Part 3

Questions about the Future

23

Technology in Mathematics Education: Tapping into Visions of the Future

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MORE than a quarter of a century ago, the first microcomputers for use in schools were being marketed; almost twenty years ago, the first graphing calculator hit the market; and fifteen years ago, we saw the first World Wide Web activity. What will technology look like in mathematics classrooms ten years from now? How will technologies be used to affect school mathematics content and teaching? What are the principal changes in mathematics teaching and learning that we might expect over the next decade? The goal of this article is to look forward to what technology-supported mathematics learning environments might look like in the future. To find answers to this question, I interviewed each of the following individuals, all of whom are noted for leading-edge thinking on the uses of technology, most of them in the use of technology in the teaching and learning of mathematics:

Daniel Chazan, mathematics education faculty, University of Maryland
J. Douglas Child, mathematics faculty, Rollins College (Winter Park, Fla.)
Jonathan Choate, mathematics faculty, Groton School (Groton, Mass.)

Jere Confrey, mathematics education faculty, Washington University (St. Louis, Mo.)
Helen Doerr, mathematics and mathematics education faculty, Syracuse University (Syracuse, N.Y.)
James T. Fey, mathematics and curriculum and instruction faculty, University of Maryland
William Finzer, software developer (Fathom), KCP Technologies (Emeryville, Calif.)
Eric Hart, mathematics and mathematics education faculty, Maharishi University of Management (Fairfield, Iowa); past director, NCTM Illuminations project
Nick Jackiw, software developer (Geometer's Sketchpad), KCP Technologies (Emeryville, Calif.)
James Kaput, mathematics faculty, University of Massachusetts Dartmouth
John Kenelly, emeritus mathematical sciences faculty, Clemson University (Clemson, S.C.)
Kenneth Koedinger, faculty, Human-Computer Interaction Institute, Carnegie Mellon University (Pittsburgh, Pa.)
Janet May, program developer, World Campus, The Pennsylvania State University
Robert McCollum, mathematics faculty, Glenbrook South High School (Glenview, Ill.)
Ricardo Nemirovsky, project director, Calculus@Museum–TERC (Cambridge, Mass.)
Kyle Peck, instructional systems faculty, The Pennsylvania State University; founder, Centre Learning Community Charter School (State College, Pa.)
Steve Rasmussen, Key Curriculum Press (Emeryville, Calif.)
Peter Rubba, professor and Director of Academic Programs, World Campus, The Pennsylvania State University
Patrick Thompson, mathematics education faculty, Vanderbilt University (Nashville, Tenn.)
Bert Waits, emeritus mathematics faculty, Ohio State University
Rose Mary Zbiek, mathematics education faculty, The Pennsylvania State University
Lee Zia, National Science Digital Library, National Science Foundation (Arlington, Va.)

I also communicated by e-mail with Gene Klotz, mathematics faculty, Swarthmore College, and founder of the Math Forum. This set of leaders[1] is,

1. Direct quotes cited in this article are taken from verbatim transcripts of either the interviews that were conducted or subsequent e-mails.

of course, hardly a complete set of those who have led the field over the past two decades, but it is a respectable, and I hope representative, subset having a wide-ranging knowledge about technology and teaching. It includes the authors of software that moved the field through interactive geometry tools, dynamic statistics, and computer algebra tools; curriculum writers who have crafted ways to use those tools in mathematics learning; and a publisher who has broken the barriers to widespread adoption of technology in mathematics. It includes the nation's leaders in the use of graphing calculators as well as those who continue to prod the field to move faster and further. It includes individuals who created groundbreaking technology-intensive curricula and individuals who are leading the nation in implementing university and school visions of the use of technology. The interviewees agreed to talk for fifteen to twenty minutes. Most talked for more than twice that amount of time—an indicator of their interest and the depth and breadth of what they had to share.

Some Constructs for Thinking about Enhancing Mathematics Learning through Technology

Jim Fey observed, "Despite the fact that the world hasn't moved where we were ready to move it twenty years ago, it really has changed. We're dealing with generations of people who have different images of mathematics." Part of the reason for this change is that technology has opened up new ways to think about mathematics teaching and learning. We now have constructs for describing aspects of these images. Consider, for example, the myriad routine procedures that students are required to master in school mathematics, procedures that are now well within the capability of handheld technology. In the past, teachers and students were confined to a sequential approach to learning these procedures, with the mastery of each step in the procedure necessary before proceeding to the next step. Technology allows a different approach, with more complex procedures (or *macroprocedures*) chunked into a series of simpler procedures (*microprocedures*) (Heid 2003). For instance, prior to the widespread availability of computer algebra systems or graphing technology, students needed to master factoring polynomials or the quadratic formula before they could consider solving quadratic equations. With technology, students can solve quadratic equations using technological factoring commands before they learn to factor those polynomials by hand, and they can find approximate solutions to quadratic equations in the absence of the quadratic formula by producing calculator-generated graphs. Teachers and students now have choices regarding which procedures to do first and which to do at all. In this type of technological environment, capabilities are distributed between the user and the tool, with the user in charge of making decisions about when and how the tool should be used.

