responses (i.e. n = 1532). In total we had 13 coding categories (including combinations of error types); the six most frequently coded categories were C, CM, CT, M, ME, and T.

4.1.1 Observed comprehension errors

When making comprehension errors, students had problems with understanding the meaning of a task. This was because they misunderstood the instruction or a particular keyword, or they had difficulties in selecting the correct information. Errors in selecting information included half of the 653 comprehension errors and indicate that students had difficulty in distinguishing between relevant and irrelevant information provided in the task or in gathering required information which was not provided in the task (see Table 5).

Table 5.

Frequencies of sub-types of comprehension errors

Sub-type of comprehension error	N	%
Misunderstanding the instruction	227	35
Misunderstanding a keyword	100	15
Error in selecting information	326	50
Total of observed errors	653	100

Figure 2 shows an example of student work which contains an error in selecting information. The student had to solve the Staircase task, which was about finding the height of each step of a staircase consisting of 14 steps.

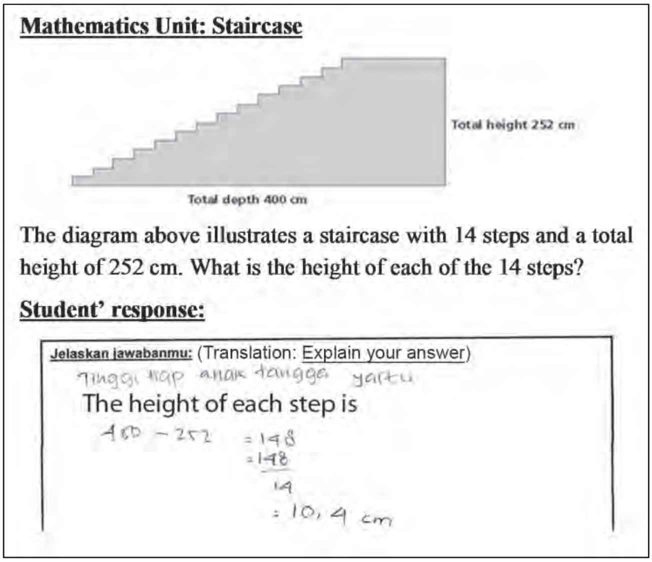


Figure 2. Example of *comprehension error*

The student seemed to have deduced correctly that the height of the staircase had to be divided by the number of steps in order to get the height of each step. However, he did not divide the total height 252 (cm) by 14. Instead, he took the 400 and subtracted 252 from it and then he divided the result of it, which is 148, by 14. So, the student made a calculation with the given total depth of the staircase, though this was irrelevant for solving the task.

4.1.2 Observed transformation errors

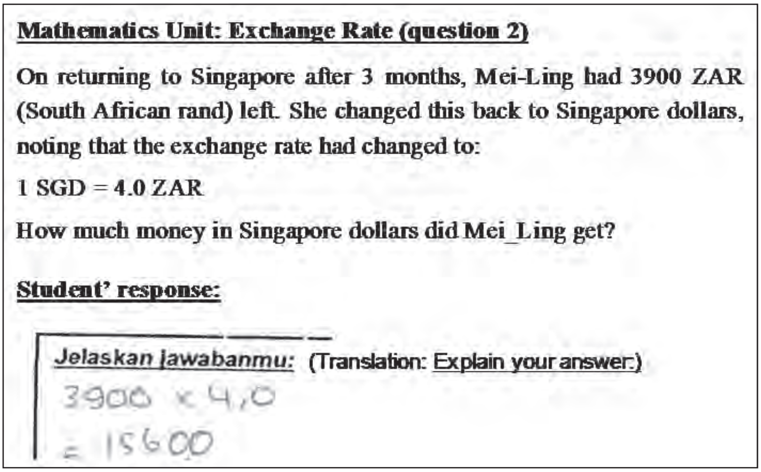
Within the transformation errors, the most dominant sub-type was using a wrong mathematical operation or concept. Of the 723 transformation errors, two thirds belonged to this sub-type (see Table 6).

Table 6.

Frequencies of sub-types of transformation errors

Sub-type of transformation error	n	%
Procedural tendency	90	12
Taking too much account of the context	56	8
Wrong mathematical operation/concept	489	68
Treating a graph as a picture	88	12
Total of observed errors	723	100

Figure 3 shows the response of a student who made a transformation error. The task was about the concept of direct proportion situated in the context of money exchange. The student was asked to convert 3900 ZAR to Singapore dollars with an exchange rate of 1 SGD = 4.0 ZAR. Instead of dividing 3900 by 4.0, the student multiplied 3900 by 4.0. This means the student chose the wrong mathematical procedure for solving the task.



Mathematics Unit: Exchange Rate (question 2)

On returning to Singapore after 3 months, Mei-Ling had 3900 ZAR (South African rand) left. She changed this back to Singapore dollars, noting that the exchange rate had changed to:

1 SGD = 4.0 ZAR

How much money in Singapore dollars did Mei_Ling get?

Student' response:

Jelaskan jawabanmu: (Translation: Explain your answer:)

3900 × 4,0

= 15600

Figure 3. Example of *transformation error*

4.1.3 Observed mathematical processing errors

Mathematical processing errors correspond to students' failure in carrying out mathematical procedures (for an overview of these errors, see Table 3). This type of errors is mostly dependent on the mathematical topic addressed in a task. For example, errors in interpreting a graph do not occur when there is no graph in the task. Consequently, the frequencies of the sub-types of mathematical processing errors were calculated only for the related tasks, i.e. tasks in which such errors may occur (see Table 7).

Table 7.
Frequencies of sub-types of mathematical processing errors

Sub-type of mathematical processing error	Related tasks	All errors in related tasks	Mathematical processing errors in related tasks	
	n	n	n	%
Algebraic error	8	243	33	14
Arithmetical error	20	956	94	10
Error in interpreting graph	6	155	43	28
Measurement error	1	74	15	20
Error related to improper use of scale	1	177	49	28
Unfinished answer	26	1125	79	7

An example of a mathematical processing error is shown in a task in Figure 4. The task is about finding a man's pace length (P) by using the formula $\frac{n}{P} = 140$ in which n, the number of steps per minute, is given. The student correctly substituted the given information into the formula and came to $\frac{70}{P} = 140$. In the next step, however, instead of dividing 70 by 140 the student subtracted 140 by 70. This response indicates that the student had difficulty to work with an equation in which the unknown was the divisor and the dividend is smaller than the quotient.

4.1.4 Observed encoding errors

Encoding errors were not divided into sub-types. They comprise all errors that are related to students' inability to interpret a mathematical answer as a solution that fits to the real-world context of a task. As mentioned earlier only 3% of the total errors belonged to this category. The response of the student in Figure 4, discussed in the previous section, also contains an encoding error. His answer of 70 is, within the context of this task, an answer that does not make sense. A human's pace length of 70 meter is a rather unrealistic answer.

Mathematics Unit: Walking

For men, the formula, $\frac{n}{P} = 140$, gives an approximate relationship between n and P where,
n = number of steps per minute, and
P = pacelength in meters

Question 1:
If the formula applies to Heiko's walking and Heiko takes 70 steps per minute, what is Heiko's pacelength? Show your work.

Students' responses:

Jelaskan jawabanmu: (Translation: Explain your answer:)

Figure 4. Example of *mathematical processing error* and *encoding error*

4.2 The relation between the types of errors and the types of tasks

In agreement with the PISA findings (OECD, 2009a), we found that the reproduction tasks were the easiest for the students, whereas the tasks with a higher cognitive demand, the connection and the reflection tasks, had a higher percentage of completely wrong answers (which means no credit) (see Figure 5).

To investigate the relation between the error types and the tasks types we performed a chi-square test of independence based on the six most frequently coded error categories (see the note in Table 4). The test showed that there was a significant relation ($\chi^2(10, N = 1393) = 91.385, p < .001$). Furthermore, we found a moderate association between the error types and the types of tasks (Phi coefficient = .256; Cramer's V = .181).

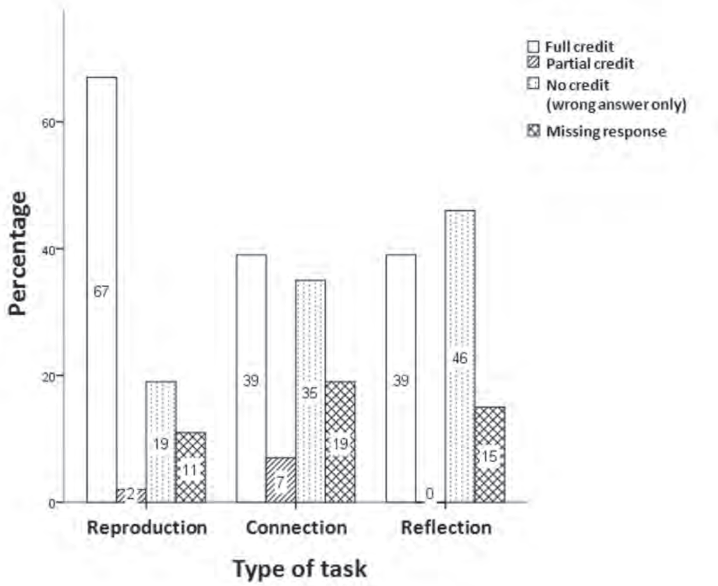


Figure 5. Percentage of full credit, partial credit, no credit, and missing response per task type

When examining the proportion of error types within every task type, it was found that in the reproduction tasks, mostly comprehension errors (37%) and transformation errors (34%) were made (see Table 8). Also in the connection tasks students made mostly comprehension errors (41%) and transformation errors (43%). However, in the reflection tasks mostly transformation errors (66%) were made. Furthermore, the analysis revealed that out of the three types of tasks the connection tasks had the highest average number of errors per task.

Table 8.

Error types within task type

Type of task	Tasks n	Incorrect responses n[a]	Compre-hension error		Transfor-mation error		Mathematical Processing error		Encoding error		Total errors		
			n	%	n	%	n	%	n	%	n	%[a]	average number of errors per task
Reproduction	15	518	211	37	194	34	136	24	23	4	564	99	38
Connection	15	852	404	41	424	43	140	14	28	3	996	101	66
Reflection	4	162	38	24	105	66	15	9	0	0	158	99	40
Total	34	1532	653		723		291		51		1718[b]		

[a] Because of rounding off, the total percentages are not equal to 100%

[b] Because of multiple coding, the total errors exceeds the total number of incorrect responses

4.3 The relation between the types of errors, the types of tasks, and the students' performance level

When testing whether students on different performance levels differed with respect to the error types they made, it was found, for all task types together, that the low performing students (Level 1 and Level 2) made more transformation errors than the high performing students (Level 3 and Level 4) (see Figure 6)[3]. For the mathematical processing errors the pattern was opposite. Here, more errors were made by the high performing students than by the low performing students. With respect to the comprehension errors there was no such a difference. The low and high performing students made about the same number of comprehension errors.

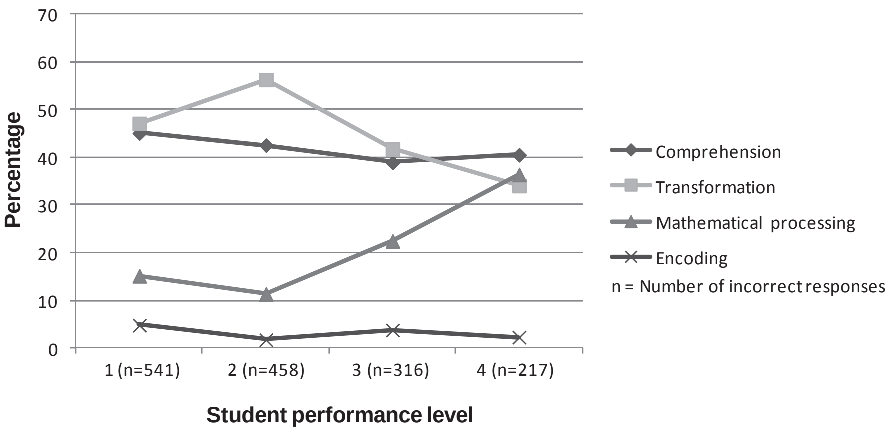


Figure 6. Types of error in all tasks for different performance levels

A nearly similar pattern of error type and performance level was also observed when the analysis was zoomed in on the connection tasks (see Figure 7). For the reproduction tasks (see Figure 8) the pattern was also quite comparable. The only difference was that the high performing students made less comprehension errors than the low performing students.

[3] The diagram in Figure 6 (and similarly in Figure 7, Figure 8, and Figure 9) can be read as follows: the students at Level 1 gave in total 541 incorrect responses of which 45% contained comprehension errors, 47% transformation errors, 15% mathematical processing errors, and 5% encoding errors. Because of multiple coding, the total percentage exceeds 100%.

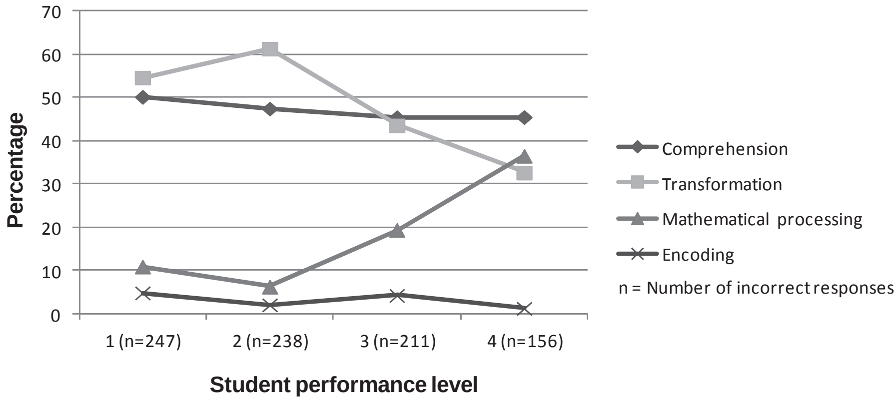


Figure 7. Types of errors in connection tasks for different performance levels

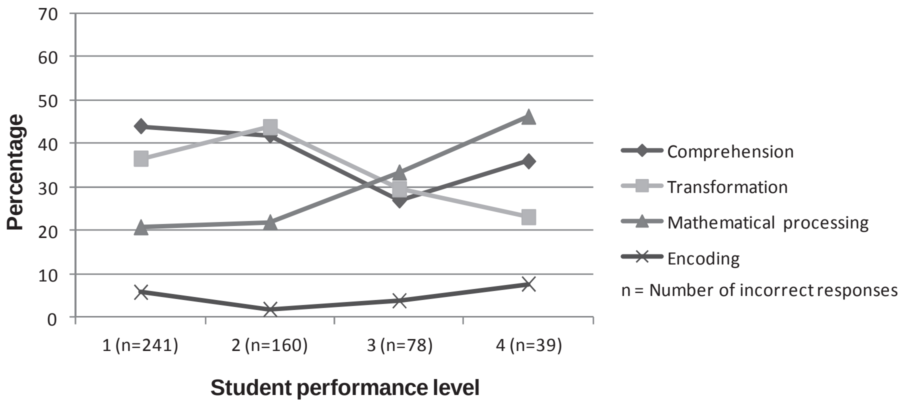


Figure 8. Types of errors in reproduction tasks for different performance levels

For the reflection tasks it was found that the high performing students made more mathematical processing errors than the low performing students (see Figure 9). For the other error types, there was no remarkable difference across student performance levels.

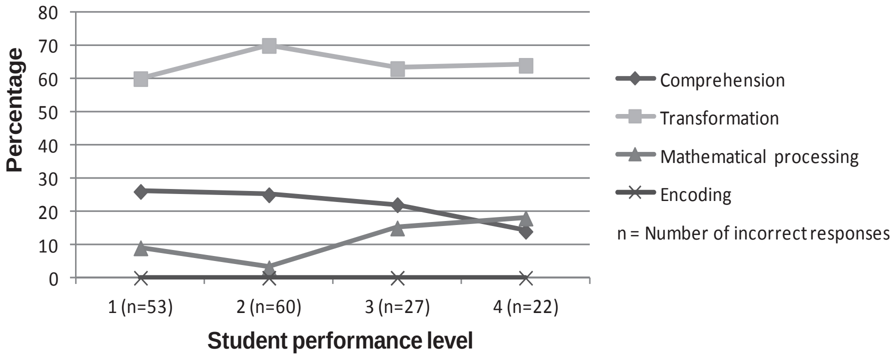


Figure 9. Types of errors in reflection tasks for different performance levels

5 Conclusions and discussion

The present study was aimed at getting a better understanding of students' errors when solving context-based mathematics tasks. Figure 10 summarizes the findings regarding the types of errors Indonesian nine- and ten-graders made when solving these tasks. Out of the four types of errors that were derived from Newman (1977; 1983), it was found that comprehension and transformation errors were most dominant and that students made fewer errors in mathematical processing and in the interpretation of the mathematical solution in terms of the original real-world situation. This implies that the students involved in the present study mostly experienced difficulties in the early stages of solving context-based mathematics tasks as described by Blum and Leiss (cited in Maass, 2010) and PISA (OECD, 2003b), i.e. comprehending a real-world problem and transforming it into a mathematical problem.

Furthermore, within the category of comprehension errors our analysis revealed that most students were unable to select relevant information. Students tended to use all numbers given in the text without considering their relevance to solving the task. This finding provides a new perspective on students' errors in understanding real-world tasks because previous studies (Bernardo, 1999; Cummins et al., 1988) mainly concerned students' errors in relation to the language used in the presentation of the task. For example, they found that students had difficulties to understand the mathematical connotation of particular words.

	Type of errors							
	Comprehension Inability to understand the meaning of a task		Transformation Inability to transformation the context problem into a mathematical model		Math. Processing Inability to perform the mathematical procedures		Encoding Inability to interpret the mathematical solution in terms of the real situation	
All tasks	All students		All students		All students		All students	
	L	H	L	H	L	H	L	H
Reproduction tasks Recalling mathematical properties, performing routine procedures or standard algorithms, and applying technical skills	All students		All students		All students		All students	
	L	H	L	H	L	H	L	H
Connection tasks Integrating and connecting different mathematical curriculum strands, or linking different representations of a problem	All students		All students		All students		All students	
	L	H	L	H	L	H	L	H
Reflection tasks Using complex problem situations in which the relevant mathematical procedures are not obvious	All students		All students		All students		All students	
	L	H	L	H	L	H	L	H

	Percentage of errors
	57% - 70%
	43% - 56%
	29% - 42%
	15% - 28%
	0 < 14%

L = low performing students
H = high performing students

[a] The percentages of errors range from 0% to 70% because the maximum percentage of students' errors is 66%. This range is divided into five equal levels. In the figure, these levels are represented in cells with different degrees of shading.

Figure 10. Summary of research findings

The above findings suggest that focusing on the early stages of modeling process or mathematization might be an important key to improve students' performance on context-based tasks. In particular for comprehending a real-world problem, much attention should be given to tasks with lacking or superfluous information in which students have to use their daily-life knowledge or have to select the information that is relevant to solve a particular task.

A further focus of the present study was the relation between the type of tasks and the types of students' errors. In agreement with the PISA findings (OECD, 2009a) we found that the cognitive demands of the tasks are an important factor influencing the difficulty level of context-based tasks. Because we did not only look at the correctness of the answers but also to the errors made by the students, we could reveal that most errors were not made in the reflection tasks, i.e. the tasks with the highest cognitive demand, but in the connection tasks. It seems as if these tasks are most vulnerable for mistakes. Furthermore, our analysis revealed that in the reflections tasks students made less comprehension errors than in the connection tasks and the reproduction tasks. One possible reason might be that most reflection tasks used in the present study did not provide either more or less information than needed to solve the task. Consequently, students in the present study did hardly have to deal with selecting relevant information.

Regarding the relation between the types of errors and the student performance level, we found that generally the low performing students made more comprehension errors and transformation errors than the high performing students. For the mathematical processing errors the opposite was found. The high performing students made more mathematical processing errors than the low performing students. A possible explanation for this result is that the low performing students, in contrast to the high performing students, might get stuck in the first two stages of solving context-based mathematics tasks and therefore are not arriving at the stage of carrying out mathematical procedures. These findings confirm Newman's (1977) argument that the error types might have a hierarchical structure: failures on a particular step of solving a task prevents a student from progressing to the next step.

In sum, the present study gave a better insight into the errors students make when solving context-based tasks and provided us with indications for improving their achievement. The results signify that paying more attention to comprehending a task, in particular selecting relevant data, and to transforming a task, which means identifying an adequate mathematical procedure, both might improve low performing students' ability to solve context-based tasks. For the high performing students, our results show that they may benefit from paying more attention to performing mathematical procedures.

However, when making use of the findings of the present study this should be done with prudence, because this study has some limitations that need to be taken into account. What we found in this Indonesian sample does not necessarily apply to students in other countries with different educational practices. In addition, the classification of task types – reproduction, connection and reflection – as determined in the PISA study might not always be experienced by the students in a similar way. For example, whether a reproduction task is a reproduction task for the students also depends on their prior knowledge and experiences. For instance, as described by Kolovou, Van den Heuvel-Panhuizen, and Bakker (2009), students who have not learned algebra cannot use a routine algebraic procedure to split a number into several successive numbers (such as splitting 51 into 16, 17, and 18). Instead, they might use an informal reasoning strategy with trial-and-error to solve it. In this case, for these students the task is a connection task, whereas for students who have learned algebra it might be a reproduction task.

Notwithstanding the aforementioned limitations, the results of the present study provide a basis for further research into the possible causes of students' difficulties in solving context-based mathematics tasks. For finding causes of the difficulties that students encounter, in addition to analyzing students' errors, it is also essential to examine what opportunities-to-learn students are offered in solving these kinds of tasks. Investigating these learning opportunities will be our next step in the CoMTI project.

References

Bates, D., Maechler, M., & Bolker, B. (2011). lme4: Linear mixed-effects models using S4 classes. R package version 0.999375-42.

Bernardo, A. B. I. (1999). Overcoming obstacles to understanding and solving word problems in mathematics. *Educational Psychology, 19*(2), 149–163.

Biembengut, M. S. (2007). Modelling and applications in primary education. In W. Blum, P. L. Galbraith, H.-W. Henn, & M. Niss (Eds.), *Modelling and applications in mathematics education* (pp. 451–456). New York: Springer.

Boaler, J. (1993a). Encouraging transfer of 'school' mathematics to the 'real world' through the integration of process and content, context and culture. *Educational Studies in Mathematics, 25*(4), 341–373.

Boaler, J. (1993b). The role of contexts in the mathematics classroom: Do they make mathematics more real? *For the Learning of Mathematics, 13*(2), 12–17.

Clarkson, P. C. (1991). Language comprehension errors: A further investigation. *Mathematics Education Research Journal, 3*(2), 24–33.

Clements, M. A. (1980). Analyzing children's errors on written mathematical task. *Educational Studies in Mathematics, 11*(1), 1–21.

De Lange, J. (1987). *Mathematics, insight and meaning.* Utrecht: OW & OC, Rijksuniversiteit Utrecht.

Graumann, G. (2011). Mathematics for problem in the everyday world. In J. Maasz & J. O'Donoghue (Eds.), *Real-world problems for secondary school mathematics students: case studies* (pp. 113–122). Rotterdam: Sense Publishers.

Gravemeijer, K.P.E. & Doorman, L.M. (1999). Context problems in realistic mathematics education: A calculus course as an example. Educational Studies in Mathematics, 39(1-3), pp. 111–129.

Greer, B., Verschaffel, L., & Mukhopadhyay, S. (2007). Modelling for life: Mathematics and children's experience. In W. Blum, P. L. Galbraith, H.-W. Henn & M. Niss (Eds.), *Modelling and applications in mathematics education* (pp. 89–98). New York: Springer.

Grek, S. (2009). Governing by numbers: The PISA 'effect' in Europe. *Journal of Education Policy, 24*(1), 23–37.

Henning, P. H. (2004). Everyday cognition and situated learning. In D. H. Jonassen (Ed.), *Handbook of research for educational communications and technology: A project of the Association for Educational Communications and Technology (2nd)* (pp. 143–168). New Jersey: Lawrence Erlbaum Associates, Inc.

Kolovou, A., Van den Heuvel-Panhuizen, M., & Bakker, A. (2009). Non-routine problem solving tasks in primary school mathematics textbooks – A Needle in a Haystack. Mediterranean Journal for Research in Mathematics Education, 8(2), 31–68.

Landis, J. R., & Koch, G. G. (1977). The measurement of observer agreement for categorical data. *Biometrics, 33*(1), 159–174.

Lave, J. (1988). *Cognition in practice: Mind, mathematics, and culture in everyday life*. New York: Cambridge University Press.

Leinhardt, G., Zaslavsky, O., & Stein, M. K. (1990). Functions, graphs, and graphing: Tasks, learning, and teaching. *Review of Educational Research, 60*(1), 1–64.

Liang, X. (2010). Assessment use, self-efficacy and mathematics achievement: Comparative analysis of PISA 2003. Data of Finland, Canada and the USA. *Evaluation & Research in Education, 23*(3), 213–229.

Lumley, T. (2004). Analysis of complex survey samples. *Journal of Statistical Software, 9*(1), 1–19.

Lumley, T. (2012). *Survey: Analysis of complex survey samples. R Package version 3.28-1.*

Maass, K. (2010). Classification scheme for modelling tasks. *Journal für Mathematik-Didaktik, 31*(2), 285–311.

Mardapi, D., & Kartowagiran, B. (2009). *Dampak Ujian Nasional.* Yogyakarta: Universitas Negeri Yogyakarta.

Newman, M. A. (1977). An analysis of sixth-grade pupils' errors on written mathematical tasks. *Victorian Institute for Educational Research Bulletin, 39*, 31–43.

Newman, M. A. (1983). *Strategies for diagnosis and remediation*. Sydney: Harcourt Brace Jovanovich.

Nunes, T., Schliemann, A. D., & Carraher, D. W. (1993). *Street mathematics and school mathematics*. New York: Cambridge University Press.

Nunez, R. E., Edwards, L. D., & Matos, J. F. (1999). Embodied cognition as grounding for situatedness and context in mathematics education. *Educational Studies in Mathematics, 39*(1–3), 45–65.

OECD. (2003a). *Literacy skills for the world of tomorrow. Further results from PISA 2000*. Paris: Author.

OECD. (2003b). *The PISA 2003 assessment framework – Mathematics, reading, science, and problem solving knowledge and skills*. Paris: Author.

OECD. (2004). *Learning for tomorrow's world. First results from PISA 2003*. Paris: Author.

OECD. (2005). *PISA 2003. Technical report*. Paris: Author.

OECD. (2007). *PISA 2006: Science competencies for tomorrow's world*. Paris: Author.

OECD. (2009a). *Learning mathematics for life. A view perspective from PISA*. Paris: Author.

OECD. (2009b). *Take the test. Sample questions from OECD's PISA assessments*. Paris: Author.

OECD. (2010). *PISA 2009 results: What students know and can do. Student performance in reading, mathematics, and science (Vol. I)*. Paris: Author.

Pusat Kurikulum. (2003). *Kurikulum 2004. Standar Kompetensi mata pelajaran matematika Sekolah Menengah Pertama dan Madrasah Tsanawiyah*. Jakarta: Departemen Pendidikan Nasional.

Roth, W.-M. (1996). Where is the context in contextual word problems? Mathematical practices and products in grade 8 students' answers to story problems. *Cognition and Instruction, 14*(4), 487–527.

Schleicher, A. (2007). Can competencies assessed by PISA be considered the fundamental school knowledge 15-year-olds should possess? *Educational Change, 8*(4), 349–357.

Schwarzkopf, R. (2007). Elementary modelling in mathematics lessons: The interplay between "real-world" knowledge and "mathematical structures". In W. Blum, P. L. Galbraith, H.-W. Henn, & M. Niss (Eds.), *Modelling and applications in mathematics education* (pp. 209–216). New York: Springer.

Van den Heuvel-Panhuizen, M. (1996). *Assessment and Realistic Mathematics Education*. Utrecht: CD-β Press, Utrecht University.

Van den Heuvel-Panhuizen, M. (2005). The role of context in assessment problems in mathematics. *For the Learning of Mathematics, 25*(2), 2–9, and 23.

Warm, T. A. (1989). Weighted likelihood estimation of ability in item response theory. *Psychometrika, 54*(3), 427–450.

Wu, M. L., Adams, R. J., Wilson, M. R., & Haldane, S. (2007). *ACER conquest version 2.0*. Mulgrave: ACER Press.

Yore, L. D., Anderson, J. O., & Hung Chiu, M. (2010). Moving PISA results into the policy arena: Perspectives on knowledge transfer for future considerations and preparations. *International Journal of Science and Mathematics Education, 8*(3), 593–603.

Yusuf, S. (2004). *Analisis tes PISA. Literasi membaca, matematika, dan sains*. Jakarta: Pusat Penelitian Pendidikan. Departemen Pendidikan Nasional.

Chapter 3

Opportunity-to-learn context-based tasks provided by mathematics textbooks

Wijaya, A., Van den Heuvel-Panhuizen, M., & Doorman, M.. Opportunity-to-learn context-based tasks provided by mathematics textbooks. Will be published in *Educational Studies in Mathematics*.

Opportunity-to-learn context-based tasks provided by mathematics textbooks

1 Introduction

Students' ability to apply mathematics in various contexts in daily life is seen as a core goal of mathematics education (e.g., Boaler, 1993; De Lange, 2003; Graumann, 2011; Muller & Burkhardt, 2007; Niss, Blum, & Galbraith, 2007). This core goal of mathematics education is also reflected in the Programme for International Student Assessment (PISA) (OECD, 2003b) in which the assessment of students' mathematics achievement focuses on students' ability to solve mathematics problems situated in real-world contexts. According to the PISA Framework (OECD, 2003b), such ability should be an educational core goal because today and in the future every country needs mathematically literate citizens to deal with complex everyday surroundings and rapidly changing professional environments.

Contexts from daily life can also be used as a didactical tool to support the learning of mathematics. Students' experiences with these contexts give a meaningful basis to the mathematical concepts they have to learn (Cooper & Harries, 2002; Van den Heuvel-Panhuizen, 1996). Furthermore, contexts can provide context-connected solution strategies (Van den Heuvel-Panhuizen, 1996, 2005). However, this does not mean that context-based tasks are always easy to solve for students. Several studies revealed that many students have low performance on such tasks (e.g., Clements, 1980; Cummins, Kintsch, Reusser, & Weimer, 1988;Schwarzkopf, 2007; Verschaffel, Greer, & De Corte, 2000). When solving context-based tasks, students have difficulties in (1) understanding what a problem is about (Bernardo, 1999; Cummins et al., 1988), (2) distinguishing between relevant and irrelevant information (Cummins et al., 1988; Verschaffel et al., 2000), and (3) identifying the mathematical procedures required to solve a problem (Clements, 1980; Verschaffel et al., 2000).

As shown by the PISA surveys (OECD, 2003a, 2004, 2007, 2010), one country in which students have low performance on context-based mathematics tasks is Indonesia. For example, in PISA 2009 (OECD, 2010) only one third of Indonesian students could answer mathematics tasks embedded in familiar contexts and less than 1% of the students could work with context-based tasks in complex situations which require well-developed thinking and reasoning skills. Furthermore, in a recent study (Wijaya, Van den Heuvel-Panhuizen, Doorman, & Robitzsch, 2014) – which was the first study of the Context-based Mathematics Tasks Indonesia (CoMTI) project – we found that the errors of Indonesian students when solving context-based tasks were mostly related to comprehending the tasks, particularly to selecting relevant information. Many mistakes were also made in choosing the correct mathematical operation or concept when transforming a context-based task into a mathematical problem.

The present study was set up to find an explanation for the difficulties Indonesian students experience when solving context-based tasks. The approach taken in the study was investigating what opportunity-to-learn Indonesian mathematics textbooks offer Indonesian students to develop the ability to solve context-based tasks. Although the study was situated in Indonesia, we think it is relevant for an international audience because it contributes to existing knowledge about the relation between textbooks' content and what students learn. Moreover, this study brings in a new aspect of this relation by focusing on the difficulties students experience when solving context-based tasks.

2 Theoretical background and research questions

2.1 The concept of opportunity-to-learn

A plausible question when particular educational goals are not achieved by students is whether they have received the education enabling them to reach the competences expressed in these goals. Therefore, it is no wonder that the concept of opportunity-to-learn (OTL) came into being. Fifty years ago this concept was coined by Carroll (1963) when referring to sufficient time for students to learn (Liu, 2009). OTL was also introduced to ensure the validity of international comparative studies of students' achievement. Researchers became aware that when comparing the achievement of students from different countries curricular differences across national systems had to be taken into account (Liu, 2009; McDonnell, 1995). In the report of the First International Mathematics Studies (Husén, 1967), OTL was defined as "whether or not […] students have had the opportunity to study a particular topic or learn how to solve a particular type of problem" (pp. 162-163).

2.2 Assessing opportunity-to-learn

To examine OTL several approaches are possible for which various aspects of OTL can be assessed. Liu (2009) considered four OTL variables: (a) content coverage, that is the match between the curriculum taught and the content tested, (b) content exposure, that is the time spent on the content tested, (c) content emphasis, that is the emphasis the teachers have placed on the content tested, and (d) quality of instructional delivery, that is the adequacy of teaching the content. Another approach was offered by Brewer and Stasz (1996) who distinguished three categories of concern when assessing OTL. The first category is the curriculum content, implying the assessment of whether the students have been taught the subjects and topics that are essential to attain the standards. The second category includes the instructional strategies, assessing whether students have experience with particular kinds of tasks and solution processes. The third category refers to the instructional resources. Here are assessed, for example, issues of teacher preparation and quality of instructional materials.

Besides different aspects of OTL that can be assessed, different methods can also reveal the OTL students receive. These methods vary from teacher and student surveys through questionnaires, to carrying out classroom observations, and to analyzing instructional materials. For example, the first measurements of OTL are based on questionnaires in which teachers have to indicate whether particular mathematical topics or kinds of problems have been taught to students. Such questionnaires were used in the international comparative studies FIMS, SIMS and TIMSS (Floden, 2002). Furthermore, the TIMSS video studies are an example of revealing the OTL based on classroom observations (Hiebert et al., 2003). Another approach applied in TIMSS was to look at what content curricula offer. Schmidt et al. (1997, p. 4) see the curriculum – and in connection with this – textbook series as "a kind of underlying 'skeleton' that gives characteristic shape and direction to mathematics instruction in educational systems around the world" and that provides "a basic outline of planned and sequenced educational opportunities."

2.3 Opportunity-to-learn and the role of textbooks

Compared to the influence of curricula, textbooks play an even more direct role in what is addressed in instruction. Teachers' decisions about the selection of content and teaching strategies are often directly set by the textbooks teachers use (Freeman & Porter, 1989; Reys, Reys, &Chavez, 2004). Therefore, textbooks are considered to determine largely the degree of students' OTL (Schmidt et al., 1997;

Tornroos, 2005). This means that if textbooks differ students will get a different OTL (Haggarty & Pepin, 2002). As a result, different student outcomes will appear, which is confirmed by several studies which found a strong relation between the textbook used and the mathematics performance of the students (see, e.g. Tornroos, 2005; Xin, 2007).

The recognition of the role of textbooks in mathematics learning in students' chances to be taught particular mathematical topics and skills has recently led to a large amount of studies examining OTL offered in textbooks. For example, textbook analysis was applied to the distributive property (Ding & Li, 2010), the equal sign (Li, Ding, Capraro, & Capraro, 2008; McNeil et al., 2006), fractions (Charalambous, Delaney, Hsu, & Mesa, 2010), subtraction up to 100 (Van Zanten & Van den Heuvel-Panhuizen, 2014), non-routine problem solving (Kolovou, Van den Heuvel-Panhuizen, & Bakker, 2009; Xin, 2007), and mathematical modeling (Gatabi, Stacey, & Gooya, 2012).

2.4 Textbook analysis to identify the opportunity-to-learn

To disclose what content is intended to be taught to students, textbooks can be analyzed in several ways. In TIMSS, textbook analysis initially focused on investigating the content profiles of textbooks (Schmidt et al., 1997). Later, they were examined based on five measures (Valverde et al., 2002). The first measure is the classroom activities proposed by the textbook. The second measure corresponds to the amount of content covered in textbooks and the mode of presentation, whether abstract or concrete. The third measure deals with the sequencing of content. The fourth measure focuses on physical characteristics of textbooks, for example the size of the book and the number of pages. The fifth measure characterizes the complexity of the demands for student performance.

Another approach to textbook analysis was proposed by Pepin and Haggarty (2001). They distinguished four areas on which a textbook analysis can focus, namely (a) the mathematics topics presented in textbooks and the beliefs about the nature of mathematics that underlie the textbooks' content, (b) the methods suggested in textbooks to help students understand the textbooks' content, (c) the sociological contexts of textbooks which examines whether textbooks are adaptive to students with different performance levels, and (d) the cultural traditions in textbooks, focusing on how textbooks reflect the cultural traditions and values.

Charalambous et al. (2010) classified the approaches to textbook analysis in three categories, namely horizontal, vertical, and contextual. The horizontal analysis examines the general characteristics of textbooks, such as physical characteristics and the organization of the textbooks' content. This analysis gives a first impression of the OTL because it can provide information about the quantity of exposure of textbooks' content. However, information about the quality and the didactical aspects of the textbooks' content is not revealed by a horizontal analysis. Therefore, a vertical analysis is needed to address how textbooks present and treat the content. Such an analysis offers an in-depth understanding of the mathematical content. The third category, the contextual analysis, focuses on how textbooks are used in instructional activities. Therefore, Charalambous et al. (2010) argued that, in fact, only the first two categories are appropriate to analyze the characteristics of textbooks.

2.5 Opportunity-to-learn required for solving context-based tasks

The primary requirement for students' learning to solve context-based tasks is that students should be offered experiences to deal with essential characteristics of context-based tasks and should be given the necessary practice in handling these characteristics.

2.5.1 Nature of the context

A critical characteristic of context-based tasks is the nature of the context. Concerning mathematical problems, there are several views on what a context means (De Lange, 1995; OECD, 2003b; Van den Heuvel-Panhuizen, 2005). In the present study the focus is on real-world contexts, which in PISA are called 'extra-mathematical contexts' (OECD, 2003b). In PISA context-based tasks are defined as problems presented within a 'situation' which can refer to a real-world or fantasy setting, can be imagined by students, and can include personal, occupational, scientific and public information. This interpretation of a context matches what De Lange (1995) called a 'relevant and essential context', which he contrasted with a 'camouflage context'. Tasks with the latter context are merely dressed-up bare problems, which do not require modeling because the mathematical operations needed to solve the task are obvious.

