5 Practices for Orchestrating Productive Mathematics Discussions

Mary Kay Stein
University of Pittsburgh

Overview

- Cognitive Demand
  - Comparing two mathematical instructional tasks
  - Levels of Cognitive Demand and the Mathematics Task Framework
- Five Practices
  - Discuss the importance and challenge of facilitating a discussion around (CDMT)
  - Describe 5 practices that you can learn in order to facilitate discussions of CDMT more effectively
  - Discuss how the 5 practices could help improve teaching with CDMT

Comparing Two Mathematical Tasks

Martha's Carpeting Task

Martha was recarpeting her bedroom which was 15 feet long and 10 feet wide. How many square feet of carpeting will she need to purchase?

Stein, Smith, Henningsen, & Silver, 2000, p. 1

The Fencing Task

Ms. Brown's class will raise rabbits for their spring science fair. They have 24 feet of fencing with which to build a rectangular rabbit pen in which to keep the rabbits

a) If Ms. Brown's students want their rabbits to have as much room as possible, how long would each of the sides of the pen be?
b) How long would each of the sides of the pen be if they had only 16 feet of fencing?
c) How would you go about determining the pen with the most room for any amount of fencing? Organize your work so that someone else who reads it will understand it.

Stein, Smith, Henningsen, & Silver, 2000, p. 2

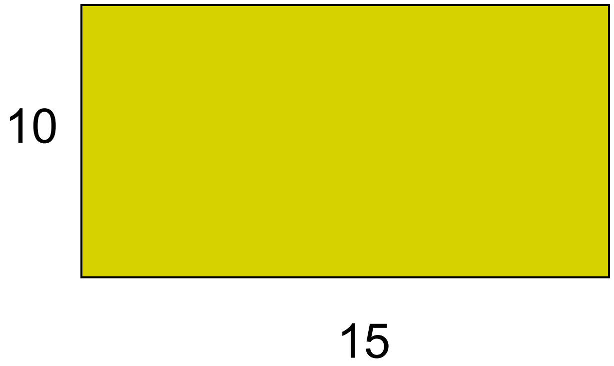
Martha's Carpeting Task Using the Area Formula

A = l x w

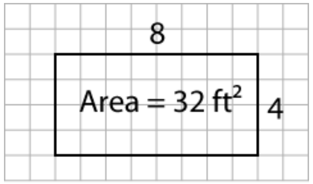
A = 15 x 10

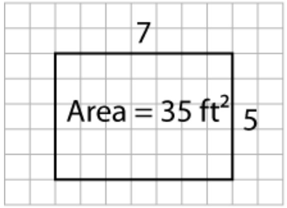
A = 150 square feet

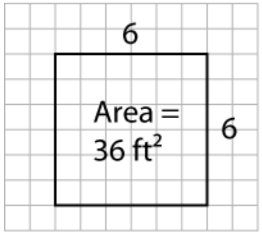
Martha's Carpeting Task Drawing a Picture