One needs to be careful not to give the impression that technology itself makes the difference in teaching and learning. It is, of course, not the technology that makes the difference but rather how it is used and by whom. Those who have studied the use of technology in mathematics teaching and learning have noted that technology mediates learning. That is, learning is different in the presence of technology. The representations that students access may conceal or reveal different features of the mathematics, and the procedures students assign to the technology (as opposed to doing them by hand) may affect what students process and learn. Moreover, how a student uses technology is dependent on his or her ever-changing relationship to the technology. When a user first encounters a particular technological tool, his or her uses of the technology may be confined to rote application of the specific keystrokes or procedures that had been introduced. As the student develops facility with, and an understanding of, the capabilities of the technology, the technology becomes an instrument that the student can tailor flexibly to specific needs. This development of a working relationship between the user and the tool is called *instrumental genesis.*

Emerging Technologies

Today's new tools and approaches are harbingers of technology in tomorrow's classrooms. A look at some of these emerging technologies through the eyes of the innovators gives some indication of likely futures. Computational tools have the greatest potential for catalyzing specific changes in mathematics classrooms. Tools for communication and information are suited for general technology use in educational settings.

Computational Tools

The interviews identified an array of computational tools for mathematics classes that are likely to influence the future. These tools include geometric construction tools, data analysis tools, computer algebra tools, cognitive tutors, and microworlds. Hardware innovations that will shape the future include a range of handheld configurations of tools, three-dimensional printers, motion detectors, and lighter and more portable "tablet" computers.

Dynamic tools

The type of computational tool that epitomizes visions of the future is the dynamic tool. The most popular dynamic tools of the past decade are the interactive geometric-construction tools like Geometer's Sketchpad (GSP) and Cabri. More recent entries onto the dynamic-tool scene include dynamic data analysis tools—most notably, Fathom. Dynamic tools are characterized by the principles of dynamic manipulation (dragging) and continuous visualization (nearly instantaneous change in the objects on the screen). Rose Zbiek noted that the development of a dragging capacity was a key to the future: "I anticipate advances in dynamic tools in terms of what representations are available and what can be dragged within each representation." Complementing her emphasis on different representations within a tool is an essential difference among dynamic tools in the types of objects that can be changed. Nick Jackiw explained that difference:

> [I]n Sketchpad what is fundamental is the definition of the relationships between objects, and the objects exist as abstractions glued together by these relationships, [whereas] in Fathom … it's instead the collection of data which has a sort of constitutional identity. So you can change the data within that collection and that's ... akin to changing the coordinates of something in Sketchpad....

Dynamic tools are likely to be essential components of the technology toolkit in the school mathematics instruction of the future. As dynamic tools take their place in more and more mathematics classrooms, there will be an increasing push for those tools to be combined with other utilities. For example, Jon Choate advocates blending an interactive geometric construction tool with a computer algebra system, and Ken Koedinger sees an intelligent tutor in which an interactive geometric construction tool is embedded. Rose Zbiek identified the importance of this combination of tools:

> It seems to me that the development of hybrid technologies that capitalize on dynamic features and incorporate a blend of current and emerging communication and mathematics utilities is the essence of the future. These tools will combine several of the types of technologies and do so in ways that continue to blur the already deteriorating lines between different areas of mathematics and statistics.

Intelligent tutors and computer algebra systems (CAS)

The computing tools that seem to be most underused in schools today are ones that might be characterized as artificial intelligence tools and ones that perform symbolic manipulation. These tools include computer algebra systems—tools that perform symbolic manipulation as well as generate graphs and perform numerical calculations—and intelligent tutors like the Algebra

Tutor, the Geometry Tutor, and those embedded in the Carnegie Learning curriculum. There is a growing interest among school mathematics leaders in finding optimal ways to incorporate CAS in high school mathematics, exemplified by the annual national conference initiated by a group of Chicago-area teachers collaborating with university faculty (www4.glenbrook.k12.il.us/USACAS/2004.html).

The leaders with whom I spoke had a range of suggestions about what was needed in order for CAS to become a more integral part of the future. Many hoped for a future in which CAS was universally available, pointing out that some CAS capability is already available on the Web. The innovators identified several factors necessary for the promise of CAS to be realized. First, there is a need for curricula that incorporate CAS, not as a dispensable option, but as an integral tool in the development of the mathematics. A start in that direction is Module 9 of the CAS-Intensive Mathematics project (www.ed.psu.edu/casim), a module that uses CAS to focus students on reasoning with and about symbolic representations. (As Rose Zbiek points out, there is a concomitant need for technology-intensive curricula that make use of the range of other emerging technologies.)