2.5.2 Mathematization and modeling process

To solve tasks which include relevant and essential contexts, students need to transform the context situation into a mathematical form through the process of mathematization (OECD, 2003b). Therefore, it is important that context-based tasks use setting or situation that gives access to and supports the process of mathematization. In other words, it is crucial that the tasks provide information that can be organized mathematically and offer opportunities for students to use their knowledge and experiences (Van den Heuvel-Panhuizen, 2005).

The process of solving context-based tasks requires interplay between the real world and mathematics (Schwarzkopf, 2007) and is often described as a modeling process (Blum & Leiss, 2007), which in general contains the following steps (1) understanding the problem situated in reality; (2) transforming the real-world problem into a mathematical problem; (3) solving the mathematical problem; and (4) interpreting the mathematical solution in terms of the real situation.

2.5.3 Adequate mathematical procedures

A further characteristic of a context-based task is that the task cannot be solved by simply translating the task into a mathematical procedure (Verschaffel, Dooren, Greer, & Mukhopadhyay, 2010). This means that standard problems which have a straightforward relation between the problem context and the necessary mathematics do not help students building up experience to transform a real-world problem into a mathematical problem. Therefore, students should be set tasks in which the necessary mathematical procedures are more implicit.

2.5.4 Different information types

Solving a context-based task is not just combining all the information given in the task (Verschaffel et al., 2010). A context-based task may contain more information than needed for solving the problem or may even lack necessary information. Providing more or less information than needed for solving a context-based task is a way to encourage students to consider the context used in the task and not just take the numbers out of the context and process them mathematically in an automatic way (Maass, 2007). Therefore, students should be offered opportunities to deal with different types of information such as matching, missing, and superfluous information (Maass, 2010). In this way they can learn to select relevant information, and to add and ignore information.

2.5.5 Cognitive demands

A final requirement to support students' learning to solve context-based tasks is that students can build experience with tasks covering the full range of levels of cognitive demands, including reproduction, connection, and reflection tasks

(OECD, 2009a). Reproduction tasks require the recall of mathematical properties and the application of routine procedures or standard algorithms. Such tasks do not require mathematical modeling. Connection tasks require integrating and linking different mathematical curriculum strands or different representations of a problem. These tasks also require interpreting a problem situation and engaging students in simple mathematical reasoning. Reflection tasks include complex problem situations in which it is not obvious in advance what mathematical procedures have to be carried out. In fact, this latter category of tasks is the closest to our definition of context-based tasks. What competences students will eventually master depends on the cognitive demands of mathematics tasks they have been engaged in (Stein &Smith, 1998), therefore these reflection tasks should not be lacking in instruction.

2.5.6 Crucial aspects of opportunity-to-learn context-based tasks

In sum, three aspects of OTL are crucial to develop the competence of solving context-based tasks. The first aspect is giving students experience to work on tasks with real-world contexts and implicit mathematical procedures. The second aspect is giving students tasks with missing or superfluous information. The last aspect is offering students experience to work on tasks with high cognitive demands.

2.6 Difficulties of Indonesian students in solving context-based tasks

The aforementioned aspects of OTL also emerged as relevant in the first study of the CoMTI project (Wijaya et al., 2014) in which we investigated Indonesian students' difficulties when solving context-based tasks. In the study, which involved a total of 362 Indonesian students (233 ninth graders and 129 tenth graders), four types of students' errors were identified, that is comprehension, transformation, mathematical processing, and encoding errors. Comprehension errors correspond to students' inability to understand a context-based task, including the inability to select relevant information. Transformation errors are related to students' inability to identify the correct mathematical procedure to solve a problem. The mathematical processing errors refer to mistakes in carrying out mathematical procedures. Encoding errors refer to answers that are unrealistic and do not fit the real-world situation described in the task.

Table 1 shows that comprehension and transformation errors were the most dominant errors made by the Indonesian ninth- and tenth-graders when solving context-based mathematics tasks. Within the former category most errors were made in selecting the relevant information; whereas in the latter category the students mostly used wrong procedures.

Table 1.

Frequency of students' error when solving context-based mathematics tasks (Wijaya et al., 2014)

Type of error	Frequency (total errors = 1718)	Sub-type of error	Frequency
Comprehension	38%	- Misunderstanding the instruction	35%
		- Misunderstanding a keyword	15%
		- Error in selecting relevant information	50%
		Total comprehension error = 653	100%
Transformation	42%	- Using a common mathematical procedure that does not apply to the problem situation	12%
		- Taking too much account of the context	8%
		- Treating a graph as a picture	12%
		- Otherwise using wrong mathematical procedure	68%
		Total transformation error = 723	100%
Mathematical processing	17%	--[a]	
		Total mathematical processing error = 291	
Encoding	3%	--[b]	
		Total encoding error = 51	

[a] The sub-categories of mathematical processing error are task-specific.
[b] There is no sub-type for the encoding error.

In addition, in agreement with the results of the PISA study 2003 (OECD, 2009a), we found in our study that the reproduction tasks were the easiest for the students (67% of the responses to these tasks got a full credit), whereas the tasks with higher cognitive demand, the connection and the reflection tasks, had lower percentages of correct answers (in both types of tasks 39% of the responses got a full credit).

2.7 Research questions

The purpose of the present study was to disclose what OTL Indonesian mathematics textbooks offer to Indonesian students for developing the ability to solve context-based tasks. Based on the literature review the focus was on four aspects of OTL: the exposure to context-based tasks, the purpose of the context-based tasks, the type of information provided in tasks, and the type of cognitive demand required by tasks. Therefore, the following research questions were addressed:

1. *What are the amount of exposure to and the purpose of context-based tasks in Indonesian mathematics textbooks?*
2. *To what extent are different types of information provided in tasks in Indonesian mathematics textbooks?*
3. *What are the cognitive demands of tasks in Indonesian mathematics textbooks?*

The reason for this study was to find an explanation for the difficulties Indonesian students experience when solving context-based tasks. Therefore, we also investigated the connection between students' difficulties when solving context-based tasks with the OTL in Indonesian mathematics textbooks. This resulted in the next research question:

4. *What is the connection between students' errors when solving context-based tasks with the characteristics of tasks in Indonesian mathematics textbooks?*

3 Method

3.1 Mathematics textbooks analyzed

To answer the research questions we carried out a textbook analysis in which we focused on grade 8. Although according to the National Curriculum (Pusat Kurikulum, 2003a, 2003b), the topics dealt with in the PISA tasks included in the CoMTI test are taught from grade 7 to 10, the main emphasis on these topics lies in grade 8. Of the topics, almost half are taught in grade 8 and the remaining part is distributed over the three other grades. Moreover, grade 8 can be considered as a relevant grade year to prepare students for being able to solve context-based tasks as assessed in the PISA studies.

The mathematics textbooks chosen for this analysis are shown in Table 2. These are the textbook series that were used in the schools involved in the first study of the CoMTI project (Wijaya et al., 2014). All the schools either used a combination of two of these textbook series or used all three. Of the textbook series MJHS, which is bilingual, we only analyzed the Indonesian pages.

Table 2.

Analyzed textbooks and material

Textbook series	Abbreviation	Material involved in analysis	Publisher
Matematika: Konsep dan Aplikasinya	MKA	"For Junior High School grade VIII"	Indonesian Ministry of National Education
Matematika	MS	"For Junior High School grade VIII: 2A & 2B"	Private publisher
Mathematics for Junior High School	MJHS	"Part 2"	Private publisher

The three textbooks have a similar main structure. All have nine chapters, each dealing with one mathematics topic. These chapters contain several sub-chapters discussing specific aspects of the mathematics topic covered in the chapter. For example, the chapter on 'Equations of straight lines' contains the sub-chapters: (a) the general form of equations of straight lines, (b) the gradient, (c) the relation between a gradient and the equation of a straight line, and (d) the application of equations of straight lines.

Each sub-chapter consists of an explanation section, followed by one or more worked example sections, and a task section with regular tasks which the students have to solve themselves. The explanation section discusses a particular mathematics concept, for example, how a rule or formula is obtained (see Figure 1). The worked example section contains one or more tasks for which an answer is given (see Figure 2). This section serves as a bridge between the explanation section and the task section (Figure 3).

Although the three textbooks have the same number of chapters, they have different numbers of sub-chapters because they discuss the mathematics topics in different levels of detail. For example, MJHS discusses the topic of 'Equations of straight lines' in three sub-chapters, whereas the other textbooks spend four sub-chapters on this topic.

b. The equation of line $y = mx + c$

Equation	Gradient	Point on Y axis
$y = \frac{3}{4}x$	$\frac{3}{4}$	(0, 0)
$y = \frac{3}{4}x + 2$	$\frac{3}{4}$	(0, 2)
$y = \frac{3}{4}x + 5$	$\frac{3}{4}$	(0, 5)
$y = \frac{3}{4}x - 3$	$\frac{3}{4}$	(0, –3)

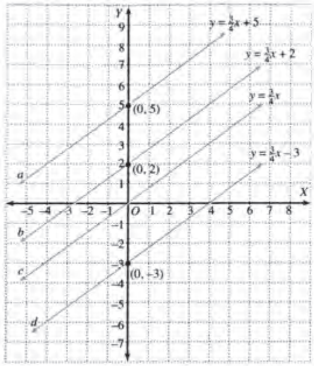


Figure 3.9

From the table above we have the following relations:

Line $y = \frac{3}{4}x + 2$ has a gradient of $\frac{3}{4}$ and passes (0, 2)

Line $y = \frac{3}{4}x - 3$ has a gradient of $\frac{3}{4}$ and passes (0, –3)

Line $y = \frac{3}{4}x + 5$ has a gradient of $\frac{3}{4}$ and passes (0, 5)

Based on the aforementioned explanation, the conclusion is:

> Line $y = mx + c$ has a **gradient of** m and **passes** $(0, c)$.
> Point $(0, c)$ is the **intersection** of line $y = mx + c$ and the **Y axis**.

Figure 1. Explanation section (MS 2A, p. 86).

Example

1. Determine the equations of the following lines:
   a. A line has a gradient of 4 and passes (0,-7)
   b. A line has a gradient of $-3\frac{1}{2}$ and passes (0,5)

   Answer:

   a. Gradient = 4, then $m = 4$
   Passing point (0,-7), then c = -7
   The equation is:
   $y = mx + c$
   $y = 4x - 7$

   b. Gradient = $-3\frac{1}{2}$, then $m = -3\frac{1}{2}$
   Passing point (0,5), then c = 5
   The equation is:
   $y = mx + c$
   $y = -3\frac{1}{2}x + 5$

2. Sketch the graph of $y = -1\frac{1}{2}x + 4$

   Answer:

   $y = -1\frac{1}{2}x + 4$
   Gradient $= -1\frac{1}{2} = -\frac{3}{2} = \frac{-3}{2}$
   It means that the change of $y = -3$ and the change of $x = 2$.
   The y-intercept is (0,4).
   The line passes (0,4), then from (0,4) we move 2 units to the right and move down 3 units. We get the second point.
   Sketch a line passing through (0,4) and the second point.

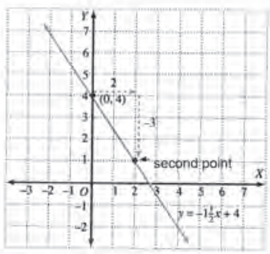
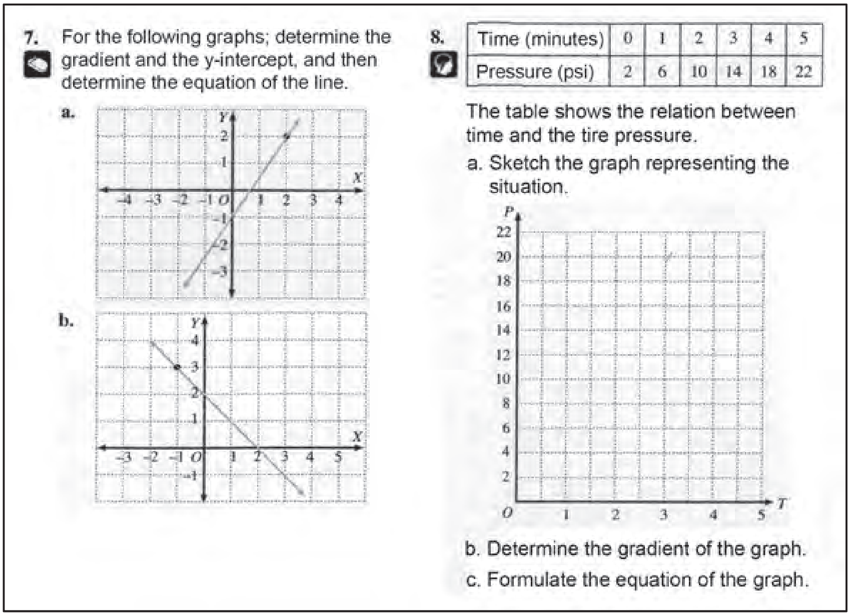


Figure 2. Worked example section (MS 2A, p. 87).



7. For the following graphs; determine the gradient and the y-intercept, and then determine the equation of the line.

a.

b.

8.

Time (minutes)	0	1	2	3	4	5
Pressure (psi)	2	6	10	14	18	22

The table shows the relation between time and the tire pressure.

a. Sketch the graph representing the situation.

b. Determine the gradient of the graph.

c. Formulate the equation of the graph.

Figure 3. Task section (MS 2B, p. 88).

3.2 Procedure of textbook analysis

Following Charalambous et al. (2010), we analyzed the textbooks from two perspectives, namely their physical characteristics and instructional components (horizontal analysis) and the characteristics of the tasks (vertical analysis).The physical characteristics and instructional components of the textbooks were investigated to provide information about the quantity of exposure to textbook content. We collected data about the page size, the number of pages, and the page surface area. Furthermore, we counted the number of explanation sections, the worked example sections, the task sections with regular tasks, the tasks in worked example sections, and the regular tasks in task sections. In this analysis we considered as a task every question or problem with the answer provided (tasks in the worked example sections) or for which the students have to give an answer (regular tasks in the task sections). For example, although the task section in Figure 3 contains two task numbers (7 and 8), in our approach this section has five tasks (7a, 7b, 8a, 8b, and 8c). The tasks in the worked example sections were counted in a similar way; therefore Figure 2 contains three tasks (1a, 1b, and 2). The tasks in the worked example sections and in the task sections could be in a bare format (with only symbols) or context-based format.

Table 3

Analysis framework for textbook analysis

Task characteristic	Sub-category	Explanation
Type of context	No context	- Refers only to mathematical objects, symbols, or structures.
	Camouflage context	- Experiences from everyday life or common sense reasoning are not needed. - The mathematical operations needed to solve the problems are already obvious. - The solution can be found by combining all numbers given in the text.
	Relevant and essential context	- Common sense reasoning within the context is needed to understand and solve the problem. - The mathematical operation is not explicitly given. - Mathematical modeling is needed.
Purpose of context-based task	Application	- The task is given after the explanation section
	Modeling	- The task is given before the explanation section.
Type of information	Matching	- The tasks contain exactly the information needed to find the solution.
	Missing	- The tasks contain less information than needed so students need to find missing information.
	Superfluous	- The tasks contain more information than needed so students need to select information.

Task characteristic	Sub-category	Explanation
Type of cognitive demand	Reproduction	- Reproducing representations, definitions or facts. - Interpreting simple and familiar representations. - Memorization or performing explicit routine computations/procedures.
	Connection	- Integrating and connecting across content, situations or representations. - Non-routine problem solving. - Interpretation of problem situations and mathematical statements. - Engaging in simple mathematical reasoning.
	Reflection	- Reflecting on, and gaining insight into, mathematics. - Constructing original mathematical approaches. - Communicating complex arguments and complex reasoning. - Making generalizations.

3.2.1 Framework for textbook analysis focusing on OTL context-based tasks

The vertical analysis was meant to investigate the OTL to solve context-based tasks. For this purpose an analysis framework (see Table 3) was developed addressing the task characteristics: type of context, type of information, and type of cognitive demand. To investigate the amount of exposure to context-based tasks, first we identified the type of context used in the tasks in Indonesian mathematics textbooks. We used the categories distinguished by De Lange (1995) including 'no context', 'camouflage context', and 'relevant and essential context'. As an extension to the type of context, we also included the 'purpose of context-based task'. The reason for including this characteristic was to distinguish whether a context-based task is used for applying mathematics or for mathematical modeling (Muller & Burkhardt, 2007; Niss, Blum, & Galbraith, 2007). In the former the solvers know the mathematics they should apply because the task is given after an explanation section. In the latter, the solvers start with a real-world problem and have to identify what mathematics is suitable to solve the problem.

For types of information, we used three types described by Maass (2010): 'matching information', 'missing information', and 'superfluous information'. The categories for the cognitive demands of the tasks were established based on PISA's competence clusters (OECD, 2003b) which included reproduction, connection, and reflection tasks. These categories were used to identify the characteristics of both the tasks in the worked example sections and in the task sections.

3.2.2 Coding procedure

All tasks in the three textbooks were coded by the first author using the analysis framework as shown in Table 3. Afterwards the reliability of the coding was checked through an additional coding by an external coder who coded a random selection of about 15% of the tasks. This extra coding resulted in a Cohen's Kappa of .75 for the type of context, 1.00 for the purpose of the context-based task, .74 for the type of information, and .84 for the type of cognitive demands. These results indicate the coding was reliable (Landis & Koch, 1977).

4 Results

4.1 Physical characteristics and instructional components of the textbooks

As shown in Table 4 MS has the largest page number and page surface area; whereas MJHS has the lowest number for these physical characteristics. MKA takes a middle position and has a page number and page surface area close to the average of mathematics textbooks from other countries. As reported by Valverde et al. (2002), the international average number of pages for grade eight mathematics textbooks is 225 pages and the international median of page surface area is 115,000 cm^2.

Table 4.
Physical characteristics and instructional components of Indonesian mathematics textbooks

	Textbook		
	MJHS[a]	MKA	MS
Physical characteristic			
Page size (in mm)	205 × 277	176 × 250	176 × 250
Number of pages	146	252	336
Page surface area[b] (in cm^2)	82,906	110,880	147,840
Instructional components			
Number of explanation sections	25	42	56
Number of worked example sections	80	73	90
Number of task sections	79	63	68
Total number of tasks	437	531	1187
Number of tasks in worked example sections	119	91	218
Number of tasks in task sections	318	440	969

[a] Only the Indonesian pages of this textbook were analyzed.
[b] Multiplication of the page number and the area of a page (Valverde et al., 2002).

Regarding instructional components, MS has the largest number of explanation sections, which is more than double that in MJHS. The largest worked example sections are also in MS, but for this instructional component the difference between the three textbooks is small. For the task sections, MJHS has slightly more than the two other textbooks. We found a large difference between the textbooks for the number of tasks. MS has a total of 1187 tasks, which is double the number in MKA and triple that in MJHS. A similar ratio was found for the regular tasks in the task sections, with respectively 318, 440, and 969 tasks for MJHS, MKA, and MS.

4.2 The amount of exposure and the purpose of context-based tasks

In relation to Research question 1 it was found that only 8% to 16% of the tasks in the three textbooks are context-based, with the highest proportion of these tasks in MS (see Table 5). In MS the proportion of context-based tasks in the worked example sections and the task sections is about the same, whereas in MJHS and MKA context-based tasks are used more often in the task sections.

Table 5.

Frequency of types of context

	Type of context	Textbook					
		MJHS		MKA		MS	
		n	%	N	%	n	%
Tasks in worked example sections	Relevant and essential context	1	1	2	2	10	5
	Camouflage context	4	3	1	1	28	13
	No context	114	96	88	97	180	83
	Total	119	100	91	100	218	101
Regular tasks in task sections	Relevant and essential context	7	2	17	4	29	3
	Camouflage context	24	8	26	6	127	13
	No context	287	90	397	90	813	84
	Total	318	100	440	100	813	100
All tasks	Relevant and essential context	8	2	19	4	39	3
	Camouflage context	28	6	27	5	155	13
	No context	401	92	485	91	993	84
	Total	437	100	531	100	1187	100

In all three textbooks most of the contexts belong to the category of camouflage context. Regarding the purpose of the context-based tasks, we found that all these tasks were intended for application as indicated by their position after the explanation sections.

Examples of each type of context are given in Figure 4, Figure 5, and Figure 6. Both tasks in Figure 4 and Figure 5 are related to the concept of gradient. The task in Figure 4 only uses mathematical objects and symbols.

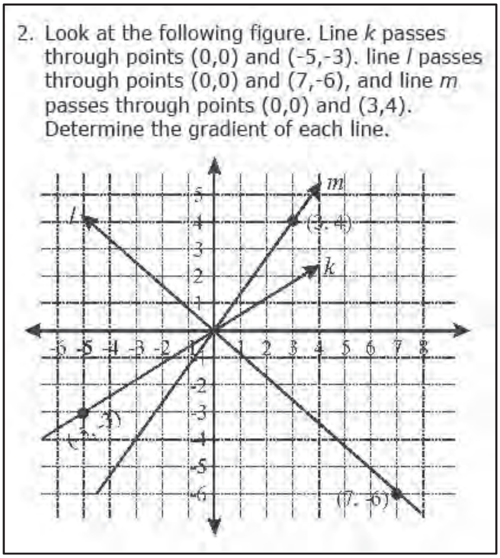

2. Look at the following figure. Line *k* passes through points (0,0) and (-5,-3). line *l* passes through points (0,0) and (7,-6), and line *m* passes through points (0,0) and (3,4). Determine the gradient of each line.

Figure 4. Task with no context (MKA, p. 70).

The task in Figure 5 is set in the context of a ski slope, which is not a daily-life situation for Indonesian students. This context is an example of a camouflage context, because it can be neglected in solving the problem. Although the task includes a real world situation, the photograph of the ski slope is cut and arranged so that it exactly resembles a straight and the arrows informing the vertical and horizontal differences also resemble a coordinate system. Furthermore, the words 'slope' and 'gradient' are mentioned explicitly and the numbers in the picture are given in such a way that students can immediately interpret the problem as a mathematical problem and follow the common procedure for calculating the slope or gradient.

2 **Determine the slope or the gradient of the ski slope shown in the following figure.**

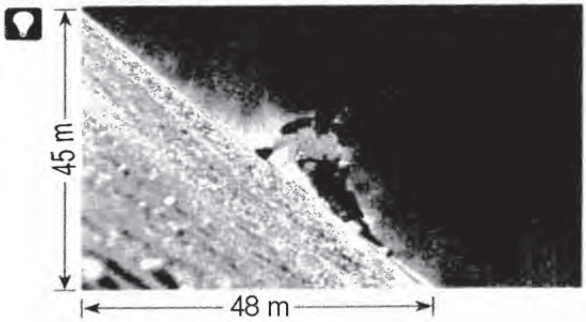


Figure 5. Task with camouflage context (MS 2A, p. 79).

4.

The price of 2 pairs of shoes and 3 pairs of sandals is 270,000 IDR. For the same models, 3 pairs of shoes and 4 pairs of sandals cost 390,000 IDR. Determine the price of 4 pairs of shoes and 5 pairs of sandals.

Figure 6. Task situated in relevant and essential context (MS 2A, p. 132).

A task with a relevant and essential context is shown in Figure 6. This task asks to determine the price of four pairs of shoes and five pairs of sandals. To solve this task students are expected to transform the 'price problem' into linear equations with two variables. However, this mathematical procedure is not explicitly mentioned in the task, nor are the numbers presented so that a solution procedure is afforded. The variables for setting up the equations, that is, the number of pairs of shoes and the number of pairs of sandals, are not explicitly indicated in the task. Consequently, students need to identify the relevant information and a solution strategy for solving the task.

4.3 Types of information provided in tasks

As an answer to Research question 2 we found that for the bare tasks , that is tasks with no context, almost all (98%, 99%, and 99% of the bare tasks in MJHS, MKA, and MS respectively) give students exactly the information needed to solve the problems. However, for the context-based tasks the proportions of tasks with matching information are lower (i.e. 86%, 85%, and 89% of the context-based tasks in respectively MJHS, MKA, and MS) (see Table 6). The remaining tasks have missing information (i.e., 14%, 15%, and 10% of the context-based tasks in MJHS, MKA, and MS respectively), whereas, except one task in MS, no tasks with superfluous information were found.

Table 6.

Frequency of types of information in context-based tasks

	Type of information	Textbook					
		MJHS		MKA		MS	
		n	%	n	%	n	%
Context-based tasks in worked example sections	Matching	4	80	2	67	35	92
	Missing	1	20	1	33	2	5
	Superfluous	0	0	0	0	1	3
	Total	5	100	3	100	38	100
Context-based tasks in task sections	Matching	27	77	37	86	138	88
	Missing	4	23	6	14	18	12
	Superfluous	0	0	0	0	0	0
	Total	31	100	43	100	156	100
All context-based tasks	Matching	31	86	39	85	173	89
	Missing	5	14	7	15	20	10
	Superfluous	0	0	0	0	1	1
	Total	36	100	46	100	194	100

An example of a task with matching information is shown in Figure 7. This task involves finding the width of a river. The width of the river is mathematically the leg of a right-angled triangle which can be found by applying the Pythagorean Theorem. All the information required to apply the Pythagorean Theorem, that is the lengths of the hypotenuse and another leg, was given.

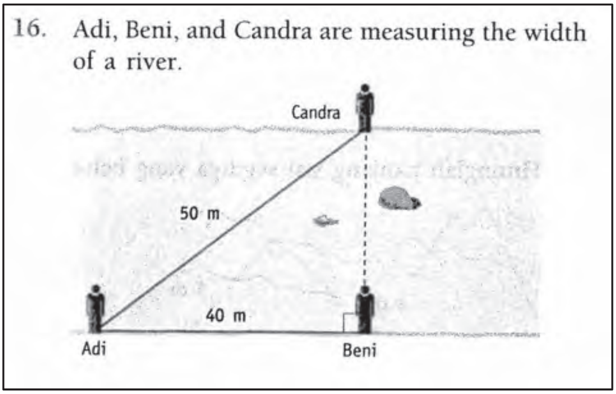


Figure 7. Task with matching information (MJHS, p. 156).

In the task in Figure 8 the students are asked to find the minimum length of rope needed to tie up the three pipes. This task is an example of a task with missing information, in the sense that not all data are directly given to carry out a mathematical procedure leading to the answer. In fact, for solving this task students have to add the lengths of the three external tangents and the three arcs. However, these lengths are not given. Thus the students must first generate this data by using further knowledge that can be derived from the contextual situation, such as the three arcs together constituting precisely a full circle and that one tangent equals two radii, that is, equals the diameter of a pipe.

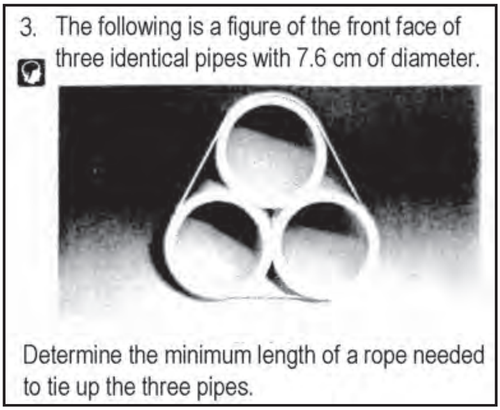


Figure 8. Task with missing information (MS 2B, p. 143).

The task in Figure 9 is an example of a task with superfluous information. The task was about finding the height of a flashlight. The task provided the diameter of the top circle and the bottom circle of the flashlight, and the shape and volume of the flashlight box. The shape of the flashlight box was cuboid; therefore the diameter of the bottom of the flashlight was not needed to solve the task. Noteworthy is that we only found one task with superfluous information, which is in the worked example section.

3. A flashlight, which its dimension is shown in the figure, will be put in its box. Determine the height of the flashlight if the volume of the box is 648 cm².

Answer:

The dimension of the box is 6 cm long, 6 cm wide, and *t* cm high.

$$V = plt$$
$$648 = 6 \times 6 \times t$$
$$648 = 36t$$
$$t = \frac{648}{36}$$
$$t = 18$$

The height of flashlight = the height of the box = 18 cm

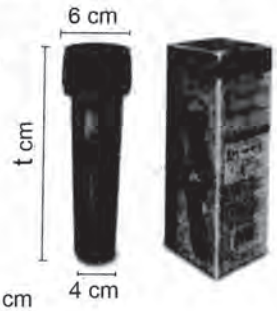


Figure 9. Task with superfluous information (MS 2B, p. 106).

4.4 Cognitive demands required for solving the tasks

Table 7 shows the answers to Research question 3. In all three textbooks almost all bare tasks were identified as reproduction tasks (95%, 93%, and 93% of the bare tasks in MJHS, MKA, and MS respectively).The three textbooks have a small proportion of connection tasks, ranging from 5 to 7%. Only MS includes reflection tasks, that is 1% of bare tasks.

Table 7.

Frequency of types of cognitive demands of bare tasks

	Type of cognitive demands	Textbook					
		MJHS		MKA		MS	
		N	%	n	%	n	%
Bare tasks in worked example sections	Reproduction	110	96	83	94	172	96
	Connection	4	4	5	6	8	4
	Reflection	0	0	0	0	0	0
	Total	114	100	88	100	180	100
Bare tasks in task sections	Reproduction	271	96	370	93	756	93
	Connection	16	4	27	7	48	6
	Reflection	0	0	0	0	9	1
	Total	287	100	397	100	813	100
All bare tasks	Reproduction	381	95	453	93	928	93
	Connection	20	5	32	7	56	6
	Reflection	0	0	0	0	9	1
	Total	401	100	485	100	993	100

Focusing only on the context-based tasks (see Table 8), including a relevant and essential context or a camouflage context, the proportions of task types according to their cognitive demand changed remarkably. The proportions of reproduction tasks in the context-based tasks (47%, 33%, and 56% of the context-based tasks in MJHS, MKA, and MS respectively) are much lower than in the bare tasks. In the context-based tasks substantial proportions of connection tasks were found (50%, 67%, and 42% of the context-based tasks in MJHS, MKA, and MS respectively). However, reflection tasks are still either a minority or absent. Only one such task was found in MJHS and three in MS. Differences between the three textbooks were also found for the balance in the type of tasks. MKA contains more connection tasks than reproduction tasks, whereas MJHS and MS have about the same proportion of reproduction and connection tasks.

Table 8.
Frequency of types of cognitive demands of context-based tasks

	Type of cognitive demands	Textbook					
		MJHS		MKA		MS	
		n	%	n	%	n	%
Context-based tasks in worked example sections	Reproduction	2	40	1	33	22	58
	Connection	3	60	2	67	16	42
	Reflection	0	0	0	0	0	0
	Total	5	100	3	100	38	100
Context-based tasks in task sections	Reproduction	15	48	14	33	87	56
	Connection	15	48	29	67	66	42
	Reflection	1	3	0	0	3	2
	Total	31	99	43	100	156	100
All context-based tasks	Reproduction	17	47	15	33	109	56
	Connection	18	50	31	67	82	42
	Reflection	1	3	0	0	3	2
	Total	36	100	46	100	194	100

Figure 4 and Figure 5 show examples of reproduction tasks. The word 'gradient' was explicitly mentioned in the tasks so students could easily identify the required mathematics procedure and apply it by using all the information. Figure 8, the task about the three pipes, is an example of a connection task situated in a camouflage context. The main focus of this task was finding the minimum length of rope to tie up three pipes, which mathematically was related to the concept of common tangent of two circles. To find the minimum length of rope students need to find not only the length of the three common tangents, but also the length of the arcs. Here, a connection to the concept of an equilateral triangle is needed to find the measure of the central angle required to calculate the length of the arcs.

A connection task could also be assigned to a bare task, for example the task in Figure 10. This task was about determining the perimeter and the area of a non-regular shape. Connecting across representations, that is, circles with different sizes was required to solve this task.

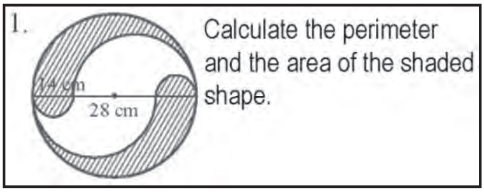


Figure 10. Connection task situated in no context (MKA, p. 168).

Figure 11 shows an example of a reflection task in a relevant and essential context. The task involved gaining insight into the mathematical meaning of 'horizontal floor'. Although the figures of triangles and their measures are provided, the mathematical concept related to 'horizontal floor' was not explicitly given. Here, students needed to identify that the task was related to the Pythagorean Theorem.

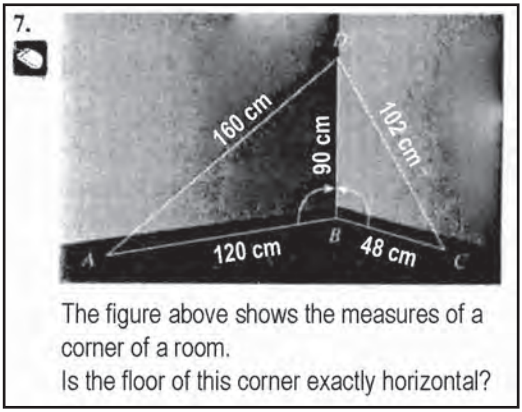


Figure 11. Reflection task situated in a relevant and essential context (MS 2A, p. 151).

4.5 Opportunity-to-learn provided in textbooks and students' errors

To answer Research question 4 we related the results from the textbook analysis to the errors students made when solving context-based tasks, as found in our first study in the CoMTI project (Wijaya et al., 2014). Because we only knew that the group of ninth-graders used the textbooks involved in our analysis, we decided to focus on the results from the ninth-graders and exclude the tenth-graders. Moreover, because the schools in our first study used different combinations of textbooks, we could not exactly classify students' errors according to the particular textbooks with which they were taught. Therefore, in Table 9 we included the total of errors over all schools and for each textbook the proportions of tasks that relate to these errors.

Table 9.
Relation between students' errors and task characteristics in textbooks

Ninth-graders' errors found in Wijaya et al. (2014) (934 errors)	Task characteristic		Textbook MJHS	MKA	MS
			Proportion of tasks (%)		
Comprehension errors: Proportion of errors in selecting relevant information: 21% of all errors	Type of information in context-based tasks	Matching	86	85	89
		Missing	14	15	10
		Superfluous	0	0	1
Transformation errors: Proportion of these errors: 45% of all errors	Type of context in all tasks	No context	92	91	84
		Camouflage context	6	5	13
		Relevant and essential context	2	4	3
Percentage of wrong answers for each type of cognitive demand - Reproduction: 32% - Connection: 61% - Reflection: 61%	Type of cognitive demand in context-based tasks	Reproduction	47	33	56
		Connection	50	67	42
		Reflection	3	0	2

Combining the findings from the textbook analysis and the error analysis, a recognizable similarity between these two findings emerged. In 21% of the total of 934 errors (made by 233 ninth-graders) students made comprehension errors by not selecting the relevant information for solving the tasks. Correspondingly, the textbook analysis revealed that the textbooks mainly provide context-based tasks with matching information, and only 11% to 15% of the context-based tasks had missing or superfluous information.

In relation to the high proportion of transformation errors, the textbook analysis disclosed that the proportions of context-based tasks in the three textbooks only range from 8% to 16%. It was also found that only 2% to 4% of these tasks use relevant and essential contexts. Furthermore, all context-based tasks are located after the explanation sections in which a particular mathematics topic is discussed.

This means these tasks are just meant to apply what has been demonstrated in the explanation section. Thus, from the textbooks' content, students can scarcely build up experience in constructing a mathematical model by mathematizing a real-world situation.

Lastly, regarding the percentage of wrong answers for the context-based tasks of the various types of cognitive demand we also found a match with what is offered in the textbooks. The lowest percentage of wrong answers was obtained for the reproduction tasks which cover 33% to 56% of the context-based tasks in the three textbooks. For the connections tasks, which form 42% to 67% of the context-based tasks, about half of the answers were wrong. The largest percentage of wrong answers belonged to the reflection tasks which include only 0% to 3% of the context-based tasks in the textbooks.

5 Discussion

As the PISA studies (OECD, 2003a, 2004, 2007, 2010) have shown and our first CoMTI study (Wijaya et al., 2014) has confirmed, Indonesian students have difficulties in solving context-based tasks. The present study was meant to disclose a possible reason for these difficulties by conducting an analysis of three Indonesian mathematics textbooks. This analysis focused on the OTL to solve context-based tasks offered by the textbooks. For identifying the characteristics of the tasks in the textbooks we developed an analysis framework including four perspectives: the types of contexts in tasks, the purpose of the context-based tasks, the information used in tasks, and the types of cognitive demands in tasks.

5.1 Opportunity-to-learn and students' difficulties

Our analysis revealed that context-based tasks are hardly available in Indonesian mathematics textbooks. The textbooks mostly provide tasks without a context, which do not require mathematization or modeling activities from the students. Furthermore, the few context-based tasks in the textbooks do mostly not have a real relevant and essential context. In addition, the context-based tasks in the Indonesian textbooks are all located after the explanation sections. This means the mathematics procedure to be applied is more or less given and students do not have to identify an appropriate mathematics procedure to solve the tasks; and consequently not getting enough experience to develop their ability to transform a context-based task into a mathematical problem. This lack of experience is a plausible explanation for the high number of transformation errors made by Indonesian students. The foregoing conclusion is in agreement with several studies which showed that students' lack of experience in a particular type of task corresponds to their difficulties with the task. For example, Haines and Crouch (2007) mentioned that particular difficulties of students in mathematical modeling are due to unfamiliarity with tasks in which students have to identify what mathematics is appropriate to solve the problem. Similarly, Stein and Smith (1998) reported that students' lack of prior experience with open-ended tasks leads to difficulties when solving tasks in which the mathematical procedure is implicit. A lack of particular experience is also directly related to a missing OTL in textbooks as shown in the studies by Li et al. (2008) and McNeil et al. (2006) which revealed that students have difficulties in interpreting the equal sign as a relation because the textbooks they use rarely provide equal signs with operations on both sides.