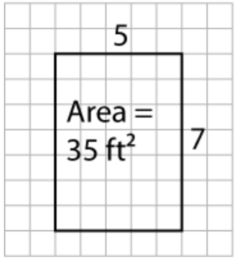


The Fencing Task Diagrams on Grid Paper





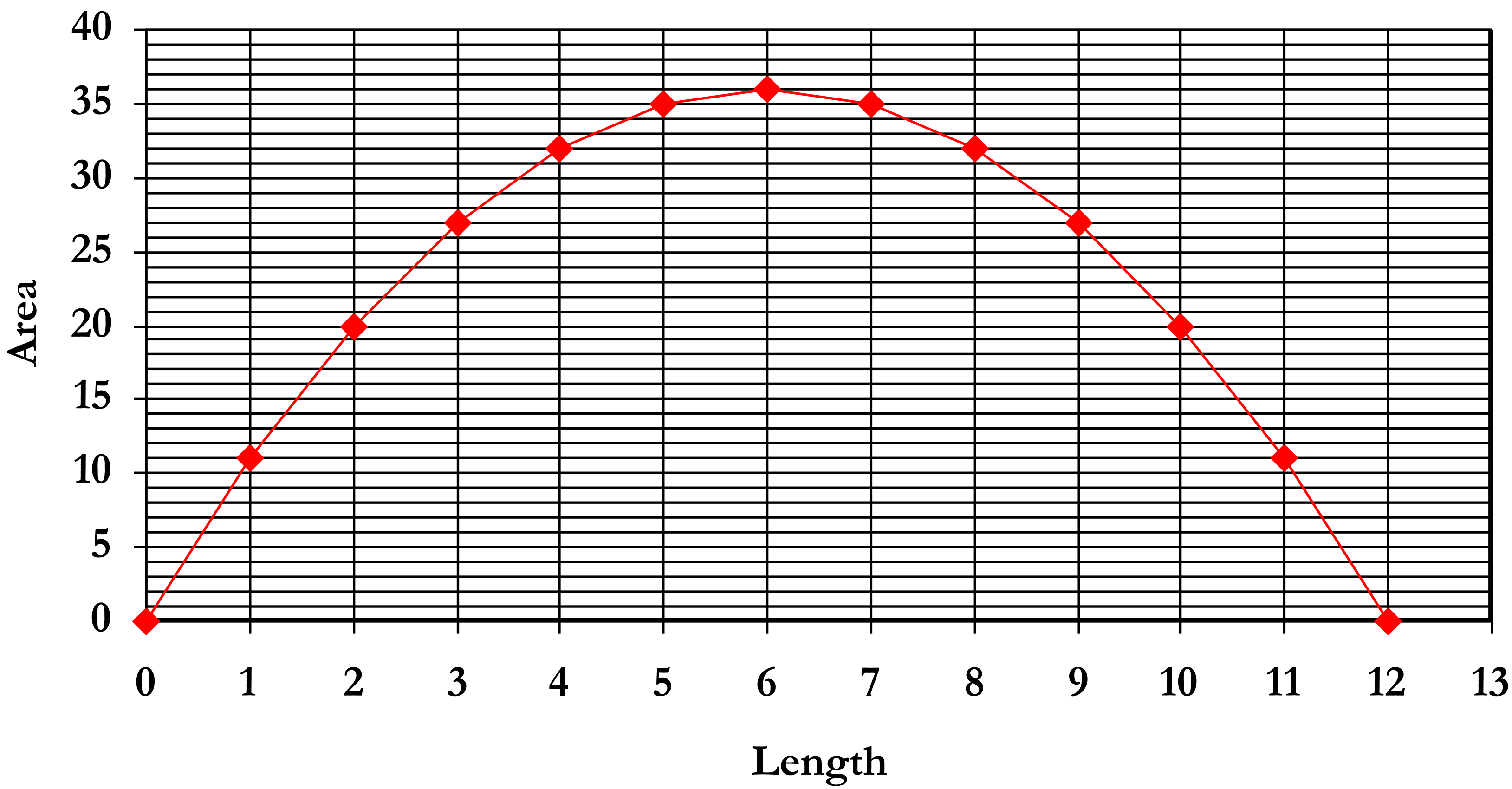




The Fencing Task Using a Table

Length (feet)	Width (feet)	Perimeter (feet)	Area (square feet)
1	11	24	11
2	10	24	20
3	9	24	27
4	8	24	32
5	7	24	35
6	6	24	36
7	5	24	35

The Fencing Task
Graph of Length and Area



Comparing Two Mathematical Tasks

How are Martha's Carpeting Task and the Fencing Task the same and how are they different?

Similarities and Differences

Similarities

- Same topic (area)
- Both require prior knowledge of area

Similarities and Differences

Similarities

- Same topic (area)
- Both require prior knowledge of area

Differences

- Way in which the area formula is used
- The need to generalize
- The amount of thinking and reasoning required
- The number of ways the problem can be solved
- The range of ways to enter the problem

Similarities and Differences

Similarities

- Same topic (area)
- Both require prior knowledge of area

Differences

- Way in which the area formula is used
- The need to generalize
- The amount of thinking and reasoning required
- The number of ways the problem can be solved
- The range of ways to enter the problem

Mathematical Tasks: A Critical Starting Point for Instruction

Not all tasks are created equal, and ***different tasks will provoke different levels and kinds of student thinking***.

Stein, Smith, Henningsen, & Silver, 2000

Mathematical Tasks: A Critical Starting Point for Instruction

The level and kind of thinking in which students engage determines what they will learn.

Hiebert, Carpenter, Fennema, Fuson, Wearne, Murray, Oliver, & Human, 1997

Mathematical Tasks: A Critical Starting Point for Instruction

There is no decision that teachers make that has a greater impact on students' opportunities to learn and on their perceptions about what mathematics is than the selection or creation of the tasks with which the teacher engages students in studying mathematics.

Lappan & Briars, 1995

Mathematical Tasks: A Critical Starting Point for Instruction

If we want students to develop the capacity to think, reason, and problem solve then we need to start with high-level, cognitively complex tasks.

Stein & Lane, 1996

High Level vs. Low Level Tasks: Different Opportunities for Thinking

- Vignettes 1 (Mr. Patrick) and 2 (Mrs. Fox) (**page 3 of your handout**)

- Consider the following question:

  What opportunities did students have to think and reason in each of the two classes?

Opportunities for Thinking and Reasoning

Vignette 1 - Mr. Patrick

- Teacher tells students what to do and how to do it
- Students solve the problems using the formula that has been provided and repeating what the teacher has demonstrated
- No thinking or reasoning needed to solve the assigned tasks

Opportunities for Thinking and Reasoning

Vignette 1 - Mr. Patrick

- Teacher tells students what to do and how to do it
- Students simply solve the problems using the formula that have been provided and modeling what the teacher has demonstrated
- No thinking or reasoning needed to solve the assigned tasks

Vignette 2 - Mrs. Fox

- Teacher leaves it up to students to *figure out* what to do and how
- Students must generate and test different pen configurations and look for a pattern in order to arrive at a generalization
- Considerable thinking and reasoning is needed to solve the assigned task