Improvements in the CAS tools themselves would make them more viable in the classroom. Jon Choate suggested the development of a CAS that would allow individual teachers to build toolkits tailored to what they were teaching. He described a vector calculator that he has built within Mathematica, with primitives that students can use (e.g., find the length of a vector, find the angle between two vectors, find the cross product, find the dot product). Tools like this would allow the teacher to focus students on particular areas of mathematics at the teacher's discretion. Jon claims that this sort of access to CAS has changed the "playability" of calculus (the extent to which students can get their hands on and investigate calculus concepts) in a somewhat similar fashion to the way that interactive geometry construction tools have changed the playability of geometry. He describes the experience he and his students had with an open-ended but mathematically intense project centering on analyzing carnival rides:

> We got into analyzing carnival rides. We wanted to talk about carnival rides so we got on the web and we went to the company that makes the Mad Hatter ride in Disney World. And there were circles around circles around circles around circles around.... We wondered, "What's going on here?" Well, you come up with functions that describe the situation, but that doesn't tell you why people scream. So that made us think, when are you turning the fastest, when are you getting snapped? With these sorts of questions, you could go to technology and use curvature to analyze what was going on.

This tailoring of the CAS to particular purposes has been a long-term project of Doug Child. He points to his Math T/L (a front end for Maple) and to the home screen on the TI-92 calculator as examples. Both furnish the user with a list of options for viable symbolic manipulations from which the student can choose. This approach focuses the student on a limited range of options. He explains his rationale for tools like this: "So the Math TL and the home screen on the TI calculators can be used to basically focus learning energy. When a teacher wants to teach something, they can think about what the most important aspects of it are and focus on those. The mathematics they want to teach may involve a lot of calculations, but they can factor that out if they want." The calculator can perform the microprocedures (e.g., taking limits, computing derivatives, factoring algebraic expressions) while the student focuses on the higher-level processes, the macroprocedures. Doug explains: "I kept seeing students get focused on little details when they were trying to learn something which I thought was a lot more important and they never even thought about." These tailored and "tailorable" computer algebra systems would allow teachers and students to target instruction more precisely on the specific learning goal.

One of the current features of many CAS is that they do not allow the user to determine the form of the output—a desirable feature, since different symbolic forms reveal different information. The tailorable CAS of the future would also allow users to specify the type of equivalent form in which they would like to see their output. Since tools like this would allow teachers to offer students automated access to any given subset of symbolic manipulations and tools of the future are likely to include access to a range of equivalent but different forms, their use would allow teachers to experiment with different configurations and to learn more about the development of understanding of symbolic representations. Experimentation with these different forms might address the concern expressed by Bert Waits that we do not yet know enough about what paper-and-pencil skills are needed before students can learn mathematics using a CAS. Rose Zbiek deemed the extension of this issue of paper-and-pencil skill to technology use with and beyond CAS as "the number one research issue for the twenty-first century."

The availability of intelligent tutors and CAS tutors inevitably raises the issue of how much guidance to offer students through these tools. Doug Child describes his Symbolic Math Guide (SMG is embedded in TI calculators) as an expert doing symbolic calculations and allowing students the choice of what calculations to do. Ken Koedinger's group works on determining production rules that emulate expert problem solving. With either approach, the question remains concerning the optimal tutoring configuration for student learning.

Microworlds

Ever since Papert's (1993) classic *Mindstorms,* microworlds like Logo have been a staple in the innovative use of computing technology. The extent to which microworlds can become an integral part of mathematics instruction, according to Pat Thompson, depends on the readiness of teachers to adopt a much different view of teaching and learning. He points out:

> Microworlds don't seem to have caught on much. And I think one reason is because the use of microworlds demands a very different conception of mathematical inquiry than most teachers have, because microworlds are typically designed to be experimented with, much like you experiment with some physical system. You poke it here and you poke it there and you try to make it do things and you try to predict how it's going to behave when you try and make it do things. And according to what happens, you increase your understanding of how it works. And that notion of inquiry doesn't fit very well with conceptions of mathematics teaching that most teachers have. Many teachers didn't know what to do with microworlds. They didn't know how to pull mathematics out of them.

This awareness about views of teaching, learning, and mathematics that are needed in order to capitalize on technology is a recurring theme.

Hardware

As the past few decades have shown, the vehicles for implementing software (that is, the hardware) will also be influential. Issues of portability, accessibility, interconnectibility, and interface will be central.

Handheld technology. Chief among hardware products that have made a difference in mathematics teaching is the graphing calculator. John Kenelly hypothesizes: "In terms of fundamentally changing the way people teach, the handheld graphing calculator has been the most influential instrument of all time." It was both what calculators did (expanded visualization capability) and how they did it (in portable, affordable form) that accounts for their unprecedented success. Visions for the future of technology in mathematics education are seeded with ideas about new features being built into handheld devices. These features are likely to have their greatest effect on the dynamics of teaching. Bert Waits believes that the USB ports on TI-84s and wireless computing will have a great impact on the nature of teaching: "The idea of the teacher being able to poll what the students are doing instantly, send stuff back, and perhaps do it wirelessly, that is all very powerful." Others see great potential impact from the data-collection and data-sharing capabilities of handheld devices. Helen Doerr points out that, as mobile computational tools, handheld devices can allow the collection of data that might not otherwise have been collected. Ricardo Nemirovsky sees major impact in the near future from handheld motion detectors with real-time graphical displays. Not all projections are without concern—Lee Zia's optimism about the future potential of handheld and mobile computing is tempered by a recognition of physical limitations. In particular, he points out that "the screen real estate on handheld devices is relatively small."