A specific characteristic of context-based tasks that we found missing in the three Indonesian textbooks is the use of incomplete or irrelevant information, which, according to Forman and Steen (2001), and Greer, Verschaffel, and Mukhopadhyay (2007), is crucial for developing students' ability to apply mathematics in real-world problems. Of the 276 context-based tasks in the three textbooks, only 32 have missing information and just one task contains superfluous information. This means Indonesian students who worked with these textbooks could not really build up experience in selecting relevant information or using knowledge of the context to add missing information. Moreover, taking the freedom to include one's own knowledge or to neglect given information is something with which the students should be familiar. Chapman (2006) emphasized that when students are encouraged to use their own real-world experiences and to relate school experiences to life outside school, they will consider contextual information in a task as important to comprehend and solve the task. The lack of tasks which give students such experiences could explain why the students made so many comprehension errors.

The substantial number of errors made by Indonesian students in tasks with high cognitive demands can also be traced back to the content of the Indonesian textbooks. Of the context-based tasks, less than 3% were reflection tasks. However, the connection tasks were found in about half the context-based tasks. This proportion is similar to the proportion of connection tasks in the PISA study 2003 (OECD, 2009a), which might give an impression that Indonesian students have OTL to solve these tasks. However, we should take into account that the context-based connection tasks in Indonesian textbooks are low in number. Over the three textbooks, only 131 context-based tasks (see Table 8; MJHS: 18; MKA: 31; MS: 82) out of all the 2155 tasks together (see Table 4; MJHS: 437; MKA: 531; MS: 1187) ask for integrating and connecting different mathematical curriculum strands, or linking different representations of a problem.

In sum, the results from our analysis of three Indonesian textbooks provide evidence of a relation between the errors Indonesian students make when solving context-based tasks and the content offered by the textbooks they use. This conclusion adds to earlier studies which showed a positive relation between OTL provided in textbooks and student achievement: the students learn what is "taught" by the textbook. For example, Tornroos (2005) found a high correlation between student achievement in a test and the amount of textbook content related to the test items. Also, Xin (2007) revealed that the algorithmic strategy used by Chinese students to solve word problem tasks was the strategy suggested in their textbooks.

5.2 Educational implications

Based on our findings, we recommend including more context-based tasks in textbooks. Moreover, these tasks should not only be given after an explanation section, because then the mathematical procedure to be chosen is more or less fixed (see also De Lange, 2003). The quality of the context-based tasks should also be of concern. Textbooks should include context-based tasks which offer students opportunities for mathematization (Freudenthal, 1986; Van den Heuvel-Panhuizen, 2005). This means that instead of camouflage contexts, relevant and essential contexts should be used which demand mathematical organization or ask for mathematization. The context-based tasks to be included in textbooks should also have superfluous or missing information. Giving such tasks will provide students not only OTL to select relevant information (Maass, 2010), but also to identify appropriate mathematics procedures (Greer et al., 2007). Lastly, attention should also be paid to the cognitive demands of context-based tasks. The investigated textbooks contain too few reflection tasks to make it possible for students to develop their ability in complex reasoning. Including more reflection tasks is essential because they stimulate mathematical thinking and reasoning related to authentic settings (OECD, 2003b).

Although this study was situated in Indonesia, the results of the study may also be beneficial for other countries where students have a low performance in context-based tasks. For such countries, our study gives strong indications for examining the OTL to solve context-based tasks as offered in the textbooks in use in these countries.

5.3 Limitations of the study and directions for future research

To be able to provide strong indications for the relation of OTL as offered in textbooks and the achievements of students, it is necessary to know at least of every student which textbook was used to teach him or her. Only then it is possible to obtain a direct proof of this relation. However, in our study we did not have a one-to-one link between students' errors and the textbook the students worked with because the schools involved in our study used a combination of textbooks. Not knowing which textbook(s) each student used can be seen as a limitation of our study; yet we found that in the three textbooks the number of context-based tasks offered and the nature of these tasks were quite similar. However, for getting more robust evidence for our findings, it would be necessary to conduct a further study in which it is known for all students with which textbook(s) they are taught. In addition, including more grades in a textbook analysis would provide a better overview of the OTL in textbooks and the relation with students' achievements. Moreover, for the purpose of generalizability, our study could be repeated in other countries where students have low performance on, for example, context-based tasks used in the PISA studies.

A final limitation of our study is that the relation between student difficulties and OTL was only investigated from the perspective of OTL in textbooks. Although textbooks might be used as the main teaching and learning resources in the classroom (see, e.g., Reys et al., 2004; Valverde et al., 2002), it is clear that teachers also have an important role. For example, Pepin and Haggarty (2001) found that how textbooks are used in classrooms is determined by the teachers. Teachers determine the textbook sections used for students' exercises, when the textbooks should be used, and the ways students work with the textbooks. Regarding the use of context-based tasks for modeling, Ikeda (2007) emphasized the important role of teachers by arguing that the obstacle in teaching modeling is not only a lack of modeling tasks in textbooks, but also teachers' perceptions of mathematics and understanding of modeling. As noted by Bishop (1988), teachers have a crucial role in integrating students' experiences and cultures in mathematics learning. Furthermore, following Lampert (1990) and Boaler (1993), if teachers assume that mathematics is a static body of knowledge and learning is the reproduction of facts, procedures and truths, teachers will fail to engage students with context-based problems or problems with missing or superfluous information. Considering these facts, we believe that revealing possible reasons for students' difficulties in solving context-based tasks should also include investigations into teachers' teaching practice and teachers' beliefs and knowledge about context-based tasks. Consequently, the OTL offered by teachers will be the next focus of the CoMTI project. Another issue that also needs further attention is the influence of the classroom culture on the OTL offered to students (see Pepin & Haggerty, 2001).

Acknowledgements

This research was supported by the Indonesian Ministry of National Education under the *Better* Education through Reformed Management and Universal Teacher Upgrading (BERMUTU) Scholarship (No. 7476-IND Grant Ref. TF090784).

References

Bernardo, A. B. I. (1999). Overcoming obstacles to understanding and solving word problems in mathematics. *Educational Psychology, 19*(2), 149-163.

Bishop, A. J. (1988). Mathematics education in its cultural context. *Educational Studies in Mathematics, 19*(2), 179-191.

Blum, W., & Leiss, D. (2007). How do students and teachers deal with mathematical modelling problems? The example "Sugarloaf". In Haines, C., Galbraith, P., Blum, W., & Khan, S., *Mathematical modelling (ICTMA 12): Education, engineering and economics* (pp. 222-231). Chichester: Horwood Publishing.

Boaler, J. (1993). Encouraging the transfer of 'school' mathematics to the 'Real world' through the integration of process and content, context and culture. *Educational Studies in Mathematics, 25*, 341-373.

Brewer, D. J., & Stasz, C. (1996). *Enhancing opportunity to learn measures in NCES data*. Santa Monica, CA: RAND.

Carroll, J. (1963). A model of school learning. *Teachers College Record, 64*, 723-733.

Chapman, O. (2006). Classroom practices for context of mathematics word problems. *Educational studies in mathematics, 62*(2), 211-230.

Charalambous, C. Y., Delaney, S., Hsu, H.-Y., & Mesa, V. (2010). A comparative analysis of the addition and subtraction of fractions in textbooks from three countries. *Mathematical Thinking and Learning, 12*(2), 117-151.

Clements, M. A. (1980). Analyzing children's errors on written mathematical task. *Educational Studies in Mathematics, 11*(1), 1-21.

Cooper, B., & Harries, T. (2002). Children's responses to contrasting 'realistic' mathematics problems: Just how realistic are children ready to be? *Educational Studies in Mathematics, 49*(1), 1-23.

Cummins, D. D., Kintsch, W., Reusser, K., & Weimer, R. (1988). The role of understanding in solving word problems. *Cognitive Psychology, 20*(4), 405-438.

De Lange, J. (1995). Assessment: No change without problems. In T. A. Romberg (Ed.), *Reform in school mathematics* (pp. 87-172). Albany: SUNY Press.

De Lange, J. (2003). Mathematics for literacy. In B. L. Madison & L. A. Steen (Eds.), *Quantitative literacy: Why numeracy matters for schools and colleges* (pp. 75-89). Princeton, NJ: National Council on Education and Disciplines.

Ding, M., & Li, X. (2010). A Comparative analysis of the distributive property in U.S. and Chinese elementary mathematics textbooks. *Cognition and Instruction, 28*(2), 146-180.

Floden, R. E. (2002). The measurement of opportunity to learn. In A. C. Porter & A. Gamoran (Eds.), *Methodological advances in cross-national surveys of educational achievement* (pp. 231-266). Washington, D.C.: National Academy Press.

Forman, S. L., & Steen, L. A. (2001). Why math? Applications in science, engineering, and technological programs. *Research Brief, American Association of Community Colleges.*

Freeman, D. J., & Porter, A. C. (1989). Do textbooks dictate the content of mathematics instruction in elementary schools? *American Educational Research Journal, 26*(3), 403-421.

Freudenthal, H. (1986). Didactical principles in mathematics instruction. In J. A. Barroso, *Aspects of mathematics and its applications* (pp. 351-357). Amsterdam: Elsevier Science Publishers BV.

Gatabi, A. R., Stacey, K., & Gooya, Z. (2012). Investigating grade nine textbook problems for characteristics related to mathematical literacy. *Mathematics Education Research Journal, 24*(4), 403-421.

Graumann, G. (2011). Mathematics for problem in the everyday world. In J. Maasz & J. O'Donoghue (Eds.), *Real-world problems for secondary school mathematics students: case studies* (pp. 113-122). Rotterdam: Sense Publishers.

Greer, B., Verschaffel, L., & Mukhopadhyay, S. (2007). Modelling for life: Mathematics and children's experience. In W. Blum, P. L. Galbraith, H.-W. Henn & M. Niss (Eds.), *Modelling and applications in mathematics education: The 14th ICMI Study* (pp. 89-98). New York: Springer.

Haggarty, L., & Pepin, B. (2002). An investigation of mathematics textbooks in England, France and Germany: Some challenges for England. *Research in Mathematics Education, 4*(1), 127-144.

Haines, C., & Crouch, R. (2007). Mathematical modelling and applications: Ability and competence frameworks. In W. Blum, P. L. Galbraith, H.-W. Henn & M. Niss

(Eds.), *Modelling and Applications in Mathematics Education: The 14th ICMI Study* (pp. 89-98). New York: Springer.

Hiebert, J., Gallimore, R., Garnier, H., Givvin, K. B., Hollingsworth, H., Jacobs, J., et al. (2003). *Teaching mathematics in seven countries: Results from the TIMSS 1999 Video Study*. NCES 2003-013. Washington, D.C.: U.S. Department of Education, National Center for Education Statistics.

Husén, T. (Ed). (1967). *International study of achievement in mathematics: A comparison of twelve countries* (Vol. II). New York: John Wiley & Sons.

Ikeda, T. (2007). Possibilities for, and obstacles to teaching applications and modelling in the lower secondary levels. In W. Blum, P. L. Galbraith, H.-W. Henn & M. Niss (Eds.), Modelling and applications in mathematics education: The 14th ICMI Study. New York: Springer.

Kolovou, A., Van den Heuvel-Panhuizen, M., & Bakker, A. (2009). Non-routine problem solving tasks in primary school mathematics textbooks – A Needle in a Haystack. *Mediterranean Journal for Research in Mathematics Education, 8*(2), 31-68.

Lampert, M. (1990). When the problem in not the question and the solution is not the answer: Mathematical knowing and teaching. *American Educational Research Journal, 27*, 29-63.

Landis, J. R., & Koch, G. G. (1977). The measurement of observer agreement for categorical data. *Biometrics, 33*(1), 159-174.

Li, X., Ding, M., Capraro, M. M., & Capraro, R. M. (2008). Sources of differences in children's understandings of mathematical equality: Comparative analysis of teacher guides and student texts in China and the United States. *Cognition and Instruction, 26*(2), 195-217.

Liu, X. (2009). *Linking competence to opportunities to learn: Models of competence and data mining*. New York: Springer.

Maass, K. (2007). Modelling tasks for low achieving students – first results of an empirical study. In D. Pitta-Pantazi & G. Philippou (Eds.), *Proceedings of the Fifth Congress of the European Society for Research in Mathematics Education CERME 5* (pp. 2120-2129). Larnaca, Cyprus.

Maass, K. (2010). Classification scheme for modelling tasks. *Journal für Mathematik-Didaktik, 31*(2), 285-311.

McDonnell, L. M. (1995). Opportunity to learn as a research concept and a policy instrument. *Educational Evaluation and Policy Analysis, 17*(3), 305-322.

McNeil, N. M., Grandau, L., Knuth, E. J., Alibali, M. W., Stephens, A. C., Hattikudur, S., & Krill, D. E. (2006). Middle-School students' understanding of the equal sign: The books they read can't help. *Cognition and Instruction, 24*(3), 367-385.

Muller, E., & Burkhardt, H. (2007). Applications and modelling for mathematics - Overview. In W. Blum, P. L. Galbraith, H.-W. Henn & M. Niss (Eds.), *Modelling* *and applications in mathematics education: The 14th ICMI study* (pp. 267-274). New York: Springer.

Newman, M. A. (1977). An analysis of sixth-grade pupils' errors on written mathematical tasks. *Victorian Institute for Educational Research Bulletin, 39,* 31-43.

Niss, M., Blum, W., & Galbraith, P. (2007). Introduction. In W. Blum, P. L. Galbraith, H.-W. Henn & M. Niss (Eds.), *Modelling and Applications in Mathematics Education: The 14th ICMI Study* (pp. 3-32). New York: Springer.

OECD. (2003a). *Literacy skills for the world of tomorrow. Further results from PISA 2000.* Paris: OECD.

OECD. (2003b). *The PISA 2003 assessment framework - Mathematics, reading, science, and problem solving knowledge and skills.* Paris: OECD.

OECD. (2004). *Learning for tomorrow's world. First results from PISA 2003.* Paris: OECD.

OECD. (2007). *PISA 2006: Science competencies for tomorrow's world.* Paris: OECD.

OECD. (2009a). *Learning mathematics for life. A view perspective from PISA.* Paris: OECD.

OECD. (2009b). *Take the test. Sample questions from OECD's PISA assessments.* Paris: OECD.

OECD. (2010). *PISA 2009 Results: What students know and can do. Student performance in reading, mathematics, and science* (Vol. I). Paris: OECD.

Pepin, B., & Haggarty, L. (2001). Mathematics textbooks and their use in English, French and German classrooms: A way to understand teaching and learning cultures. *ZDM – The International Journal on Mathematics Education, 33*(5), 158-175.

Pusat Kurikulum. (2003a). *Kurikulum 2004. Standar Kompetensi mata pelajaran matematika Sekolah Menengah Pertama dan Madrasah Tsanawiyah.* Jakarta: Departemen Pendidikan Nasional.

Pusat Kurikulum. (2003b). *Kurikulum 2004. Standar Kompetensi mata pelajaran matematika Sekolah Menengah Atas dan Madrasah Aliyah.* Jakarta: Departemen Pendidikan Nasional.

Reys, B. J., Reys, R. E., & Chavez, O. (2004). Why mathematics textbooks matter. *Educational Leadership, 61*(5), 61-66.

Schmidt, W. H., McKnight, C. C., Valverde, G. A., Houang, R. T., & Wiley, D. E. (1997). *Many visions, many aims: A cross-national investigation of curricular intentions in school mathematics.* Dordrecht: Kluwer Academic Publishers.

Schwarzkopf, R. (2007). Elementary modeling in mathematics lessons: The interplay between "real-world" knowledge and "mathematics structures". In W. Blum, P. L. Galbraith, H.-W. Henn, & M. Niss (Eds), *Modelling and applications in mathematics education: The 14th ICMI Study* (pp. 209-216). New York: Springer.

Stein, M. K., & Smith, M.S. (1998). Mathematical tasks as a framework for reflection: From research to practice. *Mathematics Teaching in the Middle School, 3*(4), 268-275.

Tornroos, J. (2005). Mathematics textbooks, opportunity to learn and student achievement. *Studies in Educational Evaluation, 31*(4), 315-327.

Valverde, G. A., Bianchi, L. J., Wolfe, R. G., Schmid, W. H., & Houang, R. T. (2002). *According to the book. Using TIMSS to investigate the translation of policy into practice through the world of textbooks.* Dordrecht: Kluwer Academic Publishers.

Van den Heuvel-Panhuizen, M. (1996). *Assessment and realistic mathematics education.* Utrecht: CD-β Press, Utrecht University.

Van den Heuvel-Panhuizen, M. (2005). The role of context in assessment problems in mathematics. *For the Learning of Mathematics, 25*(2), 2-9 + 23.

Van Zanten, M., & Van den Heuvel-Panhuizen, M. (2014). Freedom of design: The multiple faces of subtraction in Dutch primary school textbooks. In Y. Li & G. Lappan (Eds.), *Mathematics curriculum in school education* (pp. 231-259). Heidelberg /Dordrecht /London /NewYork: Springer.

Verschaffel, L., Dooren, W. V., Greer, B., & Mukhopadhyay, S. (2010). Reconceptualising word problems as exercises in mathematical modelling. *Journal für Mathematik-Didaktik, 31*(1), 9-29.

Verschaffel, L., Greer, B., & De Corte, E. (2000). *Making sense of word problems.* Lisse: Swets & Zeitlinger.

Wijaya, A., Van den Heuvel-Panhuizen, M., Doorman, M., & Robitzsch, A. (2014). Difficulties in solving context-based PISA mathematics tasks: An analysis of students' errors. *The Mathematics Enthusiast, 11*(3), 555-584.

Xin, Y. P. (2007). Word problem solving tasks in textbooks and their relation to student performance. *Journal of Educational Research, 100*(6), 347-359.

Chapter 4

Teachers' teaching practices and beliefs regarding context-based tasks and their relation with students' difficulties in solving these tasks

Wijaya, A., Van den Heuvel-Panhuizen, M., & Doorman, M. (submitted). Teachers' teaching practices and beliefs regarding context-based tasks and students' difficulties in solving these tasks.

Teachers' teaching practices and beliefs regarding context-based tasks and their relation with students' difficulties in solving these tasks

1 Introduction

Currently, the ability to apply mathematics is considered a core goal of mathematics education all around the world (see, e.g., Graumann, 2011; NCTM, 2000; OECD, 2003; Scardamalia, Bransford, Kozma, & Quellmalz, 2012; Schleicher, 2007; Tomlinson, 2004). In the United States this goal is explicitly stated in the Principles and Standard for School Mathematics (National Council of Teachers of Mathematics [NCTM], 2000). According to the NCTM (2000, p. 64), mathematics teaching should enable students to "recognize and apply mathematics in contexts outside of mathematics." In the United Kingdom, mathematics education is aimed at ensuring that young people acquire functional mathematics (Tomlinson, 2004, p. 29), which includes the knowledge and the capacity to apply mathematics. This goal is similar to what, for example, in the Programme for International Student Assessment (PISA) is called mathematical literacy, which refers to students' ability "to identify, and understand, the role that mathematics plays in the world, to make well-founded judgments and to use and engage with mathematics in ways that meet the needs of that individual's life as a constructive, concerned, and reflective citizen" (Organisation for Economic Co-operation and Development [OECD], 2003, p. 24).

However, despite the significance attributed to applying mathematics in various contexts, several studies have revealed that student performance in solving context-based tasks is rather low (Cooper & Dunne, 2000; Sam, Lourdusamy, & Ghazali, 2001). The PISA 2012 study (OECD, 2013) showed that about 32% of the total group of students in the 65 participating countries had a performance below Level 2 and could only answer "questions involving familiar contexts where all relevant information is present and the questions are clearly defined" (OECD, 2013, p. 61). A further result from the PISA 2012 study (OECD, 2013) was that there were fifteen countries in which more than half the students performed below this level.

With respect to students' difficulties when solving context-based tasks, various studies showed that students have problems with understanding the wording of context-based tasks and identifying relevant information (Klymchuk, Zverkova, Gruenwald, & Sauerbier, 2010; Prakitipong & Nakamura, 2006). It was also found that students struggle to select adequate mathematical procedures (Clements, 1980; Klymchuk et al., 2010) and often just apply a routine procedure without taking realistic considerations into account (Sepeng, 2013; Verschaffel, Greer, & De Corte, 2000; Xin, Lin, Zhang, & Yan, 2007).

Indonesia, similar to many other countries, attaches high value to applying mathematics as a core goal of mathematics education. This is clearly stated in the Indonesian national curriculum (Pusat Kurikulum, 2003). Nevertheless, in Indonesia there also is an apparent discrepancy between this goal and outcomes in student achievement. The PISA results repeatedly showed that Indonesian students perform low in solving context-based tasks. For example, in the PISA 2009 study (OECD, 2010) 77% of Indonesian students did not reach Level 2.

The result from this PISA study prompted us to set up a project called "Context-based Mathematics Tasks Indonesia" (CoMTI) to investigate how Indonesian students' performance can be improved. The first step in this project was a study into Indonesian students' difficulties when solving context-based tasks. We found that the students mostly got stuck in the early stages of the solving process, i.e., they had particular difficulties with comprehending what a problem, situated in a context, was about and made errors in transforming a context-based problem into a mathematical problem (Wijaya, Van den Heuvel-Panhuizen, Doorman, & Robitzsch, 2014). The next step in the CoMTI project was identifying possible causes of these difficulties. For this, we took the concept of opportunity-to-learn (OTL) into account, because this is seen as crucial in investigating possible reasons for students' low performance (Brewer & Stasz, 1996; Hiebert & Grouws, 2007). Our first focus was on the OTL offered by textbooks. An analysis of Indonesian mathematics textbooks revealed that only 11% of tasks were context-based (Wijaya, Van den Heuvel-Panhuizen, & Doorman, submitted).

Although this low percentage of context-based tasks in textbooks gave a possible explanation for Indonesian students' low scores on these tasks, it might not be the only reason for the low scores, because what is offered in textbooks is not always identical to what is taught by teachers (Pepin & Haggarty, 2001). For instance, teachers may leave out and add content, emphasize aspects of applying mathematics that are not addressed in textbooks, or neglect specific textbook guidelines. This means that students' performance can be influenced by teachers' teaching practices (Grouws & Cebulla, 2000; Hiebert & Grouws, 2007). In other words, the teaching practice of teachers can also contribute to the students' OTL. Moreover, teaching practice is often found to be affected by what teachers think about mathematics (Ernest, 1989; Stipek, Givvin, Salmon, & MacGyvers, 2001; Wilkins, 2008). It means teachers' beliefs may also play a role in the OTL offered by teachers.

Therefore, to further investigate the OTL to solve context-based tasks given to students, the present study addressed teachers' teaching practices in relation with teachers' beliefs. In addition, we analyzed whether our findings regarding the OTL offered by Indonesian teachers correspond to the kinds of errors that Indonesian students make when solving context-based tasks.

2 Theoretical background

2.1 OTL offered by teachers

To find an explanation for the differences in students' mathematics performance in different countries Husén (1967, p. 162-163) introduced the concept of OTL, which was defined as "whether or not […] students have had the opportunity to study a particular topic or learn how to solve a particular type of problem." This definition is specified by Brewer and Stasz (1996) who distinguished three aspects of OTL. The first is the curriculum content which focuses on the scope of the topics offered to students. The second refers to the teaching strategies teachers use to present the topics and engage students. The third aspect concerns the instructional resources, for instance textbooks used to teach the students.

Indeed, several studies (e.g., Husén, 1967; Tornroos, 2005) have found that curriculum and instructional materials are important for students' OTL. Yet, there were also studies (e.g., Grouws & Cebulla, 2000; Hiebert & Grouws, 2007) which revealed that students' mathematical performance is largely influenced by teachers' teaching practices. These studies showed that the strategies used by teachers to teach particular topics, the kinds of tasks they presented to students, and the nature of the discussions they organized in class are important factors influencing students' OTL.

2.2 Teaching practices contributing to students' OTL to solve context-based tasks

From the perspective of mathematical modeling, Antonius, Haines, Jensen, Niss, and Burkhardt (2007, p. 295) argued that teaching context-based tasks requires more than an "explanation-example-exercise ritual", i.e. teacher explains a concept, gives an example of a problem in which this concept is applied and finally offers students exercises for practicing problems with this concept. Such a directive approach does not offer students the opportunity to develop strategic competences which are necessary to solve context-based tasks (Antonius et al., 2007), such as understanding the text of problems and selecting relevant information, identifying and applying appropriate mathematical procedures, connecting different representations or mathematics concepts, and interpreting solutions (Blum, 2011; OECD, 2003). To achieve these competences, several authors (Antonius et al., 2007; Forman & Steen, 2001) emphasized that teachers, instead of using a directive teaching method, should teach context-based tasks through a student-centered and investigative teaching approach in which students are actively involved and the teacher's role is consultative rather than directive. Similarly, Blum (2011, p. 25) pleaded for consultative teaching, which he called "operative-strategic teaching" that emphasizes guiding students to actively and independently construct new knowledge by using their prior knowledge and experiences. A key aspect of such teaching is that teachers should keep a balance between providing guidance and fostering students' independence for which they can use flexible interventions and metacognitive prompts to elicit students to reflect on their own understanding of the problem and on how they selected the mathematical procedures to solve the problem. Promoting students to reflect can also be done by providing them with opportunities to assess and (if necessary) revise their own work (Bell & Pape, 2012).

Further specific recommendations can be given when zooming in on the four stages of solving context-based tasks: comprehension stage, transformation stage, mathematical processing stage, and encoding stage (see Blum, 2011).

2.2.1 OTL connected to the comprehension stage

In the comprehension stage, students have to figure out what the problem is about and to identify the information that is relevant to solving it. Directly telling students what a context-based task means and what information is required is, according to Barnes (2000, p. 41), not supportive for their learning because such practice causes students to "have less need to struggle and less occasion to make efforts of their own to achieve understanding."

Several studies (e.g., Hagaman, Casey, & Reid, 2013; Karbalei & Amoli, 2011) have shown that the so-called three-step RAP (Read-Ask-Paraphrase) strategy can stimulate students' active involvement in getting to know what is asked in the problem and can improve reading comprehension. The first step of this RAP strategy is in agreement with Blum's (2011, p. 24) suggestion to ask students to "read the text precisely and imagine the situation clearly." In the second RAP step, students are encouraged to figure out what the problem is really asking them. This approach is similar to Kramarski, Mevarech, and Arami's (2002, p. 228) suggestion to train students to formulate and answer self-addressed metacognitive comprehension questions such as "what is the problem about?" The last RAP step implies the teacher should ask students to formulate the problem in their own words. Paraphrasing is helpful for students because it makes problems more familiar and, consequently, more understandable for them (Karbalei & Amoli, 2011; Kletzien, 2009).

Another important aspect of comprehending a problem is awareness of the information needed to solve the problem. To achieve this, several authors (Blum, 2011; Forman & Steen, 2001; Lingefjard & Meier, 2010) suggested letting students discuss the information presented in a problem in relation to what is asked in the problem. Other approaches are asking students to figure out whether particular information is missing (Forman & Steen, 2001) or to formulate a self-addressed question like "Do I already have enough information to solve the problem?"

2.2.2 OTL connected to the transformation stage

In the transformation stage students have to transform a context-based task into a mathematical problem. Again, directly telling the students what to do might not offer them an OTL. It is more helpful when students are involved in this process and can themselves explore different ways of transforming a context-based task into a mathematical problem which they can use to solve this task. To stimulate this exploration, students can be asked to formulate and answer questions such as "What might be a possible mathematical procedure to solve this problem?" Another approach to achieve this exploration is to call up earlier experiences of the students when solving similar context-based tasks. In line with this, Kramarski et al. (2002, p. 228) suggested that teachers should encourage students to formulate and answer self-addressed questions such as "What are the similarities or differences between this problem and the problems I have ever solved?" What all these approaches have in common is that OTL in the transformation stage needs to orient students toward identifying relevant mathematical procedures for solving the problem (Galbraith & Stillman, 2006).

2.2.3 OTL connected to the mathematical processing stage

In the mathematical processing stage, students do not in fact have to deal with the context-based character of a problem, but only carry out the mathematical procedure(s) resulting from transforming a context-based problem into a mathematical problem. Therefore, it is not surprising that for the mathematical processing stage, none of the aforementioned studies – e.g. Forman and Steen (2001); Kramarski et al. (2002); Lingefjard and Meier (2010) – gave suggestions that offer students OTL. Furthermore, the mathematical processing stage can cover various mathematics topics which might make it difficult to provide a fixed suggestion or direction. Nevertheless, having fewer mistakes in performing mathematical procedures would eventually also help students to become better in solving context-based tasks. Therefore, a teaching practice in which the teacher stimulates students to check their mathematical procedures can also be considered an OTL to solve context-based tasks.

2.2.4 OTL connected to the encoding stage

In the encoding stage students have to interpret a mathematical solution in terms of the situation of the context-based task and take realistic and critical considerations into account. For this, the students should be encouraged to link their solution to the situation of the task and to verify the reasonableness of the solution (Blum, 2011; Forman & Steen, 2001). Kramarski et al. (2002, p. 228) proposed that teachers should stimulate their students to ask themselves whether the solution makes sense. Such a teaching practice contrasts with just focusing on the correctness of the mathematical solution.

2.3 Teacher beliefs contributing to students' OTL to solve context-based tasks

Several studies (Beswick, 2005; Ernest, 1989; Stipek, Givvin, Salmon, & MacGyvers, 2001; Wilkins, 2008) showed the influence of teachers' beliefs on teachers' practice. For example, Beswick found a relation between teachers' positive beliefs toward the importance of problem solving in mathematics and their constructivist teaching practice in which students are actively involved in the teaching-learning process. According to Ernest (1989), a teacher's belief system consists of three key components; i.e. beliefs (1) about the nature of mathematics, (2) about the nature of mathematics teaching, and (3) about the process of learning mathematics. With respect to the belief about the nature of mathematics, Lakatos (1976) distinguished two philosophical views on mathematics: the absolutist and the fallibilist. The absolutist view considers that mathematical truth is absolute and represents objective and certain knowledge (Ernest, 1991) that "somehow exists apart from everyday human affairs" (Toumasis, 1997, p. 319). Within this view, mathematical truth is seen as resulting from a deduction process and consisting of a priori knowledge without observations of the real world. In contrast, the fallibilist view considers mathematical truth as fallible (Ernest, 1991). According to this view, mathematical knowledge is open to revision and is related to everyday life, because it results from human activity in trying to organize patterns and structures in phenomena outside of and within mathematics (Ernest, 1991; Toumasis, 1997).

Toumasis (1997) and Tuge (2008) extended these different views on mathematics to views on teaching and learning of mathematics. Their studies pointed out that teachers with an absolutist view believe mathematics teaching is primarily oriented toward mathematical content and the practice of procedural skills in which teachers have the role of demonstrating how students should proceed. In contrast, teachers with a fallibilist view focus their teaching of mathematics on mathematical reasoning, the process of organizing patterns and structures, rather than on practicing procedures and dealing with mathematics as a fixed system. Regarding beliefs about the process of learning mathematics, within the absolutist view it is argued that learning mathematics depends on drill or hard exercise of mathematical procedures (Toumasis, 1997) and that mathematical concepts do not need to be connected to the real-world context (Tuge, 2008). Teachers with a fallibilist view are student-centered and believe learning mathematics should involve students in investigation, exploration, problem solving and discussion (Toumasis, 1997; Tuge, 2008).

Comparing the characteristics of these two views, we may conclude that teachers with a fallibilist view are well-placed to create teaching practices which provide students OTL to solve context-based tasks: these teachers see mathematics as related to everyday life; they emphasize student-centered learning through investigation, exploration, and discussion, which is also in line with the consultative teaching described earlier. In contrast, the absolutist view is at risk of not giving students the chance to solve context-based tasks, because it considers mathematics as a set of abstract concepts and focuses on teaching practices which offer students opportunities to exercise mathematical procedures without connection to a real-world context.

The relation between teachers' conceptions on context-based tasks and how teachers use such tasks in their teaching is also emphasized by Chapman (2009). Teachers who are mostly oriented toward mathematics as computational and algorithmic problem solving conceptualize context-based tasks as word problems meant to practice earlier learned procedures. According to Chapman (2009), these teachers prefer context-based tasks which have a clear question, contain only the information that is relevant for solving the task, and have an explicit suggestion about the mathematical procedure to use.

A different conception on context-based tasks is reflected by teachers who use contexts which have realistic value to students. Such teachers use context-based tasks as a tool to help students "experience the world" and develop in-depth mathematical thinking (Chapman, 2009, p. 232). These teachers favor context-based tasks that offer various opportunities for students to create models to structure the problems, to explore and connect different mathematical procedures or concepts, and to select information. In line with this, the teachers in Chapman's study (2009) emphasized that students can play with a large amount of information, which is also in agreement with Verschaffel, Van Dooren, Greer, and Mukhopadhyay (2010), who pointed out that context-based tasks which do not include irrelevant information and do not require students to look for additional information will not support the development of students' modeling competences and in-depth thinking.

2.4 Research questions

After first investigating the kinds of errors Indonesian students make when solving context-based tasks (Wijaya et al., 2014), and then looking for a possible explanation for these errors by examining the OTL provided by Indonesian textbooks (Wijaya et al., submitted), the present study is researching students' OTL from the perspective of teachers; focusing on teaching practices and underlying beliefs. Although the main emphasis was on teaching practices we started with teachers' beliefs, because having beliefs that support using context-based mathematics tasks can be considered necessary for a teaching practice that offers students OTL to solve such tasks. In the present study we addressed the following research questions:

1. *What beliefs do Indonesian teachers have regarding mathematics, teaching and learning of mathematics, and context-based tasks?*
2. *What OTL to solve context-based tasks do teachers offer students in their classroom practice?*
   2.a. What kinds of context-based tasks do Indonesian teachers offer their students?

   2.b. What teaching approach do Indonesian teachers use to teach context-based tasks?
3. *Is there a relationship between the OTL to solve context-based tasks offered by Indonesian teachers and the errors Indonesian students make when solving such tasks?*

3 Method

3.1 Design of the study

To investigate the students' OTL to solve context-based tasks offered by teachers, a teacher survey study was carried out through a written questionnaire and a series of classroom observations. The results from from the questionnaire and the observations were compared with the findings from our earlier study (Wijaya et al., 2014) in which we identified the kinds of errors Indonesian students made when solving context-based tasks.

3.2 Participants

The participants of the study were Junior High School mathematics teachers working at seven schools in rural and urban areas in the province of Yogyakarta, Indonesia. The seven schools were the same schools in which we collected the data for investigating students' errors when solving context-based tasks (see Wijaya et al., 2014). We chose the province of Yogyakarta for the CoMTI project for reasons of convenience (the first author originates from this province). All Junior High School mathematics teachers of the participating schools filled in the questionnaire. This resulted in a sample of 27 teachers (14 male and 13 female), including nine Grade 7 teachers, twelve Grade 8 teachers, and eleven Grade 9 teachers[1]. The teachers had two to 34 years of teaching experience (M = 19, SD = 10.02).

[1] The total number is more than 27, because some teachers taught in two grades.

For the classroom observations we asked all Grade 8 teachers[2] whether they were willing to be observed and video recorded during two mathematics lessons in which they had to address context-based tasks for which we would provide them a set of tasks. In total, four of the twelve Grade 8 teachers volunteered. The other teachers either did not feel confident to be observed or argued that spending two lessons on additional tasks would not fit their time schedule. The teachers whose lessons were observed were from three different schools, had moderate to long teaching experience, and all had a Bachelor degree in mathematics education (see Table 1). They also used the same textbook, *Matematika* (Textbook for Junior High School, Grade VIII: 2A & 2B).

Table 1

Teachers whose lessons were observed

Name[1]	School	Gender	Age (year)	Teaching experience (year)	Education background
Siti	A	Female	47	26	B.A. (math. education)
Ihsan	B	Male	44	17	B.A. (math. education)
Leni	B	Female	42	17	B.A. (math. education)
Ratih	C	Female	30	4	B.A. (math. education)

[1] These names are pseudonyms.

3.3 Teacher questionnaire

To gather information about teachers' beliefs and teaching practices we developed a written questionnaire. As an addition to five questions about general background information (age, gender, years of teaching experience, grade taught, educational background), this questionnaire contained (1) a six-item section regarding teachers' beliefs about mathematics, mathematics teaching, and mathematics learning, (2) a four-item section addressing teachers' beliefs about context-based tasks and one item on how teachers think about the number of context-based tasks in the textbooks they use, and (3) a seven-item section on how teachers see their teaching practices related to context-based tasks.

[2] Although Grade 9 students (the 15-year-olds) are the target group of the PISA studies, we decided to do the observations in Grade 8 because that is where the basis for the performance in Grade 9 is laid. Moreover, the schools did not give permission to do observations in Grade 9 classes because the preparations for the National Exam took place in these classes.

3.3.1 Teachers' beliefs about mathematics and teaching and learning mathematics

Of the six items concerning teachers' beliefs about mathematics and its teaching and learning (see Table 2), items 1-3 were derived from a questionnaire developed by Adamson et al. (2002), who investigated teachers' beliefs by focusing on their tendency toward an absolutist or fallibilist view. We took these three items because an absolutist or fallibilist view on mathematics and its teaching might be related to whether teachers offer students OTL to solve context-based tasks.

We developed items 4-6 ourselves, based on the aforementioned studies of Toumasis (1997) and Tuge (2008). Item 4 addresses teachers' beliefs about the teaching of mathematics and items 5-6 concern beliefs about the learning of mathematics.

For all six items in this section we used the format of paired opposite statements as employed by Adamson et al. (2002). We asked the teachers to rate their position on a five-point rating scale ranging from 1 to 5, in which 1 represents a strong absolutist view and 5 a strong fallibilist view.

3.3.2 Teachers' beliefs and reported teaching practice regarding context-based tasks

The four items in the questionnaire that was used to measure teachers' beliefs about context-based mathematics tasks focused on the explicitness of mathematical procedures and the type of information provided in these tasks. In addition to these four items, we included an item addressing teachers' opinions about the sufficiency of context-based tasks in the textbooks they used. All five items in this section (see Figure 4) contained a five-point rating scale (ranging from 1 "strongly disagree" to 5 "strongly agree").

The last part of our questionnaire, which contained seven items, investigated how teachers think about their teaching practices (see Figure 5). The first three items focused on how frequently teachers give context-based tasks, make their own context-based tasks, and on modifying teaching materials. The last four items examined the characteristics of the context-based tasks teachers offer to their students. The teachers were asked to indicate on a five-point rating scale the frequency of presenting particular context-based mathematics tasks (ranging from 1 "never", 2 "once or twice in a semester", 3 "monthly", 4 "weekly", to 5 "every lesson").

