Realistic Mathematics Education - An introduction

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RME: an “old” theory developed at FI ….

Treffers 1987

Freudenthal 1991
... but still alive and respected today!


Marja Van den Heuvel-Panhuizen (Ed.), 2020
Aims of this presentation

• To introduce some key aspects of the theory of Realistic Mathematics Education (RME)

• To set up a shared vocabulary for this summer school

• To reflect on RME task design and the role of contexts
Outline

• An introduction to RME
• Four RME key concepts
  o Mathematization
  o Didactical phenomenology
  o Use of models
  o Guided reinvention
• Hands-on task analysis
• Summary
What is Realistic Mathematics Education?
What is Realistic Mathematics Education?

• Realistic Mathematics Education (RME) is a domain-specific instruction theory on the teaching and learning of mathematics...

• ... that has been elaborated into a number of local instruction theories for different mathematical topics, student ages, and achievement levels
Starting point

Hans Freudenthal (1905-1990): Mathematics as human activity

“What humans have to learn is not mathematics as a closed system, but rather as an activity, the process of mathematizing reality and if possible even that of mathematizing mathematics.” (Freudenthal, 1968, p. 7)
Why RME?

Freudenthal’s opposition against “anti-didactical inversion”: don’t take the end point of the mathematician’s work as a starting point for teaching!

As a reaction to the obvious limitations of mechanistic and structuralistic approaches to mathematics education
The iceberg metaphor:
Realistic Mathematics Education

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Keywords

Domain-specific teaching theory; Realistic contexts; Mathematics as a human activity; Mathematization

What is Realistic Mathematics Education?

Realistic Mathematics Education – hereafter abbreviated as RME – is a domain-specific instruction theory for mathematics, which has been developed in the Netherlands. Characteristic of RME is that rich, “realistic” situations are given a prominent position in the learning process. These situations serve as a source for initiating the development of mathematical concepts, tools, and procedures and as a context in which students can in a later stage apply their mathematical knowledge, which then gradually has become more formal and general and less context specific.

(Van den Heuvel-Panhuizen & Drijvers, 2020)
Six RME principles and key concepts

1. The activity principle
2. The reality principle
3. The level principle
4. The intertwinement principle
5. The interactivity principle
6. The guidance principle

(Van den Heuvel-Panhuizen & Drijvers, 2020)
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Mathematization

Mathematics as human activity:
Doing mathematics = mathematizing

Treffers (1979): distinction between horizontal and vertical mathematization.
Mathematization

'Realistic' context

Mathematical objects, structures, methods

Translate

Abstract

Vertical mathematization

Mathematical model

Horizontal mathematization
Example horizontal / vertical mathematization

Horizontal:
Translating a problem on fixed and variable costs (e.g., mobile phone offers) in two linear equations

Vertical:
The development of a method / theory for solving systems of two linear equations in general
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What is Realistic?
“I prefer to apply the term ‘reality’ to what at a certain stage common sense experiences as real.”

Freudenthal (1991, p. 17)
The realistic view [...] takes the reality as a point of departure, i.e., the world of the child, which implies that it tries to identify the appearances of mathematical phenomena that fit the world of the child, so to which the child can attach meaning.

Treffers (1979, p. 12-13, my translation)
What do we mean by “Realistic”?

“Realistic” may have different meanings:

- Realistic in the sense of *feasible* in educational practice
- Realistic in the sense of related to *real life* (real world, phantasy world, math world)
- Realistic in the sense of *meaningful, sense making for students*
- Realistic in the sense of “*zich realiseren*” = to realize, to be aware of, to imagine
Didactical phenomenology (1)

A didactical phenomenology...
... relates mathematical thought objects to phenomena in the (physical, social, mental,...) world
... as to inform us how these mathematical thought objects may help to organize and structure phenomena in reality.
Didactical phenomenology (2)

As such, it identifies phenomena that ...  
... beg to be organized by mathematical means  
... invite students to develop the targeted mathematical concepts  
... and help teachers and designers to decide which contexts to use  

These phenomena can come from real life or can be ‘experientially real’
Didactical phenomenology (3)

In Freudenthal’s words (1983, p. ix), a didactical phenomenology of mathematics can “show the teacher the places where the learner might step into the learning process of mankind.”

(Van den Heuvel-Panhuizen, 2020)

-> Didactical phenomenology guides task design
   (cf. didactical engineering, Margolinas & Drijvers, 2015)
1. Maak de keersommen.

2. Hoeveel komt er in elk laatje?
   a. Je verdeelt 18 losse spijkers over 3 laatjes.
   b. Je verdeelt 24 grote spijkers over 6 laatjes.
   c. Je verdeelt 28 losse schroeven over 4 laatjes.