Levels of Cognitive Demand & The Mathematical Tasks Framework

Linking to Research: The QUASAR Project

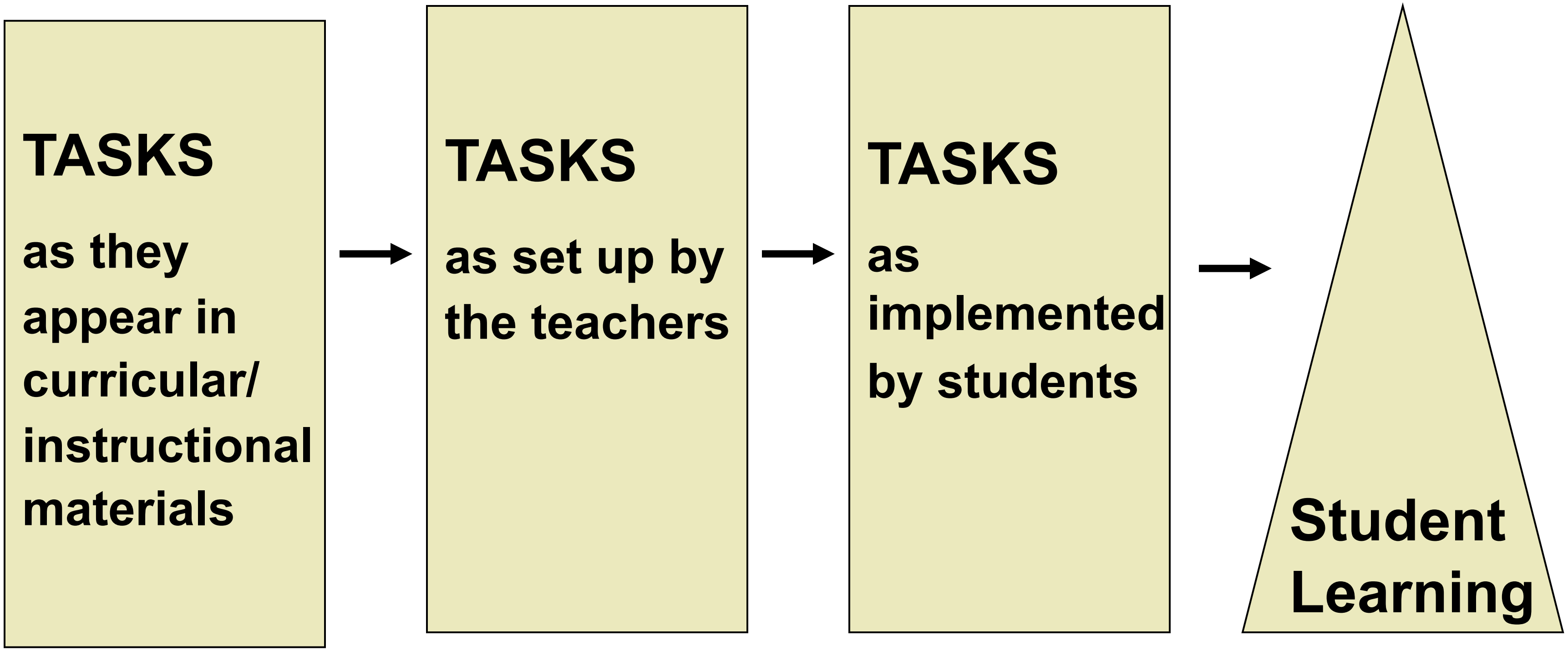
- Low-Level Tasks
  - Memorization
  - Procedures **without** connections to underlying concepts, meaning, or understanding

- High-Level Tasks
  - Procedures **with** connections to underlying concepts, meaning or understanding
  - Doing mathematics

Linking to Research: The QUASAR Project

- Low-Level Tasks
  - memorization (formulas for area and perimeter)
  - procedures without connections (Martha's Carpeting Task)
- High-Level Tasks
  - procedures with connections (use a drawing to explain why area = length x width)
  - doing mathematics (the Fencing Task)

The Mathematical Tasks Framework



Stein, Smith, Henningsen, & Silver, 2000, p. 4

The Mathematical Tasks Framework

TASKS as they appear in curricular/ instructional materials → **TASKS** as set up by the teachers → **TASKS** as implemented by students → **Student Learning**