Table 2

Questionnaire for investigating teachers' beliefs about mathematics and the teaching and learning of mathematics

Mathematics	
Mathematics consists of objects and patterns that have no existence outside the mind.	Mathematics consists of objects and patterns that can be seen in real objects and natural phenomena.
The beauty of mathematics is found mostly when complex problems are solved using abstract concepts.	The beauty of mathematics is found mostly in how it is found in the patterns of nature.
Mathematics teaching	
The most important part of instruction is the content of the curriculum.	The most important part of instruction is how it encourages thinking among students.
School mathematics is about learning skills which are needed to understand higher levels mathematics.	School mathematics is about learning skills that students will need in daily life.
Mathematics learning	
Mathematics learning is more likely to occur through drill or practices of mathematical procedures in abstract or formal forms.	Mathematics learning is more likely to occur when students actively engage in problem situated in various contexts.
Mathematics learning aims at motivating students to learn mathematics as a subject matter.	Mathematics learning aims at motivating students to learn skills they need in daily life.

3.4 Classroom Observations

3.4.1 Procedure

Classroom observations were conducted to further investigate teachers' teaching practice regarding context-based tasks. We observed two lessons for each of the four Grade 8 teachers who volunteered to participate in this part of the study. Because these classroom observations were intended to investigate how teachers helped their students to learn solving context-based tasks, rather than to examine the frequency of dealing with context-based tasks (this was covered by the questionnaire), we wanted to be sure the teachers would teach context-based tasks in their lessons. Therefore, we provided them with a set of seven context-based tasks consisting of twelve questions on graphs of linear equations. This topic was chosen because it was taught in these schools during the observation period. The tasks included different kinds of contexts such as internet fees (see Internet task in Figure 1) and travelling by bike (see Journey task in Figure 2). To offer students OTL to select relevant information most tasks contained superfluous or missing information. For example, in the Internet task the relevant information the students have to identify is 18,000 IDR (the monthly fee for the program Shine), 45 IDR (the fee per 1 MB), and 550 MB (the internet usage in May). Another characteristic of the tasks was that they did not explicitly provide suggestions about the mathematical procedure needed to solve them. For example, to solve Question b in the Internet task the students needed to decide whether making a calculation or drawing a graph would be helpful to find the answer.

To ensure the ownership of the teachers when teaching context-based tasks the teachers could employ their own teaching strategies in the observed lessons. Moreover, they were free to choose which and how many of the provided seven tasks they would use. In addition they could also include other context-based tasks, either from their textbook or designed by themselves. The observations were made by the first author and the lessons were video recorded by using two cameras, a static camera to record whole class activity and a dynamic camera to record the interaction between teacher and particular students.

An internet company offers two different programs, i.e. Smile and Shine. Program Smile charges customers 31,500 IDR for monthly fee and 30 IDR/1 Megabyte (MB). Program Shine charges customers 18,000 IDR for monthly fee and 45 IDR/1 MB. The registration fees including the price of modem for both programs are the same, i.e. 300,000 IDR. In January Doni subscribed for program Shine. In May Doni used 550 MB of internet data.

a. How much money should be paid by Doni in May?

Doni’s internet usage is increasing. He wonders whether it will be beneficial to change the internet program.

b. When Doni’s internet usage is increasing, when should he change the internet program?

Figure 1. Internet problem

Last Saturday, Joni and his friends went biking for three hours nonstop.
During the journey, Joni frequently checked the odometer on his bike. The table shows the time and the distance travelled by Joni and his friends.

a. Estimated how far Joni travelled after:
   - a half hour
   - two hours
b. Estimate when Joni reached the fastest speed.

Time (in minutes)	Distance (in km)
10	2.5
20	6
35	11
60	20.5
90	32
110	38.5
150	50

Figure 2. Journey problem

3.4.2 Data analysis

In total, we video recorded eight lessons. As the unit of analysis we chose an activity-based chunk of video data. It means that a chunk was not based on a particular time slot but on an activity that was carried out in class. In our case, this was a teaching activity related to a context-based task. Because most tasks consisted of more than one question, we treated an activity related to a question as a unit. Activities related to bare mathematics tasks were excluded from our analysis.

The approach we chose for analyzing the video data units was based on what Erickson (2006) called the part-to-whole deductive approach; an approach that is used when the analysis has a particular theoretical orientation. As discussed earlier, there is evidence from literature that teaching practices which reflect consultative teaching offer students more OTL to solve context-based tasks than directive teaching approaches. Therefore, we developed a framework for coding teaching practices (see Table 3) which included characteristics of both approaches for all four stages of solving a context-based task. When a teacher did not pay attention to a particular stage, it was coded as "no instruction".

The coding was carried out by the first author and afterwards the reliability of the coding was checked through an additional coding by a researcher of mathematics education not involved in this study. This extra coding was done based on two randomly selected lessons from the eight video-recorded lessons. The Cohen's Kappa for the coding of these two lessons was .89, which indicates that the coding was reliable (Landis & Koch, 1977).

Table 3

Framework for coding teachers' teaching practice related to context-based tasks

Stage of solving a context-based task	Teaching practice supportive for providing students OTL to solve context-based tasks (*consultative teaching*)	Teaching practice not supportive for providing students OTL to solve context-based tasks (*directive teaching*)	No instruction
COMPRE-HENSION STAGE	- Asking students to read the text precisely and imagine the context of the problem. - Stimulating students to formulate and answer comprehension questions for themselves, e.g. *"what is the problem about?"* - Asking students to paraphrase the text. - Stimulating discussion of available data in relation to what is asked; e.g. by asking students to underline only the relevant information. - Encouraging students to identify and seek out missing information; for example, by asking them to formulate and answer self-addressed metacognitive questions, such as "*Do I already have enough information to solve the problem?"*	- Reading aloud the problem by the teacher. - Telling students what the problem is about. - Telling students the information needed to solve the problem.	
TRANSFOR-MATION STAGE	- Encouraging students to explore various procedures by considering their existing knowledge and experiences; for example, by asking them to formulate and answer self-addressed metacognitive questions, such as *"What are the similarities or differences between this problem with the problems I have ever solved?"*	- Telling students the mathematics concept or procedure required to solve the problem. - Directly translating a real-world problem into a mathematical problem.	

Stage of solving a context-based task	Teaching practice supportive for providing students OTL to solve context-based tasks (*consultative teaching*)	Teaching practice not supportive for providing students OTL to solve context-based tasks (*directive teaching*)	No instruction
TRANSFOR-MATION STAGE	- Encouraging students to identify the relevant mathematical procedures; for example, by asking them to formulate and answer self-addressed metacognitive questions, such as *"What procedures are appropriate for solving the problem?"*		
MATH. PROCESSING STAGE	- Stimulating students to check the mathematical procedures they perform; e.g. by saying *"Check your calculation thoroughly"*, *"Do you already perform the algebraic operation correctly?"*	- Directly correcting students' mistake at a particular step of the mathematical procedures.	
ENCODING STAGE	- Asking students to verify the reasonableness of solutions in terms of the original problem; for example, by asking them to formulate and answer self-addressed metacognitive questions, which focus on reflecting on the solution: *"Does the solution make sense?"* - When students only give mathematical solution, asking them to interpret the solution in terms of the context of the problem; for example, by asking: *"What does this number mean?"*	- Directly telling students if their solution is unrealistic or does not make sense. - Focusing only on the correctness of the mathematical solution without connecting it to the context of the problem.	

4 Results

4.1 Teachers' beliefs

4.1.1 Beliefs about mathematics and mathematics teaching and learning

Regarding teachers' beliefs, we found the teachers had a tendency towards the fallibilist view on the nature of mathematics and the teaching and learning of it (see Figure 3). We found that 48% of the teachers (13 out of 27) rated 4 or 5, which indicates they tended to believe that mathematics consists of objects and patterns that can be seen in real objects and natural phenomena and only 7% of the teachers (two out of 27) tended to believe mathematics consists of objects and patterns that have no existence outside the mind (Item 1). Regarding beliefs about teaching mathematics (Item 3), more than 90% of the teachers (25 out of 27) tended to believe that mathematics teaching should focus on encouraging thinking and reasoning among students and not only on teaching mathematics content. A tendency toward the fallibilist view was also found for two beliefs about mathematics learning. We found that almost 60% of the teachers (16 out of 27) tended to believe that students' learning is more likely to occur when students are actively engaged in problems situated in various contexts (Item 5) and more than half of the teachers (15 out of 27) indicated believing that mathematics learning has as an objective to motivate students to learn skills needed in daily life (Item 6). However, only 26% of the teachers (seven out of 27) agreed that school mathematics is about teaching skills that students will need in daily life and almost half (13 out of 27) believed that school mathematics is about teaching pure mathematics (Item 4). Here, the teachers showed a rather absolutist view on learning mathematics.

Item	Absolutist view on	Fallibilist view on
	Mathematics	
1	Mathematics consists of objects and patterns that have no existence outside the mind.	Mathematics consists of objects and patterns that can be seen in real objects and natural phenomena.
2	The beauty of mathematics is found mostly when mathematics problems are solved using abstract concepts.	The beauty of mathematics is found mostly in how it is found in the patterns of nature.
	Mathematics teaching	
3	The most important part of teaching is the mathematics content.	The most important part of teaching mathematics is how it encourages thinking and reasoning among students.
4	School mathematics is about teaching topics, skills, and procedures which are needed to understand pure mathematics.	School mathematics is about teaching skills that students will need in daily life.
	Mathematics learning	
5	Mathematics learning is more likely to occur through drill or practices of mathematical procedures in abstract form.	Mathematics learning is more likely to occur when students actively engage in problem situated in various contexts.
6	Mathematics learning is aimed at motivating students to learn mathematics as a subject matter.	Mathematics learning is aimed at motivating students to learn skills they need in daily life.

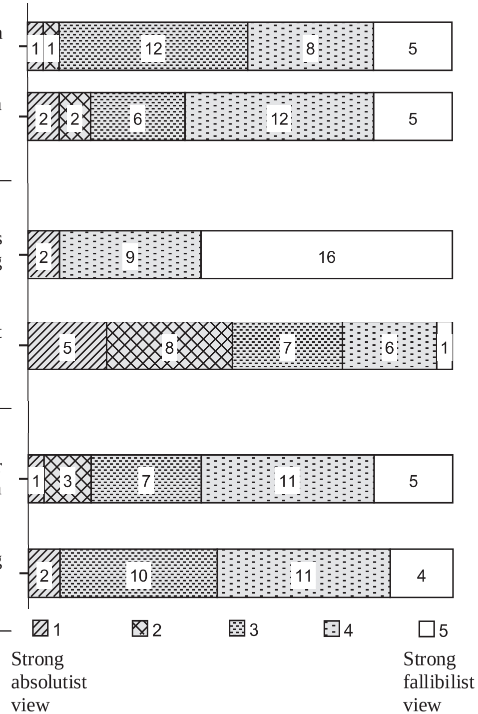


Figure 3. Teachers' tendency towards a n absolutist or fallibilist view on mathematics, mathematics teaching, and mathematics learning

4.1.2 Beliefs about context-based tasks

Regarding their beliefs about context-based tasks most teachers reflected ideas that are not considered to be supportive for students' learning to solve context-based tasks. Figure 4 shows that three quarters of the teachers (20 out of 27) (strongly) agreed with explicitly providing mathematical procedures in context-based tasks. With respect to the type information that is included in a task the teachers' beliefs seemed to be more supportive for students' learning to solve context-based tasks. Only 41% of the teachers (11 out of 27) (strongly) agreed that context-based tasks should provide precisely the information that is needed to find the solution; whereas half of the teachers (14 out of 27) (strongly) disagreed. Yet teachers were not particularly in favor of including superfluous information; barely 30% of the teachers (eight out of 27) (strongly) agreed. However, for having less information than needed the situation was different in which three quarters of the teachers (21 out of 27) (strongly) agreed.

Finally, we asked whether or not the teachers agreed with the statement that the number of context-based tasks provided in textbooks is sufficient. Almost half (13 out of 27) (strongly) disagreed and only a quarter (seven out of 27) (strongly) agreed.

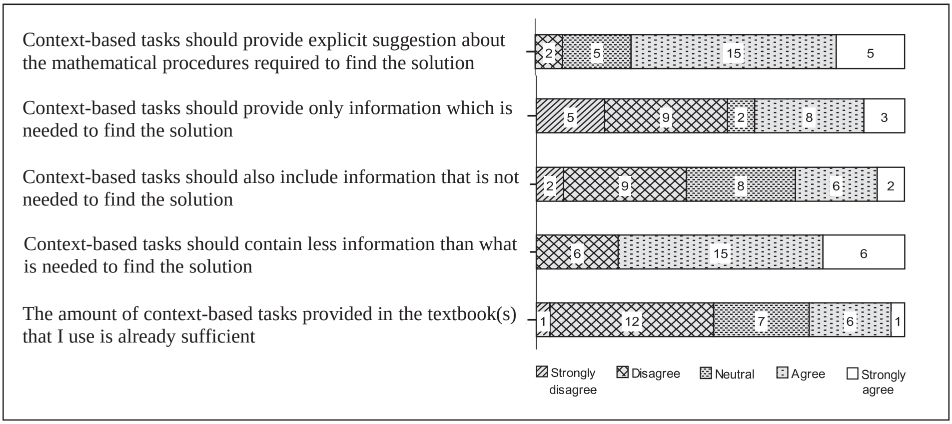


Figure 4. Teachers' beliefs about context-based tasks

4.2 Teachers’ reported teaching practices

4.2.1 Frequency of offering context-based tasks

As shown in Figure 5, according to the teachers’ own judgment, they frequently present context-based tasks to their students. 81% of them (22 out of 27) reported giving context-based tasks at least weekly. Moreover, three quarters (20 out of 27) stated they make their own context-based tasks at least weekly when such tasks are not available in the textbooks. Regarding modifying the textbook approach and supplementing it with additional problems, 81% of the teachers (22 out of 27) reported doing so at least weekly.

4.2.2 Characteristics of context-based tasks

With respect to the characteristics of context-based mathematics tasks, the questionnaire data revealed that what the teachers offer to their students are mostly only plain word problems (see the last four items in Figure 5). Two thirds (18 out of 27) stated they present context-based tasks with explicit suggestions about the mathematical procedures at least weekly. The same result was found for context-based tasks which provide only the information needed to find a solution. Furthermore, 41% (11 out of 27) reported never giving context-based tasks with superfluous information. Regarding context-based tasks with missing information, 33% (nine out of 27) reported that they never give such tasks and an equal percentage of the teachers said to give such tasks weekly.

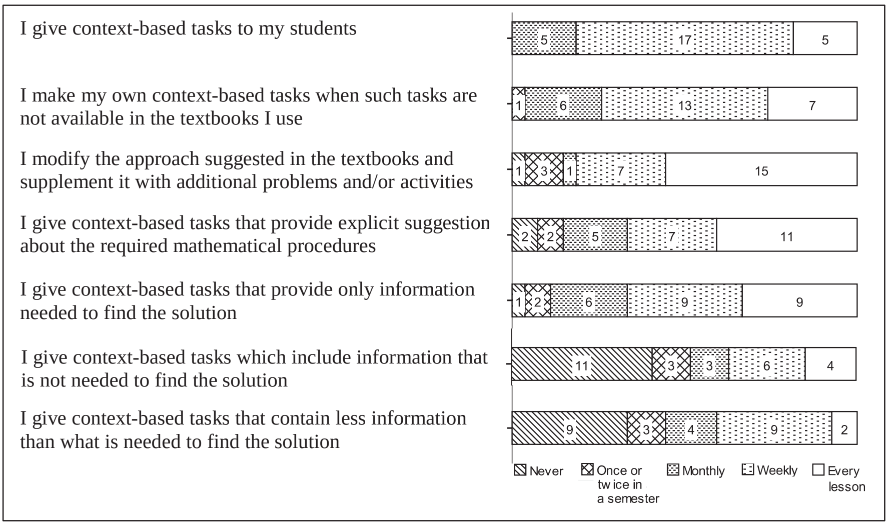


Figure 5. Teachers’ reported teaching practices

4.3 Observed teaching practices

Table 4 shows the number of cases in which mathematics tasks were dealt with in the eight lessons observed in the four classrooms. Because tasks can consist of one or more questions we took questions as our unit of analysis. A mathematics question can be used as either a worked example (a question with a given answer) or an exercise (a question to be solved by the students). Unlike worked example questions which are by definition discussed in class, exercise questions are not always discussed with the students.

Table 4

Number of Cases a Mathematics Question was Presented in the Four Observed Classrooms

Teacher	Number of cases related to a bare mathematics question		Number of cases related to a context-based mathematics question	
	Worked example question	Exercise question	Worked example question	Exercise question
Siti	3	5 (all discussed)	-	6 (4 discussed)
Ihsan	2	2 (all discussed)	2	11 (6 discussed)
Leni	3	6 (4 discussed)	-	6 (3 discussed)
Ratih	3	5 (all discussed)	-	4 (2 discussed)
Total	11	18 (16 discussed)	2	27 (15 discussed)

A total of 29 cases related to a context-based mathematics question were presented to the students, of which 27 involved exercises and 2 involved worked examples. The same number of cases was found for the bare mathematics questions, either designed by the teachers themselves or taken from the textbook. Of these 29 cases, 11 were presented as worked examples. The number of cases in which the teachers gave a context-based mathematics question ranged from 4 in Ratih's classroom, to 13 in Ihsan's classroom. Ihsan was also the only teacher who included his own context-based mathematics questions. Another observation related to teaching practice is that in only half the cases related to a context-based mathematics question, i.e. 15 cases, the question was discussed in class.

In the following we describe the observed teaching practice in these 15 cases in more detail. As shown in Figure 6, to help students learn to solve context-based tasks, the teachers more frequently used a directive than a consultative teaching approach. Directive teaching was mostly used in the comprehension stage, whereas consultative teaching was mostly used in the mathematical processing stage. We also found a substantial number of cases where no instruction was given. This happened in all stages, but most often in the encoding and transformation stages.

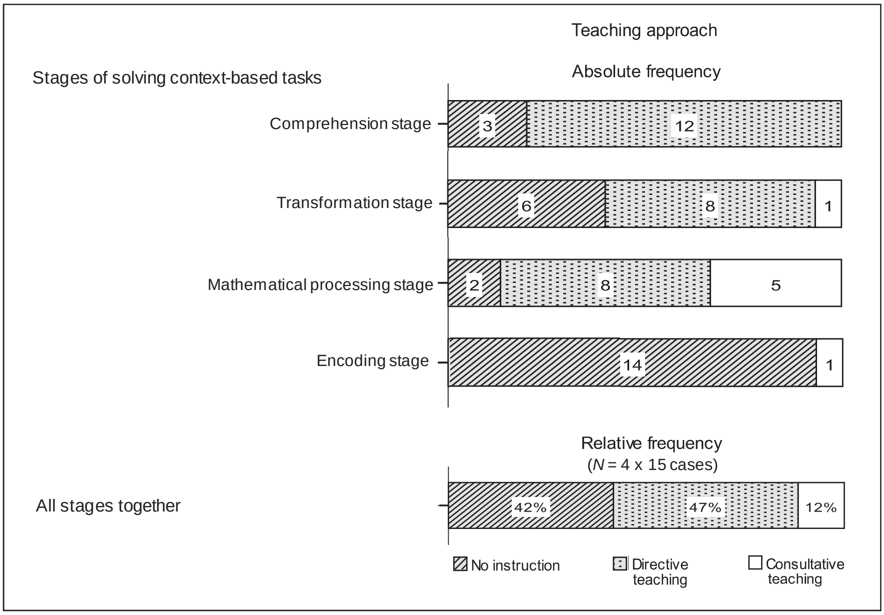


Figure 6. Overview per stage and over all stages of the teaching approaches used by the four teachers in the 15 found cases that were related to a context-based mathematics question

4.3.1 Comprehension stage

In the comprehension stage, attention was paid to 12 of the 15 cases in which a question was discussed in class. All teachers used directive teaching, i.e. reading the tasks aloud and telling students what they were about. Furthermore, no teacher asked students to paraphrase the tasks and explain what they understood from the tasks. For the remaining three cases, the teachers did not discuss what the tasks were about, but directly asked students the mathematical procedure(s) that is required to find the solution. This means the teachers skipped the comprehension stage and directly focused on the transformation stage.

An example of directive teaching in the comprehension stage is when Leni and her students were working on the *Journey* task (see Figure 2 and Excerpt 1). After distributing the student worksheet, Leni directly read aloud the text to her students (see lines 1-4) and demonstrated how to read information in the table (see lines 5-

7). The students were not given opportunities to paraphrase the task and to derive information from the table by themselves.

Excerpt 1. Teacher Leni: Journey task

Leni : For task 2 (the *Journey* task), I will read it for you. [Reading the text] [1]
Last Saturday, Joni and his friends went biking for three hours [2]
nonstop. […] The table shows the time and the distance travelled by [3]
Joni and his friends. [4]
[Reading the information in the table] After biking for 10 minutes, [5]
Joni checked his odometer showing 2.5 km. After 20 minutes, he [6]
checked his odometer again which showed 6 km; and so on. [7]

4.3.2 Transformation stage

In the transformation stage, nine of the 15 questions were discussed in class. In eight of these questions directive teaching was observed. For example, Ihsan did not ask his students to interpret the question "if water is poured into the tank in 10 minutes, how much water is in the tank?" by themselves (see Question c in Figure 7), but he translated this question into "if the value of x is 10, then what is the value of y?" (lines 1-2 in Excerpt 2).

The water in the backyard is filled in with water every day.
The relation between the time of filling water and the volume of water poured into the tank is shown in the table.

a. Let x be the time of filling water and y be the volume of Water poured into the tank. Does every pair of time and the related volume given in the table satisfy the equation $y = 5x + 2$?
b. Plot the points representing the pairs of (x,y) on the Cartesian coordinate and sketch a graph passing all these points.
c. If water is filled into the tank in 10 minutes, how much water is in the tank?

Time (x) in minutes	Volume of water in the tank (y) in litre
0	2
1	7
2	12
3	17
4	22
5	27

Figure 7. Water tank task (this task was made by Ihsan)

Excerpt 2. Teacher Ihsan: Water tank task, Question c

Ihsan : Okay, now discuss Question *c*. The question means that [1]
"if the value of *x* is 10, then what is the value of *y*?" [2]

Another kind of directive teaching observed in the transformation stage was telling the students what mathematical procedure to carry out. For example, when Leni and her students discussed a question about estimating the time Joni reached the fastest speed (see Question b in Figure 2). Although Leni encouraged her students to share their answers (see lines 3 and 5 in Excerpt 3), her focus was on the answers and not on stimulating them to identify relevant mathematical procedures. Furthermore, Leni directly told her students the fastest speed was the steepest line (see lines 5-9).

Excerpt 3. Teacher Leni; Journey task: Question b

Leni	:	Now Question *b*. Estimate when Joni reached the fastest speed.	[1]
Student 1	:	At (the period of) 60-90 minutes.	[2]
Leni	:	Any other opinion?	[3]
Student 2	:	110-150.	[4]
Leni	:	Any other (opinion)? … The fastest speed means that in a short	[5]
		time he (Joni) travelled the furthest distance. Among these lines	[6]
		[pointing at the segments on the graph], this (the fastest speed) is	[7]
		the line (segment) which is the most …? … The fastest speed is	[8]
		the steepest line. Which is the steepest line?	[9]

In the transformation stage a consultative teaching approach was observed only in Ihsan's class, when he discussed Question a of the *Water tank* task (Figure 7). Ihsan posed questions such as "How do we check it?" and "Which formula?" to stimulate students to think about strategies to solve the task (see line 2 and 5 in Excerpt 4).

Excerpt 4. Teacher Ihsan: Water tank task, Question a

Ihsan	:	For Question *a* you are asked to check whether they (the pairs of	[1]
		values in the table) satisfy the equation $y = 5x + 2$. How do we check it?	[2]
Students	:	(By using) subtraction and addition.	[3]
Other students:		Using that formula. The formula of y.	[4]
Ihsan	:	Which formula?	[5]
Students	:	The formula $y = 5x + 2$	[6]
Ihsan	:	Okay. If the x is substituted by 0, is it correct that y is 2?	[7]
Students	:	Yes, it is.	[8]
Ihsan	:	Now, let's try another value … for $x = 3$. What is the value of y?	[9]
Student 1		$5\times 3 + 2$ … 18 (this answer is incorrect; the correct answer is 17).	[10]
Ihsan	:	[Ignoring the student's answer and directly explaining the steps]	[11]
		Take $x = 3$. What is the formula? $y = 5x + 2$. This means [Writing	[12]
		on the board: $y = 5\times 3 + 2$; $y = 17$].	[13]

In the transformation stage it was further observed that when students had already found an adequate transformation into a mathematical problem – which was the case in six of the 15 questions – the teachers did not discuss with their students how they arrived at this mathematical problem and whether other procedures would have also been possible.

4.3.3 Mathematical processing stage

In comparison to the other stages, in the mathematical processing stage a consultative teaching approach was observed more frequently. In this stage the teachers posed questions to engage students in discussing mathematical procedures. Excerpt 4 illustrates interactions between Ihsan and his students discussing Question a of the *Water tank* task (Figure 7) which was about checking whether pairs of values in the table satisfy a given linear equation. In line 7 Ihsan engaged his students in the solving process by asking them to check whether the result of the substitution was correct. Furthermore, he encouraged students to do the substitution by themselves (see line 9).

4.3.4 Encoding stage

Our results clearly revealed that the teachers tended to ignore the encoding stage of solving context-based tasks. They only focused on the correctness of students' mathematical solutions without connecting the answers to the task's context. A consultative approach was observed in only one out of 15 cases, i.e. in Ihsan's classroom for Question c of the *Water tank* task (see Figure 7). When a student gave a number without any measurement unit, Ihsan asked her "Has this already solved the task?" (see line 2 in Excerpt 5). Contrary to Ihsan's teaching approaches in the other stages for this question, directive teaching was not used in the encoding stage. Here he did not directly tell the students to connect the answer to the context, but stimulated them to reflect on the answer. Furthermore, Ihsan also asked students to explain their opinion (see line 5-6). Moreover, in the end, Ihsan asked his students to conclude the correct answer in terms of the context of the task (see line 10).

Excerpt 5. Teacher Ihsan: Water tank task, Question c

Ihsan	:	Now let's check Dina's answer. This … if we substitute x	[1]
		with 0 then y is 52. Has this already solved the task?	[2]
Student 1	:	Yes.	[3]
Student 2 and 3	:	Not yet.	[4]
Ihsan	:	Some of you said "yes", but some others said "no". Please	[5]
		explain why this has not yet solved the task.	[6]
Student 2	:	Because there is no 'liter'	[7]
Ihsan	:	Yes, you are right. So, this is 52 .... what?	[8]

Students	:	Liter.	[9]
Ihsan	:	So, what is the conclusion?	[10]
Students	:	The volume of water in the tank is 52 liters.	[11]

4.4 Relation between teachers' beliefs and teaching practices and students' errors

Combining the findings of the present study with our earlier findings about the errors Indonesian ninth-graders made when solving context-based tasks (Wijaya et al., 2014)[3], we found correspondences between students' errors and teachers' beliefs and teaching practices regarding context-based tasks.

Table 5 shows that the students made a substantial number of errors in the comprehension stage, which mostly were errors in selecting relevant information. In relation to this earlier finding, 41% of the teachers (strongly) agreed that context-based tasks should only provide matching information. Furthermore, the same number of teachers (strongly) disagreed that superfluous information should be included in a context-based task. Regarding their conceived practice, 67% of the teachers reported that they provide in every lesson or weekly context-based tasks that contain only the information needed to find the solution. Finally, observed teaching practice revealed that consultative teaching was not used in this stage.

With respect to the transformation stage, the findings were quite similar. In this stage students made a high number of errors in identifying the required procedures and, correspondingly, three quarters of teachers (strongly) agreed that context-based tasks should state explicitly what mathematical procedure is required. Moreover, two thirds of the teachers offered students such context-based tasks weekly or in every lesson. The observed teaching practice showed that in 40% of the cases no instruction was given related to this stage and that in half the cases teaching was directive. Hardly any consultative teaching was provided to offer students opportunities to develop their ability to transform a real-world problem into a mathematical problem.

[3] In total, 233 ninth-graders were involved in this previous study that was carried out in school year 2011-2012. The students came from the same schools as the teachers in the present study which took place in the school year 2012-2013 and involved teachers from Grade 7 to Grade 9. The test that was administered contained 34 questions distributed over four different booklets. Every student made 13 items. The analyzed data consisted of 3027 responses (students × items). Of these responses 1855 were correct, 346 were missing and 826 were incorrect which included 934 errors (because of the multiple coding, the number of errors is larger than the number of incorrect responses).

Table 5

Teachers' beliefs and teaching practices per stage of solving context-based tasks and proportion of students' errors in this stage

Stage in solving a context-based task	Proportion of type of errors in the stage (233 ninth-graders made 934 errors)[a]		Teachers' beliefs about context-based tasks (*N* = 27 teachers)		Teachers' reported teaching practice for teaching context-based tasks (*N* = 27 teachers)		Observed type of teaching approach when teaching context-based tasks (*N* = 15 cases)	
Compre-hension stage	38% (50% of errors in this stage were errors in selecting relevant information)	Matching information	Strongly agree/Agree	41%	Weekly/Every lesson	67%	No instruction	20%
		Superfluous information	Strongly disagree/Disagree	41%	Never/Once-twice in semester	52%	Directive instruction	80%
		Missing information	Strongly disagree/Disagree	22%	Never/Once-twice in semester	44%	Consultative instruction	0%
Transfor-mation stage	39% (68% of errors in this stage were errors in identifying required procedures)	Explicitly providing required mathematical procedures	Strongly agree/Agree	74%	Weekly/Every lesson	67%	No instruction	40%
							Directive instruction	53%
							Consultative instruction	7%
Math. processing stage	20%						No instruction	13%
							Directive instruction	53%
							Consultative instruction	43%
Encoding stage	3%						No instruction	93%
							Directive instruction	0%
							Consultative instruction	7%

[a] See Note 3 and Wijaya et al. (2014)

In the mathematical processing stage the situation was different. Here consultative teaching was observed in 43% of the cases, which might explain why students made fewer mathematical processing errors than comprehension and transformation errors. Lastly, we found that in the encoding stage, where students only made a few errors, in almost all cases the teachers did not give any instruction to students. Obviously, the teachers mostly ignored the interpretation of a mathematical answer in terms of the context of a problem.

5 Conclusions and discussion

5.1 Teachers' OTL provided to students to solve context-based tasks

In the present study, the OTL to solve context-based tasks that was offered by teachers was examined in order to find possible causes of students' difficulties in solving context-based tasks. This study focused on teachers' teaching practices and the underlying beliefs. Data were collected by a teacher survey based on a written questionnaire and classroom observations.

The first focus of the study was to investigate Indonesian teachers' beliefs about the nature of mathematics, the teaching and learning of mathematics, and context-based tasks (Research question 1). It was found that in general the Indonesian teachers in the study had a tendency toward a fallibilist view on mathematics, its learning and teaching. This view suggests that they had ideas that are supportive for offering students OTL to solve context-based tasks. However, teachers were not always consistent in their responses. Almost half of them believed that school mathematics is teaching pure mathematics, which clearly reflects an absolutist view.

Regarding the teachers' beliefs about context-based tasks, we found the teachers tended to perceive context-based tasks as merely plain word problems. Most teachers thought that context-based tasks should provide only the information needed to find the solution and should explicitly provide the required mathematical procedures. In line with other researchers (Chapman, 2009; Galbraith & Stillman, 2006; Maas, 2010; Verschaffel et al., 2010), we argue that having such beliefs about context-based tasks and perceiving context-based tasks as straightforward word problems will not be supportive for providing students OTL to solve context-based tasks. Teachers who have such beliefs might only focus on the mathematical properties or structures of a context-based tasks without attaching great value to the problems' context (Chapman, 2009). Furthermore, they might abandon daily life knowledge and experiences during the solving process (Galbraith & Stillman, 2006) and might not contribute to the students' sense-making of a problem (Verschaffel et al., 2010). Moreover, word problems usually provide mathematical procedures and therefore do not offer enough opportunities for students to learn to identify an appropriate procedure or make a mathematical model of the problem situation (Maass, 2010).

When investigating the kinds of context-based tasks the Indonesian teachers offer their students (Research question 2a) the questionnaire data indicated a relation with the teachers' beliefs. In agreement with their beliefs, the teachers reported that they mostly gave context-based tasks which explicitly provide the needed procedures and contain only the information that is relevant for solving the tasks. Furthermore, most teachers stated that they rarely gave context-based tasks with superfluous information.

With respect to how context-based tasks were taught (Research question 2b) the classroom observations revealed that the Indonesian teachers in our sample mainly used a directive teaching approach. The teachers mainly told the students what the problem is about, what information they have to use, and what mathematical problem they have to solve. The teachers also immediately corrected their students' mistakes when performing a mathematical procedure, and focused on the mathematical solution without connecting it to the context of the problem. In agreement with Antonius et al. (2007) who argued that teaching context-based tasks requires more than telling what students should do and offering exercises to practice, we argue that the observed teaching practice in the investigated Indonesian classrooms cannot be considered to be supportive for providing students OTL to solve context-based tasks. More specifically, our observation that in the comprehension stage the teachers did not give their students opportunities to paraphrase the tasks, might contribute to students' difficulty in comprehending a context-based task (see Hagaman, Casey, & Reid, 2008). Paraphrasing would help students to understand the text of a task, and to get access to what they already know about the task (Kletzien, 2009). The directive teaching observed in the transformation stage is also not beneficial to teaching students to solve context-based tasks, because as Barnes (2000) stressed, this teaching discourages students to think about mathematical concepts involved in tasks. Only in the mathematical processing stage did we find the teachers using consultative teaching, which may be so because the teachers have more experience in teaching mathematical procedures than in dealing with real-world problems. Therefore they might have more flexibility in supporting their students' learning in the mathematical processing stage. Lastly, in the encoding stage the teachers tended to completely ignore the interpretation of mathematical solution(s) in terms of the context of the problem.

Our finding about teachers' preference for the directive teaching approach is in line with results from other studies which also examined teaching practices in mathematics classrooms in Indonesia (see, e.g., Human Development Department East Asia and Pacific Region, 2010; Maulana, Opdenakker, Den Brok, & Bosker, 2012). These studies revealed that Indonesian mathematics teachers tended to take a directive role in which they mostly explain while students write, listen, and answer closed questions. Maulana et al. (2012) argued that such directive practices might be caused by a cultural aspect of Indonesian society that considers the teacher profession as highly respected, so the teacher is considered as the source of knowledge, whereas students are the recipients.

Lastly, we can conclude that there is a relationship between Indonesian teachers' teaching practices and their underlying beliefs regarding context-based tasks and the errors which the Indonesian students, involved in our earlier study, made when solving these tasks (Research question 3). The teachers' beliefs about the kind of information to be included in context-based tasks and the frequent use of tasks with matching information correspond to the high occurrences of comprehension errors, especially errors in selecting information. Similarly, the teachers' beliefs and practices regarding context-based tasks with clear indications for the required mathematical procedures match to students' transformation errors. Furthermore, the dominance of directive teaching in the comprehension and transformation stages might also explain the high number of comprehension and transformation errors made by the students. The findings of this study indicate that the shortage in the OTL to solve context-based tasks offered by the teachers is a possible explanation for students' difficulties solving these tasks.

5.2 Limitations and recommendations

Due to some limitations of this study, the conclusions should be interpreted with caution. To begin with, teachers' reports about their teaching practices regarding context-based tasks should be considered with prudence because the data were based on self-report. Another limitation is that the classroom observations were only conducted in four classrooms and in each of these classrooms only two lessons were observed. This means that only a snapshot of the teachers' teaching practices was captured. Moreover, in this selection the focus was only on one mathematics topic. The teachers might have shown different teaching practices if they were observed for more lessons addressing various mathematics topics. In any case a larger sample of teachers would have given a more reliable picture of Indonesian teachers' teaching practice regarding context-based tasks. Finally, to answer the research question about the relationship between OTL offered by teachers and errors made by students we used data, as explained in the Results section, which came from the same schools, but not from the same cohorts.

These limitations make it clear that to have a more robust understanding of the teachers' role in the difficulties students have when solving context-based tasks, it is necessary to conduct a further research which has a wider scope and includes more teachers that are followed, with their students, over a long time. Nevertheless, the present study gives a first understanding of the importance of the OTL offered by the teachers and added to our earlier study about the OTL to solve context-based tasks offered in Indonesian textbooks (Wijaya et al., submitted).

Based on this first understanding we have the following recommendations for educational practice. When confronted with students' low performance in solving context-based tasks teachers (and prospective teachers) should look critically at their own role in students' learning processes. Did they really offer their students opportunity-to-learn to solve context-based tasks? And were they aware of the different stages of solving context-based tasks, each requiring specific opportunities-to-learn?

References

Adamson, S., Burtch, M., Cox, T., Banks, D., Judson, E., & Lawson, T. (2002). Nature of Mathematics Survey. Retrieved June 1, 2012, from http://www.amatyc.org/publications/Electronic-proceedings/Phoenix28/Adamson/NatureOfMathSurvey.pdf.

Antonius, S., Haines, C., Jensen, T. H., Niss, M., & Burkhardt, H. (2007). Classroom activities and the teacher. In W. Blum, P. L. Galbraith, H.-W. Henn & M. Niss (Eds.), *Modelling and applications in mathematics education* (pp. 295–308). New York: Springer.

Barnes, M. (2000). 'Magical' moments in mathematics: Insights into the process of coming to know. *For the Learning of Mathematics, 20*(1), 33–43.

Bell, C. V., & Pape, S. J. (2012). Scaffolding students' opportunities to learn mathematics through social interactions. *Mathematics Education Research Journal, 24*(4), 423–445.

Beswick, K. (2005). The beliefs/practice connection in broadly defined contexts. *Mathematics Education Research Journal, 17*(2), 39–68.

Blum, W. (2011). Can modelling be taught and learnt? Some answers from empirical research. In G. Kaiser, W. Blum, R. B. Ferri & G. Stillman (Eds.), *Trends in teaching and learning of mathematical modelling* (pp. 15–30). New York: Springer.

Brewer, D. J., & Stasz, C. (1996). Enhancing opportunity to learn measures in NCES data. In G. Hoachlander, J. E. Griffith & J. H. Ralph (Eds.), *From data to information: New directions for the National Center for Education Statistics* (pp. 3.1–3.28). Washington, D.C.: U.S. Department of Education.