Alternative hardware configurations for current computers. One can imagine future hardware configurations that address the problems teachers now recognize with current technologies—problems involving accessibility, visibility, portability, and interface. Some of these problems are likely to be addressed by extensions of existing tools. Bert Waits predicts handheld wireless personal computers with bigger and better screens, more memory, and cross-curricular sets of software. Imagine a paper-thin, $8\frac{1}{2}$-by-11-inch, wireless computer equipped with pen input capability, graphic user interfaces, touch screen capability, and powerful computing tools. This would allow students to carry their computers and to use them for a range of classes, not limited to mathematics. Textbooks might exist only as part of each student's personalized set of computer software.

Alternative hardware. Among the likely candidates for new technology in the mathematics classroom are those related to the visualization and creation of three-dimensional objects. One such object, described by Lee Zia, is the three-dimensional printer. Based on devices currently in engineering contexts, the three-dimensional printer would allow the replication of a three-dimensional object through the use of a computer code and appropriate hardware. Lee informally describes the phenomenon, inherent in one of the NSF-funded Digital Library projects, as follows:

> The basic idea is that by creating some kind of high-resolution computer-aided-design (CAD) file, or alternatively, using a high-resolution X-ray computed tomography scanning process, you can produce a huge digital representation that contains all the information about a three-dimensional object: its internal geometry, surface texture, and so forth. Then through basically an inkjet kind of process, layers of material are laid down in successive passes. To create negative spaces, a special chemically treated mixture can be deposited in appropriate places or perhaps some sort of water soluble material, and this part of the object is subsequently washed away. In the end what's left is an actual 3-D object, a complete representation of the original artifact. The printer head just moves over and over, and the digital instructions determine what is deposited, where it is deposited, and how much. In this particular digital library project the artifacts being replicated are one-of-a-kind cast iron models created in the 19th century to aid the teaching and learning of kinematics—which of course provides a context for lots of interesting mathematics!

As these 3-D printers come down in price, students can explore three-dimensional objects in ways similar to how today's students use graphing calculators to explore families of functions. They can alter constraints and see the effects on the resulting objects.

A second category of three-dimension-related hardware that is likely to have an impact on school mathematics is that of 3-D motion detectors. One such device, the Motion Visualizer 3D, is described on the manufacturer's Web site (www.albertiswindow.com/) as a combined hardware-software system that tracks, records, and analyzes an object's motion in three dimensions in real time, based on live video display. Incorporating real-time data capture and immediate graphical display, this tool allows students to experiment with changing parameters in 3-space in much the same way that they now conduct two-dimensional parameter explorations. So, instead of exploring the shape of a two-dimensional graph of a function of the form $f(x) = ax^2 + bx + c$ by examining the effects of changing the values of a or b or c, students can experiment by moving their hands along a surface in order to match a given function rule, thus developing a kinematic sense of the variables (since the student controls things like phase, frequency, and amplitude). The Web site for this product shows students engaging in representing the flight of a free throw and a three-point shot in basketball. Using the 3-D motion detector, they created a graph that displayed the flight of each ball from the initial shot to the point where the ball fell through the basket.

Rose Zbiek pointed out the overarching idea that joins 3-D tools with their 2-D dynamic counterparts when she said: "Three-dimensional and dynamic tools (among other software components) will come together as a larger genre of technology that allows students to test their mathematical ideas by means other than entering syntactically correct literal codes."

Tools for Communication and Information

Much of the focus of current technological progress in education is in the arena of communication and information tools rather than in tools specific to mathematics instruction. These tools are likely to have profound effects on the teaching and learning of mathematics in the near future. This section will discuss the potential in mathematics classrooms for Web-based instruction, for the building of interactive communities, and for research tools.

Web-based instruction

To some people, Web-based instruction seems to be the wave of the future. To many it would seem to offer an economical and viable way to provide instruction to those who might not otherwise be reached. According to the Pew Internet & American Life Project (Madden and Rainie 2003, p. 47), Americans are overwhelmingly drawn to the Internet for educational purposes:

> Over 63% of adult Americans use the Internet.… An average day in 9/02 saw 12 million users taking to the Web in search of information relevant to their educational pursuits.… [S]tudents now rely heavily on the Internet to help them do their schoolwork, some teachers use the Web to facilitate their instruction, and others use the Web in conjunction with continuing education courses or job-related training. Computers with Internet connections are now commonplace in many American classrooms and some teachers now require some form of online participation from their students.