Stein, Smith, Henningsen, & Silver, 2000, p. 4

The Mathematical Tasks Framework

TASKS

as they appear in curricular/ instructional materials

→

TASKS

as set up by the teachers

→

TASKS

as implemented by students

→

Student Learning

Stein, Smith, Henningsen, & Silver, 2000, p. 4

The Mathematical Tasks Framework

TASKS

as they appear in curricular/ instructional materials

→

TASKS

as set up by the teachers

→

TASKS

as enacted by students

→

Student Learning

Stein, Smith, Henningsen, & Silver, 2000, p. 4

The Mathematical Tasks Framework

TASKS

as they appear in curricular/ instructional materials

→

TASKS

as set up by the teachers

→

TASKS

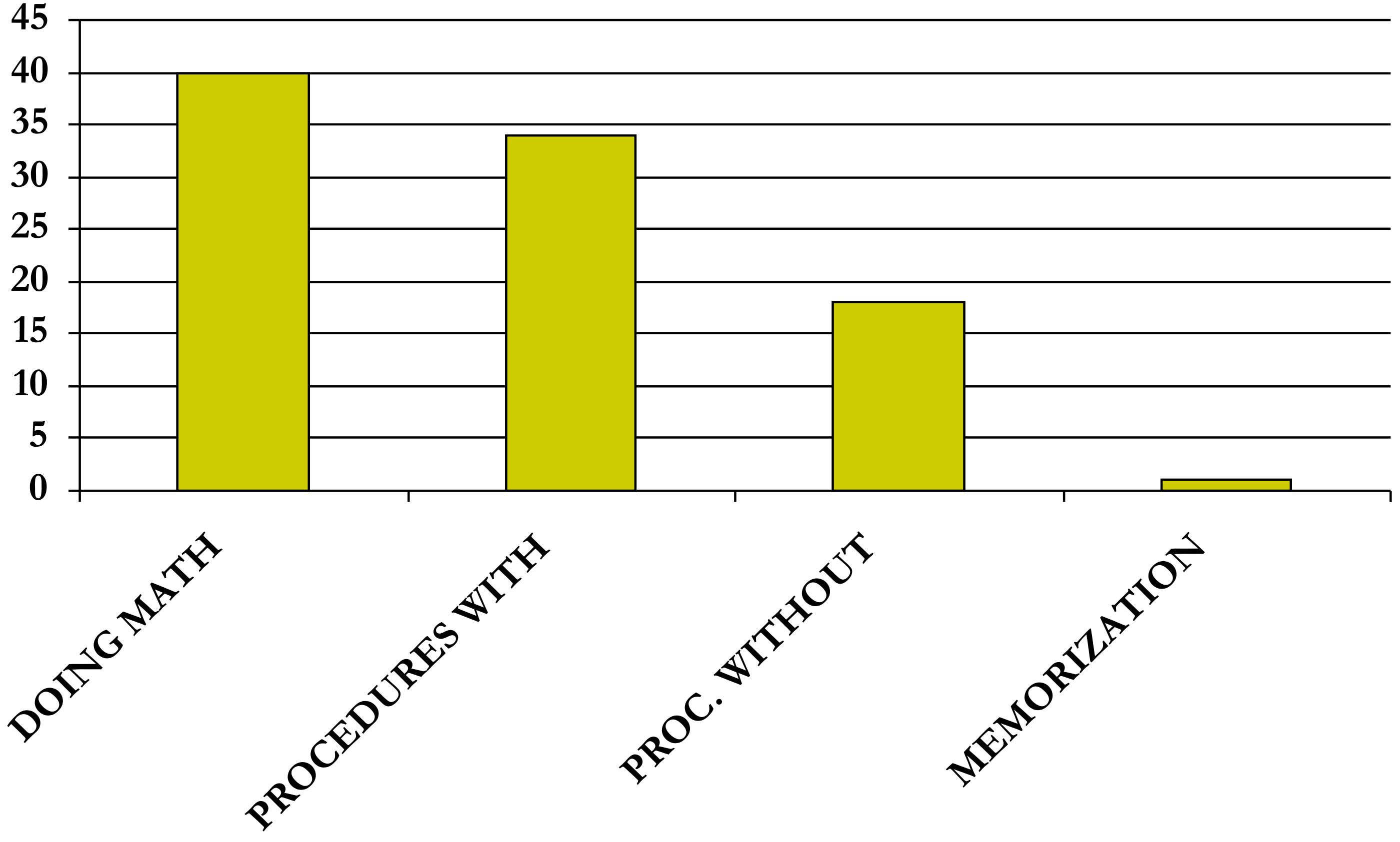
as implemented by students

→

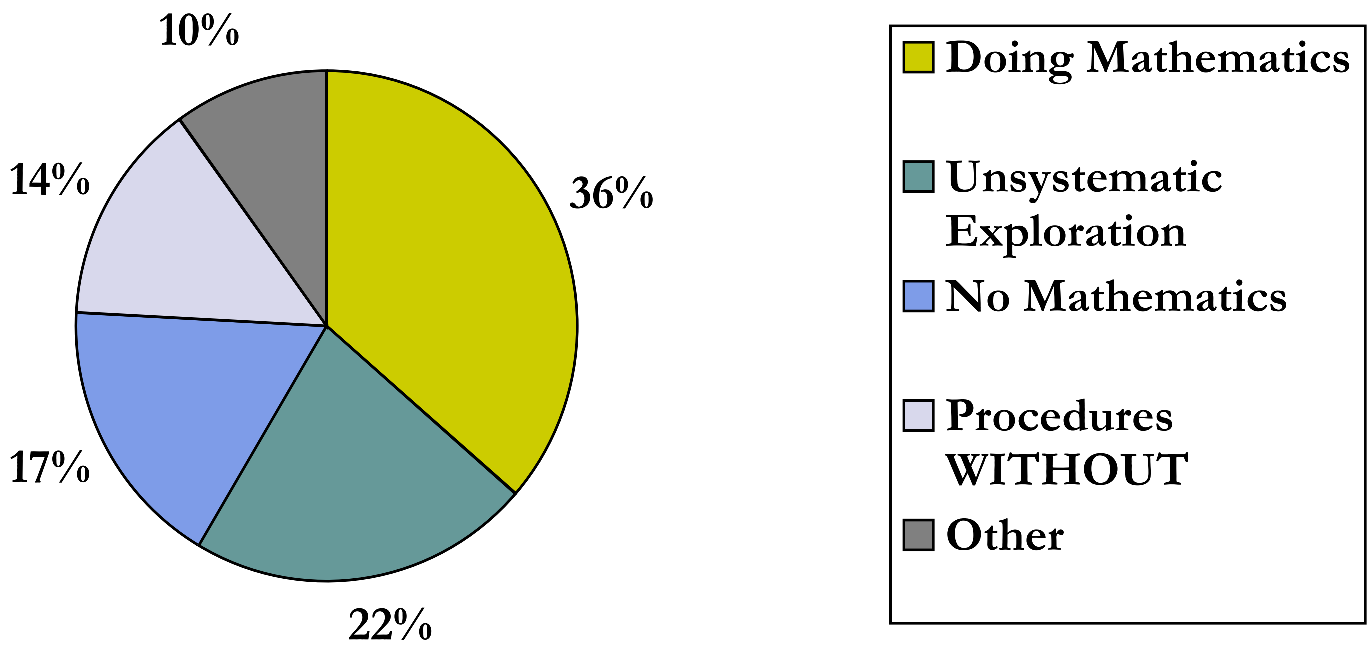
Student Learning

Stein, Smith, Henningsen, & Silver, 2000, p. 4

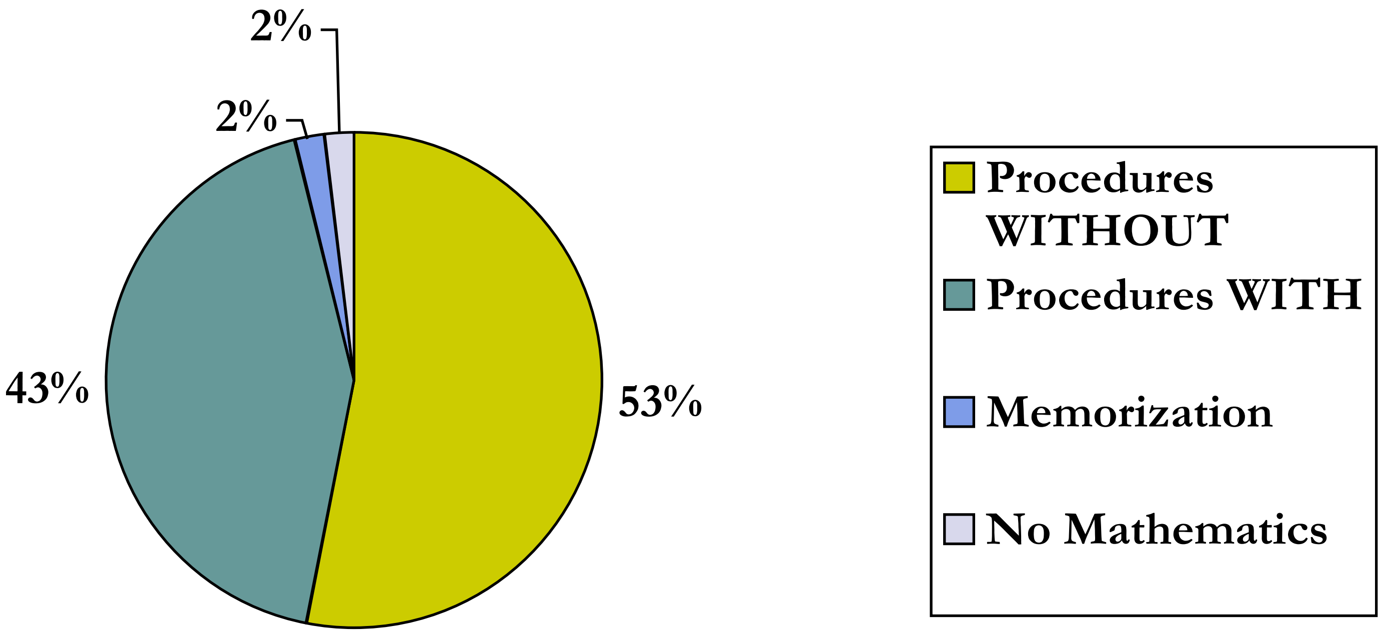
Cognitive Demands at Set Up



The Fate of Tasks Set Up as Doing Mathematics



The Fate of Tasks Set Up as Procedures WITH Connections to Meaning



Factors Associated with the Decline of High-Level Cognitive Demands

- Routinizing problematic aspects of the task
- Shifting the emphasis from meaning, concepts, or understanding to the correctness or completeness of the answer
- Providing insufficient time to wrestle with the demanding aspects of the task or so much time that students drift into off-task behavior
- Engaging in high-level cognitive activities is prevented due to classroom management problems
- Selecting a task that is inappropriate for a given group of students
- Failing to hold students accountable for high-level products or processes

Factors Associated with the Maintenance of High-Level Cognitive Demands