Chapman, O. (2009). Teachers' conceptions and use of mathematical contextual problems in Canada. In L. Verschaffel, B. Greer, W. Van Dooren & S. Mukhopadhyay (Eds.), *Words and worlds. Modelling verbal descriptions of situations* (pp. 227–244). Rotterdam: Sense Publishers.

Clements, M. A. (1980). Analyzing children's errors on written mathematical task. *Educational Studies in Mathematics, 11*(1), 1–21.

Cooper, B., & Dunne, M. (2000). *Assessing children's mathematical knowledge: Social class, sex and problem-solving*. Buckingham: Open University Press.

Erickson, F. (2006). Definition and analysis of data from videotape: Some research procedures and their rationales. In J. L. Green, G. Camili & P. B. Ellmore (Eds.), *Handbook of complementary methods in education research* (pp. 177–192). Mahwah: Lawrence Erlbaum.

Ernest, P. (1989). The impacts of beliefs on the teaching of mathematics. In P. Ernest (Ed.), *Mathematics Teaching: The state of the art* (pp. 249–253). New York: Falmer.

Ernest, P. (1991). *The Philosophy of Mathematics Education. Studies in Mathematics Education.* London: Routledge Falmer.

Forman, S. L., & Steen, L. A. (2001). Why math? Applications in science, engineering, and technological programs. *Research Brief, American Association of Community Colleges.*

Galbraith, P., & Stillman, G. (2006). A framework for identifying student blockages during transitions in the modelling process. *ZDM The International Journal on Mathematics Education, 38*(2), 143–162.

Graumann, G. (2011). Mathematics for problem in the everyday world. In J. Maasz & J. O'Donoghue (Eds.), *Real-world problems for secondary school mathematics students: Case studies* (pp. 113–122). Rotterdam: Sense Publishers.

Grouws, D. A., & Cebulla, K. J. (2000). *Improving student achievement in mathematics*. Brussels: International Academy of Education.

Hagaman, J. L., Casey, K. J., & Reid, R. (2012). The effects of the paraphrasing strategy on the reading comprehension of young students. *Remedial and Special Education, 33*(2), 110–123.

Hiebert, J., & Grouws, D. A. (2007). The effects of classroom mathematics teaching on students' learning. In F. Lester (Ed.), *Second handbook of research on mathematics teaching and learning* (pp. 371–404). Charlotte: Information Age Publishing.

Human Development Department East Asia and Pacific Region. (2010). *Inside Indonesia's mathematics classrooms: A TIMSS Video Study of teaching and practices and student achievement*. Jakarta: The World Bank.

Husén, T. (Ed). (1967). *International study of achievement in mathematics: A comparison of twelve countries (Vol. II)*. New York: John Wiley & Sons.

Karbalei, A., & Amoli, F. A. (2011). The effect of paraphrasing strategy training on the reading comprehension of college students at the undergraduate level. *Asian EFL Journal, 13*(3), 229–344.

Kletzien, S. B. (2009). Paraphrasing: An effective comprehension strategy. *The Reading Teacher, 63*(1), 73–77.

Klymchuk, S., Zverkova, T., Gruenwald, N., & Sauerbier, G. (2010). University students' difficulties in solving application problems in calculus: Student perspectives. *Mathematics Education Research Journal, 22*(1), 81–91.

Kramarski, B., Mevarech, Z. R., & Arami, M. (2002). The effects of metacognitive instruction on solving mathematical authentic tasks. *Educational Studies in Mathematics, 49*(2), 225–250.

Lakatos, I. (1976). A renaissance of empiricism in the recent philosophy of mathematics. *The British Journal for the Philosophy of Science, 27*(3), 201–223.

Landis, J. R., & Koch, G. G. (1977). The measurement of observer agreement for categorical data. *Biometrics, 33*(1), 159–174.

Lingefjärd, T., & Meier, S. (2010). Teachers as managers of the modelling process. *Mathematics Education Research Journal, 22*(2), 92–107.

Maass, K. (2010). Classification scheme for modelling tasks. *Journal fur Mathematik-Didaktik, 31*(2), 285–311.

Maulana, R., Opdenakker, M.-C., Den Brok, P., & Bosker. (2012). Teacher-student interpersonal behavior in secondary mathematics classes in Indonesia. *International Journal of Science and Mathematics Education, 10*(1), 21–47.

NCTM. (2000). *Principles and Standard for School Mathematics*. Reston: Author.

OECD. (2003). *The PISA 2003 Assessment Framework – Mathematics, Reading, Science, and Problem Solving Knowledge and Skills*. Paris: Author.

OECD. (2010). *PISA 2009 results: What students know and can do. Student performance in reading, mathematics, and science (Vo. I)*. Paris: Author.

OECD. (2013). *PISA 2012 results: What students know and can do. Student performance in mathematics, reading and science (Vol. I)*. Paris: Author.

Pepin, B., & Haggarty, L. (2001). Mathematics textbooks and their use in English, French and German classrooms: A way to understand teaching and learning cultures. *ZDM – The International Journal on Mathematics Education, 33*(5), 158–175.

Prakitipong, N., & Nakamura, S. (2006). Analysis of mathematics performance of grade five students in Thailand using Newman procedure. *Journal of International Cooperation in Education, 9*(1), 111–122.

Pusat Kurikulum. (2003). *Kurikulum 2004. Standar kompetensi mata pelajaran matematika Sekolah Menengah Pertama dan Madrasah Tsanawiyah*. Jakarta: Departemen Pendidikan Nasional.

Sam, L. C., Lourdusamy, A., & Ghazali, M. (2001). Factors affecting students' abilities to solve operational and word problems in mathematics. *Journal of Science and Mathematics Education in Southeast Asia, 24*(1), 84–95.

Scardamalia, M., Bransford, J., Kozma, B., & Quellmalz, E. (2012). New assessments and environments for knowledge building. In P. Griffin, B. McGraw & E. Care (Eds.), *Assessment and teaching of 21st century skills* (pp. 231–300). New York: Springer.

Schleicher, A. (2007). Can competencies assessed by PISA be considered the fundamental school knowledge 15-year-olds should possess? *Educational Change, 8*(4), 349–357.

Sepeng, P. (2013). Use of unrealistic contexts and meaning in word problem solving: A case of second language learners in Township schools. *International Journal of Research in Mathematics, 1*(1), 8–14.

Stipek, D. J., Givvin, K. B., Salmon, J. M., & MacGyvers, V. L. (2001). Teachers' beliefs and practices related to mathematics instruction. *Teaching and Teacher Education, 17*(2), 213–226.

Tomlinson, M. (2004). *14-19 Curriculum and Qualifications Reform. Final report of the Working Group on 14-19 Reform*. London: Department for Education and Skills.

Tornroos, J. (2005). Mathematics textbooks, opportunity to learn and student achievement. *Studies in Educational Evaluation, 31*(4), 315–327.

Toumasis, C. (1997). The NCTM standards and the philosophy of mathematics. *Studies in Philosophy and Education, 16*(3), 317–330.

Tuge, C. (2008). Mathematics curriculum, the philosophy of mathematics and its implications on Ethiopian schools mathematics curriculum. *Ethiopian Journal of Education and Science, 4*(1), 109–120.

Verschaffel, L., Greer, B., & De Corte, E. (2000). *Making sense of word problems*. Lisse: Swets & Zeitlinger.

Verschaffel, L., Van Dooren, W., Greer, B., Mukhopadhyay, S. (2010). Reconceptualising word problem as exercises in mathematical modelling. *Journal fur Mathematik-Didaktik, 31*(1), 9–29.

Wijaya, A., Van den Heuvel-Panhuizen, M., Doorman, M., & Robitzsch, A. (2014). Difficulties in solving context-based PISA mathematics tasks: An analysis of students' errors. *The Mathematics Enthusiast, 11*(3), 555-584.

Wijaya, A., Van den Heuvel-Panhuizen, M., & Doorman, M. (submitted). Opportunity-to-learn context-based tasks provided by mathematics textbooks and the relation with students' difficulties in solving these tasks.

Wilkins, J. L. M. (2008). The relationship among elementary teachers' content knowledge, attitudes, beliefs, and practices. *Journal of Mathematics Teacher Education, 11*(2), 139–164.

Xin, Z., Lin, C., Zhang, L., & Yan, R. (2007). The performance of Chinese primary school students on realistic arithmetic word problems. *Educational Psychology in Practice, 23*(2), 145–159.

Chapter 5

Opportunity-to-learn to solve context-based mathematics tasks and students' performance in solving these tasks – lessons from Indonesia

Wijaya, A., Van den Heuvel-Panhuizen, M., Doorman, M., & Veldhuis, M. (submitted). Opportunity-to-learn to solve context-based mathematics tasks and students' performance in solving these tasks – lessons from Indonesia.

Opportunity-to-learn to solve context-based mathematics tasks and students' performance in solving these tasks – lessons from Indonesia

1 Introduction

The broad recognition of the importance of mathematics for coping with the demands of the 21st century has led to an emphasis on developing students' ability to apply mathematics in a real-world context as an important goal of mathematics education (Eurydice, 2011; Graumann, 2011; National Council of Teachers of Mathematics [NCTM], 2000; Tomlinson, 2004). This ability is also the focus in the framework used in the Programme for International Student Assessment (PISA) of the Organisation for Economic Co-operation and Development (OECD, 2003). However, notwithstanding the high importance attached to teaching students to apply mathematics, there is a discrepancy between this intended goal and the achievement of students. A number of studies (e.g., Cooper & Dunne, 2000; Sam et al., 2001) have shown students' low performance in solving mathematics tasks that are situated in real-world contexts. In the PISA studies (OECD, 2010; 2013) it was found that many students cannot solve real-world problems that require reasoning skills and mathematical modeling of complex situations.

Also in Indonesia, the application of mathematics in real-world situations is considered a relevant aspect of the mathematics curriculum (Pusat Kurikulum, 2003). Yet the results of the PISA studies show that Indonesian students perform low on context-based tasks (e.g., OECD, 2013). This situation prompted us to set up the Context-based Mathematics Tasks Indonesia (CoMTI) project. The aim of this project was to identify ways to improve Indonesian students' ability to apply mathematics in a real-world context. We began the project with a study which was aimed at clarifying the difficulties Indonesian students have when dealing with context-based tasks. In this first study (Wijaya et al., 2014), we analyzed students' errors in which we found that Indonesian students mainly have difficulties in comprehending context-based problems and in transforming them into mathematical problems. Our second study (Wijaya et al., submitted a) revealed that one of the reasons for these difficulties might be that Indonesian textbooks do not offer the opportunity-to-learn how to solve context-based tasks. Indonesian textbooks contain a low number of context-based tasks. Furthermore, these context-based tasks are mostly plain word problems, which use dressed-up contexts, are often very explicit about what mathematical procedures have to be carried out, and provide precisely the information that is needed to solve the task. Finally, in our third study (Wijaya et al., submitted b) classroom observations were carried out to identify what opportunities teachers offer their students to learn to solve context-based tasks. Here we noticed that Indonesian teachers mostly used directive and teacher-centered teaching and did not give students opportunities to actively get involved in and reflect on the process of solving context-based tasks.

In the present study, the findings of these three earlier studies were synthesized and used to develop an intervention that offers students opportunity-to-learn to solve context-based tasks. This intervention was put to the test in a field experiment in which we investigated whether student performance can be enhanced by giving students experience in solving context-based tasks that require the selection of relevant information and an adequate mathematical procedure, and by offering them a teaching approach that involves them actively in the process of solving context-based tasks.

2 Theoretical background

2.1 Students' difficulties when solving context-based tasks

Solving context-based tasks involves a complex process in which students pass through several stages. According to Blum and Ferri (2009) and Blum (2011), this process comprises four main stages. The first stage relates to the process of comprehending what a context-based task is about. In the second stage students need to look for a mathematical concept or procedure required to solve the task. In this stage the real-world problem is transformed into a mathematical problem. The third stage is carrying out the mathematical procedure to solve the mathematical problem. Finally, in the fourth stage the mathematical solution obtained in the third stage is interpreted in terms of the context of the task.

In each of these stages students can face difficulties and make errors. Research has shown that in the first stage students often misunderstand the meaning of the tasks and misinterpret the terms used in the tasks (Bernardo, 1999; Klymchuk et al., 2010). In the second stage students are struggle to identify the mathematical concept or procedure that is needed to solve the tasks (Clements, 1980; Klymchuk et al., 2010). This difficulty relates to students' tendency either to apply routine mathematical procedure without realistic considerations (Verschaffel et al., 2000; Xin et al., 2007) or to take too much account of the context of the tasks so that no mathematical concept or procedure is used (Boaler, 1994). In the third stage where the mathematical processing is carried out, students can make errors in carrying out mathematical procedures. At the end of the solving process, in the fourth stage, students often have difficulties in interpreting a solution in terms of the context, and give solutions that are not relevant to the context of the tasks (Greer, 1997).

2.2 Opportunity-to-learn to solve context-based tasks

An initial question that can be asked when students perform low on a particular competence is whether they have received the necessary education to develop this competence. To cover this question, in the 1960s, the concept of opportunity-to-learn emerged. Researchers involved in comparative studies became aware that when comparing the achievements of students in different countries, students' opportunity-to-learn resulting from curricular differences had to be taken into account (McDonnell, 1995). In the report of the First International Mathematics Studies (Husén, 1967), opportunity-to-learn was defined as "whether or not […] students have had the opportunity to study a particular topic or learn how to solve a particular type of problem" (pp. 162-163). Presently, as a result of increasing requests for accountability and higher achievement standards there is concern whether students have access to the instruction required to reach these standards. Therefore, there is a renewed interest for investigating students' opportunity-to-learn (Abedi & Herman, 2010). This can generally be done in two ways: by examining the instructional resources such as textbooks and by surveying the instructional strategies employed by teachers (Brewer & Stasz, 1996; Grouws & Cebulla, 2000; Herman et al., 2000).

2.2.1 Opportunity-to-learn offered by textbooks

Textbooks are considered as the main instructional material for teachers (Brewer & Stasz, 1996), and mediate between the intended and the implemented curriculum (Valverde et al., 2002). Research has shown that textbooks have a strong influence on students' learning (Schmidt et al., 1997; Tornroos, 2005). Therefore, what is offered in textbooks can be regarded as an important measure for students' opportunity-to-learn. In this respect several aspects of textbooks may be taken into account. First, the degree of exposure to particular content included in a textbook can influence students' performance. Tornroos (2005) found a relation between student achievement on a test and the amount of textbook content related to the test items. Second, the method used in textbooks to help students understand the content is also an important aspect. As found by Xin (2007), students tend to solve word problems by using the solution strategies suggested in the textbooks. Third, students' opportunity-to-learn is also determined by the characteristics of the tasks in textbooks. In this respect, Charalambous et al. (2010) mention the cognitive demands of tasks and the required types of responses in tasks. For example, it makes a difference whether students can build up experience with tasks in which they just have to recall facts and procedures, or that they are offered tasks in which they have to model a problem situation or reflect on a solution. With respect to opportunity-to-learn to solve real-world problems, Maass (2007) highlighted the importance of giving students tasks that have superfluous and missing information. Such tasks are necessary to direct students to pay attention to the context of the tasks and to teach students to distinguish between relevant and irrelevant information. Furthermore, it is also essential to provide students with real-worlds problems that do not provide explicit suggestions about the required procedures to develop students' modeling competence (Maass, 2010).

2.2.2 Opportunity-to-learn offered by teachers' instructional strategies

In addition to research that shows the influence of textbooks on students' learning, there are also many studies (e.g. Eurydice, 2011; Grouws & Cebulla, 2000; Hiebert & Grouws, 2007) that have highlighted that student performance is affected by the instructional strategies that teachers use. How teachers teach mathematics and engage their students influences how well students learn. With respect to the teaching of context-based tasks, several researchers (Antonius et al., 2007; Blum, 2011; Forman & Steen, 2001) suggested teachers using a teaching approach in which they take a consultative role and give students opportunities to actively build new knowledge and reflect on their learning process. Such consultative teaching offers students opportunity to develop strategic competences that are necessary to solve context-based tasks, such as understanding the text of problems, selecting relevant information, identifying appropriate mathematical procedures, and interpreting solutions. A key aspect of consultative teaching is to keep a balance between teacher guidance and students' independence. Both Antonius et al. (2007) and Blum (2011) recommended the use of metacognitive prompts to create this balance and Montague (2007) emphasized that these metacognitive prompts help students to become active learners.

Metacognitive prompts can be provided in the form of self-addressed questions; i.e. students are asked to question themselves while solving a problem. Self-addressed questions are an important stimulus to help students regulate their solving process and reflect on it (Kramarski et al., 2002; Montague, 2007; 2008). Another kind of metacognitive prompt is giving a verbal prompt or instruction to help students focus attention on particular aspects of the solving process and to assist them in carrying out the solving process (Goldman, 1989; Montague, 2007; Montague et al., 2000). For example, the instruction to underline the important information in a task (see e.g. Montague, 2007; 2008) can be used to guide students to focus on identifying relevant information. Asking students to paraphrase a task is also an important prompt. According to Karbalei and Amoli (2011) and Kletzien (2009), students who explain in their own words what the task is about, gain a better understanding of the task while doing this.

3 Results from our previous studies and the research question

3.1 Indonesian students' difficulties when solving context-based tasks

To investigate students' difficulties when solving context-based tasks we (Wijaya et al., 2014) analyzed the errors made by students. Four error types – comprehension, transformation, mathematical processing, and encoding – were distinguished by associating Newman's (1977) error categories with Blum and Leiss' (2007) modeling process and PISA's (OECD, 2003) mathematization stages. Comprehension errors refer to students' inability to understand a context-based task, which includes the inability to select relevant information. Transformation errors correspond to students' failure in identifying the mathematical procedure required to solve a task. Mathematical processing errors are related to mistakes in carrying out mathematical procedures; for example, errors in calculating, solving algebraic formulae, or interpreting a graph. Encoding errors refer to answers that are unrealistic and do not match the context of the task.

Our error analysis revealed that 38% of Indonesian students' errors when solving context-based tasks were comprehension errors. Students often misunderstood the question of a context-based task. Students also made remarkable errors in selecting relevant information in which they tended to use all information given in the text without considering the relevance of this information. Transformation errors were found in 42% of students' errors. Many students used the wrong procedure or concept when solving a context-based task in which they tended to use a familiar procedure. Regarding mathematical processing errors which are not directly related to the context-based character of the tasks, but rather to the mathematical topic addressed in the tasks, we found that 17% of all errors were of this type. In our study, students frequently made errors in interpreting graphs and in dealing with algebraic formulas or expressions. Finally, encoding errors were found to be in the minority, with only 3% of students' errors belonging to this type. An example of this error type is students giving 70 meter as the pace length of a human.

3.2 Opportunity-to-learn offered in Indonesian textbooks

To find possible explanations for Indonesian students' difficulties when solving context-based tasks, we first investigated the opportunity-to-learn to solve context-based tasks as offered in Indonesian textbooks. For this investigation we carried out a textbook analysis to get an overview of the characteristics of the tasks included in the textbooks (see Wijaya et al., submitted a). The framework we developed for this analysis contained four analysis entries. First, we examined the type of context used in a task, for which we distinguished three types of context: no context, camouflage context, and relevant and essential context. Second, a further analysis was done for the context-based tasks to identify the purpose of the tasks; which could be application or modeling. Third, we investigated the type of information that was provided in a task. Three types of information were considered: matching, missing, and superfluous information. The fourth characteristic was the cognitive demand of a task. For this the tasks' types of PISA (OECD, 2003) were taken into account: reproduction, connection, and reflection.

Our textbook analysis revealed that only 10% of the tasks in the textbooks were context-based tasks. When zooming in on the characteristics of these context-based tasks, we found that three quarters of them used a camouflage context, which means that the context can be neglected when solving the task and that the procedures are explicitly provided. It also means that the students did not have to think about transforming the task into a mathematical problem. Furthermore, this lack of stimulus for transformation was amplified by the fact that all the context-based tasks were given after the explanation sections in which a particular concept or procedure was explained. With respect to the type of information, 85% of the context-based tasks contained matching information, meaning that only the relevant information was provided. Lastly, only 2% of the context-based tasks in Indonesian textbooks were reflection tasks, i.e. tasks with the highest cognitive demands that require constructing original mathematical approaches and communicating complex arguments and reasoning.

Combining these findings with our findings regarding the types of students' errors strongly indicates that both sets of findings are related. First of all, when only 10% of the tasks in Indonesian textbooks are context-based one cannot expect Indonesian students to be good at solving such tasks. Also, the characteristics of the tasks included in the textbooks are clearly reflected in the students' errors. For example, the lack of opportunity students that get in dealing with superfluous and missing information can be recognized in the large number of errors Indonesian students made in selecting information. Similarly, the high percentage of students' transformation errors corresponds to the low number of context-based tasks in textbooks in which the students had to figure out the mathematical procedure by themselves.

3.3 Opportunity-to-learn offered by the teaching practices of Indonesian teachers

Our next step was to investigate to what degree students are offered opportunity-to-learn to solve context-based tasks by Indonesian teachers in their teaching practice. For this, we collected data about this teaching practice through a written questionnaire and classroom observations (see Wijaya et al., submitted b). In the questionnaire the focus was on the characteristics of context-based tasks that were offered by teachers. It was found that 18 of the 27 Indonesian mathematics teachers who were involved in our study gave context-based tasks with explicit procedures in every lesson or weekly. The same number of teachers gave context-based tasks with matching information in every lesson or weekly. Regarding context-based tasks with superfluous information, 11 of the 27 teachers reported that they never gave such tasks or did so once or twice in a semester. Context-based tasks with missing information were never given or given only once or twice in a semester by 9 of the 27 teachers.

To examine in more detail the teaching practice, observations were done in four classrooms. As studies by Anthonius et al. (2007) and Blum (2011) highlighted that for teaching mathematical modeling a consultative teaching practice is more helpful than a directive teaching practice we took this distinction as the perspective of our classroom observations. In consultative teaching a teacher plays a consultative role and involves students to actively carry out and reflect on the process of solving context-based tasks, while in directive teaching a teacher mainly tells students what they should do to solve a context-based task. Our observations covered in total eight lessons carried out in four classrooms of four teachers.

During these observations a total of 15 questions was discussed in class, but we found consultative teaching for only two questions. Directive teaching in which the teachers directly told the students what the context-based task was about, or explained what procedure was needed to solve the task was observed in 7 of the 15 questions. In addition, it was also observed that on some occasions the teachers did not give attention to a particular stage of the solving process. This happened in 6 of the 15 questions, and mostly applied to the encoding stage in which the teachers only focused on the correctness of students' mathematical solutions without connecting the solutions to the context of the task.

3.4 Research questions

The aim of the present study was to test whether students' performance in solving context-based tasks can be improved by offering them the opportunity-to-learn these tasks. The ingredients for creating this opportunity-to-learn were derived from what we found in research literature and our three earlier studies. The opportunity-to-learn comprised context-based tasks with particular characteristics, which were found to be lacking in Indonesian textbooks, and a consultative teaching approach that is considered to be supportive in developing students' ability to solve context-based tasks. To investigate whether offering students this opportunity-to-learn is an effective way to improve their ability to solve context-based tasks we looked into the effect of this opportunity-to-learn on both the correctness of students' answers and the errors made by them. This led to the following research question:

> *Does providing students with an opportunity-to-learn to solve context-based tasks contribute to students' performance when solving these tasks and, more specifically, is there any effect on the correctness of the answers and on the types of errors?*

4 Method

4.1 Design of the study

To answer the research question, we carried out a field experiment with a pretest-posttest control-group design. In the experimental group, the teachers used an intervention program for their teaching, intended to offer students opportunity-to-learn to solve context-based tasks. In the control group the students followed a teaching program that was developed on the basis of the textbook that they regularly use.

4.2 Participants

The study took place in six junior high schools located in the province of Yogyakarta, Indonesia. This province was chosen for reasons of convenience; the first author originates from this part of the country. In every school an experimental class and a control class were selected. In four schools there was only one teacher for each grade; therefore in these schools one teacher taught both the experimental and the control class. In the other two schools the experimental class and the control class were taught by different teachers. We left the decision about the allocation of these teachers to one of the two conditions to the school principal. All eight participating teachers had a bachelor degree in mathematics education and considerable teaching experience, ranging from 5 to 32 years (M = 18.9 years; SD = 9.9 years). In each school several textbooks were in use, but all schools also had one textbook in common, which was *Matematika* (Textbook for Junior High School, Grade VIII: 2A & 2B).

In total, in the six schools 311 eight graders (M = 13.8 years; SD = 0.5 year) participated in the study, involving 146 students in the experimental group and 165 in the control group. However, the data analysis was based on the 299 students (M = 13.7 years; SD = 0.5 year) who were present during both the pretest and the posttest. Of these students, 144 students were in the experimental group and 155 students in the control group.

4.3 Intervention program

4.3.1 The components of the intervention

The purpose of the intervention was to offer students opportunity-to-learn to solve context-based tasks. The intervention program was designed based on the findings of our earlier studies and comprised two components: a set of context-based tasks with particular characteristics and a consultative teaching approach with metacognitive prompts (see Figure 1). This intervention is illustrated using one of the context-based tasks used in the intervention program: the Internet task (see Figure 2).

Results from Study 1[a]:
Analysis of Indonesian students' errors when solving context-based tasks

Of all the errors made:
- 38% of them were **comprehension** errors; in particular, errors in selecting relevant information.
- 42% of them were **transformation** errors; in particular, errors identifying the required mathematical procedures.
- 17% of them were **mathematical processing** errors.
- 3% of them were **encoding** errors.

[a] Wijaya, Van den Heuvel-Panhuizen, Doorman, & Robitzsch, in press

Results from Study 2[b]:
Analysis of Indonesian textbooks

Exposure of context-based tasks:
- Only about 10% of all tasks are context-based.

Characteristics of the included context-based tasks:
- About 75% of them use a *camouflage context*, i.e. the context can be neglected when solving the task.
- About 85% of them provide *matching information*, i.e. only the information that is needed to solve the task.
- In all of them it is apparent what *mathematical procedure(s)* has(have) to be applied (because of the wording of the tasks and/or because the tasks are placed after an explanation section).
- Only 2% of them are *reflection tasks*, i.e. tasks with highest cognitive demands which require constructing original mathematical approaches and communicating complex arguments and complex reasoning.

[b] Wijaya, Van den Heuvel-Panhuizen, & Doorman, submitted a

Results from Study 3[c]:
An investigation into Indonesian mathematics teachers' teaching practices

Teachers' report about the **characteristics of context-based tasks** offered to the students:
- 67% of them give weekly or every lesson tasks with *explicit procedures*.
- 67% of them give weekly or every lesson tasks with *matching information*.
- 52% of them never give or give once or twice in a semester tasks with *superfluous information*.
- 44% of them never give or give once or twice in a semester tasks with *missing information*.

Observed **teaching practice approach**:

Over all stages of solving context-based tasks:
- *No instruction* was given in 42% of the questions
- *Directive teaching* (the teacher told the students what the task is about, translated the task into a mathematical problem, explained what mathematical procedure has to be carried out) was applied in 47% of the questions
- *Consultative teaching* (the teachers asked a question to stimulate reflection) was applied in only 12% of the questions.

Specified for the stages:
- Directive teaching was frequently applied in the comprehension, the transformation, and the mathematical processing stage.
- When consultative teaching was applied then it was mostly in the mathematical processing stage.
- Almost no attention was paid to the encoding stage.

[c] Wijaya, Van den Heuvel-Panhuizen, & Doorman, submitted b

The components of the intervention:

Context-based tasks which have the following characteristics:
- relevant and essential contexts that require modeling
- missing or superfluous information
- no apparent indications for the procedure(s) to be applied.

Consultative teaching approach:
- Giving students *metacognitive prompts* that are gradually faded out to encourage internalization by the students:
  * inviting the students to formulate the tasks in their own words
  * focusing students on the information provided in a task
  * stimulating self-questioning.
- Giving room for whole-class and peer-to-peer *discussion* and promote *reflection* in all stages of solving context-based tasks.

Figure 1. Intervention program informed by the findings of our previous studies

An internet provider Inter-NET offers two different programs. Program Smile charges customers 30.500 IDR as monthly fee and 40 IDR/1 Megabyte (MB). Program Shine charges customers 20.000 IDR as monthly fee and 52 IDR/1 MB. The registration fees including the price of modem for both programs are the same, namely 300.000 IDR. In January Doni subscribed to the program Shine. In May Doni used 550 MB of internet data. How much money did he pay in May?

a. *Underline all information given in the task and circle only the information we need to answer the question.*

b. *Reformulate the task with less words by leaving out unnecessary information.*

c. How much money did Doni pay in May?

Doni's internet usage is increasing. Now, he has a problem to decide whether it will be wise to change the internet program.

d. *What strategy can we use to solve Doni's problem?*

e. When Doni's internet usage is increasing, is it better for him to change the internet program? If so, when should he do it?

Figure 2. Internet task

4.3.2 Context-based tasks

To compensate what was lacking in Indonesian textbooks the intervention program offered students context-based tasks that had three characteristics. The first characteristic was that the context-based tasks should have *relevant and essential contexts* that require modeling. The tasks should be a real problem for students. For example, in the Internet task students encounter a problem that is rather authentic to them. Figuring out what internet fee Doni has to pay is a meaningful context for students because there is really something at stake. Therefore students are stimulated to take the context into account when solving the task instead of just using the numbers involved to carry out more or less automatically a particular calculation.

The second characteristic concerned the type of information provided in the context-based tasks. To offer students opportunity-to-learn selecting relevant information, context-based tasks were designed with *missing or superfluous information*, i.e. tasks that have less or more information than needed to find the solution. For example, to decide how much Doni has to pay in May, students do not need to use the monthly fee for the program Smile (30,500 IDR), the fee per 1 MB (40 IDR), or the registration fee (300,000 IDR).

The third characteristic required for the context-based tasks was that they *do not contain apparent indications about the procedures* that can be used to solve the problem. If explicit information is given about what procedure to apply, then students are not offered opportunity-to-learn to decide what would be a suitable mathematical procedure. Therefore, we designed the Internet task so that students are not put directly on the track of starting with a particular mathematical procedure, such as making a calculation, formulating and solving an equation, or drawing a graph. Instead, students are just asked to decide whether it is better for Doni to change the internet program and, if yes, when it is better to change. This means that students have to come up with a procedure by themselves.

4.3.3 Consultative teaching approach

The second component of the intervention program was a consultative teaching approach. To conduct this approach, we provided the teachers with suggestions to give metacognitive prompts to students and to stimulate whole-class and peer-to-peer discussions that promote reflection for all the stages of solving context-based tasks.

The *metacognitive prompts* were meant to point students to important aspects of the tasks and the solving process. A first metacognitive prompt was asking students to underline all the information included in a context-based task and to discuss the included information. For example, in the Internet task this prompt was given in the first assignment. As a second metacognitive prompt, students were asked to use their own words to explain the Internet task. This paraphrasing of a given problem was a second type of metacognitive prompt to help students to get a better understanding of what the problem is about. Finally, a third type of metacognitive prompt was to elicit self-questioning. For example, in the Internet task students were stimulated to ask themselves questions, such as "What strategy can we use to solve Doni's problem?".

To make the metacognitive prompts an integral part of the intervention program they were embedded in the context-based tasks, yet the special character of these prompts was indicated by printing them in italic. In this way the students (and teachers) were made aware that the questions differed from the mathematical questions. In the first two lessons metacognitive prompts were provided for every question in the context-based tasks, but in the later lessons the prompts gradually faded out.

The *suggestions for discussions to promote reflection* were not embedded in the tasks, but were only included in the teacher guide. This means that suggestions were provided for additional instructions or guiding questions. For example, when students had difficulties with only circling the relevant information in the Internet task, the teacher could suggest students moving away from the question they had to answer in this task ("How much money did he pay in May?") and starting with identifying the keywords of this question. Based on these keywords students can look for relevant information in the remaining text.

4.3.4 Outline of the intervention program

The complete intervention program consisted of five 80-minutes lessons each consisting of an explanation section and an exercise section. In fact, the actual intervention took place in the exercise sections, which each lasted about 20 to 30 minutes. The intervention was conducted from September 18, 2013 to October 12, 2013. As all schools in Indonesia follow the national curriculum, during this time period the teachers taught graphical representation of linear equations. Therefore we chose this topic in designing the intervention program. An outline of the intervention program is given in Appendix 1. It gives an overview of the sub-topics of graphical representation of linear equations which were dealt with in the five lessons, the mathematical goals addressed in each lesson, the goals related to learning solving context-based tasks, and the context-based tasks that were included in the program. In total, the program included nine context-based tasks; in every lesson two and in the last lesson one. Moreover, in order not to lessen too much the experience of students in solving tasks with bare mathematics, the lesson series also contained five of such tasks.

4.3.5 Teacher training

Two weeks prior to the intervention, the experimental teachers were trained to conduct the intervention. The teacher training was done per school because the six schools were located in different districts and the teachers were busy with their regular job, so that it was difficult to gather the teachers in one training. The training consisted of two meetings of 90 to 120 minutes. During these meetings, the overall goal of the intervention program, the structure of the program, and the material belonging to this program (student material and teacher guide) were explained. After this, attention was paid to the importance of the competence of solving context-based tasks and the characteristics of context-based tasks. Finally, examples were discussed of how teachers could help students to deal with superfluous or missing information and select relevant information, how teachers could give metacognitive prompts, and how to stimulate students to discuss and reflect on the solving process.

4.3.6 Fidelity of the implementation of the intervention program

To monitor whether the teachers conducted the intervention in the intended way, classroom observations were carried out by the first author. Because the intervention in the six schools took place in the same time period, not all lessons could be observed. In general, two of each teacher's five lessons were observed and for one teacher observations were made in four lessons. After each observed lesson, a discussion was held with the teacher to reflect on how the intervention was carried out and to prepare for the intervention in the next lesson. For the lessons that were not observed the teachers were asked to keep a log and report how many tasks they discussed in class.

The observations and the teachers' reports gave the impression that with respect to the consultative teaching in general the teachers did the intervention according to the plan. However, for the exposure of context-based tasks, on average the teachers discussed only six out of the nine tasks that we gave them. Furthermore, the observations revealed that during the lessons the teachers frequently read the teacher guide before giving a metacognitive prompt or stimulating discussion and reflection.

4.3.7 Regular program

Students in the control classes were taught as they were usually taught on the basis of the textbook that the six schools had in common. This means that the teachers were using a teacher-centered approach in which they mainly explained and demonstrated how to solve tasks. In order to make the mathematics content the same in all the control classes, the control teachers were asked to follow a program consisting of 19 bare mathematics tasks and 3 context-based tasks. All these tasks were taken from the common textbook used by the teachers. Consequently, the context-based tasks all used a camouflage context and explicitly mentioned the mathematical concepts related to the task. For example, the two tasks shown in Figure 3 include a staircase and a ski slope, which are real-world contexts, yet students do not have to think about a staircase and a ski slope in reality. They can just do the calculation that is asked for, based on the figure of the staircase and the numbers provided in the picture of the mountainside (e.g. slope $= \frac{45}{48} = 0.9$). Furthermore, these two tasks explicitly mention the mathematical concept gradient, which is relevant to solve these tasks.

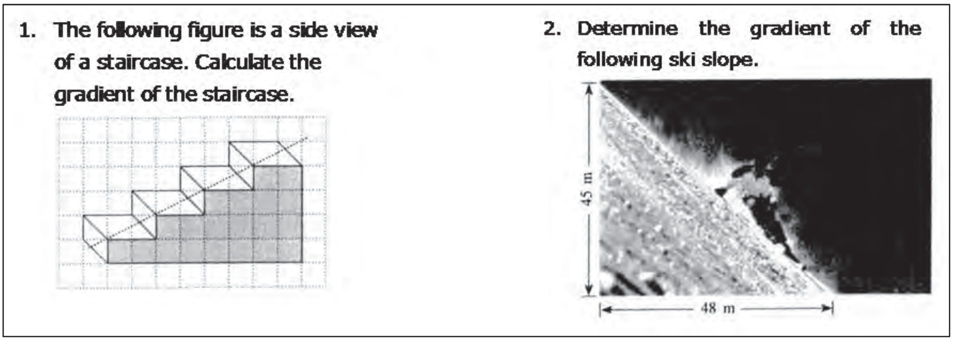


Figure 3. Context-based tasks included in the regular program

4.4 Test for measuring students' performance in solving context-based tasks

4.4.1 Composition of the test

The test used for measuring students' performance in solving context-based tasks was made from released PISA tasks (OECD, 2009b). We selected a total of 17 PISA mathematics units[9] consisting of 30 items. The 17 mathematics units were divided over two different booklets. Booklet A contained eight mathematics units (consisting of 15 items) and Booklet B contained nine mathematics units (also consisting of 15 items). The items were equally distributed over the two booklets according to: (1) the cognitive demand of the items as established in the PISA studies, including reproduction, connection, and reflection (see OECD, 2003; 2009b), (2) the difficulty level of the items as indicated by the percentage of correct answers found in the 2003 PISA study (OECD, 2009a), and (3) the mathematical topics involved in the items. Although the intervention involved the topic of graphical representation of linear equations, only 5 out of 30 items included in the test were related to graphs, including non-linear graphs.

[9] A 'mathematics unit' consists of one or more questions that can be answered independently (see OECD, 2003, p. 52). These questions are based on the same context which is generally presented by a written description and/or a graphic or another representation.

To avoid a re-test effect due to administering two times the same items, the group of students in each class was randomly split in half, leading to two groups of which one got Booklet A as a pretest and Booklet B as a posttest, with the other group getting these booklets in the reverse order.

4.4.2 Coding of the correctness of the answers

To code students' responses the scoring scheme of the PISA studies (OECD, 2009b) was used. Of the total of 30 items, 24 items were coded as correct (1), incorrect (0) or no answer (9). The other items had a partial credit scoring, including 5 items that were coded as correct (2), partially correct (1), incorrect (0) or no answer (9) and 1 item that was coded as correct (3), partially correct level 2 (2), partially correct level 1 (1), incorrect (0) or no answer (9). Based on this coding, we calculated the students' scores in the two booklets. The maximum score for Booklet A was 18 and the maximum score for Booklet B was 19.