Not only is the Internet a part of our everyday lives, but it is destined to continue to be so. According to the Pew Internet Project (p. 78):

> The Internet has been irrevocably woven into everyday life for many Americans. While there was once a time when the Internet was interesting because it was dazzling, it is now a normalized part of daily life for about two-thirds of the U.S. population. For some, it has become an integral part of work or school. For others it is a primary means to stay in touch with family and friends. All the trends set out here seem destined to continue, if not evolve, as the technology gets better, the applications become simpler, the appliances that use the Internet become omnipresent, and the technology fades into the background of people's lives—as powerful, ubiquitous, commonplace, and "invisible" as electricity.

It is no surprise, then, to see universities turning in droves to exploring the educational and economic feasibility of Web-based instruction. That universities are headed toward delivering complete undergraduate programs on the Web is inevitable, according to Pete Rubba. Because of the lack of availability of an intuitively easy and stable math editor, however, mathematics-based courses are among the last to be converted. The math editor must be easy to use, since many mathematics students have difficulty with the production of symbolic notation. Rose Zbiek points out an important related learning issue: "Student production of symbols does not immediately follow the conventions of formal mathematics. There is a nontrivial task that lies between here and there: creating a math editor that is conducive to student expression in open-ended exploration and problem solving." Other questions that accompany the conversion of mathematics instruction to online include the following: Is instruction online "as good" as it is face-to-face? Will students be able to afford the necessary software and hardware to pursue online mathematics courses? Will online courses adequately address the problems of teaching mathematics in home-school settings or in very small school districts? Will Web-based courses lead to reliance on online quizzes and low-level testing? In spite of these and other issues, Pete Rubba is optimistic about the future of online courses, remarking: "I think we have got to ask ten years from now how much face-to-face instruction will be done at the high school level."

Interactive community building

The use of technology can contribute to interactive community building. For example, with tools like LessonLab, teachers from different districts could collaborate in planning and critiquing mathematics lessons. They could watch the lessons of one another, annotate the videos to highlight areas of potential mutual interest, and meet using videoconferencing equipment. With the capability of completing all this over the Internet, such cross-district collaborations can spread to different states and even to different countries.

The Effects of Emerging Technologies on the Content of School Mathematics

Data analysis and mathematical modeling, now minimal in the school mathematics curriculum, are likely to play a much larger and intertwined role in the school mathematics of the future, partly because of increasingly easy access to increasingly powerful data analysis and function-modeling tools. Moreover, the use of technology in the learning of these areas of mathematics is likely to meet with less resistance than, for example, the incorporation of CAS in the learning of algebra. Jere Confrey offers a possible explanation: "In many ways, that is exactly why the use of software in statistics meets much less resistance—simply because you are dealing with issues of relative precision and accuracy, and people don't think the only story in town is proof."

Many statisticians and mathematicians have carefully distinguished between statistics as a field and mathematics as a field. Bill Finzer explains the role of the technology he has developed in making connections between these fields:

> One way of viewing our work with Fathom is that we are attempting to provide accessible tools that integrate data analysis and mathematics. What does it take to make these tools accessible? We believe that dynamic manipulation (dynamic statistics) is part of it; that when you can reach in and smoothly change the display from one state to another, you have a chance of

> understanding how the representation works and of forming a bond between the data and whatever mathematical model you are starting to make.

He also points to the mathematical work entailed in data analysis of seeing patterns and relationships in data and of building models to describe those relationships. With tools like Fathom, students can download and display data, calculate best-fitting curves, and graph those curves on top of the data. Like other tools (e.g., GSP sketches in CAS-Intensive Mathematics), Fathom supplies dynamically draggable sliders that can serve as parameters for the models being tested.

Jere Confrey points out that the tools are now available for data analysis but that the potential users may not yet be ready: "In some sense, the future is now. Because the tools and data resources are there. The problem is that we're not." She points to the need to develop a kind of reasoning not usually underpinning mathematics courses:

> I think the idea of probabilistic reasoning and reasoning that is much more contingent is going to be of major proportions in the future and people are going to have to begin to grapple with complexity in the sense of not just what is, in a descriptive sense. I think many more people are going to have to begin to understand inferential reasoning and what you can say and not say relative to these complex systems. They will need to develop reasoning that says, "This is the best I can—this is the best guess at this time. And that it may be proven wrong in the future, but I need to go with that kind of contingent reasoning right now."

Jere and others point out that one of the reasons students may not become engaged in mathematics is that they do not generate, or "own," many of the problems on which they work. Jere Confrey describes a project she conducted with her graduate students, examining issues centered on high-stakes testing and on learning enough statistics to investigate the relationships in the data on the basis of looking at distributions and comparisons among groups. With the increasing power and convenience of available data analysis tools, having students generate and pursue their own questions will become viable in a larger number of settings. Fathom, for example, now has a tool for generating surveys and collecting responses.