- Scaffolding of student thinking and reasoning
- Providing a means by which students can monitor their own progress
- Modeling of high-level performance by teacher or capable students
- Pressing for justifications, explanations, and/or meaning through questioning, comments, and/or feedback
- Selecting tasks that build on students' prior knowledge
- Drawing frequent conceptual connections
- Providing sufficient time to explore
- Connecting student thinking to important mathematical ideas

Factors Associated with the Maintenance and Decline of High-Level Cognitive Demands

Decline

- Routinizing problematic aspects of the task
- Shifting the emphasis from meaning, concepts, or understanding to the correctness or completeness of the answer
- Providing insufficient time to wrestle with the demanding aspects of the task or so much time that students drift into off-task behavior
- Engaging in high-level cognitive activities is prevented due to classroom management problems
- Selecting a task that is inappropriate for a given group of students
- Failing to hold students accountable for high-level products or processes

Maintenance

- Scaffolding of student thinking and reasoning
- Providing a means by which students can monitor their own progress
- Modeling of high-level performance by teacher or capable students
- Pressing for justifications, explanations, and/or meaning through questioning, comments, and/or feedback
- Selecting tasks that build on students' prior knowledge
- Drawing frequent conceptual connections
- Providing sufficient time to explore
- Connecting student thinking to important mathematical ideas

Does Task Fidelity Matter?