4.4.3 Psychometric properties of the test

To check the reliability of the two sets of items, which formed our test for measuring students' performance in solving context-based tasks, we calculated Cronbach's alpha. For the complete test, i.e. booklet A and B combined, this gave a good α of .80. For the booklets separately we found $\alpha = .75$ for booklet A, and $\alpha = .69$ for booklet B. As the complete sample was split into two groups and each group got a different booklet as pretest and posttest, we also checked whether the reliability estimates per booklet changed for whether it was used as a pretest or as a posttest. Cronbach's alpha for booklet A as pretest was $\alpha = .64$ and as a posttest $\alpha = .79$, for booklet B as a pretest $\alpha = .68$ and as a posttest $\alpha = .69$. These changes are relatively small and all alphas indicate acceptable to good reliabilities.

4.4.4 Coding of the errors made by students

To code students' errors when solving the test items we used an analysis framework that was developed in our earlier study (Wijaya et al., 2014) (see Appendix 2). The coding was done for students' an incorrect answer. Furthermore, more than one code could be given to a student's work because students could make more than one type of error.

The coding was carried out by the first author. The interrater reliability of the coding was checked through an extra coding by a mathematics teacher who was not part of this study. The extra coding was done on the basis of 12% of the coded responses, which were randomly selected. With a Cohen's Kappa of .78, the agreement between the first author and the second coder was substantial (Landis & Koch, 1977).

4.4.5 Statistical analysis

To investigate whether students in the experimental condition improved more than those in the control condition we used a univariate analysis of variance (ANOVA). We calculated a gain score for every student to use as dependent variable in the ANOVA. In order to be able to compare students' gain scores, taking into account the different total scores they could obtain on the two booklets (A and B) they got as pretest or posttest, we standardized the scores per booklet and per order of presentation of the booklets. So the mean of all students that got booklet A as a pretest was used to standardize the scores of the students who were in this group, and for the students who got booklet B as a pretest the mean of all students who were in this group was used for standardizing the scores. This same procedure was repeated for the scores on the posttest. The obtained standardized scores were used to calculate a standardized gain score for every student ("score on A – score on B" for B as pretest and A as posttest; and "score on B – score on A" for A as pretest and B as posttest).

5 Results

5.1 Effect of opportunity-to-learn on the correctness of students' answers

To investigate the effect of the intervention program we carried out a univariate ANOVA with the gain score (posttest score minus pretest score) as dependent variable and intervention as a fixed factor. Contrary to our expectations, the difference in gain scores between the students in the experimental group ($M_{experimental} = 0.11$, $SD_{experimental} = 0.99$) and the students in the control group ($M_{control} = -0.09$, $SD_{control} = 0.95$) was only marginally significant and the effect of the intervention was small ($p = .068$; $\eta_p^2 = .011$).

We also took an exploratory look at the effect of the school, finding a significant main effect for the school students were in ($p < .001$, $\eta_p^2 = .114$). In Figure 4 the different gains for the students in the control and experimental condition in the six schools are displayed.

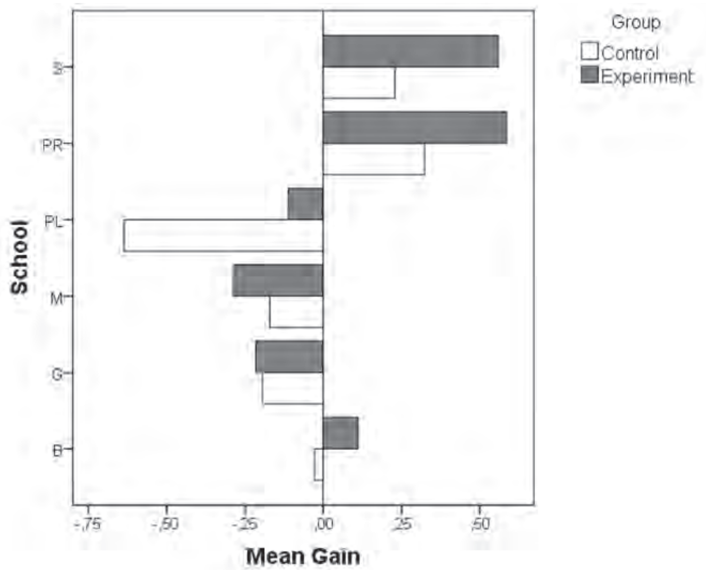


Figure 4. Mean gain scores for students in the control (white) and experimental (gray) condition in the six schools

Furthermore, we also examined exploratorily whether there was an effect of the booklet order, and whether there was a relation between the school and the order in which the booklets were presented. The booklet order by itself did not turn out to have an effect on the students' gain scores (p = .884, η_p^2 = .000), but for the school and the booklet order we observed a significant interaction effect (p = .004, η_p^2 = .06). Depending on the school the students were in, they had either more difficulty in solving the items in booklet A or those in booklet B (see Figure 5). For example, in school S both groups showed a gain in performance when the booklet order was A-B but not when it was B-A. For this school, booklet B was easier than booklet A, whereas in school PR the opposite was the case. This difference in difficulty level of the booklets is remarkable because they were constructed in such a way that we expected them to be of comparable difficulty.

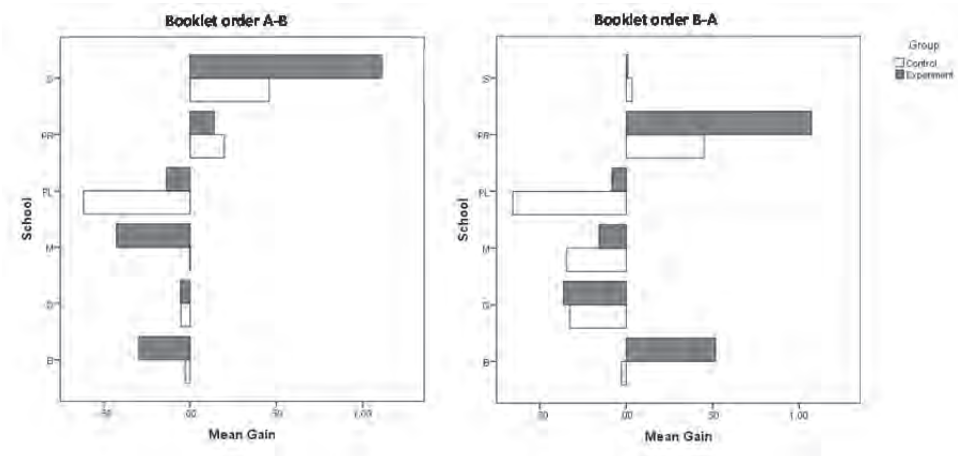


Figure 4. Mean gain scores for students in the control (white) and experimental (gray) condition in the six schools for both booklet orders

5.2 Effect of opportunity-to-learn on students' errors

To investigate whether the intervention had an effect on the types of errors, we carried out an error analysis based on the students' incorrect responses. In total, we found 1942 incorrect responses in the pretest (892 in the experimental group and 1050 in the control group) and 1705 incorrect responses in the posttest (744 in the experimental group and 961 in the control group). In accordance with our finding on the correctness of the students' answers the decrease in incorrect responses was not significant (χ^2 (1, n = 3647) = 1.934, p = .164).

Table 1 presents the results of the error analysis. Looking at the changes in the total number of errors between the experimental group and the control group we found that the decrease in the number of errors made by the students in the experimental group was about ten percent points larger than in the control group: in the experimental group the decrease in errors was 18%, whereas in the control group this was 7%. This showed that there was a significant difference between the experimental group and the control group on the decrease in the total number of errors (χ^2 (1, n = 4127) = 4.149, p = .042).

Table 1

The number of errors made by the students in the pretest and the posttest

Types of errors	Sub-types of errors	Group	Number of errors		Percent of change
			Pre-test	Post-test	
Compre-hension	Errors in understanding instruction	Exp.	68	51	- 25% [a]
		Control	84	93	11%
	Errors in understanding a keyword	Exp.	22	36	64%
		Control	14	39	179%
	Errors in selecting information	Exp.	125	86	- 31%
		Control	125	127	2%
	Total	Exp.	215	173	- 20%
		Control	223	259	16%
Transfor-mation	Procedural tendency	Exp.	7	15	114%
		Control	2	9	350%
	Taking too much account of context	Exp.	11	8	- 27%
		Control	20	10	- 50%
	Wrong mathematical procedure	Exp.	487	376	- 23%
		Control	582	442	- 24%
	Treating a graph as a picture	Exp.	68	59	- 13%
		Control	69	87	26%
	Total	Exp.	573	458	- 20%
		Control	673	548	- 19%
Math. processing[b]		Exp.	195	165	- 15%
		Control	230	239	4%
Encoding[c]		Exp.	32	33	3%
		Control	58	53	- 9%
Total[d]		Exp.	1015	829	- 18%
		Control	1184	1099	- 7%

[a] A negative value means a decrease

[b] The sub-types of mathematical processing error depend on the mathematics topics addressed in the test items

[c] No sub-type for encoding error

[d] Because of a multiple coding, the number of errors were greater than the number of incorrect responses.

Regarding comprehension errors the intervention had a positive influence. In the experimental group the number of errors decreased by 20%, whereas in the control group the occurrence of these errors increased by 16%. Particularly, the finding that there was a reduction of 31% in the number of selecting information errors and of 25% in the number of understanding the instruction errors, provides evidence that the metacognitive prompts helped. This was also supported by the fact that in the posttest work of the students in the experimental group showed clear signs of underlining and circling information. This was, for example, the case in the Skateboard task in which students were asked to calculate the minimum and the maximum price for self-assembled skateboards. This task provided a price list that included irrelevant information. A typical error when solving this task was that students included irrelevant information in their calculation. In the pretest this was done by 19 out of 72 students in the experimental group and by 21 out of 80 students in the control group. In the posttest this error decreased to 6 in the experimental group and to 12 in the control group.

For the transformation errors the results were different. Here, for both the students in the experimental and the control group there was on average a decrease of 20% in the number of errors, but at item level the change in errors differed between the two groups. For example, in a follow-up question in the Skateboard task students were asked to determine the number of different skateboards that can be assembled based on a number of different skateboard components. The transformation error that most students made was to simply add up the number of components. This error decreased by 20% in the control group, but by only 3% in the experimental group. However, for other items we found that for students in the experimental group, in contrast with the students in the control group, the number of transformation errors in the posttest was remarkably smaller than in the pretest. This appeared to be the case for context-based tasks addressing the interpretation of graphs. For example, in the Speed task in which a graph of the varying speed of a racing car that drives along a flat 3-kilometer-track is presented, students were asked to choose from five alternative race tracks along which the car could have produced the speed graph that was shown. A usual transformation error for this task was treating a graph as a literal picture of a situation. In the experimental group the occurrence of this error decreased by 17%, whereas in the control group it stayed the same. This result can be explained by the fact that in the intervention program less explicit procedures were given for solving tasks related to graphs than in the control group. In the regular program the tasks were mostly cast in a particular mathematical language by which the students were immediately directed to the procedure of using linear equations to create tables and use the numbers in the tables to draw graphs (with x horizontally and y vertically). Consequently, students in the experimental group had had more opportunities to learn to interpret and reason about graphs in context-based tasks.

Similar to comprehension errors, for mathematical processing errors the number of errors decreased in the experimental group while it increased in the control group. When taking a closer look at the sub-types of the mathematical processing errors, we found that the largest difference between the two groups was in arithmetical errors: in the posttest the number of arithmetical errors in the experimental group barely changed, whereas in the control group the number increased by 70%. Lastly, regarding the encoding errors we found only a small number of errors and in both groups the number of these errors hardly changed between pretest and posttest.

6 Discussion

Although our analysis showed that the students in the experimental group made more progress than the students in the control group, the difference between the two groups was only marginally significant and the effect was small. This result might be due to the short duration of our intervention, which contained only five 30-minute lessons over a two-week period. In such a short time the students might not have internalized the metacognitive prompts. As highlighted by Veenman et al. (2006), prolonged training is fundamental for a successful metacognitive instruction. Furthermore, because the teachers only offered and discussed six of the nine tasks we provided, the students might not have received enough chances to deal with the context-based tasks. Finally, following Kramarski et al. (2002), who indicated that teacher experience in metacognitive instruction influences student performance, the Indonesian teachers experience with this type of instruction might have played a role. Several studies have shown that directive teaching is still the dominant approach in mathematics lessons in Indonesia (Maulana et al., 2012; Wijaya et al., submitted b; World Bank, 2010).

Based on a closer examination of students' errors we found a positive effect of our intervention on students' ability to comprehend context-based tasks. This finding, which is in agreement with other studies (e.g., Karbalei & Amoli, 2011; Kletzien, 2009), indicates the potential of the paraphrasing strategy to develop students' task comprehension. The improvement in students' task comprehension was also reflected in the progress students made in selecting information. This result signifies the power of context-based tasks with missing or superfluous information to develop students' ability to select information. Furthermore, we would also like to highlight the benefit of asking students to circle only the relevant information. This metacognitive prompt seems to be effective in guiding students to thoroughly look at the information provided in the task.

Despite the promising result regarding task comprehension, in general no effect of the intervention program on students' performance in transforming a real-world problem to a mathematical problem was found. A possible explanation for this finding is that the intervention took place in teaching a particular topic. The students might already have been put on the track of the procedure or concept that is needed to solve the context-based tasks. Therefore, the students in the experimental group might not have had enough chances to deal with identifying the needed procedure. Also, Howson (2013) underlined recently that including context-based tasks in a particular chapter of a textbook will discourage students from thinking about the required procedure before they solve the tasks.

Because of limited financial resources available for this study there were restrictions in the way we could set up our research. This resulted in some limitations that should be taken into account in the interpretation of our findings. A first limitation is that we could not include a large number of schools to achieve sufficient statistical power. Furthermore, because of the limited number of schools that we could take on board, we could not compose two groups of matched schools followed by a random allocation to the experimental or the control group. In our design, the two conditions were situated within every school. However, most schools had only one teacher for each grade, which means that in these schools the experimental students and the control students were taught by the same teacher. This is not an ideal situation, because what the teachers did in the experimental group might have had an influence on their teaching practices in the control group. Another shortcoming of the study is that we could not include a measure of the students' general achievement level in mathematics in the analysis, because the different districts where the schools were located administered their own tests for Grade 8, which made the scores of students in different schools not comparable.

These limitations ask for further research to get a more thorough understanding of whether and how students' ability in solving context-based tasks can be improved by offering them a particular opportunity-to-learn. Such research should include a large sample of schools and teachers. In addition to our study, new research should also include data on the quality of the schools, the teachers' teaching experience and the students' achievement level in mathematics. All these factors might have an influence on the effect of an intervention program aimed at improving students' performance in context-based tasks.

7 Conclusion and policy implications

In Indonesia great attention is paid to developing students' ability to apply mathematics. This is reflected not only in the Indonesian national curriculum, but also in Indonesia's regular participation in the PISA study and in the nationwide educational movement implementing Realistic Mathematics Education (see e.g., Sembiring, Hadi, & Dolk, 2008). With respect to the application of mathematics as an educational goal, we would like to highlight two relevant recommendations based on the findings of our study. First, our results showed the need for a textbook quality improvement program. Because textbooks are the primary resources in the Indonesian classroom practice and are found to be of influence on what students learn, textbooks give the Indonesian government a powerful steering instrument for improving students' achievement in the application of mathematics. Through the publication of freely available (electronic) textbooks the government can reach all teachers, and offer students more opportunity-to-learn. However, this necessitates that textbook authors include more context-based tasks in textbooks, especially more tasks that have superfluous or missing information, and more tasks with non-explicit procedures. With respect to the latter we also advice to revise the chapter structure of textbooks somewhat by including in the exercise sections not only context-based tasks that are related to the topic discussed in the chapter, but also context-based tasks that refer to topics discussed in other chapters. Of course, commercial publishers are also invited to improve their textbooks in the same direction.

Our second recommendation is related to one of the quality requirements of the Teacher Law set by Indonesian government, i.e. the need for higher standards in classroom teachings, in which teachers are required to engage students in the learning process (Jalal et al., 2009). Based on our experiences with the consultative teaching approach (giving metacognitive prompts and room for discussion and reflection) as included in the intervention program, we found that such an approach is helpful to get students involved in the learning process of solving context-based tasks. Moreover, it turned out to be also feasible for teachers to carry out as an alternative for their teaching practices. However, as was the case in our project, professional development will be necessary to guarantee an appropriate implementation and a widespread and prolonged practice of consultative teaching. Therefore, we suggest including consultative teaching as a subject in the two major teacher training programs in Indonesia, i.e. in-service teacher profession training, which is called PLPG, and professional education for pre-service teachers that is called PPG.

With the abovementioned recommendations we would like to underline the significance of the availability of appropriate instructional resources and professional development for teachers as key aspects for a sound implementation of a national curriculum and the improvement of student achievement.

References

Abedi, J., & Herman, J. L. (2010). Assessing English language learners' opportunity to learn mathematics: Issues and limitations. *Teacher's College Record 112*(3), 723–746.

Antonius, S., Haines, C., Jensen, T. H., Niss, M., & Burkhardt, H. (2007). Classroom activities and the teacher. In W. Blum, P. L. Galbraith, H.-W. Henn & M. Niss (Eds.), *Modelling and Applications in Mathematics Education: The 14th ICMI Study* (pp. 295–308). New York: Springer.

Bernardo, A. B. I. (1999). Overcoming obstacles to understanding and solving word problems in mathematics. *Educational Psychology, 19*(2), 149–163.

Blum, W. (2011). Can modelling be taught and learnt? Some answers from empirical research. In G. Kaiser, W. Blum, R. B. Ferri & G. Stillman (Eds.), *Trends in teaching and learning of mathematical modelling* (pp. 15–30). New York: Springer.

Blum, W., & Ferri, R. B. (2009). Mathematical modelling: Can it be taught and learnt? *Journal of Mathematical Modelling and Application, 1*(1), 45–58.

Blum, W., & Leiss, D. (2007). How do students and teachers deal with mathematical modelling problems? The example "Sugarloaf". In C. Haines, P. Galbraith, W. Blum & S. Khan (Eds.), *Mathematical modelling (ICTMA 12): Education, engineering and economics* (pp. 222–231). Chichester: Horwood Publishing.

Boaler, J. (1994). When girls prefer football to fashion? An analysis of female under achievement in relation to realistic mathematics contexts. *British Educational Research Journal, 20*(5), 551–564.

Brewer, D. J., & Stacz, C. (1996). *Enhancing opportunity to learn measures in NCES data*. Santa Monica, CA: RAND.

Charalambous, C. Y., Delaney, S., Hsuc, H.-Y., & Mesac, V. (2010). A comparative analysis of the addition and subtraction of fractions in textbooks from three countries. *Mathematical Thinking and Learning, 12*(2), 117–151.

Clements, M. A. (1980). Analyzing children's errors on written mathematical task. *Educational Studies in Mathematics, 11*(1), 1–21.

Cooper, B., & Dunne, M. (2000). *Assessing children's mathematical knowledge: Social class, sex and problem-solving*. Buckingham: Open University Press.

Eurydice. (2011). *Mathematics education in Europe: Common challenges and national policies*. Brussels: Education, Audiovisual and Culture Executive Agency.

Forman, S. L., & Steen, L. A. (2001). Why math? Applications in science, engineering, and technological programs. *Research Brief, American Association of Community Colleges*.

Goldman, S. R. (1989). Strategy instruction in mathematics. *Learning Disability Quarterly, 12*(1), 43–55.

Graumann, G. (2011). Mathematics for problem in the everyday world. In J. Maasz & J. O'Donoghue (Eds.), *Real-world problems for secondary school mathematics students: Case studies* (pp. 113–122). Rotterdam: Sense Publishers.

Greer, B. (1997). Modeling reality in mathematics classrooms: The case of word problems. *Learning and Instruction, 7*(4), 293–307.

Grouws, D. A., & Cebulla, K. J. (2000). *Improving student achievement in mathematics*. Brussels: International Academy of Education.

Herman, J. L., Klein, D. C. D., & Abedi, J. (2000). Assessing students' opportunity to learn: Teacher and student perspectives. *Educational Measurement: Issue and Practice, 19*(4), 16-24.

Hiebert, J., & Grouws, D. A. (2007). The effects of classroom mathematics teaching on students' learning. In F. Lester (Ed.), *Second handbook of research on mathematics teaching and learning* (pp. 371–404). Charlotte: Information Age Publishing.

Howson, G. (2013). The development of mathematics textbooks: historical reflections from personal perspective. *ZDM – The International Journal on Mathematics Education, 45*(5), 647–658.

Husén, T. (Ed). (1967). *International study of achievement in mathematics: A comparison of twelve countries (Vol. II)*. New York: John Wiley & Sons.

Jalal, F., Samani, M., Chang, M. C., Stevenson, R., Ragatz, A. B., & Negara, S. D. (2009). *Teacher Certification in Indonesia: A strategy for teacher quality improvement*. Jakarta: Indonesian Ministry of National Education and the World Bank.

Karbalei, A., & Amoli, F. A. (2011). The effect of paraphrasing strategy training on the reading comprehension of college students at the undergraduate level. *Asian EFL Journal, 13*(3), 229–344.

Kletzien, S. B. (2009). Paraphrasing: An effective comprehension strategy. *The Reading Teacher, 63*(1), 73–77.

Klymchuk, S., Zverkova, T., Gruenwald, N., & Sauerbier, G. (2010). University students' difficulties in solving application problems in calculus: Student perspectives. *Mathematics Education Research Journal, 22*(1), 81–91.

Kramarski, B., Mevarech, Z. R., & Arami, M. (2002). The effects of metacognitive instruction on solving mathematical authentic tasks. *Educational Studies in Mathematics, 49*(2), 225–250.

Landis, J. R., & Koch, G. G. (1977). The measurement of observer agreement for categorical data. *Biometrics, 33*(1), 159–174.

Maass, K. (2007). Modelling tasks for low achieving students – first results of an empirical study. In D. Pitta-Pantazi & G. Philippou (Eds.), *Proceedings of the Fifth Congress of the European Society for Research in Mathematics Education CERME 5* (pp. 2120–2129). Larnaca, Cyprus.

Maass, K. (2010). Classification scheme for modelling tasks. *Journal fur Mathematik-Didaktik, 31*(2), 285–311.

Maulana, R., Opdenakker, M.-C., Den Brok, P., & Bosket, R. J. (2012). Teacher-student interpersonal behavior in secondary mathematics classes in Indonesia. *International Journal of Science and Mathematics Education, 10*(1), 21–47.

McDonnell, L. M. (1995). Opportunity to learn as a research concept and a policy instrument. *Educational Evaluation and Policy Analysis, 17*(3), 305-322.

Montague, M. (2007). Self-regulation and mathematics instruction. *Learning Disabilities Research and Practice, 22*(1), 75–83.

Montague, M. (2008). Self-regulation strategies to improve mathematical problem solving for students with learning disabilities. *Learning Disability Quarterly, 31*(1), 37–44.

Montague, M., Warger, C., & Morgan, T. H. (2000). Solve it! Strategy instruction to improve mathematical problem solving. *Learning Disabilities Research and Practices, 15*(2), 110–116.

National Council of Teachers of Mathematics. (2000). *Principles and Standard for School Mathematics*. Reston: Author.

Newman, M. A. (1977). An analysis of sixth-grade pupils' errors on written mathematical tasks. *Victorian Institute for Educational Research Bulletin, 39*, 31–43.

OECD. (2003). *The PISA 2003 Assessment Framework - Mathematics, Reading, Science, and Problem Solving Knowledge and Skills*. Paris: Author.

OECD. (2009a). *Learning Mathematics for Life. A View Perspective from PISA*. Paris: Author.

OECD. (2009b). *Take the Test. Sample Questions from OECD's PISA Assessments*. Paris: Author.

OECD (2010). *PISA 2009 Results: What students know and can do. Student performance in reading, mathematics, and science (Vol. I)*. Paris: Author.

OECD. (2013). PISA 2012 *Results: What students know and can do. Student performance in mathematics, reading and science*. Paris: Author.

Pusat Kurikulum (2003). *Kurikulum 2004: Standar kompetensi mata pelajaran matematika Sekolah Menengah Pertama dan Madrasah Tsanawiyah*. Jakarta: Departemen Pendidikan nasional.

Sam, L. C., Lourdusamy, A., & Ghazali, M. (2001). Factors affecting students' abilities to solve operational and word problems in mathematics. *Journal of Science and Mathematics Education in Southeast Asia, 24*(1), 84–95.

Schmidt, W. H., McKnight, C. C., Valverde, G. A., Houang, R. T., & Wiley, D. E. (1997). *Many visions, many aims: A cross-national investigation of curricular intentions in school mathematics*. Dordrecht: Kluwer Academic Publishers.

Sembiring, R. K., Hadi, S., & Dolk, M. (2008). Reforming mathematics learning in Indonesian classrooms through RME. *ZDM The International Journal on Mathematics Education, 40*(6), 927–939.

Tomlinson, M. (2004). 14-19 *Curriculum and Qualifications Reform. Final report of the Working Group on 14-19 Reform*. London: Department for Education and Skills.

Tornroos, J. (2005). Mathematics textbooks, opportunity to learn and student achievement. *Studies in Educational Evaluation, 31*(4), 315–327.

Valverde. G. A., Bianchi, L. J., Wolfe, R. G., Schmidt, W. H., & Houang, R. T. (2002). *According to the book. Using TIMSS to investigate the translation of policy into practice through the world of textbooks*. Dordrecht: Kluwer Academic Publishers.

Veenman, M. V. J., Van Hout-Wolters, B. H. A. M., & Afflerbach, P. (2006). Metacognition and learning: conceptual and methodological considerations. *Metacognition Learning, 1*(1), 3–14.

Verschaffel, L., Greer, B., & De Corte, E. (2000). *Making sense of word problems*. Lisse: Swets & Zeitlinger.

Wijaya, A., Van den Heuvel-Panhuizen, M., Doorman, M., & Robitzsch, A. (2014). Difficulties in solving context-based PISA mathematics tasks: An analysis of students' errors. *The Mathematics Enthusiast, 11*(3), 555-584.

Wijaya, A., Van den Heuvel-Panhuizen, M., & Doorman, M. (submitted a). Opportunity-to-learn context-based tasks provided by mathematics textbooks and the relation with students' difficulties in solving these tasks.

Wijaya, A., Van den Heuvel-Panhuizen, M., & Doorman, M. (submitted b). Teachers' teaching practices and beliefs regarding context-based tasks and students' difficulties in solving these tasks.

World Bank. (2010). *Inside Indonesia's mathematics classrooms: A TIMSS Video Study of teaching practices and student achievement*. Jakarta: Author.

Xin, Y. P. (2007). Word problem solving tasks in textbooks and their relation to student performance. *Journal of Educational Research, 100*(6), 347–359.

Appendix 1

Outline of the intervention program

Lesson	Mathematical sub-topic	Goal related to mathematical content	Goal related to solving context-based tasks	Context-based task	Bare math. task
1	Sketching graph of straight line	- Making table of ordered pairs of linear equation - Sketching graph of equation on Cartesian coordinate by plotting ordered pairs	- Identifying and completing missing information. - Selecting relevant information. - Paraphrasing the tasks. - Identifying appropriate mathematics procedure.	- *Taxi task*: Comparing taxi tariffs - *Internet task*: Comparing internet fees	1
2	Gradient (1)	- Understanding definition of gradient - Determining gradient of a straight line through two points	- Identifying and completing missing information. - Selecting relevant information. - Identifying appropriate mathematics procedure.	- *Experiment task*: Determining the fastest speed of bike simulation - *Price task*: Determining the increase rate of oil price	1
3	Gradient (2)	- Determining gradient of parallel lines - Determining gradient of perpendicular lines	- Identifying and completing missing information. - Identifying appropriate mathematics procedure. - Interpreting the solution in terms of the problem situation.	- *Bus task*: Comparing bus speed - *Water pump task (1)*: Visualizing the rates of filling water tanks	1

Lesson	Mathematical sub-topic	Goal related to mathematical content	Goal related to solving context-based tasks	Context-based task	Bare math. task
4	Equation of straight line (1)	Determining the equation and sketching the graph of: - A line with a gradient of m and through point (x_1,y_1) - A line through points (x_1,y_1) and (x_2,y_2)	- Selecting relevant information	- *Olympic task*: Predicting the number of future participants - *Hospital task*: Estimating drug dosage	-
5	Equation of straight line (2)	Determining the equation and sketching the graph of: - A line through point (x_1,y_1) and parallel to another line - A line through point (x_1,y_1) and perpendicular to another line	- Selecting relevant information - Interpreting the solution in terms of the problem situation	- *Water pump task (2)*: Estimating filling rates	2

Appendix 2

Coding scheme for error types when solving context-based mathematics tasks

Error type	Sub-type	Explanation
Comprehension	Misunderstanding the instruction	Student incorrectly interprets what (s)he is asked to do.
	Misunderstanding a keyword	Student misunderstands a keyword, which is usually a mathematical term.
	Error in selecting information	Student is unable to distinguish between relevant and irrelevant information (e.g. using all information provided in a task or neglecting relevant information) or is unable to gather required information which is not provided in the task.
Transformation	Procedural tendency	Student tends to directly use a mathematical procedure such as formula or algorithm without analyzing whether or not it is needed.
	Taking too much account of the context	Student's answer only refers to the context or real-world situation without taking the perspective of the mathematics.
	Wrong mathematical operation/concept	Student uses mathematical procedures or concepts which are not relevant to the tasks.
	Treating a graph as a picture	Student treats a graph as a literal picture of a situation. (S)he interprets and focuses on the shape of the graph, instead of on the properties of the graph.
Mathematical Processing	Algebraic error	Error in solving algebraic expression or function.
	Arithmetical error	Error in calculation.

Error type	Sub-type	Explanation
Mathematical Processing	Error in mathematical interpretation of graph:	
	- Point-interval confusion	Student mistakenly focuses on a single point rather than on an interval.
	- Slope-height confusion	Student does not use the slope of the graph but only focuses on the vertical distance.
	Measurement error	Student cannot convert between standard units (e.g. from m/minute to km/h) or from a non-standard unit to a standard unit (e.g. from step/minute to m/minute).
	Improper use of scale	Student cannot select and use the scale of a map properly.
	Unfinished answer	Student uses a correct procedure, but (s)he does not finish it.
Encoding		Student is unable to correctly interpret and validate the mathematical solution in terms of the real-world problem. This error is reflected by an impossible or not realistic answer.

Chapter 6

Conclusion

Conclusion

1 Gains from the study

The main purpose of this PhD research was to gain a better insight into Indonesian students´ low performance on context-based mathematics tasks and to generate recommendations for improving student performance. To achieve this goal, three interrelated issues regarding context-based mathematics tasks in Indonesia were studied. First, it was investigated *what* difficulties Indonesian students experience when solving context-based tasks. Second, it was examined *why* Indonesian students struggle with context-based tasks, for which the focus was on analyzing textbooks and investigating teachers' teaching practices. Lastly, to study *how* student performance can be improved, students were provided with opportunity-to-learn to solve context-based tasks and afterward the effects of this opportunity-to-learn on students' performance was examined.

1.1 Indonesian students' difficulties when solving context-based tasks

In the first study of this PhD research the difficulties experienced by Indonesian students when solving context-based tasks were examined through an analysis of students' errors. This approach was chosen because students' errors provide access to students' reasoning (Brodie, 2014) and are considered a powerful source to diagnose learning difficulties (Batanero, Godino, Vallecillos, Green, & Holmes, 1994; Borasi, 1987; Seng, 2010).

The error analysis revealed that Indonesian students primarily had difficulties in comprehending context-based tasks and in transforming real-world problems into mathematical problems. In the process of comprehending a context-based task, students often made errors in selecting relevant information, i.e. they tended to use all information provided in the task. This finding adds to the results of the PISA studies (e.g. OECD, 2013) that most Indonesian students can solve context-based tasks if all relevant information is given and the questions are clearly stated. In line with Jupri, Drijvers, and Van den Heuvel-Panhuizen (2014), it was found that when Indonesian students had to transform a real-world problem into a mathematical problem many of them were unable to identify the required mathematical concept or procedure. The students had a tendency to apply a familiar procedure or calculation without considering its relevance for the context of the task.

In addition to these specific results, this study showed how analyzing students' difficulties can be a crucial preliminary step in the process of improving student performance because it sheds light on key aspects of solving context-based tasks that need to be developed. For example, the findings of this study suggest that improving the task comprehension of Indonesian students requires a focus not only on students' language competence, but also on the ability to select relevant information. Furthermore, the ability to identify the required procedure or concept was found to be another key competence that needs to be improved.

1.2 Identifying why Indonesian students struggle with context-based tasks: The opportunity-to-learn as a key concept

The next step to find ways to improve student performance on context-based tasks was identifying possible explanations for students' difficulties. The concept of 'opportunity-to-learn' was used as a key reference to investigate whether students have received education and experiences that are supportive for them to learn to solve context-based tasks. This investigation was focused on two aspects: the opportunity-to-learn that is provided in Indonesian mathematics textbooks and the opportunity-to-learn that is offered by teachers through their teaching practices.

1.2.1 Context-based tasks in Indonesian mathematics textbooks

In order to find possible explanations for Indonesian students' difficulties, a second study was conducted to analyze the mathematics textbooks that were used in the schools participating in the first study. This textbook analysis study focused on the mathematics tasks that were provided in the textbooks.

The analysis revealed that Indonesian mathematics textbooks contained an insufficient number of context-based tasks. Only 10% of the tasks in the textbooks were context-based. To get a better picture of the relation between textbooks and student performance, an in-depth analysis of the characteristics of these context-based tasks was carried out. The results of this analysis provided indications for a relation between the characteristics of the context-based tasks in Indonesian mathematics textbooks and the types of errors made by students when solving such tasks. First, most of the context-based tasks in the textbooks used camouflage context and provided explicit suggestions about the procedure required to solve the tasks. This characteristic corresponds to the large number of students' errors in transforming a real-world problem into a mathematical problem that includes errors in identifying the required procedure. As underlined by Haines and Crouch (2007), students' unfamiliarity with tasks that require identifying the necessary procedure leads to students' difficulties in mathematical modeling. Second, it was found that most of the context-based tasks in the textbooks provided the exact information that was necessary to solve the tasks; so students had no experience in selecting relevant information. This finding matches the high number of students' errors in selecting information.

In conclusion, the findings of this study indicate that Indonesian mathematics textbooks do not provide students enough opportunity-to-learn to solve context-based tasks. It also means that the textbooks do not seem to optimally put into practice the mandate of the national curriculum with regard to developing students' ability to apply mathematics.

1.2.2 Indonesian teachers' teaching practices of context-based tasks

In the third study, teachers' teaching practices were investigated to identify from this perspective the opportunity-to-learn to solve context-based tasks that is offered to Indonesian students. Moreover, considering the potential influence of teachers' beliefs on teaching practices, the beliefs of Indonesian teachers were also investigated.

Teachers' beliefs

The findings of this study showed that the involved teachers tended to perceive context-based tasks merely as plain word problems. The teachers focused only on the use of simple real-world contexts without really taking mathematical modeling into account and argued that the mathematical procedure required to solve a context-based task should be given explicitly. Furthermore, the teachers also did not consider missing and superfluous information as an important characteristic of a context-based task. In sum, the teachers did not hold beliefs that are supportive towards providing students with opportunity-to-learn to solve context-based tasks.

Teachers' teaching practices

Two aspects of teachers' teaching practices were investigated in this third study. The first aspect is the characteristics of the context-based tasks offered by teachers to students in which an indication for the influence of teachers' beliefs on teaching practices was found. In line with their beliefs about context-based tasks, the teachers mostly gave students context-based tasks that had explicit indications about the required procedure and provided only the relevant information. The second aspect is the teaching approach used by teachers to help students learn to solve context-based tasks. It was revealed that the teachers mostly used directive teaching. In directive teaching, students are not actively involved in reflecting on and carrying out the stages of solving context-based tasks. With respect to the process of comprehending a context-based task the teachers frequently told students what the task was about and what information was needed to solve it. This directive teaching approach was also observed in the process of transforming a real-world problem into a mathematical problem. It was visible in how the teachers directly told their students the mathematical procedure required to solve the task. In contrast to directive teaching, a consultative teaching approach, in which students get opportunities to actively perform and reflect on the solving process, was rarely used by the teachers. This teaching approach was mostly observed in the process of carrying out mathematical procedures, which, in fact, is a process in which students do not have to deal with the contexts of a task.

Correspondences were indicated between teachers' teaching practices and students' difficulties in solving context-based tasks. A lack of opportunities for students to paraphrase a context-based task seems to be related to students' difficulty in comprehending a context-based task because, as pointed out by Kletzien (2009), paraphrasing helps students understand the text of a task. Teachers' direct advice regarding the procedures to be carried out might correspond to students' transformation errors because, according to Barness (2000), it will discourage students from thinking about the mathematics concepts addressed in the task.

In conclusion, the observed teaching practices of the teachers seem to be not supportive for students to learn solving context-based tasks because, according to Antonius, Haines, Jensen, Niss, and Burkhardt (2007) and Forman and Steen (2001), teaching context-based tasks requires more than telling students what they should do. This finding suggests that improving the teaching practices of context-based tasks might contribute to improving student performance on such tasks.

1.3 Opportunity-to-learn to solve context-based mathematics tasks and students' performance in solving such tasks

In the fourth and final study of this PhD research, a field experiment was carried out to investigate whether providing students opportunity-to-learn to solve context-based tasks can help them improve their performance on such tasks. For this field experiment an intervention program was developed to compensate what was considered to be lacking in Indonesian mathematics textbooks and in teachers' teaching practices. The intervention comprised two components of opportunity-to-learn; first, context-based tasks that have superfluous and missing information and do not explicitly provide the required procedures, and second, a consultative teaching approach that uses metacognitive prompts. This opportunity-to-learn was offered to students, and its effects on students' performance in solving context-based tasks were examined from the perspectives of students' gain scores and students' errors.

The field experiment showed that there was only a small and marginally significant effect of the opportunity-to-learn on students' gain scores. Nevertheless, a closer examination of the effect of the opportunity-to-learn on students' errors revealed a positive influence. Students who received the opportunity-to-learn could better understand the instruction for a context-based task and had improved performance in selecting relevant information. With respect to transforming a real-world problem into a mathematical problem and identifying the required procedure in general no influence of the opportunity-to-learn was found. However, a positive influence was found for context-based tasks addressing graphs – i.e. the topic taught during the intervention period – in which students who got the opportunity-to-learn were better able to explain a mathematical interpretation of a graph. Reflecting upon this finding and referring to Howson (2010), it can be learned that to improve students' ability to identify the required procedure it is essential to provide not only context-based tasks that are related to the topic being taught, but also context-based tasks that address other topics.