Technology is likely to change not only the content of school mathematics but also the processes of school mathematics and the nature of mathematical understandings. Students in technologically rich classrooms are likely to develop multirepresentational views of mathematics. Some technologies will enable them to develop almost a kinematic understanding of functional relationships. Ricardo Nemirovsky refers to this kinematic understanding as the development of bodily intuition:

> There is still a bias toward thinking that instruction that is based in formal statements or things that are not sensorial is more elevated or more mathematical or more profound.… There is a huge overlap between what is activated in a brain by thinking about an activity and what is activated when you actually perform that activity. And so I think that for example imagining that a cube rotates in space is deeply rooted in the physical act of rotating cubes with your hand.

Ricardo envisions a future in which there would be a plethora of activities in which students develop a sense of the differences in how mathematical relationships feel.

Technology and the Practice of Teaching

Ideas from the interviews about the potential impact of technology on the mathematics classrooms of the future ranged from claims about the types of teaching and learning afforded by particular technologies to descriptions of classrooms of the future enabled by technologically rich settings and generalization about the types of mathematical thinking supported through particular technological approaches.

Changes in Schools and School Days

Experts came down on both sides of the fence with respect to changes in schools and school days. Pete Rubba, as reported earlier, suggested that Web-based instruction would, in the future, replace much of face-to-face instruction. Bob McCollum, however, is adamant in pointing out the necessity of the teacher in front of students:

> No matter what we do, I feel the most important thing is the teacher in front of the students. I try to be careful to warn people not to get lost in that. In fact, someone just shared a Japanese proverb that says something like: "Better one day with the teacher than a thousand days studying." I just really think that that needs to be at the crux of what we do. But, given that, I certainly recognize and believe that the technology can play a really important role in helping kids to understand things better. That is really what we try to do with it.

Similarly, Kyle Peck makes clear his belief that schools will continue to exist but school days and activities will be different from what they are now:

> You know, some people are probably expecting me to say, well, there won't be any school. There will be school; we have a day-care responsibility—and there is a lot to be said for the collaboration that happens face-to-face.

> Schools should be the most exciting place because there are other kids there and because you can be wrestling with cool problems and because you can be using tools that you can't necessarily have at every home.

Kyle's work in reforming schools includes the Centre Learning Community Charter School (the charter school he founded), which he describes as "kids actively engaged in project-based learning, multidisciplinary, theme-based, problem-based learning in a technology-rich environment." Continuing to work in this area, Kyle gives the rationale for the project he is currently developing:

> So what I envision happening is a school day that's kind of like our day in that we have big things we're trying to do and our day is punctuated by meetings. So we have these things we're working on and we get up and we go and do something else and come back and pick up where we left off. I think the student's day is going to be that way in terms of his multidisciplinary projects. They have these big things that they really are into, using cool tools, and we bring them lots of great resources. That way they can create things that they are proud of, and their day will be punctuated by skill-based lessons—where somebody is trying to teach them something for which they have the prerequisites and that they don't already know. Right now when we [have] group-based education, we group for the whole semester or the whole year. And we put people in groups that turn out to be self-fulfilling prophecies and so on. So I think what we'll do instead is we'll have just enough just-in-time instruction based on an actual understanding of the knowledge and skills that people have. So there will be a profile on each student maintained by technology that keeps track of what students know, like medical records. I sometimes use a metaphor of a dentist office, because when I go to the dentist, they don't say, well, Kyle, you're fifty-one years old, and most fifty-one-year-olds are having trouble with their molars, so we're going to drill your molars. They say, now let's take a look, and they figure out what I need and give me what I need. So I think we've got to do the same thing with mathematics skills, but I think the big breakthrough is that the students will see math and use math in the context of doing other things.

Kyle sums up the success of CLC by describing comments from one special-needs student:

> One special-needs kid from the charter school said, "At this school they don't do everything for me, because at my last school they used to do everything for me. But here they don't. They did a little at first but now they make me do everything myself. And I like it that way, and they're much cooler things." And to me that quote—"They make me do everything myself, and I like it that way, and they're much cooler things."—that really kind of captured it all. I think that's what we're going to be able to do with education. We're going to have

> kids do things, rather than I tell it to you and you tell it back to me and I give you a grade for it. They're going to have to do things that are cooler things with tools that people outside school are using, with tools that they will take home with them so they will get better and better at these tools because they will do things that they care about in addition to things that we care about. So I think that's what it's about. And math becomes maybe even transparent. They don't even realize when they're doing math and when they're not doing math. They're just building something. And so the math just happens.

Changes in the Role of Students in Defining the Tasks

The issue of students doing tasks they define for themselves is a theme that also arose in other conversations. Steve Rasmussen sees this approach as endemic to dynamic tools. He sees interactive geometry tools as blurring the distinction between the learner and the author of the activities, and he points out that every activity created with Sketchpad can be extended by the user. He contrasts this use of tools with that of microworlds, describing dynamic tools as addressing objectives from the learner's point of view and microworlds as addressing objectives from the teacher's point of view. For example, he sees teachers using Java Sketchpad creating activities from a teacher's point of view, whereas an open-ended use of GSP allows students to define their own tasks. Far less interest is generated in students through these Java Sketchpad predefined types of activities than through GSP. He paints a picture of classrooms in which authority is shared between teacher and students and in which students have the tools to express their mathematical ideas visually. In a school community using GSP, Steve points out that teachers can see what their students can do. In such school settings, there is a breakdown of the expert/nonexpert divide, and the curricula blurs the lines between what students are capable of and what teachers ask them to do. Possible impediments to such an approach in the future are limitations on the availability of the technology or on the willingness of the schools to participate.