3. Maak de sommen.
   27 = 3 × ...
   24 = 6 × ...
   18 = 2 × ...
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(Van den Heuvel-Panhuizen & Drijvers, 2020)
Broad meaning and important role for models

Within RME, models are seen as representations of problem situations, which necessarily reflect essential aspects of mathematical concepts and structures that are relevant for the problem situation, but that can have different manifestations. (Van den Heuvel-Panhuizen, 2003, p. 13)

A model may be material, a situation, a sketch, a diagram, ...

The meaning and role of these models may shift during the learning process, from being situation-related to becoming more general.
Examples of didactical models

- Empty number line (for arithmetic operations)
- Chocolate bar (for ratios)
- Ratio table (for operations with ratios)
- Pizza model (for fractions)
- Arrow chains (for functions)
- Tree model (for expressions)
- Abacus (for calculations)

...
Model of – model for: emergent modeling

Models of informal mathematical activity develop into models for mathematical reasoning

(Streefland, 1985; Gravemeijer et al., 2000; Van den Heuvel-Panhuizen, 2003)
Emergent modelling

*Situational level*
Activity in the task setting. Interpretations and solutions depend on understanding of how to act in the (often out of school) settings
Emergent modelling (ctnd)

Referential level
Referential activity, in which models refer to activity in the setting of instructional activities (posed mostly in school)
Emergent modelling (ctnd)

*General level*
General activity, in which models focus on situation-independent interpretations and solutions.
Emergent modelling (final)

Formal level
Reasoning with conventional symbolizations, which is no longer dependent on the support of models
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Guided reinvention

(Van den Heuvel-Panhuizen & Drijvers, 2020)
Guided reinvention

**Reinvention:**
Reconstructing and developing a mathematical concept in a natural way in a given problem situation.

**Guidance:**
Students need guidance (from books, peers, teacher) to ascertain convergence towards common mathematical standards

Tension between reinvention and guidance?
Guided reinvention heuristics

Think how you would approach a problem situation if it were new to you, ‘think how you might have figured it out yourself’ (Gravemeijer 1994, p. 179)

See what you can learn from the historical development of a mathematical concept for educational design
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Task A: Extending the lawn

The lawn in Mr. Jones’ garden measures 15 by 20 meters. Mr. Jones decides to extend the lawn. To two sides he adds a strip of equal width of $x$ meters. See Figure 7.16.

a. Show that the area of the enlarged lawn is represented by

\[ \text{Area} = x^2 + 35x + 300 \]

b. The new lawn has an area of 374 $m^2$. Set up an equation and calculate the width of the strip.
Task B: Melting ice

An ice cube with edges of 30 mm long starts to melt down slowly. Every minute, the edges get 1.5 mm shorter. The volume of the ice cube is described by the formula \( V = (30 - 1.5t)^3 \), where \( V \) stands for the volume in mm\(^3\) and \( t \) for the time in minutes.

a. Calculate the volume of the ice cube when \( t=0 \).

b. What are meaningful values for \( t \)? And for \( V \)?

c. Plot and sketch that part of the graph for which the variables are meaningful.

d. Trace the graph with the cursor and investigate after how many minutes the volume is less than 10 000 mm\(^3\). Provide your answer with a precision of one decimal.
Task C: Cutting a parabola

A parabola is intersected by a straight line. The line is moved upwards. The midpoint of the intersection points seems to move over a vertical line. Is this really the case?
Hoe deelnemen?

Klik op het geprojecteerd scherm om de vraag te activeren.

wooclap
The LAWN task: do you consider the task realistic?
The ICE CUBE task: do you consider the task realistic?
The PARABOL
A task: do you consider the task realistic?
Discussion on the tasks

What is your opinion on the realistic qualities of the contexts and the tasks A, B and C?
Contexts in mathematics education ...

- can be quite artificial
- can be quite confusing, for example from a science perspective
- may lack opportunities for mathematization
- should not necessarily be taken from daily life

*Misunderstanding:* “RME means that tasks start with a real life story”
Realistic contexts in RME

An appropriate context or problem situation ...

• is meaningful for students

• can be a real-life situation, but can also emerge from the world of science or mathematics itself

• should take into account the skills, competences and interests of the students
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Summary (1): What is RME?

• RME is a domain specific instruction theory on the teaching and learning of mathematics
• ‘Reality’ refers to what at a certain stage common sense experiences as real, in the sense of meaningful
• Mathematics is a human activity, you do mathematics through mathematization
Summary (2): Four key words in RME

Students’ learning of mathematics can be fostered through:

- Mathematization
- Didactical phenomenology
- Use of models
- Guided reinvention
Summary (3): Caution on contexts

• Please mind not using artificial problem situations in textbooks and assessments that may puzzle students and don’t invite the mathematics at stake!

• Real life is not the main criterion; opportunities for meaning making is the challenge!
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Some seminal past RME publications