To sum up, offering students context-based tasks that have non-explicit procedures and provide superfluous and missing information, and a consultative teaching approach with metacognitive prompts is promising to support students' learning of context-based tasks. However, in agreement with OECD (2014), it should be noted that a prolonged and continuous implementation of such practice might be needed in order to have optimum results.

1.4 Results at a glance

In summary, the results of this PhD research cover three interrelated issues regarding context-based mathematics tasks in Indonesia (see Figure 1). First, the investigation into students' difficulties provided information about key competences that need to be improved. Second, possible reasons for the students'

STUDENTS' DIFFICULTIES
(*Study 1*)

Analysis of Indonesian students' errors when solving context-based tasks

The most dominant error types:
- **comprehension** errors; in particular, errors in selecting relevant information.
- **transformation** errors; in particular, errors identifying the required mathematical procedures.

POSSIBLE REASONS FOR STUDENTS' DIFFICULTIES: OTLS AS A KEY CONCEPT
(*Study 2* and *Study 3*)

Analysis of Indonesian mathematics textbooks

Exposure of the context-based tasks:
- Only about 10% of all tasks were context-based.

Characteristics of the context-based tasks:
- most of the tasks used *camouflage contexts* and provide explicit indications about the required mathematical procedures.
- most of the tasks provide *matching information*, i.e. only the information that is needed to solve the tasks.
- almost no *reflection tasks*, i.e. tasks with highest cognitive demands which require constructing original mathematical approaches and communicating complex arguments and complex reasoning.

An investigation into Indonesian mathematics teachers' teaching practices

Teachers' report about the **characteristics of context-based tasks** offered to students:
- most of the teachers frequently give tasks with *explicit procedures*
- most of the teachers frequently give tasks with *matching information*
- a half of the teachers never or rarely give tasks with *superfluous information*
- a half of the teachers never or rarely tasks with *missing information*.

Teachers' **teaching approach**:
Over all stages of solving context-based tasks:
- *No instruction* was given in 42% of all questions discussed in the lessons.
- *Directive teaching* was applied in 47% of all questions discussed in the lessons.
- *Consultative teaching* was applied in only 12% of all questions discussed in the lessons.

Specified for the stages of solving context-based tasks:
- Directive teaching was most frequently applied in the comprehension and the transformation stages.
- Consultative teaching was mostly applied in the mathematical processing stage.
- Almost no attention was paid to the encoding stage.

OTL AND STUDENT PERFORMANCE
(*Study 4*)

Offering students opportunity-to-learn (OTL)		**Effects of the OTL on students' performance**
Context-based tasks:	**Consultative teaching approach with metacognitive prompts:**	
- Context-based tasks with missing or superfluous information.	- Paraphrasing: asking students to formulate a task in their own words. - Underlining all information and circling only the relevant information - Self- questioning; e.g. *"Do we have enough information to solve the task?"*	A positive effect of the OTL on students' task comprehension was found: - Students could understand better the instruction of the task - Students' ability to select relevant information improved
- Context-based tasks with a relevant context that requires modeling - Context-based tasks with non-explicit procedure	- Self-questioning; e.g. *"What are possible strategies to solve the task?"*	- In general no effect of the OTL on students' ability to transform a real-world problem into a mathematical problem. However, a positive effect was found for tasks addressing an interpretation of a graph, which in fact was related to the topic taught during the intervention. - This finding leads to a recommendation to offer students 'mixed exercises', i.e. a set of context-based tasks that address various topics.

Figure 1. Results at a glance

difficulties were identified when the difficulties were connected to opportunity-to-learn offered in Indonesian textbooks and teachers' teaching practices. Lastly, these possible reasons served as an important basis for designing an intervention program to offer students opportunity-to-learn to solve context-based tasks. This intervention program was found to have a positive influence on students' ability to comprehend a context-based task.

2 Toward a better performance of Indonesian students on context-based tasks

It is widely acknowledged that there are different ways to improve educational achievement (see e.g. Pearson, 2014; Stacey, 2011). For example, as mentioned by Stacey (2011), Finland attributes its success in PISA to teacher quality, whereas Singapore and the Netherlands point to their curriculum. The way a lesson is constructed can also contribute to improving student achievement. As a case in point, Japan is a country that refers its success in education to carefully constructed lessons and its culture of lesson study.

Reflecting upon these different ways to improve educational achievement, in this final chapter I would like to raise and answer the following question:

How could the performance of Indonesian students on context-based tasks be improved?

Improving Indonesian students' performance cannot be simply done by applying an educational practice that is used in other countries because, according to Pearson (2014) and Stacey (2011), what works in one particular country will not necessarily give the same result in other countries. Careful thought about what is missing in current educational practices and what might be needed by students is necessary. In this respect, this PhD research provides a better insight into educational practices in Indonesia regarding context-based tasks. Although the research took place in one province in Indonesia, I argue that its findings can be taken into consideration for a broader scope because the issues addressed in the study were quite general; for example, the textbooks that were analyzed are widely used in Indonesia. Furthermore, the finding about the dominance of directive teaching is in agreement with other studies on mathematics classrooms in Indonesia (e.g. Maulana, Opdenakker, Den Brok, & Bosker, 2012; World Bank, 2010). Therefore, on the basis of the results of this PhD research I would like to offer some recommendations for the practice of teaching context-based tasks in Indonesia that might contribute to improving student performance.

2.1 Learning materials: Adequate and appropriate context-based tasks

My first recommendation addresses learning materials, for which I recommend providing students with adequate and appropriate context-based tasks. 'Adequate' refers to the exposure to context-based tasks. Indonesian students need to get more experiences and chances to deal with context-based tasks; therefore it is essential to increase the quantity of such tasks in classroom practices.

What I mean with 'appropriate' is the quality of context-based tasks. Although some studies (OECD, 2014; World Bank, 2010) report an increased connection to real-life problems in mathematics lessons in Indonesia, this PhD research revealed that what students received were mainly tasks with dressed-up contexts and explicit indication about the required procedure. Such tasks do not really require students to transform a real-world problem into a mathematical problem and are therefore not sufficient to support students learning to solve context-based tasks. Therefore, as an addition to increasing the exposure of context-based tasks, I recommend improving the quality of the tasks by considering the following characteristics:

Relevant and essential context. A context-based task should use a real-world context that is essential and needs to be considered in the solving process. Such tasks offer students more opportunities to transform a real-world problem to a mathematical problem, which is an important aspect of applying mathematics.

Non-explicit procedures. As an addition to relevant contexts, I recommend giving students context-based tasks that do not imply the mathematical procedures or concepts needed to find the solution. Such tasks are important to develop students' ability to identify the procedure that is required to solve a context-based task. The results of this research suggest that not implying the required procedures is not only a property of a context-based task, but also a matter of how and when a context-based task is used. It was found that students did not really benefit from context-based tasks with non-explicit procedures when the tasks were given after an explanation of particular mathematics topics. Therefore, my further recommendation is to provide students with a 'mixed exercises' section that includes context-based tasks related to various mathematics topics.

Superfluous and missing information. The ability to distinguish between relevant and irrelevant information is a key aspect of solving problems in real life. However, this PhD research shows that most students were not able to select information and, correspondingly, received mainly tasks that contained only the information required to find the solution. With respect to this finding, I recommend offering students context-based tasks that have superfluous or missing information; i.e. tasks that have more or less information than what is needed to find the solution.

Lastly, I would like to emphasize that the abovementioned recommendation is not only for teachers, but also for textbook authors because OECD (2014) reported an increased use of textbooks as a primary resource for mathematics lessons in Indonesia. This means that improving the quality of textbooks might be followed by improved teaching practice, at the very least with respect to the quality of tasks offered to students. Furthermore, textbooks are essential for a widespread dissemination of an educational innovation because textbooks are widely used in many schools.

2.2 Teaching practices: Consultative teaching approach with meta-cognitive prompts

Several researchers (e.g. Antonius et al., 2007; Forman & Steen, 2001; Lingefjard & Meier, 2010) underlined the importance of a student-centered approach for teaching students mathematics problems situated in real-world contexts. However, the term 'student-centered' is still rather general and, therefore, concrete suggestions about how to conduct student-centered teaching might be necessary for teachers. Here I would like to recommend the consultative teaching approach developed in this PhD research as an alternative for student-centered teaching. In order to have student-centered teaching, metacognitive prompts can be used to keep a balance between teacher guidance and students' independence. Self-questioning, which is a kind of metacognitive prompt, is important not only to guide students to independently carry out the process of solving context-based tasks, but also to reflect on their work and understanding.

Teaching approach is not the only important aspect of teaching practices, because in order to optimally support their students' acquiring particular competences, teachers also need to know students' learning difficulties. This research suggests that diagnosing students' difficulties could provide essential information for improving student performance. Therefore, I recommend teachers including diagnosing students' difficulties as an integral part of their teaching practices.

2.3 Teachers

In relation to the previous two recommendations, I would like to highlight three essential roles of a teacher. First, a teacher is required to select tasks that match the learning goals and the students' ability level, because not every task in a textbook might be appropriate for his/her classroom practice. Second, a teacher should design tasks when textbooks do not accommodate all learning goals set in the curriculum or when the teacher has his/her own specific learning goals. The third role of a teacher is to convert tasks to learning opportunities; which means the teacher should use the tasks to support students' learning.

The findings about the low exposure of context-based tasks in textbooks and that teachers mostly offered students mere plain word problems indicate a need to increase the teachers' role in selecting and designing context-based tasks. Designing one's own tasks could also lead to better task ownership for teachers and also contribute to teachers' flexibility and creativity in using the tasks to support students' learning, which is in fact related to the third role of a teacher. To increase the three roles of a teacher I recommend paying attention to teachers' beliefs about context-based tasks. A correspondence between teachers' beliefs about context-based tasks and the characteristics of tasks they offered to students indicates that having appropriate beliefs might be crucial for selecting and developing good context-based tasks. Furthermore, appropriate beliefs, according to Blum (2011), are also necessary for teaching context-based tasks, which is the third role of a teacher. As an addition to beliefs, knowledge is also an important element for increasing the roles of a teacher in teaching context-based tasks. A teacher needs to have knowledge of context-based tasks, which includes: (1) the type of context; (2) non-explicit procedures; (3) the type of information; and (4) cognitive demands of tasks.

Above all, as my final note I would like to highlight that we cannot rely solely on teachers to improve student performance. Teachers need professional development for which support from other parties, especially teacher training institutions, is highly required. Professional development for teachers has been recognized as a key aspect to improve the quality of education and student achievement (Guskey, 2002; Porter, Garet, Desimone, Yoon, & Birman, 2000; Supovitz & Turner, 2000). Therefore, I would like to recommend teacher training institutions to consider beliefs and knowledge about context-based tasks in their training programs.

2.4 Recommendations at a glance

To sum up, in Figure 2 I illustrate how the aforementioned recommendations can contribute to improving student performance. Learning materials and teaching practices are key elements to improve students' performance. Here, the role of teachers is to select and design learning materials and to convert the learning materials into learning opportunities through the use of appropriate teaching practices. Despite the importance of their roles, teachers are not the only key actor to improve students' performance. Support from textbook authors and teacher training centers is required.

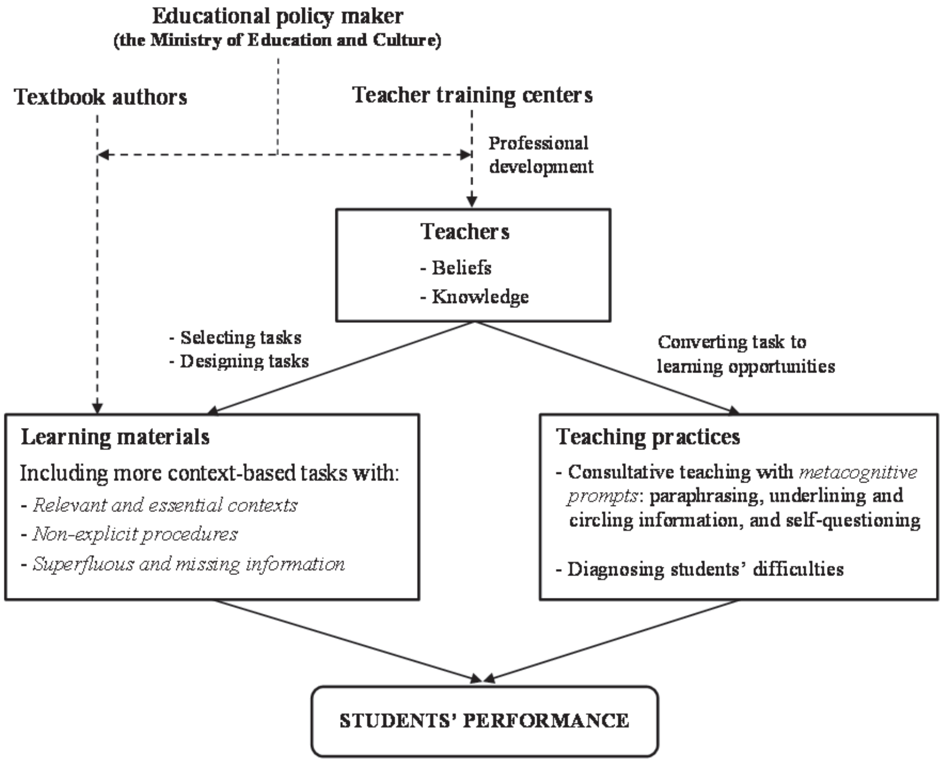


Figure 2. Recommendations at a glance

3 Suggestions for further research

Reflecting upon the results and the processes of this PhD research, it is realized that there is a need for further study. The research took place only in the Province of Yogyakarta. Therefore, to get a more thorough understanding of how the performance of Indonesian students on context-based tasks can be improved, it is necessary to extend this research to a broader area by involving more schools in more provinces. Having more schools might provide greater chances to investigate more crucial aspects such as school quality, teacher quality, school facility, and student background.

The investigation into possible reasons for student difficulties only emphasized factors that are related to cognitive aspects, i.e. textbooks and teachers' teaching practices. However, as pointed out by Leron and Hazzan (1997), students' thinking is influenced not only by cognitive factors, but also by affective factors. Therefore, investigating affective factors such as students' motivation might provide a comprehensive picture about possible factors that influence student performance on context-based tasks.

With regard to teachers' teaching practices the focus was only on teachers' beliefs and teaching approaches. Although the research findings regarding these two aspects already provide recommendations for improving educational practices in Indonesia, it is important to take teachers' knowledge about context-based tasks into consideration. Teachers' knowledge about context-based tasks might also have influence on teaching practices and, therefore, on student performance.

Another issue that needs further consideration is teacher quality, because teachers play a key role in the learning process of students. In attempting to improve teacher quality, the Indonesian government, through the Teacher Law 2005, sets minimum academic and professional requirements for teachers. One of the requirements is the need for higher standards in classroom teaching, in which teachers are required to engage students in the learning process (Jalal et al., 2009). I argue that the consultative teaching that was developed in this PhD research meets this requirement. Therefore, as a further study it might be valuable to incorporate and collaborate the knowledge generated by this research into two major teacher training programs that are positioned by the Indonesian government as strategies for teacher quality improvement; i.e. *Pendidikan dan Latihan Profesi Guru* (in-service teacher profession training) and *Pendidikan Profesi Guru* (professional education for pre-service teachers).

References

Antonius, S., Haines, C., Jensen, T. H., Niss, M., & Burkhardt, H. (2007). Classroom activities and the teacher. In W. Blum, P. L. Galbraith, H.-W. Henn & M. Niss (Eds.), *Modelling and applications in mathematics education* (pp. 295–308). New York: Springer

Barnes, M. (2000). 'Magical' moments in mathematics: Insights into the process of coming to know. *For the Learning of Mathematics*, 20(1), 33–43.

Batanero, C., Godino, J. D., Vallecillos, A., Green, D. R., & Holmes, P. (1994). Errors and difficulties in understanding elementary statistical concepts. *International Journal of Mathematical Education in Science and Technology, 25*(4), 527–547.

Blum, W. (2011). Can modelling be taught and learnt? Some answers from empirical research. In G. Kaiser, W. Blum, R. B. Ferri & G. Stillman (Eds.), *Trends in teaching and learning of mathematical modelling* (pp. 15 – 30). New York: Springer.

Borasi, R. (1987). Exploring mathematics through the analysis of errors. *For the Learning of Mathematics*, 7(3), 2–8.

Brodie, K. (2014). Learning about learner errors in professional learning communities. Educational Studies in Mathematics, 85(2), 221–239.

Guskey, T. R. (2002). Professional development and teacher change. *Teachers and Teaching: Theory and Practice, 8*(3-4), 381–391.

Haines, C., & Crouch, R. (2007). Mathematical modelling and applications: Ability and competence frameworks. In W. Blum, P. L. Galbraith, H.-W. Henn & M. Niss (Eds.), *Modelling and applications in mathematics education: The 14th ICMI study* (pp. 89-98). New York: Springer.

Howson, G. (2010). The development of mathematics textbooks: historical reflections from a personal perspective. *ZDM – The International Journal on Mathematics Education, 45*(5), 647–658.

Jalal, F., Samani, M., Chang, M. C., Stevenson, R., Ragatz, A. B., & Negara, S. D. (2009). *Teacher Certification in Indonesia: A strategy for teacher quality improvement*. Jakarta: Indonesian Ministry of National Education and the World Bank.

Jupri, A., Drijvers, P., & Van den Heuvel-Panhuizen, M. (2014). Difficulties in initial algebra learning in Indonesia. *Mathematics Education Research Journal, 26*(4), 683–710.

Kletzien, S. B. (2009). Paraphrasing: An effective comprehension strategy. *The Reading Teacher, 63*(1), 73–77.

Leron, U., & Hazzan, O. (1997). The world according to Johnny: A coping perspective in mathematics education. *Educational Studies in Mathematics, 32*(3), 265–292.

Maulana, R., Opdenakker, M.-C., Den Brok, P., & Bosker. (2012). Teacher-student interpersonal behavior in secondary mathematics classes in Indonesia. *International Journal of Science and Mathematics Education, 10*(1), 21–47.

OECD. (2013). *PISA 2012 results: What students know and can do. Student performance in mathematics, reading and science*. PISA: OECD Publishing.

OECD. (2014). *Measuring innovation in education: A new perspectives*. Educational Research and Innovation, OECD Publishing. doi: 10.1787/9789264215696-en.

Pearson. (2014). *The Learning Curve 2014: Education and skills for life*. Retrieved from http://thelearningcurve.pearson.com/reports/the-learning-curve-report-2014 on June 24, 2014.

Porter, A. C., Garet, M. S., Desimone, L., Yoon, K. S., & Birman, B. F. (2000). *Does professional development change teaching practice? Results from a three-year study*. Washington D. C.: U. S. Department of Education.

Stacey, K. (2011). The PISA view of mathematical literacy in Indonesia. *IndoMS Journal on Mathematics Education, 2*(2), 95–126.

Seng, L. K. (2010). An error analysis of Form 2 (Grade 7) students in simplifying algebraic expressions: A descriptive study. *Electronic Journal of Research in Educational Psychology, 8*(1), 139–162.

Supovitz, J. A., & Turner, H. M. (2000). The effects of professional development on science teaching practices and classroom culture. *Journal of Research in Science Teaching, 37*(9), 963–980.

World Bank. (2010). *Inside Indonesia's mathematics classrooms: A TIMSS Video Study of teaching practices and student achievement*. Jakarta: Author.

Summary

Samenvatting

Ringkasan

Acknowledgments

Curriculum vitae

List of presentations related to this thesis

FIsme Scientific Library

ICO Dissertation Series

Summary

The Indonesian national curriculum emphasizes that mathematics education must be relevant to the needs of life and should offer students opportunities to develop the ability to apply their knowledge in society. Furthermore, there are educational movements in Indonesia that emphasize the application of mathematics and promote the use of context-based tasks, i.e. *Pendidikan Matematika Realistik Indonesia* (Indonesian Realistic Mathematics Education) and *Pembelajaran Kontekstual* (Contextual Teaching and Learning). Nevertheless, the results of PISA studies showed that Indonesian students have poor performance on context-based tasks. Considering this situation, this PhD research was carried out to gain a better insight into Indonesian students´ low performance on context-based mathematics tasks and to generate recommendations for improving student performance.

This PhD research addresses three main research questions:

1. *What difficulties do Indonesian students have when solving context-based tasks?*
2. *What are possible reasons for Indonesian students' difficulties when solving context-based tasks?*
3. *How can Indonesian student performance on context-based tasks be improved?*

Four studies were carried out to answer these research questions. The studies are reported in chapters 2 to 5 of this thesis. *Chapter 2* contributes to answering the first research question, *Chapter 3* and *Chapter 4* address the second research question, and *Chapter 5* provides an answer to the last research question. Finally, the findings of the four studies are synthesized in *Chapter 6*.

Chapter 2 describes the study into Indonesian students' difficulties when solving context-based tasks. In this first study, a total of 362 ninth- and tenth-grade students from eleven schools from rural and urban areas in the Province of Yogyakarta in Indonesia took a paper-and-pencil test on context-based PISA mathematics tasks. To investigate the kinds of difficulties students have when solving these tasks an error analysis was performed on the students' responses. For this purpose, an analysis framework was developed by combining Newman's error categories with the stages of modeling when solving context-based tasks and the stages of mathematization as used in PISA. The analysis framework comprised four types of errors: comprehension, transformation, mathematical processing, and encoding.

The error analysis revealed that the most dominant errors made by the students were comprehension and transformation errors. This finding indicates that Indonesian students mostly have difficulties with the first two stages of solving context-based tasks: comprehending a context-based task and transforming it into a mathematical problem. With respect to the first stage it was found that the students mainly made errors in selecting relevant information; whereas for the second stage the students were mostly unable to select the procedures that are suitable to solve the tasks.

After investigating students' difficulties, the next step in this PhD research was identifying possible explanations for these difficulties. For this purpose the concept of 'opportunity-to-learn' was used as the key perspective. The investigation focused on opportunities-to-learn in the used mathematics textbooks as well as in the teaching practice of the teachers.

Chapter 3 reports on the study into the opportunity-to-learn offered in Indonesian mathematics textbooks. Three mathematics textbooks were analyzed that are used in the schools participating in the first study (see *Chapter 2*). This textbook analysis study focused on investigating the mathematics tasks that are included in the textbooks.

The analysis revealed that the investigated Indonesian mathematics textbooks have a very low number of context-based tasks. Only 10% of tasks could be labeled as context-based tasks. To get a better picture of the relation between opportunity-to-learn offered in textbooks and student performance, an in-depth analysis of the characteristics of these context-based tasks was carried out. It was found that three quarters of the context-based tasks in the textbooks used a camouflage context (which means that the tasks are merely dressed-up bare problems) and provided an explicit indication about the mathematical procedure to be used. This finding shows that Indonesian mathematics textbooks do not offer students enough opportunity-to-learn to identify a suitable mathematical procedure to solve a context-based task, which might explain the high number of transformation errors made by the students. Moreover, the textbook analysis also disclosed that most context-based tasks in Indonesian textbooks provided just precisely the information needed to solve a task. This result signifies a lack of opportunities for students to learn to select relevant information, which, therefore, might have contributed to students' comprehension errors, in particular errors in selecting information.

Chapter 4 describes the study that focused on investigating the opportunity-to-learn to solve context-based tasks offered by the teachers' teaching practice. Teachers' beliefs were also taken into account because they often influence the teaching practice. The study started by surveying 27 teachers from seven schools which also participated in the first study (see *Chapter 2*) through a written questionnaire. Then, to further examine teachers' teaching practice related to context-based tasks, four teachers of this group of 27 teachers were observed and video-recorded in two mathematics lessons in which they were asked to deal with context-based tasks. The focus of the observations was to investigate what approach the teachers used to help their students learn to solve context-based tasks.

The questionnaire revealed that the teachers tended to perceive context-based tasks merely as plain word problems. They believed that the mathematical procedure needed to solve a context-based task should be given explicitly. The teachers also did not consider missing and superfluous information as an important characteristic of a context-based task. Moreover, these beliefs were reflected in their reported teaching practice. In agreement with their beliefs, the teachers stated that they frequently offered their students context-based tasks with explicit procedures and that they rarely gave context-based tasks that provided superfluous or missing information. Such practice might explain Indonesian students' difficulties in identifying a suitable procedure and in selecting relevant information.

The classroom observations showed that the teachers mainly used a directive teaching approach in which they told the students what a context-based task is about, translated the task into a mathematical problem, and explained what mathematical procedure had to be carried out. Students were not encouraged to be actively involved in and reflect on the process of solving context-based tasks. This directive teaching approach was mostly used in the comprehension and transformation stages. Consultative teaching in which students were actively engaged in the process of solving context-based tasks was barely used by the teachers. Remarkably, this consultative teaching approach was mostly used in the mathematical processing stage; the stage in which students factually do not have to deal with the context of a task.

Chapter 5 describes the final study of this PhD research in which the findings of the first three studies were used to develop an intervention intended to offer students opportunity-to-learn to solve context-based tasks. This intervention comprised two components: a set of context-based tasks and guidelines for a consultative teaching approach including giving students metacognitive prompts. The context-based tasks used in the intervention had three important characteristics: including a relevant and essential context, containing superfluous or missing information, and not giving explicit suggestions about a suitable mathematical procedure. The effects of this opportunity-to-learn on students' performance in solving context-based tasks were examined through a field experiment involving 299 eight-graders from six schools. The students' performance was seen from the perspectives of students' gain scores and students' errors.

This study revealed that there was only a small and marginally significant effect of the opportunity-to-learn on the students' gain scores. Nevertheless, a closer examination of the effect of the intervention on the students' errors revealed a positive influence. The students who received the opportunity-to-learn could better understand the wording of a context-based task and had improved performance in selecting relevant information. With regard to the students' performance in transforming a real-world problem into a mathematical problem and in identifying a suitable procedure no influence of the intervention was found in general. However, a positive influence was found for context-based tasks addressing graphs, which was the topic taught during the intervention period. Students who were involved in the intervention were better able to give a mathematical interpretation of graphs. This finding indicates that to improve students' ability to identify a suitable mathematical procedure it is important to provide students with context-based tasks that address all kinds of topics and not only offer them context-based tasks that are related to the topic that is currently taught, because then the mathematical procedure is more or less given.

Chapter 6 provides an overview of the findings of the four studies and offers recommendations for educational practice and for further research. The main conclusions that can be drawn from this PhD research are that on the one hand Indonesian students mainly had difficulties in comprehending context-based tasks and in transforming them into a mathematical problem, and that on the other hand it was found that there was a lack of opportunity-to-learn to solve context-based tasks in textbooks and teachers' teaching practice. On the basis of these research findings three recommendations were given for improving the practice of teaching context-based tasks. The first recommendation is to include more context-based tasks in the learning materials; especially context-based tasks that use relevant and essential contexts, have superfluous or missing information, and do not explicitly signify what mathematical procedure is suitable. The second recommendation addresses the teaching practice; that is, the use of a consultative approach and make use in the teaching practice of the knowledge about students' difficulties when they have to solve context-based tasks. The third recommendation is to pay attention in teacher education and professional development to teachers' beliefs and their knowledge about context-based tasks, because these two aspects are essential for selecting good tasks (or designing them) and creating learning opportunities with them.

Samenvatting

In het Indonesische wiskundecurriculum wordt benadrukt dat het onderwijs moet aansluiten bij de behoeften van de huidige maatschappij en aan leerlingen de gelegenheid moet bieden om kennis en vaardigheden te ontwikkelen die ze in de maatschappij kunnen gebruiken. Dit is in lijn met vernieuwingsprojecten zoals *Pendidikan Matematika Realistik Indonesia* (Realistisch Wiskunde Onderwijs Indonesië) en *Pembelajaran Kontekstual* (Contextrijk Onderwijs), die beide gericht zijn op het leren toepassen van wiskunde en het gebruik van contextopgaven stimuleren. Ondanks deze initiatieven laten de resultaten van PISA (Programme for International Student Assessment) zien dat Indonesische leerlingen zwak presteren op contextopgaven. Het onderhavige promotieonderzoek is opgezet om te onderzoeken hoe dit komt en hoe die prestaties verbeterd kunnen worden.

De volgende drie vragen zijn onderzocht:

1. *Welke problemen hebben Indonesische leerlingen bij het oplossen van contextopgaven?*
2. *Wat zijn mogelijke verklaringen voor de problemen van Indonesische leerlingen bij het oplossen van contextopgaven?*
3. *Hoe kunnen de prestaties van Indonesische leerlingen bij contextopgaven verbeterd worden?*

De hoofdstukken 2, 3, 4 en 5 rapporteren over de vier studies die zijn uitgevoerd om deze vragen te beantwoorden. De eerste onderzoeksvraag en de beantwoording ervan zijn het onderwerp van hoofdstuk 2. De studies die besproken worden in hoofdstuk 3 en hoofdstuk 4 richten zich op de tweede onderzoeksvraag en hoofdstuk 5 heeft betrekking op de derde en laatste onderzoeksvraag. Hoofdstuk 6 omvat een synthese van de resultaten van deze vier studies.

Hoofdstuk 2 beschrijft het onderzoek naar de problemen die Indonesische leerlingen hebben bij het oplossen van contextopgaven. Aan deze eerste studie deden 362 leerlingen mee uit de derde en vierde klas voortgezet onderwijs van elf Indonesische scholen gelegen in de provincie Yogyakarta. Het betrof zowel scholen uit een landelijke omgeving als scholen in steden. De schriftelijk toets die de leerlingen hebben gemaakt, bestond uit een aantal contextopgaven die ontleend waren aan de vrijgegeven wiskundeopgaven van PISA. Om te onderzoeken wat nu precies de problemen zijn die Indonesiche leerlingen tegenkomen bij deze opgaven, is een foutenanalyse van het leerlingenwerk uitgevoerd. Hierbij is gebruikgemaakt van een speciaal hiervoor ontwikkeld raamwerk. Bij dit raamwerk is enerzijds uitgegaan van de foutencategorieën van Newman en anderzijds van de verschillende fasen die bij het modelleren van wiskundeopgaven en bij de door PISA gehanteerde definitie van mathematiseren onderscheiden worden. Dit raamwerk omvatte vier type fouten: fouten in het begrijpen van de (tekst van de) opgave, in het omzetten van de opgave naar een wiskundig probleem, in het uitvoeren van de wiskundige bewerking en in het terugvertalen naar de context.

De foutenanalyse liet zien dat de meeste fouten van de leerlingen gemaakt werden in het begrijpen van de opgaven en in het omzetten naar een wiskundige bewerking. Deze bevinding impliceert dat de Indonesische leerlingen vooral moeite hadden met de eerste twee fasen in het proces van het oplossen van contextopgaven. Bij het begrijpen van de opgaven maakten de leerlingen vooral fouten in het selecteren van de relevante gegevens en bij het omzetten naar een wiskundige bewerking lukte het ze vaak niet om een passende wiskundige procedure te vinden.

De volgende stap in het promotieonderzoek was het zoeken naar mogelijke verklaringen voor deze fouten. Hierbij is nagegaan of aan de leerlingen wel voldoende leermogelijkheden (opportunities-to-learn) worden geboden om contextopgaven op te lossen. Dit is op twee manieren nagegaan. Eerst is naar Indonesische wiskundemethodes gekeken en daarna naar de leermogelijkheden die Indonesische leraren in hun lessen bieden.

Hoofdstuk 3 beschrijft het onderzoek naar de leermogelijkheden die in Indonesische wiskundemethodes geboden worden voor het oplossen van contextopgaven. In dit onderzoek zijn drie wiskundemethodes geanalyseerd die gebruikt werden op de scholen die betrokken waren bij de eerste studie (zie hoofdstuk 2). De analyse richtte zich op de opgaven die in deze wiskundemethodes zijn opgenomen. De methode-analyse maakte duidelijk dat de onderzochte Indonesische methodes nauwelijks contextopgaven bevatten. Slechts 10% van de opgaven waren contextopgaven. Vervolgens zijn de kenmerken van deze contextopgaven verder geanalyseerd met de bedoeling om te onderzoeken of er een relatie kon worden vastgesteld tussen de leermogelijkheden die de wiskundemethodes bieden en de fouten van de leerlingen. Hierbij kwam naar voren dat driekwart van de contextopgaven in feite opgaven waren met een camouflagecontext (aangeklede kale opgaven) en opgaven waarin expliciet is aangegeven welke wiskundige procedure nodig is om de opgaven te kunnen oplossen. Met andere woorden, de onderzochte Indonesische methodes geven de leerlingen nauwelijks leermogelijkheden voor het identificeren van de wiskundige procedure die nodig is om een probleem op te lossen. Dit tekort in de leermogelijkheden die de methodes bieden, kan dus als een mogelijke verklaring worden gezien voor het grote aantal fouten die de Indonesische leerlingen maakten bij de omzetting van contextopgave naar wiskundige bewerking. Een ander punt dat bij de methode-analyse naar voren kwam, is dat de meeste contextopgaven in de Indonesische methodes precies die gegevens bleken te bevatten die nodig zijn om de opgaven te kunnen oplossen, waardoor leerlingen die met deze methodes werken dus weinig gelegenheid krijgen om het selecteren van de relevante gegevens te leren, hetgeen weer de vele fouten kan verklaren die de Indonesische leerlingen hierbij maakten.

Hoofdstuk 4 beschrijft de studie naar de leermogelijkheden die leraren in hun onderwijs aan leerlingen bieden voor het oplossen van contextopgaven. Hierbij werd niet alleen gekeken naar wat leraren in hun lessen doen, maar ook naar hun opvattingen en naar hoe ze zelf tegen hun lespraktijk aankijken. Van de scholen die betrokken waren bij de eerste studie (zie hoofdstuk 2), hebben 27 wiskundeleraren aan dit verdere onderzoek meegedaan. Met een vragenlijst werden hun opvattingen over allerlei aspecten van het wiskundeonderwijs en hun lespraktijk in kaart gebracht. Daarnaast zijn bij vier leraren lesobservaties uitgevoerd om een dieper inzicht in hun onderwijsaanpak te krijgen. Hierbij is vooral gekeken naar de manier waarop de leraren hun leerlingen ondersteunden bij het leren oplossen van contextopgaven.

De analyse van de vragenlijstgegevens liet zien dat de betrokken leraren de contextopgaven vooral benaderden als wiskundeopgaven die in gewone taal beschreven zijn (redactieopgaven) in plaats van met formele wiskundetaal. De leraren waren de mening toegedaan dat bij zo'n opgave de uit te voeren wiskundige procedure expliciet gegeven moet worden. De leraren zagen niet het belang in van het gebruik van contextopgaven met te weinig of met te veel gegevens. Deze opvattingen waren ook zichtbaar in datgene wat de leraren over hun lespraktijk rapporteerden. In overeenstemming met hun opvattingen meldden de lerarendat ze voornamelijk contextopgaven aan hun leerlingen gaven die expliciete aanwijzingen bevatten over de wiskundige procedures en dat ze nauwelijks opgaven gaven met ontbrekende of overbodige gegevens. Gezien deze lespraktijk is het dus heel aannemelijk dat de leerlingen bij contextopgaven problemen hadden met het identificeren van de wiskundige procedure en het selecteren van de relevante gegevens.

De lesobservaties brachten naar voren dat de leraren vooral op een directieve manier les gaven waarbij ze de leerlingen vertelden waar een contextopgave over ging, welke gegevens gebruikt moesten worden en welke wiskundige procedure uitgevoerd moest worden. De leerlingen werden niet aangemoedigd om zelf na te denken over het oplossen van de opgaven. Een onderwijsaanpak waarbij deleerlingen wel werden betrokken bij het selecteren van de relevante gegevens en het kiezen van de wiskundige procedure, en waarbij vragen werden gesteld om reflectie te stimuleren werd nauwelijks toegepast. Echter, de leraren lieten de leerlingen wel meedenken bij het uitvoeren van de wiskundige procedures, terwijl dit juist de fase is die los staat van de context.

Hoofdstuk 5 beschrijft de laatste studie die in het kader van dit promotieonderzoek is uitgevoerd. In deze studie is een veldexperiment uitgevoerd om de bevindingen van de tweede studie (leermogelijkheden in wiskundemethodes, zie hoofdstuk 3) en de derde studie (leermogelijkheden in de lespraktijk, zie hoofdstuk 4) te toetsen. Daarvoor is een interventie ontwikkeld waarin de leerlingen uitdrukkelijk leermogelijkheden werden geboden voor het oplossen van contextopgaven. De interventie bestond uit twee componenten: een verzameling contextopgaven en een onderwijsaanpak waarbij de leerlingen door meta-cognitieve hints werden gestimuleerd actief mee te denken bij het oplossen van contextopgaven. Deze contextopgaven hadden drie belangrijke kenmerken: een relevante en essentiële context, overbodige of ontbrekende gegevens, en geen expliciete aanwijzingen voor de uit te voeren wiskundige procedures. Om het effect van de interventie te meten is een voortoets en een natoets afgenomen bestaande uit contextopgaven die ontleend waren aan vrijgegeven PISA-opgaven. Het experiment is uitgevoerd met derde klassen voortgezet onderwijs van zes scholen, waarbij op elke school een experimentele klas en een controleklas heeft meegedaan. In totaal zijn de gegevens van 299 leerlingen (met een gemiddelde leeftijd van 13.7 jaar) geanalyseerd waarbij zowel gekeken is naar het verschil in goedscore tussen de voortoets en natoets als naar het type gemaakte fouten.

De analyses lieten zien dat een onderwijsaanpak waarbij de leraren de leerlingen ervaring lieten opdoen in het zelf selecteren van relevante gegevens en het zelf kiezen van de wiskundige bewerking, en waarbij de leraren vragen stelden om reflectie bij de leerlingen te stimuleren, inderdaad tot minder fouten leidde bij het maken van de contextopgaven. De leerlingen uit de experimentele groep begrepen de contextopgaven beter en waren beter in staat om de relevante gegevenste selecteren dan de leerlingen uit de controlegroep. De interventie had echter geen effect op de vaardigheid van de leerlingen om contextopgaven om te zetten naar een wiskundig probleem. Daarentegen werd wel een positieve invloed van de interventie gevonden op het oplossen van contextopgaven waarin grafieken een rol speelden. Dit heeft waarschijnlijk te maken met het feit dat de interventie betrekking had op het onderwerp grafieken en de leerlingen in de experimentele groep voor dit onderwerp dus meer actieve leermogelijkheden hebben gehad dan de leerlingen in de controlegroep. Deze bevinding geeft wel aan dat bij het bieden van leermogelijkheden voor het oplossen van contextopgaven voorkomen moet worden dat de contextopgaven per onderwerp worden aangeboden. Bij een hoofdstuk over grafieken is het voor de leerlingen duidelijk dat de contextopgave die na zo'n hoofdstuk gegeven wordt met een grafiek moet worden opgelost. De leerlingen krijgen op die manier geen ervaring in het omzetten van een contextopgave naar een wiskundig probleem. Beter is het dus om bij het bieden van leermogelijkheden voor het oplossen van contextopgaven de wiskundige onderwerpen af te wisselen.