Changes in Classroom Connectivity

With the growing popularity of wireless computing and the increasing similarities among computers, calculators, and PDAs, the mathematics classroom is becoming multiply connected—students' technological devices and teachers' technological devices will be connected in ways that allow unprecedented communication. Jim Kaput describes the potential for this kind of connectivity:

> The big old computers, including microcomputers, didn't fit into daily practice very well; they didn't fit into the physical classroom; they didn't fit into

> the social structure; and they didn't fit particularly well into the pedagogical structure of most classrooms. The handheld fits much more readily into the physical structure and to a significant extent into the pedagogical structure. While the teacher did not have any kind of direct control or knowledge of what's going on on the handhelds when each of the kids had one, with the new network capacity that is now being built into them, such as the TI Navigator, the teacher can now get what's on the student's handheld as well as broadcast stuff to all the handhelds. With this kind of a network, you can first of all do some traditional things better, more efficiently, like hand out homework, hand out quizzes, collect data, and so on, provided you have the right ancillary software to do that.

Jim describes new "activity structures" for connected classrooms. For example, he describes classroom activities in which students each construct a graph with a given set of constraints—for example, the position versus time graph for two cars in a race. When the students download their graphs to the teacher's computer, they find that the graphs have certain commonalities defined by the constraints and certain differences on characteristics that were free to vary. This activity structure can bring home to students the necessary characteristics of mathematical objects and the characteristics that may vary. The aggregation of the graphs makes salient the defining characteristics of the graphs. Students can, in this way, learn to identify essential and nonessential features of graphs.

The Effect of Technology on Mathematics Learning in Informal Settings

Mathematics learning need not be confined to students or to formal schools and classrooms, and similarly for technology in the service of mathematics learning. The development of structures for informal mathematics learning is in its infancy. Two exemplary projects that illustrate the uses of technology in informal settings are Illuminations and Calculus@Museum.

The Illuminations (illuminations.nctm.org/index.html) project has as a major goal the provision of Internet resources that will help improve the teaching and learning of mathematics for all students. It is intended to provide NCTM *Standards*-based resources for classroom use and professional development for teachers. Eric Hart points out that once these resources are freely accessible on the Internet, they are available in libraries, in community centers, for Parent Nights, and for home schooling. He observes that such sites help the general public to understand what is meant by important mathematics taught in engaging, sense-making ways, what is meant by students learning mathematics in deep ways, and what is meant by teachers teaching mathematics in richer ways. He notes that Illuminations gives examples that can meet all these goals simultaneously.

The Calculus@Museum (www.terc.edu/mathofchange/CM/home.html) project in the Science Museum in St. Paul, Minnesota, focuses on three central concepts: (1) rates of change, (2) incremental summation, and (3) the use of parametric functions to provide a simple way to describe a complex motion. A major goal of this project is to use kinesthetic experiences of physical actions to make calculus ideas accessible and informative to a general audience. Based on TERC's "Line Becomes Motion" (LBM) technology, the project gives visitors "direct experience of these motions through bodily engagement, fully involving the hand as well as the eye in the learning process." Ricardo Nemirovsky describes the exhibit: "You can either draw a graph and drive the physical event or you can move by hand and generate a graph."

Although in the past mathematics has not commonly been an integral part of museums, projects like the two just described that capitalize on emerging technologies furnish exemplars of how technology may affect informal mathematics exhibits over the next decade.

Technology to Help Teachers Do Their Jobs

Technological approaches of the near future will provide welcome assistance to teachers in their day-to-day work. Among the most promising technologies are those that will enable teachers, students, and parents to access the increasing number of technological mathematics-specific tools. Gene Klotz describes his current project, Math Tools:

> A critical challenge is to actually get needed technology materials into the hands of students and teachers. Toward this end, the Math Forum has been experimenting with an online learning community built around a digital library of mathematics software. Our new project, Math Tools, http://mathforum.org/mathtools/, collects math software and organizes it by topic within courses, pre-K–calculus. We also offer user reviews, discussions about the tools, help in using the software, user experiences, problems that use the tools, classroom activities.

Venues like Math Tools afford teachers, mathematics teacher educators, and prospective teachers the opportunity not only to access tools but also to engage in online discussion with other users as well as with the tool developers.

Technology in the mathematics classrooms of the future will not necessarily be entirely mathematics-specific. Some technology will simply be useful to teachers in carrying out the day-to-day responsibilities of their jobs. Bob McCollum and Dan Chazan commented on some of those possibilities:

Logistics of Running a Classroom

- Teachers could take attendance and check in homework on the spreadsheets on their Palm Pilots, then later sit down at their computers and load these records into their electronic grade books.
- Students could beam in their homework.
- Palm Pilots could have probe capabilities for collecting data, and so students could be ready on a moment's notice to collect data for a class experiment.

