The social psychology of mathematics education

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I would like to begin my talk by presenting to you three items of data from different pieces of research into the learning of mathematics. I have selected these because between them they seem to be to 'catch' the essence of my title. I hope also that these items will relate to your own personal store of evidence, so that you will be able to see their significance in a wider context than that within which I shall present them.

The first concerns a secondary mathematics teacher, Alec, (MacPherson, 1973) who was deliberately trying to affect what we called the 'working relationship' between himself and certain of his pupils (it was a similar idea to that of the 'didactical contract' developed by Brousseau (1981), although it was not so directly concerned with negotiating the conditions of classwork). In this case Alec had set himself some tasks to improve the working relationship with a few of his pupils with whom he felt he had not got a good relationship, e.g. he was finding out more about their hobbies, talking to them every lesson, and only asking them a question publicly when he already felt sure that they knew the answer. In his school there were class-orders kept in each subject, and these were reviewed each half-term, so that he was able to see if his behaviour had any effect on the pupils' order in class. With all pupils his behaviour had important effects and with one girl a surprising fact was revealed.
not only did she change her class-order from 20th place to the 4th place in mathematics, but also her best friend, who sat next to her, moved up as well.

In some way Alex's influence was communicated to another pupil than just the one he was trying to influence. I was reminded of Jacob Kounin's (1970) study of discipline and group management, where he explored what he called the 'ripple effect' of various 'desist techniques'.

The second piece of reported data is from Lorenz's (1982) study where he described particularly the ways in which mathematics teachers thought about their pupils and how these views manifested themselves in the classroom. Amongst various fascinating findings was one which stood out for me, because it was a paradigmatic example of a phenomenon I had seen in many classrooms of both beginning and experienced teachers. He found that teachers' behaviours which were designed to be 'helpful' were in fact directed more often towards the more-able pupils than to their less-able, but presumably more needy, peers.

The third item of phenomena is one which I have reported before (Bishop, 1979). I make no apology for reporting it again because it continues to fascinate me. The fact that I was directly involved is also important because it illustrates the point that it has for me a social, and indeed an emotional significance, as well as a cognitive significance.

It concerns me interviewing a university student in Papua New Guinea and trying to understand more about his 'local' of 'folk' mathematics. I asked him how he would find the areas of a rectangular piece of paper. He replied:

"Multiply the length by the width". "You have gardens in your village. How do your people judge the area of their gardens?" "By adding the length and the width". "But they both refer to area". "Yes, but one is about the area of a piece of paper and the other is about a garden". So I drew two (rectangular) gardens on the paper, one bigger than the other. "If these were two gardens which would you rather have?" "It depends on many things, I cannot say. The soil, the shade ...." I was then about to ask the next question "Yes, but if they had the same soil, shade ..." when I realised how silly that would sound in that context.

Now it would be relatively easy to dismiss the first item as some sort of accidental coincidence. The second item would be harder to dismiss but could be explained by criticising the accuracy of teachers' knowledge of who were the more- and less-able pupils in their classes. The third item seems set up for a perfect piece of resolution by the teacher of a learner's cognitive conflict.
However I do not want to dismiss them, nor do I want to try to find essentially cognitive explanations for them. They interest me because they are typical of many situations which have a strong social component, and which I feel have been relatively ignored by researchers. In the context of this talk they represent phenomena and problems in the area of the social psychology of mathematics education. I hope also, as I said earlier, that these three items have some personal meanings for you as well because whilst describing my ideas about social psychology I am also trying to influence you. I can only do that if you are socially involved with these problems as well as being intellectually motivated to attend to them.

A learning experience like this, and I hope this is a learning experience for you, is as much a social experience as it is a cognitive one. For example, learning from other people is different from learning from texts and a context such as this does have a social dimension to it. I know that listening to me talking today is a different experience from reading the printed text. I am not suggesting or implying that one is a better experience than the other but I hope we can agree that they are certainly different experiences. Therefore, if the aim of research into the learning and teaching of mathematics is to understand more how these happen, we must attend to this social dimension, since mathematics learning in classrooms, by definition, takes place in a social context. Mathematics classrooms are very 'public' places in which it is impossible to achieve privacy. Every act is performed in a social situation even if it involves pupils using their own individual text materials. Every interaction between a pupil and some mathematics in the classroom is socially mediated. As with the classic research of Asch (1951) even if an individual pupils believes a certain mathematical proposition to be true, the social and interpersonal influences operating in the classroom can prevent the pupil expressing that proposition 'publicly', and can also make the pupil think she is wrong.