Does Task Fidelity Matter?

YES

Mathematical Tasks and Student Learning

- Students who performed the best on project-based measures of reasoning and problem solving were in classrooms in which tasks were more likely to be set up <u>and</u> enacted at high levels of cognitive demand (Stein & Lane, 1996; Stein, Lane, & Silver, 1996).
- Higher-achieving countries implemented a greater percentage of high level tasks in ways that maintained the demands of the task (Stiegler & Hiebert, 2004).
- The success of students was due in part to the high cognitive demand of the curriculum and the teachers' ability to maintain the level of demand during enactment through questioning (Boaler & Stapes, 2008).

Conclusion

- Not all tasks are created equal -- they provided different opportunities for students to learn mathematics.
- High level tasks are the most difficult to carry out in a consistent manner.
- Engagement in cognitively challenging mathematical tasks leads to the greatest learning gains for students.
- Professional development is needed to help teachers build the capacity to enact high level tasks in ways that maintain the rigor of the task.

The Five Practices

The Importance of Discussion

Mathematical discussions are a key part of current visions of effective mathematics teaching

- To encourage student construction of mathematical ideas
- To make student's thinking public so it can be guided in mathematically sound directions
- To learn mathematical discourse practices

Leaves and Caterpillar Vignette (pages 5-7 of handout)

- What aspects of Mr. Crane's instruction would you want him to see as promising (*reinforce*)?

- What aspects of Mr. Crane's instruction would you want to help him to work on (i.e., *refine*)?

Leaves and Caterpillar Vignette *What is Promising*

- Students are working on a mathematical task that appears to be both appropriate and worthwhile
- Students are encouraged to provide explanations and use strategies that make sense to them
- Students are working with partners and publicly sharing their solutions and strategies with peers
- Students' ideas appear to be respected

Leaves and Caterpillar Vignette *What Can Be Improved*

- Beyond having students use different strategies, Mr. Crane's goal for the lesson is not clear
- Mr. Crane observes students as they work, but does not use this time to ***assess what students seem to understand*** or ***identify which aspects of students' work to feature*** in the discussion in order to make a mathematical point
- There is a "show and tell" feel to the presentations
  - not clear what each strategy adds to the discussion
  - different strategies are not related
  - key mathematical ideas are not discussed
  - no evaluation of strategies for accuracy, efficiency, etc.

Some Sources of the Challenge in Facilitating Discussions

- Reduces teachers' perceived level of control
- Requires complex, split-second decisions
- Requires flexible, deep, and interconnected knowledge of content, pedagogy, and students

Purpose of the Five Practices

To make student-centered instruction more manageable by moderating the degree of improvisation required by the teachers and during a discussion.

The Five Practices (+)

1. **Anticipating** (e.g., Fernandez & Yoshida, 2004; Schoenfeld, 1998)
2. **Monitoring** (e.g., Hodge & Cobb, 2003; Nelson, 2001; Shifter, 2001)
3. **Selecting** (e.g., Lampert, 2001; Stigler & Hiebert, 1999)
4. **Sequencing** (e.g., Schoenfeld, 1998)
5. **Connecting** (e.g., Ball, 2001; Brendehur & Frykholm, 2000)

The Five Practices (+)

0. **Setting Goals and Selecting Tasks**
1. **Anticipating** (e.g., Fernandez & Yoshida, 2004; Schoenfeld, 1998)
2. **Monitoring** (e.g., Hodge & Cobb, 2003; Nelson, 2001; Shifter, 2001)
3. **Selecting** (e.g., Lampert, 2001; Stigler & Hiebert, 1999)
4. **Sequencing** (e.g., Schoenfeld, 1998)
5. **Connecting** (e.g., Ball, 2001; Brendehur & Frykholm, 2000)

0$_1$. Setting Goals

- **It involves:**
  - Identifying what students are to know and understand about mathematics as a result of their engagement in a particular lesson
  - Being as specific as possible so as to establish a clear target for instruction that can guide the selection of instructional activities and the use of the five practices
- **It is supported by:**
  - Thinking about what students will come to know and understand rather than only on what they will do
  - Consulting resources that can help in unpacking big ideas in mathematics
  - Working in collaboration with other teachers

Mr. Crane's Lesson Goals

Implied Goal

Students will be able to solve the task correctly using one of a number of viable strategies and realize that there are several different and correct ways to solve the task.

Possible Goals

- Students will recognize that the relationship between quantities is multiplicative not additive – that the 2 quantities (leaves and caterpillars) need to grow at a constant rate.