Hoofdstuk 6 levert een overzicht van de resultaten van de vier studies en geeft aanbevelingen voor de lespraktijk en voor vervolgonderzoek. De belangrijkste conclusies van dit promotieonderzoek zijn dat Indonesische leerlingen van de ene kant problemen hebben met het begrijpen van wat precies in een contextopgave gevraagd wordt en met het vertalen van contextopgaven naar een wiskundig probleem en dat van de andere kant wiskundemethodes en de lespraktijk van leraren de leerlingen nauwelijks gelegenheid bieden om het oplossen van contextopgaven te leren. Daarom zijn op basis van dit onderzoek drie aanbevelingen geformuleerd. De eerste aanbeveling is het opnemen van meer contextopgaven in het lesmateriaal, in het bijzonder moeten daarin contextopgaven worden opgenomen met relevante en essentiële contexten, met ontbrekende of overbodige gegevens en zonder expliciete aanwijzingen voor de te gebruiken wiskundige procedure. De tweede aanbeveling heeft betrekking op de manier waarop contextopgaven onderwezen moeten worden. Leraren zouden hun leerlingen actief mee moeten laten denken bij het oplossen van de opgaven. Ook zouden leraren meer oog moeten hebben voor de fouten die de leerlingen in de verschillende fasen van het oplossingsproces maken. Hiermee zouden de leraren rekening moeten houden bij hun onderwijsaanpak. De derde aanbeveling is dat er in de opleiding en begeleiding van leraren aandacht moet zijn voor de opvattingen en kennis die leraren hebben over contextopgaven en het belang ervan, omdat deze beide uiteindelijk bepalend zijn voor het kiezen (of ontwerpen) van geschikte contextopgaven en voor het creëren van leermogelijkheden.

Ringkasan

Di dalam kurikulum nasional Indonesia ditekankan bahwa pendidikan matematika harus relevan untuk kebutuhan kehidupan dan memberikan siswa kesempatan untuk mengembangkan kemampuan dalam menerapkan pengetahuan. Sejalan dengan hal tersebut, di Indonesia terdapat gerakan pendidikan yang menekankan pada aplikasi matematika dan penggunaan soal berbasis konteks, yaitu*Pendidikan Matematika Realistik Indonesia* dan *Pembelajaran Kontekstual*. Namun demikian, hasil tes PISA menunjukkan rendahnya kemampuan siswa Indonesia dalam menyelesaikan soal berbasis konteks. Mempertimbangkan situasi tersebut, penelitian PhD ini ditujukan untuk memperoleh pemahaman yang lebih baik tentang rendahnya pencapaian siswa Indonesia dalam menyelesaikan soal berbasis konteks serta untuk merumuskan rekomendasi untuk meningkatkan pencapaian siswa.

Dalam PhD tesis ini terdapat tiga pertanyaan utama yang dibahas:

1. *Kesulitan apakah yang dihadapi siswa Indonesia saat mengerjakan soal berbasis konteks?*
2. *Apakah kemungkinan penyebab kesulitan siswa dalam mengerjakan soal berbasis konteks?*
3. *Bagaimana kemampuan siswa Indonesia dalam mengerjakan soal berbasis konteks dapat ditingkatkan?*

Bab 2 sampai 5 dalam PhD tesis ini menyajikan hasil dari empat penelitian yang dilaksanakan untuk menjawab ketiga pertanyaan utama penelitian tersebut di atas. *Bab 2* ditujukan untuk menjawab pertanyaan pertama, *Bab 3* dan *Bab 4* membahas pertanyaan kedua, dan *Bab 5* menampilkan jawaban untuk pertanyaan ketiga. Hasil dari empat penelitian tersebut dirangkum dan disintesis di *Bab 6*.

Bab 2 membahas penelitian pertama yang ditujukan untuk mengidentifikasi kesulitan siswa dalam mengerjakan soal berbasis konteks. Penelitian ini dilaksanakan di 11 sekolah di provinsi Dareah Istimewa Yogyakarta dan melibatkan 362 siswa kelas 9 dan 10. Siswa tersebut berpartisipasi dalam suatu tes yang menggunakan soal berbasis konteks. Analisis kesalahan (*error analysis*) dilakukan untuk mengidentifikasi jenis kesulitan siswa saat mengerjakan soal berbasis konteks. Untuk hal ini, suatu kerangka analisis dikembangkan dengan mengaitkan kategori kesalahan siswa yang dirumuskan Newman (*Newman's error categories*) dengan tahapan pada pemodelan (*modeling process*) dan matematisasi PISA.

Kerangka analisis ini memuat empat jenis kesalahan, yaitu: pemahaman (*comprehension*), transformasi (*transformation*), pemrosesan matematis (*mathematical processing*), dan penafsiran (*encoding*).

Analisis kesalahan mengungkapkan bahwa jenis kesalahan yang paling sering dilakukan siswa adalah kesalahan terkait pemahaman dan transformasi. Temuan ini menunjukkan bahwa kesulitan terbesar dialami siswa Indonesia di dua tahap awal penyelesaian soal berbasis konteks, yaitu dalam memahami maksud soal dan dalam mengubah soal tersebut menjadi permasalahan matematika. Terkait tahap pertama penyelesaian soal banyak siswa yang membuat kesalahan dalam memilih informasi yang relevan, sedangkan pada tahap kedua siswa tidak bisa memilih prosedur atau konsep matematika yang tepat untuk menyelesaikan soal.

Setelah meneliti kesulitan siswa, langkah selanjutnya dalam penelitian PhD ini adalah mengidentifikasi kemungkinan penyebab kesulitan tersebut. Untuk hal ini konsep 'kesempatan belajar' (*opportunity-to-learn*) digunakan sebagai perspeksi utama.

Bab 3 memuat penelitian tentang 'kesempatan belajar' mengerjakan soal berbasis konteks yang ditawarkan buku teks di Indonesia. Dalam penelitian ini dilakukan analisis terhadap tiga buku teks yang digunakan di sekolah yang berpartisipasi dalam penelitian pertama (yaitu *Bab 2*). Fokus analisis buku (*textbook analysis*) ini adalah soal-soal matematika yang tersedia dalam ketiga buku tersebut.

Hasil analisis menunjukkan kurangnya soal berbasis konteks di ketiga buku teks matematika yang dianalisis. Soal berbasis konteks hanya sebanyak 10% dari keseluruhan soal yang ada dalam buku. Untuk mendapatkan pemahaman yang lebih baik tentang hubungan antara 'kesempatan belajar' dengan tingkat pencapaian siswa, dilakukan analisis lanjutan untuk mengidentifikasi karakteristik soal berbasis konteks. Analisis ini mengungkap bahwa tiga perempat dari soal berbasis konteks yang ada di buku menggunakan konteks kamuflase (yaitu konteks dapat diacuhkan dalam penyelesaian soal) dan menampilkan prosedur penyelesaian secara jelas. Temuan ini menunjukkan bahwa buku teks matematika di Indonesia tidak menyediakan kesempatan yang cukup kepada siswa untuk belajar mengidentifikasi prosedur atau konsep matematika yang dibutuhkan untuk menyelesaikan suatu soal berbasis konteks. Hal ini mungkin merupakan salah satu alasan atas banyaknya kesalahan transformasi yang dilakukan siswa. Lebih lanjut lagi, hasil analisis buku juga mengungkap bahwa mayoritas soal berbasis konteks yang ada di buku teks hanya menyediakan informasi yang dibutuhkan untuk menjawab soal. Hal ini menunjukkan kurangnya kesempatan untuk siswa belajar memilih informasi yang relevan. Hal tersebut berperan pada banyaknya kesalahan pemahaman yang dibuat siswa, khususnya kesalahan dalam memilih informasi.

Bab 4 memuat penelitian tentang 'kesempatan belajar' menyelesaikan soal berbasis konteks yang ditawarkan melalui praktik mengajar guru. Keyakinan (*beliefs*) guru juga diteliti karena hal tersebut sering mempengaruhi bentuk praktik mengajar guru. Penelitian ini melibatkan 27 guru matematika dari sekolah yang berpartisipasi dalam penelitian pertama (*Bab 2*). Seluruh guru tersebut diminta mengisi angket tentang keyakinan dan praktik mengajar guru. Untuk meneliti lebih jauh tentang praktik mengajar guru, observasi kelas dilakukan di empat kelas dimana para guru tersebut diminta menggunakan soal berbasis konteks dalam kegiatan belajar mengajar. Fokus observasi kelas adalah meneliti strategi mengajar yang digunakan guru untuk membantu siswanya belajar menyelesaikan soal berbasis konteks.

Angket guru menunjukkan bahwa para guru menganggap soal berbasis konteks sekadar soal cerita biasa (*word problems*), yaitu soal yang menyediakan prosedur penyelesaian secara jelas. Selain itu, mereka juga menganggap bahwa soal berbasis kontek tidak perlu memuat informasi berlebih (*superfluous information*) maupun memiliki informasi terselubung (*missing information*). Keyakinan guru tersebut juga tercermin pada praktik mengajar guru. Para guru melaporkan bahwa mereka sering memberi siswa soal yang memuat prosedur penyelesaian secara jelas tetapi jarang memberi siswa soal yang memuat informasi berlebih ataupun informasi terselubung. Praktik mengajar semacam itu merupakan salah satu kemungkinan penyebab kesulitan siswa dalam mengidentifikasi prosedur matematika untuk menyelesaikan soal dan dalam memilih informasi yang relevan.

Hasil observasi kelas menunjukkan bahwa para guru lebih sering menggunakan 'pengajaran langsung' (*directive teaching*) dimana mereka: memberi tahu siswa apa yang dimaksud dalam soal, mengubah soal menjadi bentuk permasalahan matematika, dan menjelaskan prosedur atau konsep matematika apa yang dibutuhkan. Siswa tidak diminta untuk terlibat secara aktif dalam menyelesaikan dan melakukan refleksi atas proses penyelesaian soal. Praktik pengajaran secara langsung tersebut paling sering digunakan di dua tahap awal penyelesaian soal: pemahaman (*comprehension*) dan transformasi (*transformation*). Pengajaran konsultatif (*consultative teaching*), dimana siswa terlibat secara aktif dalam penyelesaian soal, sangat jarang digunakan oleh guru. Pengajaran konsultatif tersebut digunakan guru di tahap pemrosesan matematis (*mathematical processing*), yaitu tahapan dimana konteks dunia nyata (*real-world contexts*) bukan menjadi fokus utama.

Bab 5 memuat penelitian terakhir dalam proyek PhD ini dimana hasil dari tiga penelitian awal dijadikan sebagai dasar untuk mengembangkan suatu program intervensi untuk memberi siswa kesempatan belajar menyelesaikan soal berbasis konteks. Intervensi ini terdiri dari dua komponen: soal berbasis konteks dan pengajaran konsultatif (*consultative teaching*). Soal berbasis konteks yang digunakan dalam intervensi memiliki tiga karakter penting: menggunakan konteks yang relevan, memuat informasi berlebih dan informasi terselubung, serta tidak menampilkan prosedur penyelesaian secara langsung. Pengaruh pemberian kesempatan belajar ini pada kemampuan siswa dalam menyelesaikan soal berbasis konteks diteliti melalui suatu eksperimen yang melibatkan 299 siswa kelas 8 dari enam sekolah. Pencapaian siswa dilihat dari dua aspek, yaitu peningkatan skor (*score gains*) dan kesalahan siswa (*students' errors*).

Hasil eksperimen menunjukkan bahwa efek intervensi pada peningkatan skor siswa hanya signifikan secara marginal (*marginally significant*). Namun demikian, penelitian lanjutan pada efek intervensi terhadap kesalahan siswa menunjukkan adanya pengaruh yang positif. Siswa yang menerima intervensi memiliki peningkatan kemampuan dalam memahami soal dan memilah informasi. Terkait kemampuan siswa dalam melakukan transformasi soal berbasis konteks menjadi bentuk permasalahan matematika, untuk keseluruhan soal tidak ditemukan pengaruh dari intervensi tetapi untuk soal yang berkaitan dengan grafik, yaitu soal yang diajarkan saat intervensi, ditemukan pengaruh positif. Untuk soal yang terkait grafik siswa di kelas eksperimen memiliki kemampuan yang lebih baik dalam memberikan interpretasi matematis untuk suatu grafik yang terkait konteks dunia nyata. Berdasarkan temuan tersebut, dapat disimpulkan bahwa untuk meningkatkan kemampuan siswa dalam mengidentifikasi konsep atau prosedur matematis untuk menyelesaikan soal berbasis konteks tidaklah cukup hanya memberikan soal yang berkaitan dengan topik yang sedang diajarkan, melainkan juga soal yang terkait dengan berbagai macam topik berbeda.

Bab 6 menampilkan ikhtisar dari hasil penelitian PhD ini serta menawarkan rekomendasi untuk praktik pendidikan dan penelitian lanjutan. Kesimpulan utama dari hasil penelitian PhD ini adalah: (1) kesulitan terbesar yang dialami siswa Indonesia dalam menyelesaikan soal berbasis konteks adalah dalam proses memahami soal dan mengubahnya ke dalam permasalahan matematika, dan (2) kurangnya kesempatan belajar soal berbasis konteks yang diperoleh siswa, baik di buku teks maupun dalam praktik mengajar guru. Hasil tersebut menunjukkan adanya hubungan antara kesulitan siswa dengan kesempatan belajar yang diperoleh siswa. Hasil penelitian juga menunjukkan bahwa mendiagnosis kesulitan siswa dan mengidentifikasi kesempatan belajar yang diperoleh siswa merupakan dua langkah penting untuk merumuskan cara meningkatkan pencapaian siswa. Berdasarkan hasil penelitian PhD ini, dapat dirumuskan beberapa rekomendasi untuk meningkatkan praktik pengajaran soal berbasis konteks. Pertama, memberi siswa lebih banyak soal berbasis konteks, khususnya soal yang: (1) menggunakan konteks yang relevan, (2) memuat informasi berlebih dan informasi terselubung, dan (3) tidak menampilkan secara langsung prosedur matematika yang dibutuhkan untuk menjawab soal. Kedua, menggunakan pendekatan konsultatif serta memasukkan kegiatan mendiagnosis kesulitan siswa sebagai bagian terpadu dari praktik mengajar guru. Ketiga, pentingnya memberi perhatian pada keyakinan dan pengetahuan guru tentang soal berbasis konteks pada program pendidikan dan pelatihan guru karena kedua hal tersebut merupakan modal penting untuk meningkatkan kemampuan guru dalam memilih (atau mengembangkan) soal serta dalam menciptakan kesempatan belajar untuk siswa.

Acknowledgements

"The Milky Way is nothing else but a mass of innumerable stars planted together in clusters."

The abovementioned statement of Galileo Galilei has always been one of my favorite quotes because it reflects my academic journey. This quote also perfectly represents the completion of this thesis which would not have been possible without support, guidance, and help from many people around me. Therefore, I would like to dedicate this section to acknowledge these people.

First, I would like to express my sincere gratitude to my promotor, Marja van den Heuvel-Panhuizen, who has supported and guided me with her impressive knowledge and expertise. It was a great opportunity for me to do my PhD research under her guidance. She guided me to see things in a comprehensive and coherent way that was not only important for my PhD work, but will also be essential for my future professional life. The way she guided me reminds me of another quote from Galileo Galilei: "*we cannot teach people anything; we can only help them discover it within themselves.*" She gave me this opportunity to learn.

I also owe a debt of gratitude to Michiel Doorman, my copromotor, for his valuable guidance and support. He often posed questions that stimulated me to further reflect on my own thought and works. Furthermore, he did not only give me professional and academic supports, but also personal ones. He even took me to the Assen circuit – one of the places in my dream list – to watch British Superbike. Thank you very much for this experience, Michiel.

I would like to acknowledge the Indonesian Ministry of Education and Culture for the scholarship I received. I would also like to express my gratitude to the authorities of Yogyakarta State University, the university where I work as a lecturer, for giving me permission to pursue my doctoral degree.

During my PhD study I received support and help from my colleagues at the Freudenthal Institute for Science and Mathematics Education. Special thanks to my officemates, Michiel Veldhuis and Xiaoyan Zhao, for the encouragement, fruitful discussion, and also our routine 'lunch 'meeting'. I would also like to thank Al Jupri, my fellow Indonesian PhD student, for helping me with interrater reliability and many other things. I am grateful to Nathalie Kuijpers for correcting my written English and for her editorial assistance. Thanks to Mark Uwland and

Heleen Verhage for arranging the financial matter, especially in the final year of my study.

When dealing with statistical analyses of my data I also got help from Alexander Robitzsch and Michiel Veldhuis. Thanks to both of you. I am also deeply indebted to the teachers and the students who participated in my studies. Completion of my PhD research would have never been possible without their help and cooperation.

I would like to extend my gratitude to my Indonesian friends who helped me during my stay in the Netherlands. Special thanks to oom Richard and tante Nanda who have given me a lot of help since my arrival in the Netherlands. They were really like my parents in the Netherlands. I owe my gratitude to Domesia Novi for designing the cover of this PhD thesis. I am also grateful to: Eko Handoyo, Richo, Siswanto, Gustiandi, Supardi, Arie Fiandri, Bambang, Yusuf Setiyono, Bangaip, and Citra Dewi.

Above all, I would like to express my immeasurable gratitude to my wife, Endang Ilmi NH, for her love and understanding. Without her unconditional support I would never have completed my study. Although she was busy with our son and her teaching job, she was always there whenever I needed her. Furthermore, she did not only support my personal life, but also helped me with the data collection and data analysis in some of my studies. To my son, Arkapraba Wijaya, I am sorry for the time we spent apart. I was not even there when he took his first steps. I dedicate this book to the two of you. Special thanks to my parents, my parents in law, my sisters, and my brothers who always supported and encouraged me.

Curriculum Vitae

Ariyadi Wijaya was born on July 16, 1982 in Banjarnegara, Indonesia. In 2000, he completed his secondary schooling at Sandhy Putra Technical High School of Telecommunication in Purwokerto and then started his university studies in Mathematics Education at Yogyakarta State University, Indonesia. After obtaining a bachelor degree, with honors, in 2004, he immediately worked as a senior high mathematics teacher. In 2005 he started a new job as a lecturer at the Department of Mathematics Education, the Faculty of Mathematics and Natural Sciences, Yogyakarta State University (YSU). He pursued a master degree in Science, Communication and Education at Utrecht University, the Netherlands in 2006 and graduated with honors in 2008.

In 2009, he was appointed by YSU to take a short course on 'Content Language Integrated Learning' (CLIL) at the Melbourne Graduate School of Education, the University of Melbourne. In addition to his duty as a lecturer at YSU, in the period from 2009 to 2010 he also frequently gave lectures in short courses which were organized by the Southeast Asian Ministers of Education Organization (SEAMEO) Regional Centre for Quality Improvement of Teachers and Education Personnel (QITEP) in Mathematics. He published three books about mathematics education and frequently participated in conferences on mathematics education.

In 2011, he was awarded a scholarship from the Indonesian Directorate General of Higher Education. In the same year he started his PhD study at Utrecht University under supervision of prof. dr. Marja van den Heuvel-Panhuizen and dr. Michiel Doorman. His PhD researchfocused on the teaching and learning of context-based mathematics tasks in Indonesia. His research proposal was accepted by the Interuniversity Center for Educational Sciences (ICO) Research School in the Netherlands and he fulfilled all requirements to be qualified as an ICO PhD member.

In February 2015 he will go back to his position as a lecturer at the YSU to teach in the Undergraduate Program and Graduate Program of mathematics education.

List of presentations related to this thesis

Wijaya, A., Van den Heuvel-Panhuizen, M., & Doorman, M. *Identifying ways to improve student performance on context-based mathematics tasks*. Paper that will be presented at the 9th Congress of European Research in Mathematics Education (CERME 9), Prague, Czech Republic, 4-8 February 2015.

Wijaya, A., Van den Heuvel-Panhuizen, M., Doorman, M., & Robitzsch, A. (2014, November). *Identifying (Indonesian) students' difficulties in solving context-based (PISA) mathematics tasks*. Paper presented at International Seminar on Innovation in Mathematics and Mathematics Education 2014, Yogyakarta, Indonesia.

Wijaya, A., Van den Heuvel-Panhuizen, M., & Doorman, M. (2013, November). *Analyzing Indonesian mathematics textbooks. Do they provide opportunity-to-learn context-based tasks?* Paper presented at ICO National Fall School 2013, Maastricht, the Netherlands.

Wijaya, A., Van den Heuvel-Panhuizen, M., & Doorman, M. (2012, November). *An investigation of opportunities to learn solving context-based tasks in Indonesian mathematics textbooks and classroom practices*. Paper presented at ICO International Fall School 2012, Girona, in Cataluña, Spain.

FIsme Scientific Library
(formerly published as CD-β Scientific Library)

87. Klerk, Saskia (2015). *Galen reconsidered. Studying drug properties and the foundations of medicine in the Dutch Republic ca. 1550-1700.*
86. Krüger, Jenneke (2014). *Actoren en factoren achter het wiskundecurriculum sinds 1600.*
85. Lijnse, P.L. (2014). *Omzien in verwondering. Een persoonlijke terugblik op 40 jaar werken in de natuurkundedidactiek.*
84 Van Weelie, D. (2014). *Recontextualiseren van het concept biodiversiteit.*
83 Bakker, M. (2014). *Using mini-games for learning multiplication and division: A longitudinal effect study.*
82. Hăng, N. V. T. (2014). *Design of a social constructivism-based curriculum for primary science education in Confucian heritage culture.*
81. Sun, L. (2014). *From rhetoric to practice: enhancing environmental literacy of pupils in China.*
80. Mazereeuw, M. (2013). *The functionality of biological knowledge in the workplace. Integrating school and workplace learning about reproduction.*
79. Dierdorp, A. (2013). *Learning correlation and regression within authentic contexts.*
78. Dolfing, R. (2013). *Teachers' Professional Development in Context-based Chemistry Education. Strategies to Support Teachers in Developing Domain-specific Expertise.*
77. Mil, M. H. W. van (2013). *Learning and teaching the molecular basis of life.*
76. Antwi, V. (2013). *Interactive teaching of mechanics in a Ghanaian university context.*
75. Smit, J. (2013). *Scaffolding language in multilingual mathematics classrooms.*
74. Stolk, M. J. (2013). *Empowering chemistry teachers for context-based education. Towards a framework for design and evaluation of a teacher professional development programme in curriculum innovations.*
73. Agung, S. (2013). *Facilitating professional development of Madrasah chemistry teachers. Analysis of its establishment in the decentralized educational system of Indonesia.*
72. Wierdsma, M. (2012). *Recontextualising cellular respiration.*
71. Peltenburg, M. (2012). *Mathematical potential of special education students.*
70. Moolenbroek, A. van (2012). *Be aware of behaviour. Learning and teaching behavioural biology in secondary education.*
69. Prins, G. T., Vos, M. A. J. & Pilot, A. (2011). *Leerlingpercepties van onderzoek & ontwerpen in het technasium.*
68. Bokhove, Chr. (2011). *Use of ICT for acquiring, practicing and assessing algebraic expertise.*
67. Boerwinkel, D. J. & Waarlo, A. J. (2011). *Genomics education for decision-making. Proceedings of the second invitational workshop on genomics education, 2-3 December 2010.*
66. Kolovou, A. (2011). *Mathematical problem solving in primary school.*
65. Meijer, M. R. (2011). *Macro-meso-micro thinking with structure-property relations for chemistry. An explorative design-based study.*
64. Kortland, J. & Klaassen, C. J. W. M. (2010). *Designing theory-based teaching-learning sequences for science. Proceedings of the symposium in honour of Piet Lijnse at the time of his retirement as professor of Physics Didactics at Utrecht University.*
63. Prins, G. T. (2010).*Teaching and learning of modelling in chemistry education. Authentic practices as contexts for learning.*

62. Boerwinkel, D. J. & Waarlo, A. J. (2010). *Rethinking science curricula in the genomics era. Proceedings of an invitational workshop.*
61. Ormel, B. J. B. (2010). *Het natuurwetenschappelijk modelleren van dynamische systemen. Naar een didactiek voor het voortgezet onderwijs.*
60. Hammann, M., Waarlo, A. J., & Boersma, K. Th. (Eds.) (2010). *The nature of research in biological education: Old and new perspectives on theoretical and methodological issues – A selection of papers presented at the VIIth Conference of European Researchers in Didactics of Biology.*
59. Van Nes, F. (2009). *Young children's spatial structuring ability and emerging number sense.*
58. Engelbarts, M. (2009). *Op weg naar een didactiek voor natuurkunde-experimenten op afstand. Ontwerp en evaluatie van een via internet uitvoerbaar experiment voor leerlingen uit het voortgezet onderwijs.*
57. Buijs, K. (2008). *Leren vermenigvuldigen met meercijferige getallen.*
56. Westra, R. H. V. (2008). *Learning and teaching ecosystem behaviour in secondary education: Systems thinking and modelling in authentic practices.*
55. Hovinga, D. (2007). *Ont-dekken en toe-dekken: Leren over de veelvormige relatie van mensen met natuur in NME-leertrajecten duurzame ontwikkeling.*
54. Westra, A. S. (2006). *A new approach to teaching and learning mechanics.*
53. Van Berkel, B. (2005). *The structure of school chemistry: A quest for conditions for escape.*
52. Westbroek, H. B. (2005). *Characteristics of meaningful chemistry education: The case of water quality.*
51. Doorman, L. M. (2005). *Modelling motion: from trace graphs to instantaneous change.*
50. Bakker, A. (2004). *Design research in statistics education: on symbolizing and computer tools.*
49. Verhoeff, R. P. (2003). *Towards systems thinking in cell biology education.*
48. Drijvers, P. (2003). *Learning algebra in a computer algebra environment. Design research on the understanding of the concept of parameter.*
47. Van den Boer, C. (2003). *Een zoektocht naar verklaringen voor achterblijvende prestaties van allochtone leerlingen in het wiskundeonderwijs.*
46. Boerwinkel, D. J. (2003). *Het vormfunctieperspectief als leerdoel van natuuronderwijs. Leren kijken door de ontwerpersbril.*
45. Keijzer, R. (2003). *Teaching formal mathematics in primary education. Fraction learning as mathematising process.*
44. Smits, Th. J. M. (2003). *Werken aan kwaliteitsverbetering van leerlingonderzoek: Een studie naar de ontwikkeling en het resultaat van een scholing voor docenten.*
43. Knippels, M. C. P. J. (2002). *Coping with the abstract and complex nature of genetics in biology education – The yo-yo learning and teaching strategy.*
42. Dressler, M. (2002). *Education in Israel on collaborative management of shared water resources.*
41. Van Amerom, B. A. (2002). *Reinvention of early algebra: Developmental research on the transition from arithmetic to algebra.*
40. Van Groenestijn, M. (2002). *A gateway to numeracy. A study of numeracy in adult basic education.*
39. Menne, J. J. M. (2001). *Met sprongen vooruit: een productief oefenprogramma voor zwakke rekenaars in het getallengebied tot 100 – een onderwijsexperiment.*
38. De Jong, O., Savelsbergh, E. R. & Alblas, A. (2001). *Teaching for scientific literacy: context, competency, and curriculum.*
37. Kortland, J. (2001). *A problem-posing approach to teaching decision making about the waste issue.*
36. Lijmbach, S., Broens, M., & Hovinga, D. (2000). *Duurzaamheid als leergebied; conceptuele analyse en educatieve uitwerking.*

35. Margadant-van Arcken, M. & Van den Berg, C. (2000). *Natuur in pluralistisch perspectief – Theoretisch kader en voorbeeldlesmateriaal voor het omgaan met een veelheid aan natuurbeelden.*
34. Janssen, F. J. J. M. (1999). *Ontwerpend leren in het biologieonderwijs. Uitgewerkt en beproefd voor immunologie in het voortgezet onderwijs.*
33. De Moor, E. W. A. (1999). *Van vormleer naar realistische meetkunde – Een historisch-didactisch onderzoek van het meetkundeonderwijs aan kinderen van vier tot veertien jaar in Nederland gedurende de negentiende en twintigste eeuw.*
32. Van den Heuvel-Panhuizen, M. & Vermeer, H. J. (1999). *Verschillen tussen meisjes en jongens bij het vak rekenen-wiskunde op de basisschool – Eindrapport MOOJ-onderzoek.*
31. Beeftink, C. (2000). *Met het oog op integratie – Een studie over integratie van leerstof uit de natuurwetenschappelijke vakken in de tweede fase van het voortgezet onderwijs.*
30. Vollebregt, M. J. (1998). *A problem posing approach to teaching an initial particle model.*
29. Klein, A. S. (1998). *Flexibilization of mental arithmeticsstrategies on a different knowledge base – The empty number line in a realistic versus gradual program design.*
28. Genseberger, R. (1997). *Interessegeoriënteerd natuur- en scheikundeonderwijs – Een studie naar onderwijsontwikkeling op de Open Schoolgemeenschap Bijlmer.*
27. Kaper, W. H. (1997). *Thermodynamica leren onderwijzen.*
26. Gravemeijer, K. (1997). *The role of context and models in the development of mathematical strategies and procedures.*
25. Acampo, J. J. C. (1997). *Teaching electrochemical cells – A study on teachers' conceptions and teaching problems in secondary education.*
24. Reygel, P. C. F. (1997). *Het thema 'reproductie' in het schoolvak biologie.*
23. Roebertsen, H. (1996). *Integratie en toepassing van biologische kennis – Ontwikkeling en onderzoek van een curriculum rond het thema 'Lichaamsprocessen en Vergift'.*
22. Lijnse, P. L. & Wubbels, T. (1996). *Over natuurkundedidactiek, curriculumontwikkeling en lerarenopleiding.*
21. Buddingh', J. (1997). *Regulatie en homeostase als onderwijsthema: een biologie-didactisch onderzoek.*
20. Van Hoeve-Brouwer G. M. (1996). *Teaching structures in chemistry – An educational structure for chemical bonding.*
19. Van den Heuvel-Panhuizen, M. (1996). *Assessment and realistic mathematics education.*
18. Klaassen, C. W. J. M. (1995). *A problem-posing approach to teaching the topic of radioactivity.*
17. De Jong, O., Van Roon, P. H. & De Vos, W. (1995). *Perspectives on research in chemical education.*
16. Van Keulen, H. (1995). *Making sense – Simulation-of-research in organic chemistry education.*
15. Doorman, L. M., Drijvers, P. & Kindt, M. (1994). *De grafische rekenmachine in het wiskundeonderwijs.*
14. Gravemeijer, K. (1994). *Realistic mathematics education.*
13. Lijnse, P. L. (Ed.) (1993). *European research in science education.*
12. Zuidema, J. & Van der Gaag, L. (1993). *De volgende opgave van de computer.*
11. Gravemeijer, K, Van den Heuvel Panhuizen, M., Van Donselaar, G., Ruesink, N., Streefland, L., Vermeulen, W., Te Woerd, E., & Van der Ploeg, D. (1993). *Methoden in het reken-wiskundeonderwijs, een rijke context voor vergelijkend onderzoek.*
10. Van der Valk, A. E. (1992). *Ontwikkeling in Energieonderwijs.*
9. Streefland, L. (Ed.) (1991). *Realistic mathematics education in primary schools.*

8. Van Galen, F., Dolk, M., Feijs, E., & Jonker, V. (1991). *Interactieve video in de nascholing reken-wiskunde.*
7. Elzenga, H. E. (1991). *Kwaliteit van kwantiteit*.
6. Lijnse, P. L., Licht, P., De Vos, W. & Waarlo, A. J. (Eds.) (1990). *Relating macroscopic phenomena to microscopic particles: a central problem in secondary science education.*
5. Van Driel, J. H. (1990). *Betrokken bij evenwicht*.
4. Vogelezang, M. J. (1990). *Een onverdeelbare eenheid.*
3. Wierstra, R. F. A. (1990). *Natuurkunde-onderwijs tussen leefwereld en vakstructuur.*
2. Eijkelhof, H. M. C. (1990). *Radiation and risk in physics education.*
1. Lijnse, P. L. & De Vos, W. (Eds.) (1990). *Didactiek in perspectief.*

ICO Dissertation Series

In the ICO Dissertation Series dissertations are published of graduate students from faculties and institutes on educational research within the ICO Partner Universities: Eindhoven University of Technology, Leiden University, Maastricht University, Open University of the Netherlands, University of Amsterdam, University of Twente, Utrecht University, VU University Amsterdam, and Wageningen University, and formerly University of Groningen (until 2006), Radboud University Nijmegen (until 2004), and Tilburg University (until 2002). The University of Groningen, University of Antwerp, University of Ghent, and the Erasmus University Rotterdam have been 'ICO 'Network partner' in 2010 and 2011. From 2012 onwards, these ICO Network partners are full ICO partners, and from that period their dissertations will be added to this dissertation series.

(Partial list – updated in January 2014)

281. Cviko, A. (19-12-2013). *Teacher Roles and Pupil Outcomes. In technology-rich early literacy learning.* Enschede: University of Twente.
280. Kamp, R.J.A. (28-11-2013). *Peer feedback to enhance learning in problem-based tutorial groups.* Maastricht: Maastricht University.
279. Lucero, M.L. (21-11-2013). *Considering teacher cognitions in teacher professional development: Studies involving Ecuadorian primary school teachers.* Ghent: Ghent University.
278. Dolfing, R. (23-10-2013). *Teachers' Professional Development in Context-based Chemistry Education. Strategies to Support Teachers in Developing Domain-specific Expertise.* Utrecht: Utrecht University.
277. Popov, V. (8-10-2013). *Scripting Intercultural Computer-Supported Collaborative Learning in Higher Education.* Wageningen: Wageningen University.
276. Bronkhorst, L.H. (4-10-2013). *Research-based teacher education: Interactions between research and teaching.* Utrecht: Utrecht University.
275. Bezdan, E. (04-10-2013). *Graphical Overviews in Hypertext Learning Environments: When One Size Does Not Fit All.* Heerlen: Open University of the Netherlands.
274. Kleijn, R.A.M. de, (27-09-2013). *Master's thesis supervision. Feedback, interpersonal relationships and adaptivity.* Utrecht: Utrecht University.
273. Pillen, M.T. (12-09-2013). *Professional identity tensions of beginning teachers.* Eindhoven: Eindhoven University of Technology.
272. Meeuwen, L.W. van (06-09-13). *Visual Problem Solving and Self-regulation in Training Air Traffic Control.* Heerlen: Open University of the Netherlands.
271. Keuvelaar-Van den Bergh, L. (26-06-2013). *Teacher Feedback during Active Learning: The Development and Evaluation of a Professional Development Programme.* Eindhoven: Eindhoven University of Technology.
270. Hornstra, T.E. (17-06-2013). *Motivational developments in primary school. Group-specific differences in varying learning contexts.* Amsterdam: University of Amsterdam.
269. Vandyck, I.J.J. (17-06-2013). *Fostering Community Development in School-University Partnerships.* Amsterdam: VU Universtiy Amsterdam.

268. Milliano, I.I.C.M. de (24-05-2013). *Literacy development of low-achieving adolescents. The role of engagement in academic reading and writing*. Amsterdam: University of Amsterdam.
267. Taminiau, E.M.C. (24-05-2013). *Advisory Models for On-Demand Learning*. Heerlen: Open University of the Netherlands.
266. Azkiyah, S.N. (23-5-2013). *The effects of Two Interventions - on Teaching Quality and Student Outcome*. Groningen: University of Groningen.
265. Diggelen, M.R. van (21-05-2013). *Effects of a self-assessment procedure on VET teachers' competencies in coaching students' reflection skills*. Eindhoven: Eindhoven University of Technology.
264. M.H. Knol (19-04-2013). *Improving university lectures with feedback and consultation*. Amsterdam: University of Amsterdam.
263. Dekker-Groen, A. (19-04-2013). *Teacher competences for supporting students' reflection. Standards, training, and practice*. Utrecht: Utrecht University.
262. Verberg, C.P.M. (18-04-2013). *The characteristics of a negotiated assessment procedure to promote teacher learning*. Leiden: Leiden University.
261. Jong, R.J. de (11-04-2013). *Student teachers' practical knowledge, discipline strategies, and the teacher-class relationship*. Leiden: Leiden University.
260. Belo, N.A.H. (27-03-2013). *Engaging students in the study of physics*. Leiden: Leiden University.
259. Bijker, M.M. (22-03-2013). *Understanding the gap between business school and the workplace: Overconfidence, maximizing benefits, and the curriculum*. Heerlen: Open University of the Netherlands.
258. Noroozi, O. (11-01-2013). *Fostering Argumentation-Based Computer-Supported Collaborative Learning in Higher Education*. Wageningen: Wageningen University.
257. Van der Linden, P.W.J. (20-12-2012). *A design-based approach to introducing student teachers in conducting and using research*. Eindhoven: Eindhoven University of Technology.
256. Timmermans, A.C. (27-11-2012). *Value added in educational accountability: Possible, fair and useful*? Groningen: University of Groningen.
255. De Feijter, J.M. (09-11-2012). *Learning from error to improve patient safety*. Maastricht: Maastricht University.
254. Kenbeek, W.K. (31-10-2012). *Back to the drawing board. Creating drawing or text summaries in support of System Dynamics modeling*. Enschede: University of Twente.
253. Doppenberg, J.J. (24-10-2012). *Collaborative teacher learning: settings, foci and powerful moments*. Eindhoven: Eindhoven University of Technology.
252. Peltenburg, M.C. (24-10-2012). *Mathematical potential of special education students*. Utrecht: Utrecht University.
251. Phielix, C. (28-09-2012). *Enhancing Collaboration through Assessment & Reflection* [*Samenwerking Verbeteren door middel van Beoordeling en Reflectie*]. Utrecht: Utrecht University
250. Van de Pol, J.E. (28-09-2012). *Scaffolding in teacher-student interaction. Exploring, measuring promoting and evaluating scaffolding*. Amsterdam: University of Amsterdam.
249. Vrijnsen-de Corte, M.C.W. *Researching the Teacher-Researcher. Practice-based research in Dutch Professional Development*. Schools Eindhoven: Eindhoven University of Technology.