Fortunately research in this social area is growing and it is not as deserted a terrain as once it was. We have seen developments in research on topics like the fear of mathematics, sex-role stereotyping, pupils' attitudes and attributions, teachers' perceptions and epistemologies, and collaborative learning, all of which can increase our awareness and understanding of social phenomena in the learning of mathematics. What I should like to do today is to help increase the momentum of this research, to help coordinate and connect some of the developments and to help identify the significant aspects from the perspective of teacher training and teacher education.

Firstly I think it is necessary for today's talk to set the "social psychology" emphasis in context of my general views on the social
dimension in mathematics education. Research into this dimension is significant, for me, at five levels. At the cultural level research can inform us about the history and development of mathematical ideas and their relationship with one's culture (e.g. Kline, 1954). Also cross cultural studies like Lancy's (1983) and analyses like Ellul's (1980) and Weizenbaum's (1976) sensitise us to more complex aspects of this relationship.

At the societal level, the research concerns the various institutions in society and the political and ideological influences which they bring to bear on the mathematics education of our children (see for example Griffiths and Howson, 1974; Swetz, 1978). Some of these institutions are formally concerned with education of course but many are not and accounts like Fasheh's (1982) illustrate well the tensions and conflicts which exist between them.

At the institutional level research is about for example the within school influences which help to shape the intended and the implemented mathematics curriculum for the pupils (see for example, Stake and Easley, 1978). Donovan's (1983) study also concerns these influences and shows how the values and ideologies of the dominant cultural group filter into the institution of school. Marrett and Gates (1983) describe how such values determine which pupils study mathematics in which tracks (or sets) and thereby indicates another institutional mechanism for controlling the pupils' mathematical education.

At the pedagogical level we at last enter the classroom and find research some of which relates specifically to our topic today. I have added another level to the social dimension though which I call the individual level, because there is a growing amount of research of research which focusses on the learner from a social perspective. This again I shall say more about.

I hope this brief overview serves to demonstrate that the social and interpersonal influences on the learner in the mathematics classroom have strong connections with values and ideologies emerging from interactions taking place far from the classroom. An awareness of the whole social dimension reminds us I hope of these connections. If there is one thing to be learned from research into social aspects of mathematics education it is that the context, and the situation, are all important.

Concentrating now on social psychology, I want to look at three aspects today: social motivation, social cognition and social interaction.

1. Social motivation
Let us begin with a topic which has stimulated a great deal of research activity world-wide namely, the fear of mathematics. It is a topic which has been fruitfully analysed by Buxton (1981) amongst
others and which contains many ideas of importance to teachers and teacher educators. Of particular interest here is the fact that both Buxton and Skemp (1979) use the idea of goals and anti-goals (to be avoided by the learner) and their discussions of anxiety, frustration and other emotions are very helpful to our understanding of how the classroom mathematical experience appears to pupils.

Another anti-goal identified in the literature is the 'fear of success' construct found to be of great value by Leder (1980) in understanding why bright girls in mathematics deliberately avoid success and achievement in order to retain the respect and acceptance of their peers. This is of course not just a phenomenon to be seen with bright girls. It will be noticed by any teacher of mathematics particularly of adolescent children, who will apparently prefer not to succeed and indeed, not to try to succeed for fear of losing the respect of their friends. At the adolescent stage, well noted for being a time of questioning and challenging authority, goals promoted by the teacher may well be perceived as anti-goals by some pupils (hopefully not all!).

Whether the teacher-mediated goals are accepted by the pupils as goals, or converted into anti-goals, will be determined by various factors. In particular the role of Significant Others must be recognised. Although this idea (S.O.) was developed by Sullivan (1940) within the psychiatric field it does have value for us also. You do not require much observation time in mathematics classrooms to begin to identify which individuals in the group significantly affect the behaviour and the motivations of others. The situation presented at the start of the talk illustrates this. The pupil whom Alec was trying to influence was also clearly a Significant Other for her neighbour and the change in motivation and achievement in one had a very strong affect on the other pupil.