- Students will recognize that there are three related strategies for solving the task – unit rate, scale factor and scaling up.

0₂. Selecting a Task

- **It involves:**
  - Identifying a mathematical task that is aligned with the lesson goals
  - Making sure the task is rich enough to support a discussion (i.e., a cognitively challenging mathematical task)
- **It is supported by:**
  - Setting a clear and explicit goal for learning
  - Working in collaboration with colleagues

Mr. Crane's Task

A fourth-grade class needs five leaves each day to feed its 2 caterpillars. How many leaves would the students need each day for 12 caterpillars?

Use drawings, words, or numbers to show how you got your answer.

1. Anticipating

likely student responses to mathematical problems

- **It involves considering:**
  - The array of strategies that students might use to approach or solve a challenging mathematical task
  - How to respond to what students produce
  - Which strategies will be most useful in addressing the mathematics to be learned
- **It is supported by:**
  - Doing the problem in as many ways as possible
  - Discussing the problem with other teachers
  - Drawing on relevant research
  - Documenting student responses year to year

Leaves and Caterpillar

- **Unit Rate**--Find the number of leaves eaten by one caterpillar and multiply by 12 or add the amount for one 12 times
- **Scale Factor**--Find that the number of caterpillars (12) is 6 times the original amount (2) so the number of leaves (30) must be 6 times the original amount (5)
- **Scaling Up**--Increasing the number of leaves and caterpillars by continuing to add 5 to the leaves and 2 to the caterpillar until you reach the desired number of caterpillars
- **Additive**--Find that the number of caterpillars has increased by 10 (2 + 10 = 12) so the number of leaves must also increase by 10 (5 + 10 = 15)

Leaves and Caterpillar: Incorrect Additive Strategy

Missy and Kate's Solution

They added 10 caterpillars, and so I added 10 leaves.

2 caterpillars $\xrightarrow[+10]{}$ *12 caterpillars*

5 leaves $\xrightarrow[+10]{}$ *15 leaves*

2. Monitoring

students' actual responses during independent work

(page 8 of handout)

- **It involves:**
  - Circulating while students work on the problem and watching and listening
  - Recording interpretations, strategies, and points of confusion
  - Asking questions to get students back "on track" or to advance their understanding

- **It is supported by:**
  - Anticipating student responses beforehand
  - Carefully listening and asking probing questions
  - Using recording tools

Monitoring Tool

Strategy	Who and What	Order

Monitoring Tool

Strategy	Who and What	Order

List the different solution paths you anticipated

Monitoring Tool

Strategy	Who and What	Order
Unit Rate--Find the number of leaves eaten by one caterpillar and multiply by 12 or add the amount for one 12 times		
Scale Factor--Find that the number of caterpillars (12) is 6 times the original amount (2) so the number of leaves (30) must be 6 times the original amount (5)		
Scaling Up--Increasing the number of leaves and caterpillars by continuing to add 5 to the leaves and 2 to the caterpillar until you reach the desired number of caterpillars		
Additive--Find that the number of caterpillars has increased by 10 (2 + 10 = 12) so the number of leaves must also increase by 10 (5 + 10 = 15)		
OTHER		

Monitoring Tool

Strategy	Who and What	Order
Unit Rate--Find the number of leaves eaten by one caterpillar and multiply by 12 or add the amount for one 12 times		
Scale Factor--Find that the number of caterpillars (12) is 6 times the original amount (2) so the number of leaves (30) must be 6 times the original amount (5)		
Scaling Up--Increasing the number of leaves and caterpillars by continuing to add 5 to the leaves and 2 to the caterpillar until you reach the desired number of caterpillars		
Additive--Find that the number of caterpillars has increased by 10 (2 + 10 = 12) so the number of leaves must also increase by 10 (5 + 10 = 15)		
OTHER		

Make note of which students produced which solutions and what you might want to highlight

Monitoring Tool

Strategy	Who and What	Order
Unit Rate--Find the number of leaves eaten by one caterpillar and multiply by 12 or add the amount for one 12 times	Janine (number sentence) Kyra (picture)	
Scale Factor--Find that the number of caterpillars (12) is 6 times the original amount (2) so the number of leaves (30) must be 6 times the original amount (5)	Jason	
Scaling Up--Increasing the number of leaves and caterpillars by continuing to add 5 to the leaves and 2 to the caterpillar until you reach the desired number of caterpillars	Jamal (table) *Martin and Melissa did sets of leaves and caterpillars*	
Additive--Find that the number of caterpillars has increased by 10 (2 + 10 = 12) so the number of leaves must also increase by 10 (5 + 10 = 15)	Missy and Kate	
OTHER—Multiplied leaves and caterpillars	Darnell and Marcus	

3. Selecting

student responses to feature during discussion

- **It involves:**
  - Choosing particular students to present because of the mathematics available in their responses
  - Making sure that over time all students are seen as authors of mathematical ideas and have the opportunity to demonstrate competence
  - Gaining some control over the content of the discussion (no more “who wants to present next?”)
- **It is supported by:**
  - Anticipating and monitoring
  - Planning in advance which types of responses to select

4. Sequencing

student responses during the discussion

- **It involves:**
  - Purposefully ordering presentations so as to make the mathematics accessible to all students
  - Building a mathematically coherent story line
- **It is supported by:**
  - Anticipating, monitoring, and selecting
  - During anticipation work, considering how possible student responses are mathematically related