Providing individual assistance to students

- The Internet could be used as a tutorial medium for students.
- Students could take assessments on the Web, and remediation could be suggested on the basis of their specific answers.

Textbooks and teacher's editions

- Textbooks, although they will still be around, will become less and less important because of the vast array of tools and information that will be available on computers and on Palm Pilots.
- Teacher's editions will be electronic and contain a vast array of resources.
- Teacher's editions could be dynamic, with teachers adding their own activities and comments.

Accountability

- Districts could collect districtwide assessment information online.

Collaboration

- Teachers could share lesson plans and comment on one anothers' plans through shared electronic resources.
- Teachers across districts could participate in teleconferences dealing with common courses and problems.

Regarding this last suggestion, Dan Chazan comments:

> And if you are working electronically, the cost for extra pages [in a teacher's edition] is very small. So if you want to include an essay on the mathematical background behind this task, and you know not everybody is going to need it, in an electronic environment you would be more likely to include it.

> You could have links to the same document from a number of different activities in a way that right now in the teacher's guide you can't really do.

Technology also affords new opportunities for professional development. To Dan, one such role is in supporting communities of teachers: "I can imagine down the line that there will be different tools for supporting communities of teachers to do joint planning of different kinds." To Bob McCollum, professional development over the Internet helps communication among teachers in different schools, including those in remote locations:

> I just see the Internet, or whatever will come next from the Internet, as obviously being a great way for teachers to access professional development. You know, some of the chat room sort of things aren't quite dynamic enough and aren't quite real-time enough; but I think they are going to get there; and I think we will be able to have my department engage in a discussion with your department, for example, over a particular topic, over the Internet, cheaply and easily, and the benefits from that I think are huge. I think about the teleconferencing capability, too, that you can now do that over the Web, and that makes it affordable in a sense. And think about it, especially for those schools that can't afford to bring in an expert, but they might be able to talk to them over the Internet.

Dan Chazan observes: "Increasingly as people start to use the Web for different kinds of professional development with embedded video technology, for example, LessonLab and the TIMSS Tools, new ways will be created to talk about teaching and to read about teaching and to have people communicate ideas about teaching." A venue for implementing this idea is the VideoPaper (brp.terc.edu/VPB/vpb.html). In a VideoPaper, "classroom or interview-filmed episodes can be displayed and synchronized with interpretations, transcriptions, closed captions, images of student work, clarifying diagrams, or other pieces of information that expand the events, portraying their full complexity. Teachers, researchers, and other communities interested in video-based dissemination can use VideoPapers to make their conversations more grounded in actual events, more insightful, and more resistant to oversimplifications."

Some Parting Thoughts

Lest this article seem unrealistically optimistic about the technological future of the mathematics classroom, there are, of course, caveats. If technology is to fulfill its promise, the stars must be at least somewhat aligned. If the Internet is to continue to provide wonderfully rich resources, there must be financially viable organizations supporting the creation of such resources. If teachers are to make the best use of available technologies and tools, they must be able to locate and take advantage of appropriate professional development. If mathematics classrooms are to become places where students can pursue problems of their own invention and interest, the technology must be available and the curriculum must be open enough to allow such exploration. These changes will take place with teachers who themselves have experienced learning mathematics with technology.

Nevertheless, according to this amazing group of national leaders in education, the many possible futures of technology in mathematics education are bright and enticing. An essential ingredient of some visions of the future is that we will be using well and regularly the technologies that now exist (but are not used to any significant extent in today's classrooms). There were some important ideas that came to the fore in the course of my hours of conversations with these educators. I conclude this article with a list of principal predictions associated with the range of possible futures for technology and mathematics instruction:

- Wireless connectivity suggests a future in which students and teachers freely share and build on the ideas of one another.
- Students will experience mathematics kinematically.
- Web-based facilitation of learning will grow and develop.
- New kinds of reasoning (rooted in reasoning in the face of uncertainty and reasoning with contingencies) will be needed as technology-supported data analysis takes its rightful place in the school mathematics curriculum.
- Teachers will use just-in-time technology-based instruction for honing students' missing skills.
- New school configurations will allow for smooth transitions between technology-assisted group work and technology-delivered individualized instruction.
- Teachers and students will regularly use tailored and tailorable computer algebra systems and intelligent tutors.
- Technology at its best will allow students to test their mathematical ideas by means other than entering syntactically correct literal codes.

As Rose Zbiek pointed out in our conversation, the coming decade of technology use in mathematics classrooms will be punctuated by the growing need to understand the intricate relationship between doing anything by hand or by technology and the development of students' understanding at a more global level. It is this shared commitment to intense use of technology not for the sake of using tools but for the sake of developing and enhancing students' mathematical understanding that will serve the future well.

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