Of course it is likely that for many pupils the teacher will have the status of a Significant Other. But it is also true that for some pupils this will not be the case. Likewise there will often be some pupils who will become S.O. for the teacher, and will have a significant shaping effect on the teacher's motivation and behaviour. This point reminds us that teachers can also have goals and anti-goals, with e.g. the "development of mathematical understanding" being a clear goal and the "fear of confrontation" being a strong anti-goal for many teachers. Again we can understand how individual pupils, acting as Significant Others for the teacher, can affect the relative strength of the teacher's goal/anti-goal tension.

In an earlier paper (Bishop, 1981), I presented some ideas concerning mathematical involvement, a construct designed to describe affect-in-action, i.e. the observable realisation of a positive attitude towards mathematics. It concerns the extent to which pupils
demonstrate their willingness to engage in a class' mathematical activity. Given the goal of the teacher to try to increase the mathematical involvement of as many pupils as possible, it would also be an indicator of the extent to which any one pupil related to the teacher as Significant Other. Furthermore the teacher as a leader, or as a model, would also clearly be an important factor in creating mathematical involvement. Finally in the section I should like to mention exchange theory - a motivation theory which essentially proposes that individuals engage in interactions which offer them more as rewards than they are giving out as costs (Homans, 1961). In these terms it is unrealistic of the teacher to imagine that 'motivation' is a once and for all problem, e.g. that once the child is motivated to do well at mathematics that motivation will carry on through the year and through the school. Equally it offers an alternative view of motivation from that of the verb 'to motivate'. It implies instead that teachers recognise that pupils will only become involved in a mathematical activity if the perceived 'rewards' are greater than the perceived costs (potential loss of friendships, mental strain, fear of failure, etc.). Furthermore it predicts that once the costs exceed the rewards, the involvement will cease. Despite this theory's simple and perhaps 'too-mercenary' view of human nature, it does nevertheless help to explain and predict many of the paradigmatic problems of social motivation.

Perhaps we need more research which looks at pupils' goals (and anti-goals) in relation to Significant Others? Perhaps we need to resuscitate the old ideas of sociometrics and sociograms, but instead of looking merely for friendship groupings and for isolates, we should look more for the S.O.'s who influence choices of goals or anti-goals? Finally, perceptions of the 'rewards' and 'costs' of mathematical involvement by different pupils would also be of importance together with the relevant perceptions of their S.O.'s, one of whom may be the teacher. Once again this kind of analysis shows us that research which considers only the teacher as the influence on the pupil will probably miss the real influences.

2. Social cognition
This section concerns the ways in which people 'know' other people, and in relation to mathematics classrooms we are particularly interested in the ways the pupils are known. Teachers' perceptions about their pupils has been a fertile ground for research for many years and their importance was well demonstrated by studies of their 'self-fulfilling prophecy' - whereby pupil's live up to, or live down to, the perceptions and expectations of them by their teachers. It seems to me moreover that what is pedagogically significant about any psychological pupil construct is how that phenomenon is perceived by
the teacher. Even if a researcher 'establishes' that, for example, a pupil has a preference for using visual imagery in mathematics, what really matters is the teacher's perception of that situation. As another example, I found it interesting to analyse teachers' responses to pupils' errors by using the idea of teachers' perceived error (Bishop, 1976).

Mention of the word 'construct' above requires that I give due recognition to Kelly's (1955) Personal Construct Theory, a theory which many researchers now use implicitly to guide their work. At the heart of the theory lies our individual system of bi-polar constructs and one construct which, in my experience, many mathematics teachers use, both implicitly and explicitly to shape their constructions of their pupils, is that of 'mathematical ability'. Teachers' behaviour seem to be strongly affected by their perceptions of the more-able/less-able 'dimension' and I would like us to be clear that when we are discussing aspects of teaching like this with teachers, we call it perceived mathematical ability. Labels like 'mathematical ability' have a way of becoming very fixed and stable classifications in many teacher's minds, and they need reminding that they are only talking about 'perceptions' which one can, and should, be prepared to change.

If we link this idea with another we can see some important consequences. Various researchers have considered the particular problems faced by girls in learning mathematics, and amongst other ideas which have been explored is that of sex-role stereotyping. This label is put on the behaviours of teachers and others which seem to restrain girls' behaviours so that they stay close to a certain role-model for girls. Researchers like Becker (1981) have identified the obvious and not so obvious ways in which teachers do this.