Monitoring Tool

Strategy	Who and What	Order
Unit Rate--Find the number of leaves eaten by one caterpillar and multiply by 12 or add the amount for one 12 times	Janine (picture and number sentence) Kyra (picture)	
Scale Factor--Find that the number of caterpillars (12) is 6 times the original amount (2) so the number of leaves (30) must be 6 times the original amount (5)	Jason	
Scaling Up--Increasing the number of leaves and caterpillars by continuing to add 5 to the leaves and 2 to the caterpillar until you reach the desired number of caterpillars	Jamal (table) *Martin and Melissa did sets of leaves and caterpillars*	
Additive--Find that the number of caterpillars has increased by 10 (2 + 10 = 12) so the number of leaves must also increase by 10 (5 + 10 = 15)	Missy and Kate	
OTHER—Multiplied leaves and caterpillars	Darnell and Marcus	

Indicate the order in which students will share

Monitoring Tool

Strategy	Who and What	Order
Unit Rate--Find the number of leaves eaten by one caterpillar and multiply by 12 or add the amount for one 12 times	Janine (picture and number sentence) Kyra (picture)	3 (Janine)
Scale Factor--Find that the number of caterpillars (12) is 6 times the original amount (2) so the number of leaves (30) must be 6 times the original amount (5)	Jason	4 (Jason)
Scaling Up--Increasing the number of leaves and caterpillars by continuing to add 5 to the leaves and 2 to the caterpillar until you reach the desired number of caterpillars	Jamal (table) *Martin and Melissa did sets of leaves and caterpillars*	2 (Jamal) 1 (Martin)
Additive--Find that the number of caterpillars has increased by 10 (2 + 10 = 12) so the number of leaves must also increase by 10 (5 + 10 = 15)	Missy and Kate	
OTHER—Multiplied leaves and caterpillars	Darnell and Marcus	

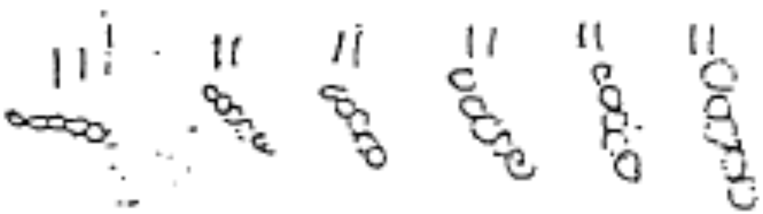
Leaves and Caterpillar Vignette

Possible Sequencing:

1. Martin – picture (scaling up)
2. Jamal – table (scaling up)
3. Janine -- picture/written explanation (unit rate)
4. Jason -- written explanation (scale factor)

Leaves and Caterpillar Vignette

1. Martin – picture (scaling up)



2. Jamal – table (scaling up)

leaves	5	10	15	20	25	30
caterpillars	2	4	6	8	10	12

3. Janine -- picture/written explanation (unit rate)

if each of the caterpillars need 2 1/2 leaves a day then you just X's 2 1/2 x's 12 = 30.

4. Jason -- written explanation (scale factor)

If it takes 5 leafs for two caterpillars, you just count by twos, until you come to half of 12. The number is six, and then you multiply 5 X 6, and it equals 30.

5. Connecting

student responses during the discussion

- **It involves:**
  - Encouraging students to make mathematical connections between different student responses
  - Making the key mathematical ideas that are the focus of the lesson salient

- **It is supported by:**
  - Anticipating, monitoring, selecting, and sequencing
  - During planning, considering how students might be prompted to recognize mathematical relationships between responses

Why These Five Practices Likely to Help

- **Provides teachers with more control**
  - Over the content that is discussed
  - Over teaching moves: not everything improvisation
- **Provides teachers with more time**
  - To diagnose students' thinking
  - To plan questions and other instructional moves
- **Provides a reliable process for teachers to gradually improve their lessons over time**

Why These Five Practices Likely to Help

- **Honors students' thinking while guiding it in productive, disciplinary directions** (Ball, 1993; Engle & Conant, 2002)
  - Key is to support students' disciplinary authority while simultaneously holding them accountable to discipline
  - Guidance done mostly 'under the radar' so doesn't impinge on students' growing mathematical authority
  - At same time, students led to identify problems with their approaches, better understand sophisticated ones, and make mathematical generalizations
  - This fosters students' accountability to the discipline

Resources Related to the Five Practices

- Stein, M.K., Engle, R.A., Smith, M.S., & Hughes, E.K. (2008).Orchestrating productive mathematical discussions: Helping teachers learn to better incorporate student thinking*. Mathematical Thinking and Learning, 10*, 313-340.

- Smith, M.S., Hughes, E.K., & Engle, R.A., & Stein, M.K. (2009). Orchestrating discussions. *Mathematics Teaching in the Middle School, 14* (9), 549-556.

Resources Related to the Five Practices

- Smith, M.S., & Stein, M.K. (2011). *5 Practices for Orchestrating Productive Mathematics Discussions*. Reston, VA: National Council of Teachers of Mathematics.

For additional information, you can contact us at

Mary Kay Stein

mkstein@pitt.edu