If however we consider the general idea of role-stereotyping, we can see how other groups of learners are made to become disadvantaged in the same way as some girls are. For example, there are undoubtedly many instances of ability-role stereotyping, whereby teachers' behaviours towards more-able pupils are markedly different from their behaviours towards less-able pupils. One would naively assume that these different behaviours are designed to improve the performance of the less-able pupils, but this (as the data from Lorenz's study show) is not what happens. The way to understand this phenomenon is to treat it as role-stereotyping, whereby the more-able pupils are encouraged to be more-able and the less-able pupils are encouraged to continue to be less-able.

One has of course also seen many instances of class-role stereotyping (upper, middle, and working classes) and of race-role stereotyping but I have come across another situation which also surprised me. I call it handicap-role stereotyping which I have seen with both blind and deaf children who are kept playing a dependent and
'appropriate' handicapped role. It is, furthermore, very difficult for any mathematics teacher who wants to break away from the stultifying effects of any of this stereotyping if the educational system continues to support it. In the U.K., for example, 'setting' the pupils into so-called homogeneous ability groups for mathematics occurs in almost all secondary schools. Such an institutionalised system clearly reinforces the ability-role stereotyping which many teachers adopt. In my personal view this is a far more serious and widespread problem than sex-role stereotyping nowadays.

One way to get beyond mere stereotyping is perhaps to make teachers more aware about how their behaviours and expectations shape pupils' attributions. The interest in attribution theory has grown in recent years and there is a well-developed literature (see Weiner, 1972, for example). One strand of the research looks at children's perceived causes of their performance and whether these causes are internal or external to the pupil. Another strand considers teachers' attributions of pupils' performance. For example, Johnson et al. (1964) taught pairs of 10 year old children some arithmetic procedures. For each pair it was organised that one child (A) would do well at the first assessment while the other (B) would do poorly. The teachers then taught each pair again and this time, while A continued to do well, it was arranged that half of the B pupils improved and the other half declined. Amongst other findings was the interesting one that the teachers attributed the improving B's performance to their teaching, but they attributed the declining B's performance to the pupils themselves.

Clearly attribution theory could help teachers to understand more about their role in pupils' development of their own self-concept. What would be important to know more about is how, and under what conditions, attributions can change. Once again Alec's story, from earlier, gives us some indications, but if all of us are not to remain trapped by our own attributions we must try to interpret this idea much more dynamically. Kelly discusses a treatment he calls 'fixed-role therapy' which I applied to the idea of teachers doing more of their own research and investigations (see Bishop, 1972). That was the context from which Alec's story arose. It was clear to me, and to him, that the 'therapy' of playing a 'fixed-role' (the researcher) for a period of time, had the effect of changing dramatically his perceptions, his constructions and his attributions. It would be useful to have more evidence of such changes.

And why not make an imaginative analogue here? If that strategy helped to change a teacher's attributions, could a similar strategy affect a pupil's attributions? But what could such a strategy look like?
3. Social interaction
We now turn to research and ideas which focus more explicitly on the processes of social interaction. All the time we have been discussing motivation and cognition the social interaction processes have had an implied presence and effect, but now we should consider some aspects a little more directly.

First of all we find that the literature sensitizes us to the distinction between 'communication' and 'influence'. I feel also that it is important to distinguish between these because the relative position of the interactors implied by the two is different, and has therefore different consequences for the teacher. For example, many teachers having asked a pupil a question, then only prepare to evaluate and judge the answer received. Indeed the position of 'evaluator' also predisposes teachers to ask certain kinds of questions rather than others. The difference between communication and influence is illustrated well by this extract from Harvey et al. (1982) in their study of language in mathematics:

D. 15's odd and a 1/2's even.
RH. 15's odd and a 1/2's even? Is it?
D. Yes
RH. Why is a 1/2 even?
D. Because erm, 1/4's odd and 1/2 must be even.
RH. Why is 1/4 odd?
D. Because it's only 3.
RH. What's only 3?
RH. A 1/4's only 3?
D. That's what I did in my division

At this point another child joined in to explain to the teacher:

R. Yes, there's three parts in a quarter like on a clock. It goes 5,10, 15.
RH. Oh, I see.
R. There's only three parts in it.
RH. Ah, so you've got three lots of 5 minutes makes a quarter of an hour
D. yes. No. Yes, yes, yes. (Harvey et al, 1982, p.28)

The teacher could have evaluated the pupil's response at various points, which would then probably have led to attempts to change the pupil's view - that is the shift from communication to influence. There is a danger, I feel, in the teacher only using the influential
mode rather than appropriately (as in the case above) using the communication mode. I have chosen this example because, in my experience, such examples of communication in mathematics classrooms are rare. They need to be better documented in research, I feel.

But communication is not only verbal, as many studies and our own experience tell us. It is also a two-way process in that both transmitter and receiver must play a part if communication is to occur. Moreover it is not always intentional on the part of the transmitter, and unintentional 'messages' are conveyed around the classroom by gesture, bodily position, facial reactions and by words. Such messages are the 'fabric' of social interaction from which we weave our constructions of others, and our views of ourselves as others see us. Unintentional messages convey to pupils the teacher's perceptions of them as much as do the intentional messages, and some would say more so.

It is sometimes a matter of what is not communicated, as Webb's (1982) research shows. She studies heterogenous small groups working on mathematical problems and found that one frequent occurrence was that questions to the rest of the group by the less-able pupils were more often ignored than accepted. One can easily predict the meanings conveyed by that message.

Moving now to the area of social and interpersonal influence, the report of Perret - Clermont and her colleagues (1984) must be quoted here. It provides us with an excellent summary of research and ideas concerning the role of other people in children's intellectual development. As well as containing many ideas of interest and value to teachers it should also provide a warning to any interviewer of any child not to ignore the social relationship between them. (My story from Papua New Guinea at the start of the talk should not just be interpreted from a cognitive standpoint either. The interactors had markedly different values and sets of cultural assumptions which clearly affected the interview). Time and again we learn of children assessing the social situation and context of the interview in their interpretation of the task, in their responses and in their judgement of the way their responses are received. Clearly, too, in a mathematics classroom, a mathematical activity is as much a social activity as it is an intellectual one, and this awareness is critical for the teacher in interpreting problems of both motivation and cognition.

Whereas Piagetian interviewers may not be intentionally influencing the child, the teacher usually is, and correctly so in my opinion. But the power given to the teacher by society, and usually achieved also in the classroom, does not necessarily dictate the kind of influence exerted by the teacher. My own analysis based on Barnes' ideas (1976) and others is that, in terms of the pupils' mathematical development
we can see an 'influence' dimension varying from 'imposition' to 'negotiation'. An imposition interaction pattern is characterised by the teacher maintaining tight controls over rules of procedure, over the kinds of acceptable contributions (unusually of a very limited nature) over the amount of talk (teacher maximum), over meanings of terms and over the methods of solution. The mathematics teacher together with the textbook would represent the mathematical authority for the validity of solutions and the transmission of ideas and meaning from teacher to pupils would be emphasised.

In a negotiation interaction pattern on the other hand, the rules of procedure are discussed and agreed on rather than imposed, the kinds of contributions from pupils will vary, there will be more equal amounts of teacher and pupil talk, and there will be discussions over meanings and over methods of solution. The mathematical context itself will offer the criteria for judging the acceptability and validity of solutions wherever possible, and in other cases the nature of he conventional criteria will be made explicit. In comparison with the transmission of ideas in the imposition pattern, here we would expect to find more of an emphasis on communication of ideas between teacher and pupils, and on establishing and developing shared meanings.

Once again it would be useful to have ideas from research about changes in interaction patterns and to know what conditions surrounded a change from imposition to negotiation, or vice versa. However, from the unintentional messages contained in my descriptions above, I am sure that you will correctly infer that my preference would be for more negotiation and less imposition!

There is much more one could say about social interaction but space only permits this brief reflection on what I feel are the most promising research developments for us in mathematics education.

Postscript
Here then are some preliminary thoughts about what I think of as the social psychology of mathematics education. I hope my reasons for my interest have also been communicated in the talk and I hope that I have persuaded others of the validity of these reasons.

I would certainly like to see this organisation take a lead in developing this area of research and I look forward to seeing more papers on this area presented in subsequent conferences. I would be pleased to convene a meeting during this conference to try to involve more people in the development of this research.

References
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